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Relation between isospin-symmetry-breaking correction to superallowed β decay and the energy of the charge-exchange giant monopole resonance

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After application of an analytical transformation, a new exact representation for the nuclear isospin-symmetry-breaking correction δ_C to superallowed β decay is obtained. The correction is shown to be essentially the reciprocal of the square of an energy parameter Ω_M which characterizes the charge-exchange monopole strength distribution. The proportionality coefficient in this relation is determined by basic properties of the ground state of the even-even parent nucleus and should be reliably calculable in any realistic nuclear model. Therefore, the single parameter Ω_M contains all the information about the properties of excited 0^+ states needed to describe δ_C. This parameter can possibly be determined experimentally by charge-exchange reactions. Basic quantities of interest are calculated within the isospin-consistent continuum random phase approximation, and the values of δ_C are compared with the corresponding results from other approaches.

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I. INTRODUCTION

Superallowed $0^+ \rightarrow 0^+$ β decays (SA β decays) allow us to test fundamental properties of the weak interaction, such as the conserved vector current (CVC) hypothesis and the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix V_{ij} (see, e.g., a recent review [1] by Towner and Hardy). The decay rates for SA Fermi transitions between $T = 1$ nuclear multiplet states have accurately been measured in a dozen nuclei. Since the CVC hypothesis is only true in the isospin-symmetry limit, an uncertainty enters into the analysis of the experimental ft values depending on the model calculation of the effect of isospin breaking in nuclei. Although the breaking is weak, the current situation is such that the theoretical uncertainties in the calculated correction terms predominate over the experimental uncertainties in the SA β decay data. This calls for better accuracy of the theory applied to interpret the experimental results.

From the 2009 survey of experimental data, Hardy and Towner [2] determined $|V_{ud}| = 0.97425 \pm 0.00022$, which, combined with the complementary experimental data on $|V_{us}|$ and $|V_{ub}|$, gave

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.99990 \pm 0.00060 \qquad (1)$$

for the norm of the first row of the CKM matrix. Thus, this test confirmed the unitarity of the CKM matrix with an accuracy of 0.06%.

Isospin symmetry is slightly broken in nuclei, mainly by the Coulomb interaction. This leads to a small reduction of the nuclear matrix element M_F for SA Fermi transitions between the ground state (g.s.) of the even-even parent nucleus and its isobaric analog state (IAS) in the odd-odd daughter nucleus:

$$|M_F|^2 = \left|M_F^0\right|^2(1 - \delta_C), \qquad (2)$$

where $|M_F^0|^2 \equiv 2T_0 = |N - Z|$ is the exact-symmetry value, with $T_0 = |T_z|$ being the isospin of the g.s. of the even-even parent nucleus, and $\delta_C > 0$ is the isospin-symmetry-breaking correction.

There have been a number of methods used recently to calculate the correction δ_C: a shell model with Woods-Saxon and Hartree-Fock (HF) radial functions [2,3], relativistic Hartree (RH) and HF approaches with the random phase approximation (RPA) [4], an isovector monopole resonance (IVMR) model [5], and self-consistent isospin- and angular-momentum-projected nuclear density functional theory [6]. Still, there is a significant spread in the obtained values of δ_C. Therefore, a better understanding of the aspects of nuclear structure that are important for more accurate evaluation of δ_C is needed.

The main purpose of this work is to derive a new exact representation for the correction δ_C, which emphasizes the role of the physical charge-exchange monopole strength distributions, that can be probed experimentally. After application of an exact analytical transformation, δ_C is shown to be essentially the reciprocal of the square of an energy parameter Ω_M which characterizes charge-exchange monopole strength distributions. The proportionality coefficient in this relation is determined by basic properties of the ground state of the even-even parent nucleus, and it should be reliably calculated in any realistic nuclear model. Therefore, the single parameter Ω_M contains all the information about the properties of excited 0^+ states needed to describe δ_C. The possibility of experimental determination of this parameter in charge-exchange reactions is discussed. Also in this paper basic quantities of interest are calculated for a few nuclei within the isospin-consistent continuum RPA, and the obtained values of δ_C are compared with the corresponding results obtained by using other approaches.

II. A NEW EXPRESSION FOR THE NUCLEAR COULOMB CORRECTION δ_C

In all the experimental cases of interest, the IAS of the g.s. $|0\rangle$ of a even-even parent nucleus with $T = 1$ is an isolated low-lying state (or, in most of the cases, even the g.s.) in

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the daughter odd-odd nucleus. This physical IAS contains, along with the major $T = 1$ component, various small isospin admixtures.

The representation for $|M_F|^2$ can identically be transformed as follows:

$$|M_F|^2 \equiv \langle 0|\hat{T}^{(+)}|\text{IAS}\rangle\langle\text{IAS}|\hat{T}^{(-)}|0\rangle = 2T_0 + S_F^{(+)} - S_F^{(-)}. \quad (3)$$

Here, $S_F^{(+)} \equiv \sum_j |\langle j|\hat{T}^{(+)}|0\rangle|^2$, $S_F^{(-)} \equiv \sum_i' |\langle i|\hat{T}^{(-)}|0\rangle|^2$, and $\sum_i' \equiv \sum_i - \sum_{i=\text{IAS}}$ runs over all physical 0^+ states, but the IAS, in the daughter nucleus, $\hat{T}^- = \sum_a \tau_a^-$ and $\hat{T}^{(+)} = (\hat{T}^{(-)})^\dagger$, are the standard isospin lowering and raising operators, respectively. Note that Eq. (3) represents simply a version of the Ikeda sum rule for Fermi transitions: $|M_F|^2 + S_F^{(-)} - S_F^{(+)} = 2T_0 = N - Z$. Hereafter we consider for definiteness the case $N > Z$, which can readily be generalized for the case $N < Z$.

Therefore, the Coulomb correction δ_C can be represented in the following form:

$$\delta_C = \frac{1}{2T_0}(S_F^{(-)} - S_F^{(+)}). \quad (4)$$

It proves very useful to further transform Eq. (4) to explicitly relate δ_C to isospin-breaking terms of the total nuclear Hamiltonian $\hat{H}$, which include the Coulomb interaction and the small isospin-violating part of the nuclear forces. For this, one introduces auxiliary operators $\hat{V}_C^{(\mp)} \equiv \pm[\hat{H}, T^{(\mp)}]$ which are determined by these isospin-breaking terms, and one uses an exact relation between the matrix elements of $\hat{V}_C^{(\mp)}$ and $\hat{T}^{(\mp)}$:

$$\langle s|\hat{V}_C^{(\mp)}|0\rangle = \omega_s\langle s|\hat{T}^{(\mp)}|0\rangle, \quad (5)$$

with $\omega_s = E_s - E_0$ being the excitation energy of a state of the isobaric odd-odd daughter nucleus measured from the g.s. of the parent nucleus. The degree of fulfillment of Eq. (5) in a nuclear model can serve as an important check of the isospin consistency of the model. In particular, Eq. (5) ensures the equalities

$$S_F^{(-)} = S_{C[-2]}^{(-)}; \quad S_F^{(+)} = S_{C[-2]}^{(+)}, \quad (6)$$

where $S_{C[L]}^{(-)} \equiv \sum_i' |\langle i|\hat{V}_C^{(-)}|0\rangle|^2\omega_i^L$ and $S_{C[L]}^{(+)} \equiv \sum_j |\langle j|\hat{V}_C^{(+)}|0\rangle|^2\omega_j^L$ are the energy-weighted Coulomb sum rules. As a result, one gets a representation equivalent to Eq. (4):

$$\delta_C = \frac{1}{2T_0}(S_{C[-2]}^{(-)} - S_{C[-2]}^{(+)}). \quad (7)$$

Although physically Eqs. (4) and (7) are equivalent, it is preferable to use Eq. (7) in a model calculation as this representation is much less sensitive to possible residual isospin inconsistencies of the model.

The charge-dependent isospin-breaking interaction is dominated by the Coulomb interaction between protons. Because the Coulomb force is of long range, the one-body Coulomb mean field is mainly determining the transition operators $\hat{V}_C^{(\pm)}$ in Eq. (7), which in this case also become one-body monopole charge-exchange operators. Therefore, most of the strength for the transition operators is exhausted by the corresponding giant isovector charge-exchange monopole resonance (IVMR), associated with the $2\hbar\omega$ particle-hole excitations of proton-neutron type with $J^\pi = 0^+$. The importance of the IVMR as a doorway state for the isospin mixing of the IAS was realized already in the early years of IAS studies [7,8] and was re-emphasized recently in Ref. [5].

Further, one can introduce an auxiliary energy Ω_M defined as

$$\Omega_M^2 \equiv \frac{S_{C[0]}^{(-)} - S_{C[0]}^{(+)}}{S_{C[-2]}^{(-)} - S_{C[-2]}^{(+)}}. \quad (8)$$

This energy characterizes the charge-exchange monopole strength distributions in odd-odd isobaric nuclei. Then the original expression for δ_C (7) can be identically rewritten as $\delta_c = \frac{1}{2T_0}\frac{1}{\Omega_M^2}(\langle 0|[\hat{V}_C^{(+)}, \hat{V}_C^{(-)}]|0\rangle - \omega_A^2|M_F|^2)$. Here, we again have used $\sum_i' \equiv \sum_i - \sum_{i=\text{IAS}}$ and $\langle\text{IAS}|\hat{V}_C^{(-)}|0\rangle \equiv \omega_A M_F$. From this expression one obtains

$$|M_F|^2 = \frac{2T_0}{1 - \frac{\omega_A^2}{\Omega_M^2}}\left(1 - \frac{1}{2T_0\Omega_M^2}\langle 0|[\hat{V}_C^{(+)}, \hat{V}_C^{(-)}]|0\rangle\right),$$

and, finally, arrives at the following expression:

$$\delta_C = \frac{1}{\Omega_M^2 - \omega_A^2}\left(\frac{1}{2T_0}\langle 0|[\hat{V}_C^{(+)}, \hat{V}_C^{(-)}]|0\rangle - \omega_A^2\right). \quad (9)$$

Thus, one sees that δ_C (9) is determined by two energies, Ω_M and ω_A, which are the only input into the problem related to the spectrum of 0^+ states in the odd-odd daughter nuclei, and by the properties of the g.s. of the parent nucleus via the g.s. expectation value of the commutator $\langle 0|[\hat{V}_C^{(+)}, \hat{V}_C^{(-)}]|0\rangle$. Both ω_A and Ω_M can be determined experimentally (with the former in fact being already very accurately known; the value of the latter can be determined by charge-exchange reactions on the parent nucleus; see below). The numerical simulations (see below) indicate that a strong inequality $\Omega_M \gg \omega_A$ is fulfilled, which is to be expected because of the high IVMR energy.

Now we would like to evaluate the expectation value $\langle 0|[\hat{V}_C^{(+)}, \hat{V}_C^{(-)}]|0\rangle$ in the dominant mean-field approximation, $\hat{V}_C = \sum_a U_C(r_a)(1 - \tau_{az})/2$, with $U_C(r)$ being the Coulomb mean field. The realistic potential $U_C(r)$ resembles very much that of a uniformly charged sphere, which is a quadratic function: $U_C(r) = \frac{Ze^2}{2R}[3 - (r/R)^2]$ inside a nucleus $r \leqslant R$, where R is the nuclear radius. It turns out that if one extends this quadratic dependence also to the outer region $r > R$ (instead of proportionality to $1/r$), this gives numerically just a small deviation in the Coulomb sum rules. Thus, the Coulomb sum rules are determined by single-particle charge-exchange operators $\hat{V}_C^{(\mp)} \equiv \hat{U}_C^{(\mp)} = \sum_a U_C(r_a)\tau_a^{\mp}$, where the term in $U_C(r)$, proportional to r^2, gives the dominant contribution to the sum rules. Further, one has

$$[\hat{V}_C^{(+)}, \hat{V}_C^{(-)}] = \sum_a U_C^2(r_a)\tau_{az} \quad (10)$$

and finally gets

$$\frac{1}{2T_0}\langle 0|[\hat{V}_C^{(+)},\hat{V}_C^{(-)}]|0\rangle = \frac{1}{2T_0}\int U_C^2(r)\varrho^{(-)}(r)\,d^3r \equiv \overline{U_C^2}, \tag{11}$$

where the bar means averaging over the neutron excess density $\varrho^{(-)}(r) = \varrho_n(r) - \varrho_p(r)$ defined as the difference between the total neutron and proton number densities $\varrho_n(r)$ and $\varrho_p(r)$, respectively.

The nominator in Eq. (9) is subject to a strong cancellation between the two terms. This can be best seen if one introduces the value of the Coulomb mean field averaged over the neutron excess density:

$$\overline{U_C} \equiv \frac{1}{2T_0}\int U_C(r)\varrho^{(-)}(r)\,d^3r. \tag{12}$$

By adding and subtracting $\overline{U_C}^2$ in the nominator of Eq. (9), one gets

$$\delta_C = \frac{1}{\Omega_M^2-\omega_A^2}\left(\overline{(U_C-\overline{U_C})^2} + \left(\overline{U_C}^2-\omega_A^2\right)\right). \tag{13}$$

Each of the two terms in the nominator of Eq. (13) is now much smaller than its counterpart in Eq. (9): $\overline{(U_C-\overline{U_C})^2} \ll \overline{U_C^2}$ as a consequence of the smoothness of the Coulomb mean field, and $(\overline{U_C}^2-\omega_A^2) \ll \omega_A^2$ since $\overline{U_C}$ provides a leading contribution to the Coulomb displacement energy (see, e.g., [7,8]). The numerical simulations below show that only for light nuclei is the term $\overline{(U_C-\overline{U_C})^2}$ comparable with $\overline{U_C}^2-\omega_A^2$; for heavier systems the inequality $\overline{U_C}^2-\omega_A^2 \gg \overline{(U_C-\overline{U_C})^2}$ holds.

Different models must give similar results for $\overline{U_C^2}$ and $\overline{U_C}$ provided that the basic nuclear geometry (such as the mean radii of proton and neutron density distributions) can reasonably be reproduced. Therefore, different values of δ_C obtained by different methods should mainly stem from differences in corresponding values of Ω_M.

Returning to the question of the possible experimental determination of Ω_M, we note that the charge-exchange IVMR was first observed in pion single-charge-exchange reactions [9,10]. Recently, the IVMR has been studied in various charge-exchange reactions: (^{3}He,t) [11], (^{3}He,tp) [11–13], and (t,^{3}He) [13]. The spin-flip IVMR was mainly excited in the experiments. Though the measurements are rather difficult to make, one may expect that the excitation of the non-spin-flip charge-exchange IVMR might be separated from its spin-flip counterpart (by means of polarized beams or by comparing measurements at different projectile energies). Also, we note that the effective one-body transition operator leading to the IVMR excitation in charge-exchange forward-scattering reactions is determined by the r^2 dependence of the Bessel function $j_0(r)$ [14], in accord with the r^2 dependence of $U_C(r)$. In such a case, Ω_M can directly be obtained from the experimental cross sections by a formula analogous to Eq. (8).

III. CALCULATION RESULTS

We consider here by way of example four decays: ^{10}C → ^{10}B, ^{38}K → ^{38}Ar, ^{66}As → ^{66}Ge, and ^{70}Br → ^{70}Se. (The two latter cases, both with $A \approx 70$, are taken to check the abrupt drop in δ_C while going from ^{70}Br to ^{66}As as has appeared in calculations of Ref. [4].) We use here a semiphenomenological nuclear mean field and apply the continuum RPA with Landau-Migdal zero-range forces to calculate the quantities of interest: ω_A, $\overline{U_C^2}$ (11), $\overline{U_C}$ (12), $S_F^{(\pm)}$, $S_{C[0]}^{(\pm)}$, $S_{C[2]}^{(\pm)}$, Ω_M (8), and finally δ_C.

TABLE I. Experimental (Expt.) and calculated (Calc.) values. Columns 2 and 3 list the experimental and calculated excitation energies ω_A, measured from the g.s. energy of the corresponding even-even nuclei in SA β decays from column 1 (with calculated RPA values being corrected for the neutron-proton mass difference). The calculated charge radii r_c of the even-even nuclei are given in column 5, and the only available experimental value for ^{38}Ar is given in column 4. In columns 6 and 7 the values of $\overline{U_C^2}$ (11) and $\overline{U_C}$ (12), respectively, are listed.

	ω_A (MeV)		r_c (fm)		$\overline{U_C^2}$	$\overline{U_C}$
	Expt.	Calc.	Expt.	Calc.	(MeV^2)	(MeV)
^{10}C → ^{10}B	−1.397	−1.66	–	2.69	−8.96	−2.91
^{38}K → ^{38}Ar	5.533	5.57	3.40	3.30	55.83	7.36
^{66}As → ^{66}Ge	9.609	8.97	–	3.93	142.80	11.91
^{70}Br → ^{70}Se	10.109	9.49	–	3.99	159.18	12.59

The first calculations of the IVMR within the self-consistent HF + continuum RPA approach were done in Ref. [15]. Here we use the relevant continuum-RPA equations from Refs. [16,17]. Note that we do not need any discretization of the single-particle (s.p.) continuum as done in Ref. [4], because the equations are written in terms of the s.p. Green's functions.

The mean field is chosen as described in Ref. [16], and it includes the fully phenomenological isoscalar part, with its parametrization tracing back to Chepurnov's potential [18], and both the symmetry potential and the mean Coulomb field calculated in the Hartree approximation. The chosen dimensionless intensity $f' = 1.0$ of the isovector part of the Landau-Migdal forces determines the symmetry potential via the isospin self-consistency condition [17]. Thus, the mean Coulomb field is the only source of isospin breaking in the present model.

Since the nuclei in question are open-shell ones, one would in principle need to take into account the pairing correlations and, better, to use the continuum quasiparticle RPA (QRPA) [19–21] instead of the continuum RPA. However, the continuum QRPA calculations are much more time consuming, and, more importantly, one can easily argue that the effect of nucleon pairing on the quantities in question must be small (since the pairing gap is much smaller than Ω_M). Also, a more modern choice of the isoscalar mean-field parameters of Ref. [21], which allows for their A dependence, is not expected to markedly affect the results.

In Table I the calculated excitation energies ω_A and charge radii r_c are listed along with the corresponding experimental data. Table I also contains calculated values of $\overline{U_C^2}$ (11) and $\overline{U_C}$ (12) (columns 6 and 7, respectively). The underestimate of the experimental IAS energy in the calculations for heavier nuclei reflects the Nolen-Schiffer anomaly [22–24].

TABLE II. Calculated quantities characterizing the IVMR strength distributions. Calculated $S_F^{(\mp)}$ and $S_{C[-2]}^{(\mp)}$ are listed in columns 2, 3, 4, and 5, respectively. In columns 6 and 7 values of $S_{C[0]}^{\mp}$ are given, and in column 8 the calculated energy Ω_M (8) is listed. The isospin-symmetry-breaking correction δ_C is listed in columns 9 (obtained directly from the RPA solution) and 10 [calculated from Eq. (13)].

Decay	S_F (%)		$S_{C[-2]}$ (%)		$S_{C[0]}$ (MeV^2)		Ω_M (MeV)	δ_C (%)	
	−	+	−	+	−	+		RPA	(13)
$^{10}C \to {}^{10}B$	0.065	0.36	0.074	0.39	1.01	1.47	12.13	0.147	0.142
$^{38}K \to {}^{38}Ar$	2.16	1.33	2.18	1.33	26.95	9.35	45.38	0.434	0.436
$^{66}As \to {}^{66}Ge$	7.38	5.44	7.43	5.43	96.51	19.77	61.90	0.992	1.007
$^{70}Br \to {}^{70}Se$	7.83	5.92	7.89	5.90	109.47	21.17	66.66	0.992	0.993

The fact that in lighter nuclei, in particular in the $A = 38$ case, the calculated Coulomb displacement energies are larger than the experimental ones is apparently related to the global parametrization of the phenomenological mean field chosen in the paper, which was fixed to fit properties of medium-heavy and heavy nuclei, and can lead to larger deviations for light nuclei. It can also be seen in Table I that the calculated Coulomb radius is smaller then the experimental one. Trying to fit the latter by an appropriate choice of the nuclear radius of the mean field, one would get a smaller calculated Coulomb displacement energy.

The other calculated quantities of interest, which characterize the IVMR strength distributions, $S_F^{(\pm)}$, $S_{C[0]}^{(\pm)}$, $S_{C[2]}^{(\pm)}$, and Ω_M (8), are listed in Table II. One can see a fairly good agreement between the corresponding entries in columns 2 and 4 and those in 3 and 5, respectively, in agreement with Eq. (6). This is clear evidence of the isospin self-consistency of the applied continuum RPA. Finally, the isospin-symmetry-breaking correction δ_C is listed in columns 9 (obtained directly from the RPA solution) and 10 [calculated from Eq. (13)]. Both ways of calculating δ_C agree well, again as a consequence of the isospin self-consistency of the applied continuum RPA.

Apart from the decay $^{66}As \to {}^{66}Ge$, the present results for δ_C are close to those of Ref. [4] calculated within the RH +RPA, and also are systematically smaller than those of Ref. [3]. The corresponding value of δ_C for the decay $^{66}As \to {}^{66}Ge$ is pretty close to the one for $^{70}Br \to {}^{70}Se$, in contrast to the conclusion of Ref. [4], but in qualitative accord with the small relative change in δ_C between these decays as observed in Ref. [3].

One can try to explain the difference between the shell model and the RPA results in terms of the difference in Ω_M. The former approach uses the differences in radial single-particle wave functions of the neutron and proton with the same quantum numbers; i.e., it employs a pure mean-field picture. In this picture the collectivity of the IVMR is missing, and the effective Ω_M must be less than Ω_M of the continuum RPA. In the latter approach a collective IVMR is formed by the repulsive residual particle-hole interaction and is thereby shifted to higher excitation energy (see also similar arguments in Ref. [5]).

Physically, the collectivization of the IVMR results in both its energy shift to higher energy and a reduction of its Coulomb strength. However, these effects are not independent and are related via an energy-weighted sum rule. Namely, the existence of such a relation allows one to relate δ_C exclusively to a single energy parameter Ω_M which characterizes the monopole strength distribution [see Eqs. (8), (9), and (13)]. Therefore, both effects of the IVMR collectivization can effectively be accumulated in a single energy parameter Ω_M.

Note that an estimate of the effect of the isospin splitting of the IVMR goes beyond the framework of the RPA. The splitting effectively pushes the monopole strength to higher excitation energies, and it is expected that Ω_M will further slightly increase. This would lead to a corresponding decrease of δ_C, bringing them closer to the estimates of Ref. [5].

IV. CONCLUSIONS

In the present work a new exact representation for the correction δ_C is derived in which the role of the physical charge-exchange monopole strength distributions is emphasized. After application of an exact analytical transformation, δ_C is shown to be essentially the reciprocal of the square of an energy parameter Ω_M which characterizes charge-exchange monopole strength distributions. The proportionality coefficient in this relation is determined by basic properties of the ground state of the even-even parent nucleus, and it should be reliably calculated in any realistic nuclear model. The possibility of experimental determination of the parameter Ω_M in charge-exchange reactions is discussed. Also in this paper basic quantities of interest are calculated for a few nuclei within the isospin-consistent continuum RPA, and the obtained values of δ_C are compared with the corresponding results obtained by using other approaches.

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