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Ground-state phase transition in odd-A and odd-odd nuclei near $N = 90$

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Ground-state phase transitions in odd nuclei are examined. Especially, the phase transitions in odd-odd nuclei are revealed. It is found that odd-even effects may reach their maximal or minimal values around the critical point at $N = 90$; the signals of phase transition in odd nuclei are greatly enhanced relative to those in the adjacent even-even species. Moreover, a pairing model analysis of the phase transition in Sm isotopes indicates that the critical behaviors related to pairing may be driven by the decrease in the pair strength G with a sudden flattening in ΔG around the critical point.

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I. INTRODUCTION

Quantum phase transitions in nuclei [1–34] have attracted a lot of attention experimentally and theoretically, as they provide new insights into and understanding of the evolution of nuclear structures, particularly in transitional regions. The interacting boson model [5] may be the most widely used model to study phase transitions in nuclei. In this model, the concept of quantum phase transition is seriously defined in the large-N limit with the help of the coherent-state technique [2–5]. Then the types and orders of phase transitions involved in the parameter space of the interacting boson model can be identified within the catastrophe theory [3,27,33]. Moreover, investigations of other interesting issues relevant to phase transitions, such as the finite-N effect [13,15,16], effective order parameter [11,20–22], triple point [8], quasidynamical symmetry [12], and partial dynamical symmetry [18], can also be carried out in the framework of the interacting boson model. In addition, studies of quantum phase transitions were further extended to the proton-neutron interacting boson model [5], in which some new characters of phase transition were identified [10]. The collective model provides another important framework in which to study quantum phase transitions in nuclei. Particularly, the critical-point symmetries [24] proposed in the collective model have attracted considerable attention [34], as they provide benchmarks of the shape phase transitions in these nuclei. Recently, the relativistic density-functional theory was exploited to determine all the parameters of the collective Hamiltonian [25,26]. It thus provided a microscopic method to study the low-lying spectrum of nuclei in the transitional region.

The quantum phase transition in nuclei is typically referred to as the ground-state phase transition, though the concept can also be applied to excited states; this is also commonly called the shape phase transition, as it describes changes in the equilibrium shape of the nuclear ground state as a function of the number of nucleons in the nucleus. Evidence of ground-state phase transitions in nuclei are signaled experimentally through a sudden change in the properties of the ground state, and as suggested above, such phase transitions can also be reflected in excited states [32]. Traditionally, the concept of phase transition is associated with an infinite system, in which a discontinuity in the order parameter or in its derivatives with respect to the control parameter at the critical point determines the order of phase transitions [5]. However, the number of nucleons in nuclei is finite, so the phase transitional behavior will be muted owing to the finiteness of the system [32]. This means that rather than a sharp discontinuity, one typically finds a rapid change in observables signaling the presence of a phase transition in atomic nuclei [11,32]. It is of practical importance to investigate the characters of the shape phase transitions modulated by the finiteness of the system [11]. Currently, most experimental and theoretical studies on shape phase transitions in nuclei focus on even-even systems [1–9,11–21,23,26–34], except for a few theoretical discussions [35–42] about odd-A nuclei. The purpose of this work is to investigate the characters of the ground-state phase transition in odd-A and odd-odd nuclei around $N = 90$. The critical behaviors of some experimental observables (the effective order parameters) will be revealed, along with possible microscopic explanations of the phenomena.

II. EFFECTIVE ORDER PARAMETERS

One way of addressing quantum phase transitions is to use the potential energy approach. To define the phase transition in theory, it is convenient to consider a schematic "Landau" potential [11], written as

$$V(\beta) = \beta^2 + x[(1-\beta^2)^2 - y\beta^3], \quad \beta \geqslant 0, \tag{1}$$

with two control parameters, $0 \geqslant x \geqslant 1$ and $y \geqslant 0$. This kind of potential may be formally derived from the interacting boson model [5], which has been widely used to study quantum phase transitions in nuclei as mentioned above. It can be proven that the system has a second-order phase transition at $x = x_c = 1/2$ when $y = 0$ because the minimum of $V(\beta)$, $V_{\min}$, and $\frac{\partial V_{\min}}{\partial x}$ are continuous, but $\frac{\partial^2 V_{\min}}{\partial x^2}$ is discontinuous. More generally, the system will show a first-order phase transition as a function of x for any fixed value of $y > 0$. For example, one can show that $V_{\min}$ is continuous but $\frac{\partial V_{\min}}{\partial x}$ is discontinuous

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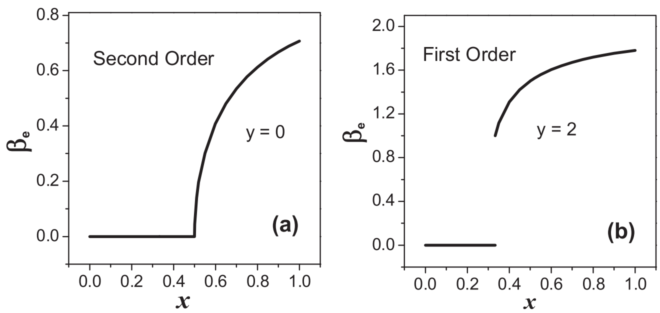

FIG. 1. Order parameter β_e as a function of x for $y = 0$ and $y = 2$.

at $x = x_c = 1/3$ when $y = 2$, which indicates the first-order phase transition occurring at x_c. The potential, (1), can be considered as a simplified phenomenological nuclear potential surface varying as a function of the deformation β, one could take $\beta_{\rm equilib} = \beta_{\rm e}$ to be the order parameter. Then, as shown for the cases considered in Fig. 1, the order parameter β_e changes continuously as a function of x, with the first derivative being discontinuous at $x_c = 1/2$ if $y = 0$, corresponding to the second-order phase transition, while β_e abruptly jumps from 0 to 1 at $x_c = 1/3$ when $y = 2$, corresponding to the first-order phase transition. In this case β_e and $V_{\rm min}$ correspond to the ground-state deformation and energy, respectively, and can be used to identify the ground-state phase transition in nuclei. However, for real nuclei, things are considerably more complicated because β_e is not an observable. In fact, instead of β_e, there are so-called effective order parameters [11]; that is, observables that are sensitive to shape phase transitions that occur within nuclei and therefore can be used to find where they occur and, in some cases, even determine their orders. The typical effective order parameters include the isomer shifts, defined as $v = c[\langle r^2\rangle_{0_2} - \langle r^2\rangle_{0_1}]$ and $v' = c'[\langle r^2\rangle_{2_1} - \langle r^2\rangle_{0_1}]$ [11], with c and c' being the scale parameters; the $B(E2)$ ratio $B(E2; (L+2)_1 \to L_1)/B(E2; 2_1 \to 0_1)$ [13,20]; and the energy ratio E_{L_1}/E_{0_2} [21,22]. Most effective order parameters are related to the quantum numbers of excited states, which make it particularly difficult to identify the phase transitions in odd nuclei. In contrast, the two-neutron separation energy [5,32,34], $S_{2n}(Z,N)$, may serve as a qualified effective order parameter to identify the phase transitions in both even-even and odd nuclei, as its value depends only on the number of nucleons and experimental data on it are also relatively abundant.

Based on the theoretical analysis within the interacting boson model [5], the first-order phase transition is expected to appear when $S_{2n}(Z,N)$ is discontinuous as a function of the neutron number N, while the second-order phase transition is expected to appear when $\Delta S_{2n}(Z,N) = S_{2n}(Z,N+2) - S_{2n}(Z,N)$ is discontinuous. The shape phase transitions in the even Nd and Ba isotopes have been recognized as the first- and the second-order phase transitions, respectively [21,34]. In Fig. 2, we take these two isotopes as examples to illustrate how $S_{2n}(Z,N)$ is applied to identify the phase transitions and their orders. It is clearly shown in Fig. 2(a) that $S_{2n}(Z,N)$ in the Nd isotopes shows a sudden flattening around $N = 90$, which is considered an experimental signal of the first-order phase transition [5]. In contrast, $S_{2n}(Z,N)$ changes as a linear function of the number of neutrons in the Ba isotopes but with a rapid increase in $\Delta S_{2n}(Z,N)$ around $N = 78$ as shown Fig. 2(b), which indicates the emergence of the second-order phase transition. It is thus verified that $S_{2n}(Z,N)$ is qualified to be taken as an effective order parameter to identify shape phase transitions in nuclei. Obviously, the experimental signal of the first-order phase transition is much clearer than that of the second-order one as shown in Fig. 2. Further investigations indicate that the characters of $S_{2n}(Z,N)$ shown in Fig. 2 also appear in other nuclei including Sm, Gd, Xe, and Ru isotopes, in which the shape phase transitions are considered as either first- or second-order [34].

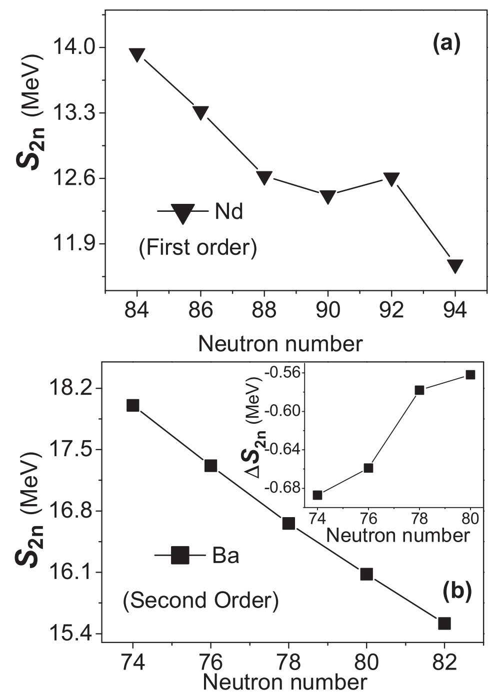

FIG. 2. (a) $S_{2n}(Z,N)$ in Nd isotopes changes as a function of neutron number. (b) The same as (a), but for Ba isotopes. Inset: $\Delta S_{2n}(Z,N) = S_{2n}(Z,N+2) - S_{2n}(Z,N)$. Experimental data are taken from [43].

A. Two-neutron separation energy

Because an odd nucleus can be considered a boson-fermion system formed by an even-even core plus one or two unpaired valence nucleons in the interacting boson-fermion model framework [44], it is believed that the basic characteristics of the ground-state phase transition shown by the even-even nuclei should be present in the adjacent odd nuclei, where the odd nuclei are odd-even nuclei with an odd proton number, even-odd nuclei with an odd neutron number, and odd-odd nuclei with an odd proton number and odd neutron number. To ensure clarity with respect to phase transition signals, we focus on the extra single-particle effect on the first-order ground-state phase transition observed around the $A \sim 150$ region [32]. To systematically investigate the ground-state phase transition in this mass region, the experimental data on

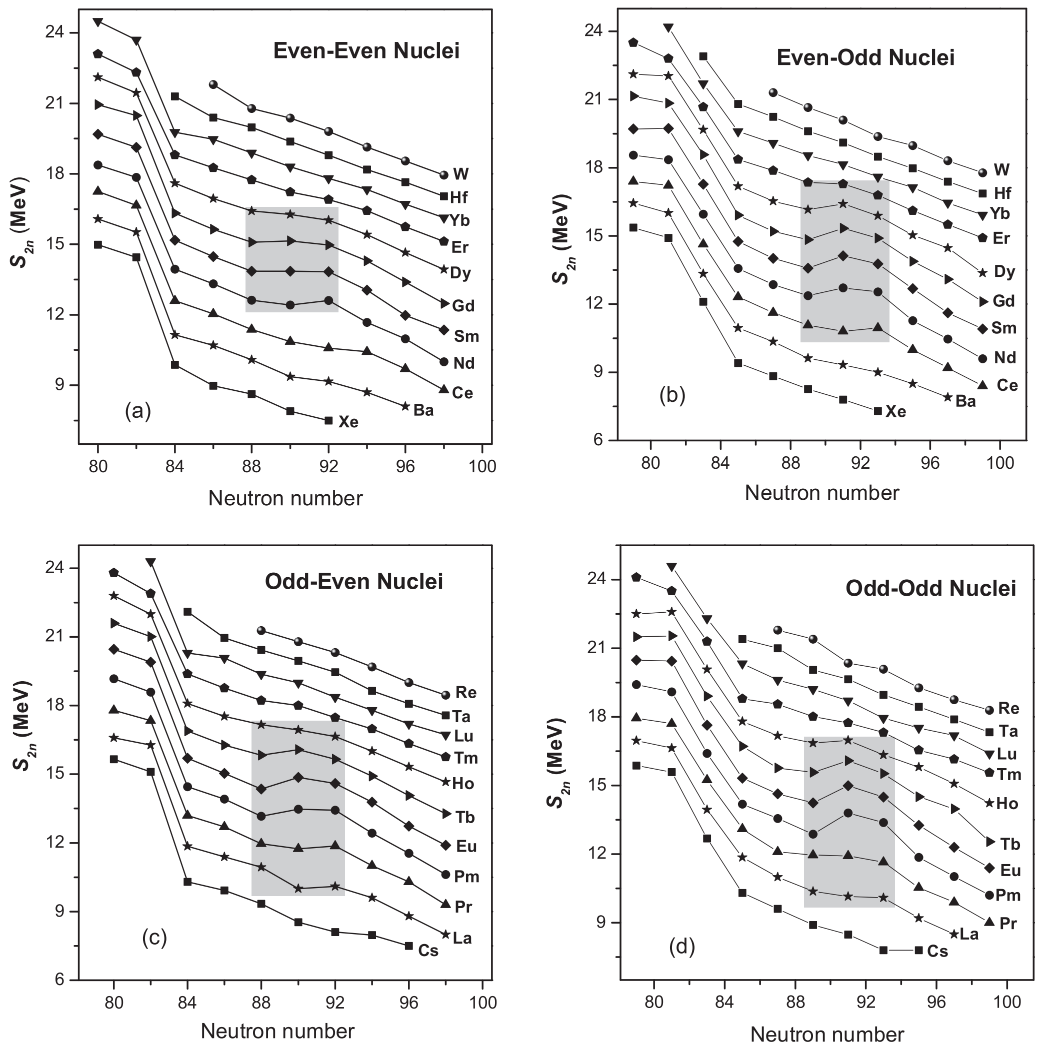

FIG. 3. Two-neutron separation energy, S_{2n}, for even-even nuclei with $Z = 54$-74 and that of the adjacent odd nuclei as functions of the neutron number N, where the shaded rectangular areas show the visible ranges of the sudden flattening around $N = 90$. Experimental data are taken from [43].

the effective order parameter, S_{2n}, for even-even nuclei with $Z = 54$–74, as well as their adjacent even-odd, odd-even, and odd-odd nuclei, as functions of the neutron number, are shown in Fig. 3. It is clearly shown in Fig. 3(a) that a noticeable feature is the sudden flattening near $N = 90$, represented by the shaded (gray) rectangle, after the striking drop around the magic number $N = 82$. According to the conclusions drawn from the discussion of Fig. 2, the sudden flattening indicates the first-order phase transition emerging near $N = 90$ in the corresponding isotopes [32]. More interestingly, a similar or even more noticeable flattening compared to that in even-even nuclei also appears in S_{2n} for odd nuclei as shown in Figs. 3(b)–3(d), which indicate that the first-order phase transition also occurs in these odd nuclei around $N = 90$. In contrast to the even-even nuclei, it seems that the visible range marked by the shaded (gray) rectangle in Fig. 3, representing the first-order phase transition, is enlarged owing to the extra single particle(s), which makes the signal of the transition much clearer.

S_{2n} can further be written as a smooth contribution that is linear in the number of valence neutron pairs, plus the contribution of the deformation [5,41,45,46]. Specifically, one can write

$$S_{2n} = -A - BN_P + S(2n)_{\text{def}}\,, \tag{2}$$

where A and B are the parameters, and N_P is the number of valence neutron pairs, as the proton number is a constant for the isotopes. In order to emphasize the single-particle effect on the phase transition in odd nuclei, we consider the Nd family, including the even-even Nd, even-odd Nd, odd-even Pm, and odd-odd Pm isotopes, and the Sm family, including the even-even Sm, even-odd Sm, odd-even Eu, and odd-odd Eu isotopes, as examples in the comparison of the deformed part of S_{2n} in even systems with that in odd systems. Similarly to the method used in [41,46], the experimental values of $S(2n)_{\text{def}}$ extracted from (2) are shown in Fig. 4. The results for the Nd family are obtained from the data [43] fitted with $A = -14.61$ MeV for even-even Nd, $A = -14.248$ MeV

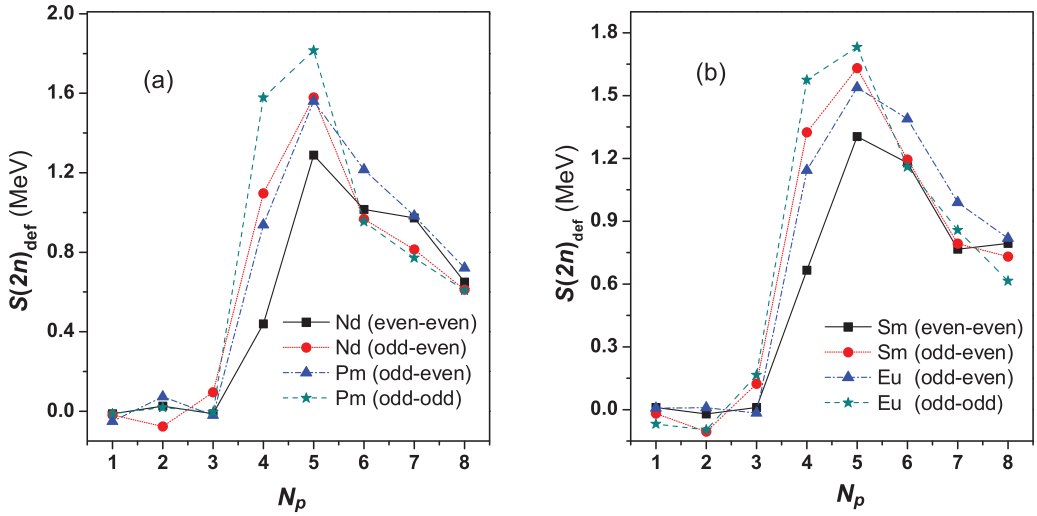

FIG. 4. (Color online) Contribution of deformation to the two-neutron separation energies, $S(2n)_{\mathrm{def}}$, for even Nd and Sm isotopes, as well as their even-odd, odd-even, and odd-odd partners.

for even-odd Nd, $A = -15.159$ MeV for odd-even Pm, and $A = -14.852$ MeV for odd-odd Pm, respectively, and $B = 0.657$ MeV for all isotopes in the family. Similarly, the results for the Sm family are obtained with $A = -15.82$ MeV for even-even Sm, $A = -15.433$ MeV for even-odd Sm, $A = -16.346$ MeV for odd-even Eu, and $A = -16.049$ MeV for odd-odd Eu, respectively, and $B = 0.659$ MeV for all isotopes in the family. In particular, we use the linear function $f(N_p) = -A - BN_P$ to fit the experimental values of S_{2n} for the spherical-like nuclei ^{144,146,148}Nd and ^{146,148,150}Sm as well as their odd partners to fix A and B, as S_{2n} in these spherical-like nuclei behaves as a linear function of the number of valence neutrons [32,34] as shown in Fig. 3. Then one can get $S(2n)_{\mathrm{def}}$ by subtracting the linear part calculated with $f(N_p)$ from the experimental value of S_{2n} according to Eq. (2). As shown in Fig. 4, the phase transitions indicated by $S(2n)_{\mathrm{def}}$ all occur around $N_P = 4$, corresponding to the neutron number $N = 90$ or $N = 91$. Thus, $N_p = 4$ can be considered the critical point of the ground-state phase transition in these isotopes. More importantly, the signal of the phase transition is greatly enhanced in odd nuclei compared to that in even-even nuclei. Specifically, there is an increase of about a factor of 1/4 in $S(2n)_{\mathrm{def}}$ for odd-even and even-odd nuclei and a 1/3 increase for odd-odd nuclei around $N_p = 4$. In addition, there is a small difference between the effect of an extra proton and that of an extra neutron on the phase transition, though both of them show approximately the same contribution to $S(2n)_{\mathrm{def}}$. As pointed out in [41,44], the former is similar to that of adding an external field to the isotopes, as the proton number is a constant for the isotopes, while the latter results in a neutron Fermi surface change in the isotopes.

B. Odd-even effects

To further investigate the effect of an extra single-particle(s) on the phase transition, we show in Fig. 5 the odd-even mass difference, defined as $D_1 = B_A - \frac{B_{A-1}+B_{A+1}}{2}$ [47], for Nd-Sm-Pm-Eu, where B_A represents the total binding energy for a nucleus with mass number A. Results of D_1 for nuclei with neutron number $N =$ odd are shown in Figs. 5(a) and 5(b), and results for nuclei with $N =$ even are presented in Figs. 5(c) and 5(d). Figures 5(a) and 5(b) show that the phase transition is implicitly shown by D_1, whose value is negative and reaches a minimum around $N = 90$ for all isotopes, but B_A itself, shown in the insets, does not present any visible signal of the transition. In addition, there is a wider valley in D_1 of Nd-Sm isotopes versus Pm-Eu isotopes, which indicates that the phase transition may be enhanced by the extra single proton. In contrast to the results shown in Fig. 5 (a) and 5(b), the value of D_1 with $N =$ even is positive and reaches a maximum around $N = 90$ as shown in Figs. 5(c) and 5(d). As we know, the pairing interactions between nucleons provide a positive contribution to the binding energy [47], which indicates that odd-A nuclei are less bound than their even-even neighbors but more bound than their odd-odd neighbors. Then it is easy to understand why D_1, defined above, shows a positive value for a nucleus with $N =$ even but a negative value for a nucleus with $N =$ odd. In addition, as shown in Figs. 5(c) and 5(d), a thinner peak is shown by D_1 for Pm-Eu isotopes compared to D_1 for Nd-Sm isotopes, which further confirms that the effect of an extra single proton may enhance the phase transition.

From the analysis of D_1, it is expected that odd-even effects on all kinds of separation energies may be widely affected by the ground-state phase transitions in nuclei. In the following, we take the Sm and Eu isotopes as examples to show how the other odd-even effects change along the isotope chains. Specifically, the experimental data [43] on two observables, including the α-decay energy $Q(\alpha)$ and double β-decay energy $Q(2\beta)$, together with the corresponding odd-even differences, denoted D_s ($s = 2, 3$), respectively, are shown in Fig. 6, in which the difference is defined as $D_s = R(Z, N-1) - R(Z, N)$ for the energy $R(Z, N)$. It should be mentioned that these kinds of quantities also include the β-decay energy, four β-decay energy, (d, α) reaction energy, and electron capture decay energy [43], which may be discussed elsewhere.

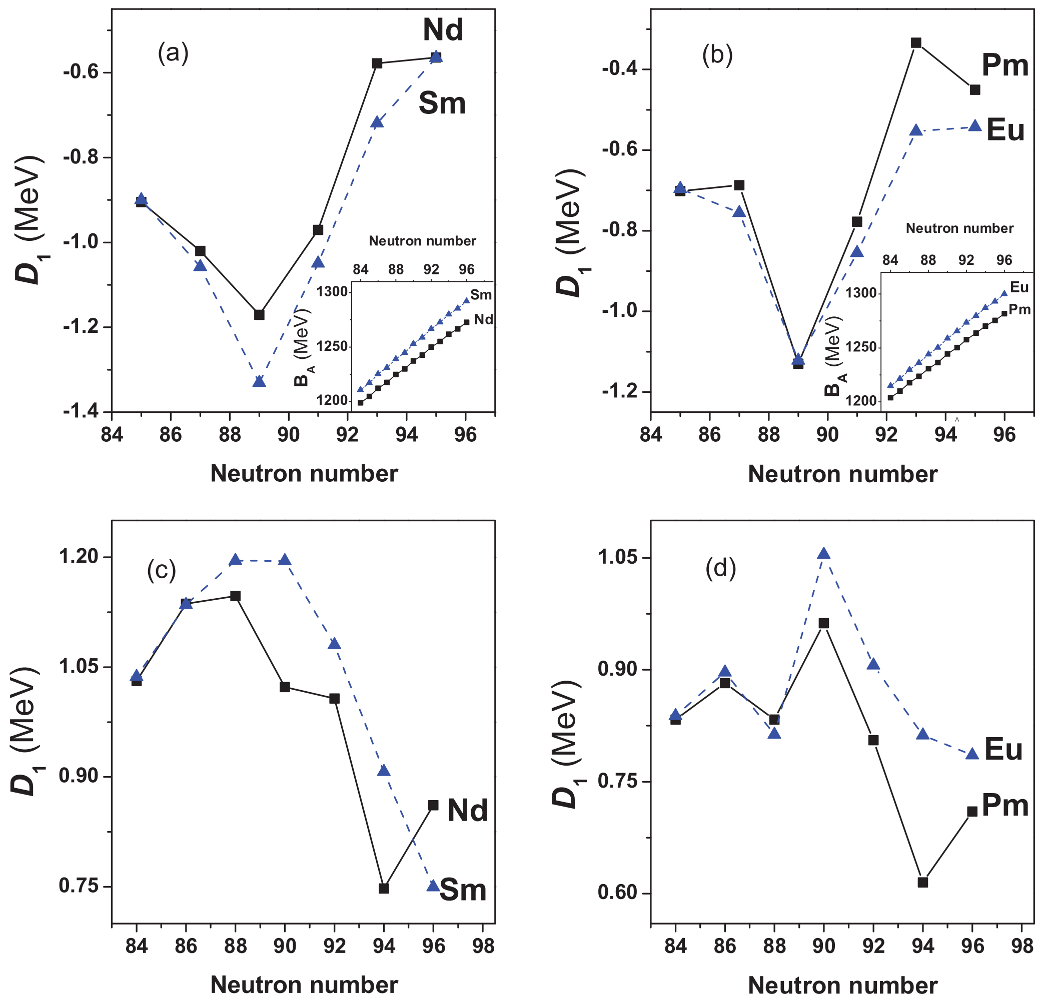

FIG. 5. (Color online) Odd-even mass difference defined by $D_1 = B_A - \frac{B_{A-1}+B_{A+1}}{2}$ shown as a function of neutron number. (a) D_1 for Nd and Sm nuclei with neutron number N = odd as well as the corresponding total binding energy, B_A, for which the values of odd and even isotopes are joined for the same element shown in the insets. (b) The same as (a), but for Pm and Eu nuclei. (c) The same as (a), but for nuclei with N = even. (d) The same as (b), but for N = even. All experimental data are taken from [43].

As shown in the insets in Figs. 6(a) and 6(b), there is a sudden decrease or flattening in $Q(\alpha)$ and $Q(2\beta)$ around $N = 90$, which can thus be taken as other signals of the ground-state phase transition in nuclei. Especially, the values of $Q(\alpha)$ in Sm and Eu isotopes are almost 0 at $N = 90$, where the α-decay energy $Q(\alpha)$ may change from a positive value when $N < 90$ to a negative one when $N > 90$. The results indicate that the α cluster in the nuclei around the critical point $N = 90$ should be less bound according to the definition [43] $Q(\alpha) = M(A, Z) - M(A-4, Z-2) - M(^4He)$. As a result, the α decay in this case may become a spontaneous process [48]. It is further shown in Figs. 6(a) and 6(b) that the corresponding odd-even differences D_2 and D_3 also present evident signals of the phase transition, i.e., the peaks at $N = 91$. Similar features are also shown by D_2 and D_3 for nuclei with N = even, but with the peaks appearing at $N = 90$, as shown in Figs. 6(c) and 6(d). In addition, the variational amplitude of $Q(\alpha)$ or $Q(2\beta)$ is almost the same order of magnitude as the corresponding odd-even difference D_2 or D_3 as shown in Figs. 6(a)–6(d). In contrast, the variational amplitude of B_A is about 10^2 times higher than that of the corresponding odd-even difference D_1 as shown in Fig. 5, which explains why the signal of the phase transition is visible in some quantities but not in others. In short, only the quantities such as $Q(\alpha)$, for which the maximal amplitude is less than 10 MeV, and their odd-even differences may be taken as the qualified effective order parameters to identify the ground-state phase transition in experiments.

III. PAIRING MODEL ANALYSIS OF THE PHASE TRANSITION

As shown in the previous section, the odd-even differences of various separation energies provide signals of the ground-state phase transition. It is well known that odd-even effects, such as those shown in the odd-even differences, are the most important evidence of pairing interactions in nuclei [47,48]. Therefore, a possible microscopic origin of the phase transition shown by the signals in odd-even effects may be revealed by a shell model study. In the following, we adopt the extended

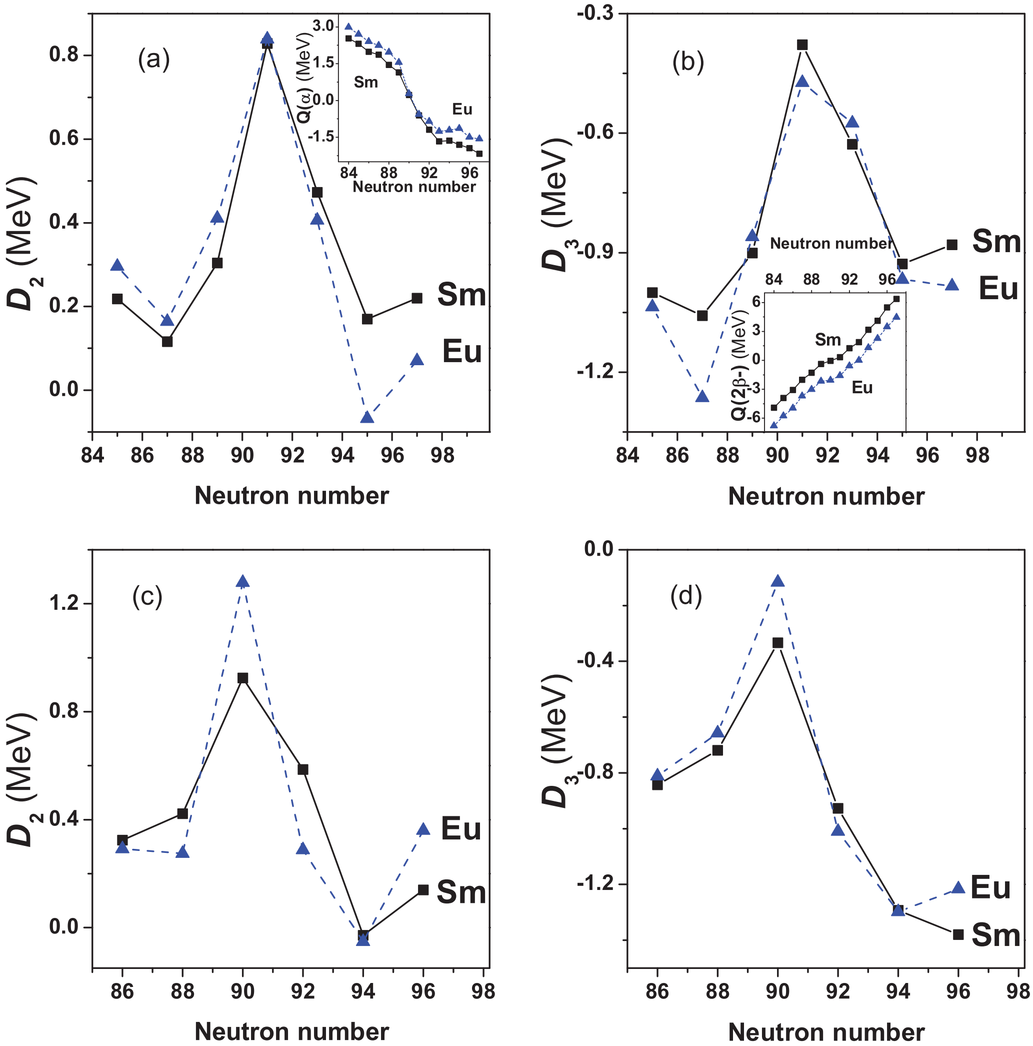

FIG. 6. (Color online) The α-decay energy, $Q(\alpha)$, and double β-decay energy, $Q(2\beta)$, together with the corresponding odd-even differences, defined as $D_s = R(Z, N-1) - R(Z, N)$, for quantity R are shown as functions of the neutron number. (a) D_2, which represents the odd-even difference in $Q(\alpha)$ shown in the corresponding inset, for Sm and Eu nuclei with $N =$ odd; (b) D_3, which represents the odd-even difference in $Q(2\beta)$ shown in the corresponding inset, for Sm and Eu nuclei with $N =$ odd; (c) the same as (a), but for nuclei with $N =$ even; (d) the same as (b), but for nuclei with $N =$ even. All experimental data are taken from [43].

pairing model proposed in [49], which is exactly solvable and, thus, is applicable to nuclei in the whole phase transitional region, ranging from weakly deformed to well-deformed cases. For both proton and neutron sectors, the Hamiltonian of the extended pairing model [49] can be written as

$$\hat{H} = \sum_{i=1}^{p} \varepsilon_i (a_i^\dagger a_i + a_{\bar{i}}^\dagger a_{\bar{i}}) - G \sum_{i,j=1}^{p} b_i^\dagger b_j - G \left(\sum_{u=2}^{\infty} \frac{1}{(u!)^2} \sum_{i_1 \neq i_2 \neq \ldots \neq i_{2u}} b_{i_1}^\dagger b_{i_2}^\dagger \ldots b_{i_u}^\dagger b_{i_{u+1}} b_{i_{u+2}} \ldots b_{i_{2u}} \right), \quad (3)$$

where ε_i denotes the corresponding Nilsson single-particle energy, and $b_i^\dagger = a_i^\dagger a_{\bar{i}}^\dagger$ ($b_i = a_{\bar{i}} a_i$) is the pair creation (annihilation) operator, with $\bar{i}$ labeling the time-reversed state of that labeled withy i. In contrast to the standard pairing model, for which the Hamiltonian involves only the first two terms in (3), the Hamiltonian form of the extended pairing model includes many-pair hopping terms that allow nucleon pairs to simultaneously scatter (hop) between and among different Nilsson levels. It has been further shown that the extended pairing model is similar to the standard pairing model with the first step approximation [50]. In our calculations, single-particle energies $\{\varepsilon_i\}$ are calculated from the Nilsson model with deformation parameters taken from [51], which were determined systematically from the corresponding experimental data [52]. For a chain of isotopes, as the number of valence protons is a constant, it is thus assumed that the odd-even difference in the binding energy $D_1 = B_A - \frac{B_{A-1}+B_{A+1}}{2}$ is only induced by the neutron pairing interactions in the present pairing model, in which no proton-neutron interactions are involved. Accordingly, one can determine the neutron pairing strength G in the pairing model by fitting the odd-even difference D_1 of the corresponding isotopes. Similarly, the

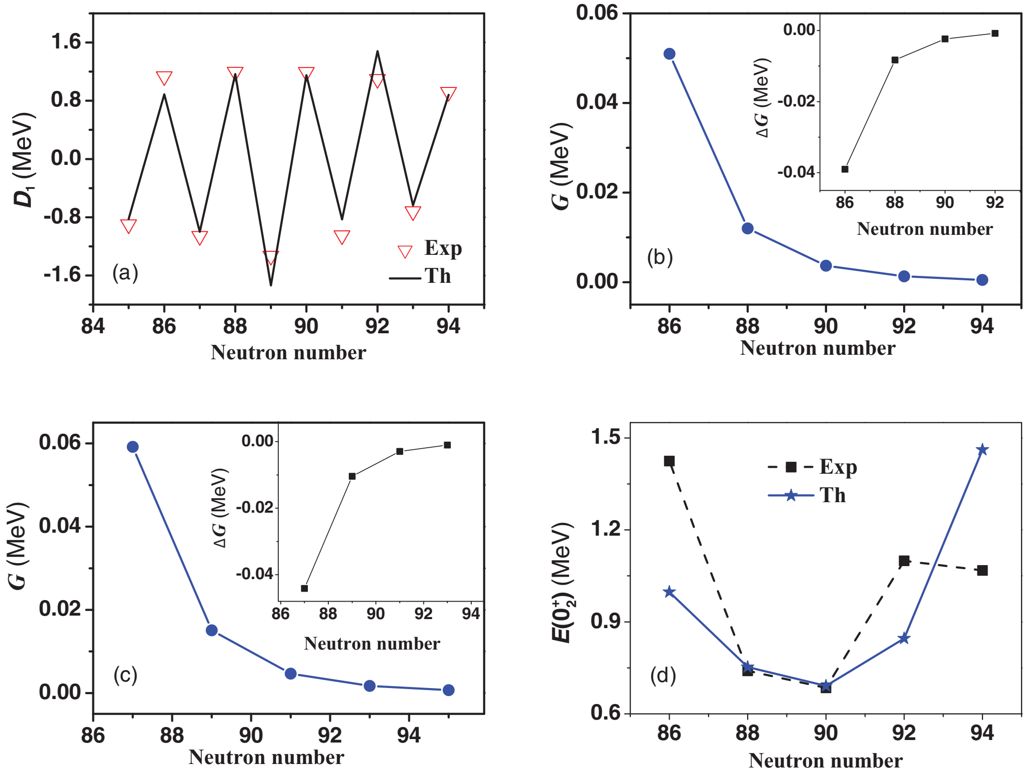

FIG. 7. (Color online) (a) Odd-even difference D_1 (in MeV) for Sm isotopes fitted by the pairing model; (b) neutron pairing strength G (in MeV) together with its difference (in MeV) $\Delta G = G(Z, N+2) - G(Z, N)$ for even Sm nuclei; (c) the same as (b), but for odd Sm nuclei; (d) pairing excitation energy (PEE; in MeV) calculated in the pairing model (stars) and corresponding experimental data on $E_{0_2^+}$ for even Sm nuclei (squares). Experimental data are taken from [43].

proton pairing strength can be obtained by fitting the odd-even differences of the corresponding isotones.

In the following, we take the Sm isotopes as an example to present a pairing model analysis of the phase transition occurring in an isotope chain discussed in Sec. II. Specifically, the odd-even difference D_1 of the isotopes is fitted by the extended pairing model to determine the neutron pairing strength G, for which the results are shown in Fig. 7. As shown in Fig. 7(a), the odd-even difference of the Sm isotopes can be well described by the extended pairing model. Particularly, the phase transitional character around $N = 90$ may be explicitly reproduced by the theoretical results. In contrast, the resulting neutron paring strength G for even Sm nuclei monotonically decreases as a function of the neutron number N as shown in Fig. 7(b). In addition, the sudden flattening around $N = 90$ is clearly demonstrated by the pairing strength difference, defined as $\Delta G = G(Z, N+2) - G(Z, N)$, as shown in the inset in Fig. 7(b). A similar situation also appears in ΔG for odd Sm nuclei, but with the sudden flattening occurring around $N = 91$ as shown in Fig. 7(c). The global change in ΔG as function of the neutron number N is similar to that in S_{2n} shown in Fig. 2. As we know, the phase transition occurring around $N = 90$ in Sm isotopes corresponds to the vibrational-to-rotational transition [5]. Meanwhile, the sudden flattening in the pairing strength difference ΔG just occurs around the vibrational-rotational transitional point (the critical point) as shown in Figs. 7(b) and 7(c). Further investigations indicate that the similar evolutional character of the pairing strength G also emerges in Nd and Gd isotopes. One can thus deduce from the above pairing model analysis that the phase transitional phenomena relevant to pairing in these isotopes may be driven by the decrease in the pairing strength with a sudden flattening in ΔG around the critical point. However, more detailed analysis of this point is still needed.

To further reveal that the phase transitional behavior is closely related to the pairing interaction, we also calculated the pairing excitation energy (PEE), which is also a quantity that is sensitive to the pairing strength G. Because the angular momentum projection along the third axis in the intrinsic frame is considered to be a conserved quantity in the model, the pairing excitation states determined by the model are thus regarded approximately as the excited states with the same spin and parity as those of the ground state of a nucleus. In particular, the results for the first PEE calculated in the theory are compared with the experimental data on $E_{0_2^+}$ of the even Sm nuclei in Fig. 7(d). It should be noted that the calculated first PEE here corresponds to the first neutron pair excitation because the energy of the first excited proton pair in an Sm nucleus is always higher than that of the first excited neutron pair in the present model under the procedure of determining the pairing strength mentioned above. As clearly shown in Fig. 7(d), an evident phase transitional signal has been demonstrated by the calculated PEE around $N = 90$, where the PEE reaches its minimum. Such a transitional character is accompanied by the sudden flattening of ΔG around $N = 90$. More importantly, the phase transitional signal shown by the PEE calculated in the theory is indeed confirmed by the corresponding experimental data, which are systematically reproduced by the pairing model. But some quantitative deviations of the results calculated by the theory from the corresponding experimental values are also obvious. Further improvements in the theoretical results may require consideration of angular momentum projection as well as other interactions among nucleons besides the pairing interaction. At any rate, the results shown in Fig. 7 indicate that the transitional characters in connection with the pairing interactions in Sm isotopes can be qualitatively described by the extended pairing model. It should be mentioned that the PEE in odd Sm nuclei are not considered here because some single-particle excitations with the same spin and parity as those of the ground state are often involved in the low-lying spectrum, which makes it difficult to pick out the PEE from the spectrum of the odd nuclei. On the other hand, it was recognized [32,34] that collective phenomena in nuclei may be caused by competition between the spherical-driving pairing interaction and the deformation-driving valence proton-neutron interaction. More rigorous microscopic calculations of the structural characters of the shape phase transition in odd nuclei should involve valence proton-neutron interactions in the model; this is, however, beyond the present simple analysis.

IV. SUMMARY

In summary, we have made a systematic investigation of the ground-state phase transition in even-even, odd-A, and odd-odd nuclei in the $A \sim 150$ mass region by the analysis of some experimental observables (the effective order parameters). We have shown that the effective order parameter S_{2n} is qualified to identity the phase transition and distinguish its order. Based on S_{2n}, we found that the signal of the ground-state phase transition in odd nuclei is greatly enhanced by an extra single particle(s) in comparison to that in adjacent even-even nuclei. Specifically, there is an $\sim 1/4$ increase in $S(2n)_{\mathrm{def}}$ for odd-even and even-odd nuclei and a $1/3$ increase for odd-odd nuclei around the critical point. Through the analysis of other separation energies and their odd-even differences, a wealth of information on the ground-phase transition has been revealed. In particular, we found that nearly all the odd-even differences reach their extreme values (maximum or minimum) around the critical point. The analysis of Sm isotopes based on the extended pairing model shows that the critical phenomena relevant to the pairing interaction may be microscopically driven by the continuous decrease in the pairing strength G with a sudden flattening in the pairing strength difference ΔG around the critical point. However, more sophisticated microscopic analysis of excited-state properties are still needed to eventually confirm or disprove the theoretical predictions.

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