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Coarse-grained potential analysis of neutron-proton and proton-proton scattering below the pion production threshold

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Using the δ-shell representation we present a successful fit to neutron-proton and proton-proton scattering data below the pion production threshold. A detailed overview of the theory necessary to calculate observables with this potential is presented. A new data selection process is used to obtain the largest mutually consistent data base. The analysis includes data within the years 1950 to 2013. Using 46 parameters we obtain $\chi^2/N_{\rm data} = 1.04$ with $N_{\rm data} = 6713$ including normalization data. Phase shifts with error bars are provided.

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I. INTRODUCTION

The determination of the nucleon-nucleon (NN) interaction has played a central role in nuclear physics [1]. So far, the only direct way to determine the interaction from first principles and in terms of the underlying quark and gluon degrees of freedom is by means of lattice QCD calculations which will eventually come to realistic scenarios (see, e.g., [2,3] and references therein). The traditional alternative to those incipient calculations is to determine a phenomenological interaction from a partial wave analysis (PWA) of the highly abundant (about 8000) scattering data. Equally important and helpful should be a credible determination of theoretical uncertainties in the interaction and its propagation to nuclear structure calculations, an aspect that applies both to lattice QCD and PWA. The necessary condition to carry out such a program is to achieve in any case a chi square per degree of freedom $\chi^2/\mathrm{DOF} \lesssim 1$ description of all available NN data when confronted either with the predictions of the lattice QCD or the fitted phenomenological interaction. Before 1990 all fits determining phenomenological potentials which were routinely used in nuclear structure calculations did not lower the value $\chi^2/\mathrm{DOF} \sim 2$ (for a historic account see, e.g., [1]). According to well-known statistical principles this prevents estimating the errors due to statistical fluctuations of the data. Only in the mid-1990's was it possible to provide high quality fits to np and pp scattering data with $\chi^2/\mathrm{d.o.f.} \lesssim 1$ mainly due to (i) the scrupulous inclusion of charge dependence (CD) including vacuum polarization, relativistic corrections, magnetic moment interactions, among other effects, and (ii) a sound rejection criterion of 3σ-inconsistent data with the validating NN interaction. Along these lines several parametrizations have been proposed to describe a continuously increasing database of np and pp experimental scattering data below the pion production threshold [4–8] and even to energies as high as 3 GeV for pp and 1.3 GeV for np [9] (in this latter case all data are included in the analysis and $\chi^2/\mathrm{DOF} \sim 1.6$). In Ref. [10] we presented a δ-shell potential fitted to pseudodata that consisted of the mean and standard deviation of the np phase shifts given by the Nijmegen PWA [4] and six other potentials with $\chi^2/\mathrm{DOF} \lesssim 1$ [5–8] and obtained an estimate of the systematic uncertainties of the NN interaction. A new PWA to pp and np data including experiments until 2013 was presented in [11] from which statistical uncertainties were extracted. Here we present the details of that work, paying special attention to the fitting procedure and the data selection process.

We note that with the total NN database, comprising about 8000 scattering data, none of the available post-1993 analyses yields an acceptable fit, i.e., $\chi^2/\mathrm{DOF} \lesssim 1$, by itself to their contemporary *complete* database (for a discussion about the pre-1993 situation see, e.g., [12]). A dedicated look at the database shows that there are experiments which measure several observables in quite similar and/or overlapping kinematical conditions. However, a closer inspection reveals that certain data are mutually incompatible within statistical errors. Clearly, this implies that at least one dataset is incorrect. Of course, the possibility of several datasets being incorrect should not be discarded *a priori*. The key question is which data or dataset should be kept and which ones should be rejected. All analyses carried out so far approach this issue from the point of view of the tension between the data and the model used to analyze them. This obviously introduces a bias in the choice of the database, which can be included as a source of systematic errors. The Nijmegen group fixed the database from that point of view in 1993 [4] and the high quality phenomenological NijmI, NijmII, Reid93, and AV18 potentials developed thereafter [5,6] use the same selected data to perform the analysis. The CD-Bonn potential analysis [7] kept the same accepted database and applied the 3σ criterion for the new pp and np data published between 1993 and 1999 but the possible tension between pre- and post-1993 data was not considered. The covariant spectator model was used to analyze np scattering data [8] resulting in a new selection of compatible data.

Following an interesting suggestion made by Gross and Stadler [8] proposing a refined 3σ criterion, in this work we analyze the tension among each pair of datasets of experiments performed and published from 1950 until 2013.

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As a consequence, a large fraction of otherwise rejected data is rescued with a statistical significance and in a model independent way. Although we use a specific representation of the unknown part of the interaction, if an unbiased analysis is carried out, all errors should be of purely statistical origin and the particular representation should not play any role. In this regard, our motivation to upgrade the PWA in a statistical meaningful fashion was the realization [13,14] that discrepancies among different high quality fits were larger than the declared statistical uncertainties.

The paper is organized as follows, Sec. II defines the potential, as well as the parametrization of the short and intermediate-range parts by the δ-shell representation and the expressions that describe the long-range part. In Sec. III the fitting procedure is laid out. Special attention is given to review the on-shell scattering amplitude computation, especially the electro-magnetic part, which appears scattered in several publications and we collect here for the benefit of the unfamiliarized readers. Section IV details the improved selection data criterion to obtain the largest database without incompatible data. Extensive tables of accepted and rejected data are also given. Section V presents the results and includes a table of the fitted parameters in the operator basis. After error propagation with the pertinent correlations as encoded in the standard covariance matrix is made, we also provide the low angular momentum partial waves' phase shifts with statistical errors. Conclusions and outlook for further implementations of the δ-shell potential are given in Sec. VI. Finally the appendixes include expressions that relate our fitting parameters with parameters in the operator basis and details for integrating the Schrödinger equation with a δ-shell potential both with central and tensor terms.

II. DESCRIPTION OF THE POTENTIAL

For our purposes the NN interaction can be decomposed into different known pieces featuring understood physical effects and unknown contributions which are constrained with the help of the currently existing data. In this paper we only considered *published np* and *pp* scattering data. To this end many possible functional forms have been proposed. In our previous works [14,15] we have motivated the use of the δ-shell representation, $V_{\rm DS}(r)$, which was first introduced in the NN context by Aviles in 1973 [16]. It consists of a sum of N Dirac δ functions, each one centered around a concentration radius r_i and multiplied by a strength coefficient V_i,

$$V_{\rm DS}(r) = \sum_{i=1}^{N} V_i \Delta r_i \delta(r - r_i). \tag{1}$$

The computational advantages of this representation in nuclear structure calculations and uncertainty estimation have already been stressed [11,14,15]. Using this representation it is possible to accurately describe the short- and intermediate-range part of the nucleon-nucleon interaction by fixing the distance between concentration radii at $\Delta r = r_{i+1} - r_i = 0.6$ fm and determining the strength coefficients by a fit to scattering data below the pion production threshold [11]. The long-range part consists of the well-known charge dependent one pion exchange (OPE) potential and electromagnetic (EM) interactions. In its complete form the potential reads

$$V(r) = \sum_{n=1}^{21} O_n \left[\sum_{i=1}^{N} V_{i,n} \Delta r_i \delta(r - r_i) \right] + [V_{\rm OPE}(r) + V_{\rm EM}(r)]\theta(r - r_c), \tag{2}$$

where O_n is a set of operators. The first 18 operators correspond to the ones used on the AV18 potential [6]; the remaining three new operators incorporate further charge dependence and are defined in Appendix A. A typo in our previous work [11] indicates 18 instead of 21 operators; an erratum about this has been published [17]. The distinction between intermediate and long range is made explicit by the cutoff radius r_c, which turns out to be 3.0 fm since the interaction above that distance is correctly described by OPE and EM terms only with no finite-size effects. Smaller values of r_c were also considered but did not come out as optimal.

Even though the strength coefficients $V_{i,n}$ can be fitted directly to the data, the partial wave decomposition of the potential

$$V_{l,l'}^{JS}(r) = \frac{1}{2\mu_{\alpha\beta}} \sum_{i=1}^{N} (\lambda_i)_{l,l'}^{JS} \delta(r - r_i), \quad r \leqslant r_c, \tag{3}$$

where $\mu_{\alpha\beta}$ is the reduced mass with $\alpha, \beta = n, p$, allows us to directly incorporate charge dependence in the $^1S_0$ parameters. To reconstruct the complete potential, the strength coefficients in the operator basis can be written as a linear combination of the $(\lambda_i)_{l,l'}^{JS}$ coefficients of low angular momentum partial waves as is shown in Appendix A. In practice the potential can be parametrized by using only 15 independent partial waves, therefore only 15 operators will have independent strength coefficients; the rest will be either fixed to zero or will be linearly dependent on other operators' coefficients. A good reason to use the lowest partial wave coefficients as primary fitting parameters is that correlations among different partial waves turn out to be much smaller than the correlations between the operator coefficients. Note that we are just making a change of basis, but the coefficients of the higher partial waves are calculated by constructing the complete potential and decomposing it into the corresponding partial waves (see Appendix A). The resulting partial wave coefficients were displayed in our previous work [11]. Here we will show the equivalent results for the operator coefficients (see Table VI below).

The charge dependent OPE potential in the long-range part of the interaction is the same as the one used by the Nijmegen group on their 1993 partial wave analysis [4] and reads

$$V_{m,\rm OPE}(r) = f^2 \left(\frac{m}{m_{\pi^\pm}} \right)^2 \frac{1}{3} m [Y_m(r)\sigma_1 \cdot \sigma_2 + T_m(r) S_{1,2}], \tag{4}$$

f being the pion coupling constant, σ_1 and σ_2 the single nucleon Pauli matrices, $S_{1,2}$ the tensor operator, $Y_m(r)$ and

$T_m(r)$ the usual Yukawa and tensor functions,

$$Y_m(r) = \frac{e^{-mr}}{mr},$$
$$T_m(r) = \left(1 + \frac{3}{mr} + \frac{3}{(mr)^2}\right)\frac{e^{-mr}}{mr}. \quad (5)$$

Charge dependence is introduced by the difference between the charged $m_{\pi^\pm}$ and neutral m_{π^0} pion mass by setting

$$V_{\mathrm{OPE},pp}(r) = V_{m_{\pi^0},\mathrm{OPE}}(r),$$
$$V_{\mathrm{OPE},np}(r) = -V_{m_{\pi^0},\mathrm{OPE}}(r) + (-)^{(T+1)}2V_{m_{\pi^\pm},\mathrm{OPE}}(r). \quad (6)$$

The neutron-proton electromagnetic potential includes only a magnetic-moment interaction

$$V_{\mathrm{EM,np}}(r) = V_{\mathrm{MM,np}}(r) = -\frac{\alpha\mu_n}{2M_n r^3}\left(\frac{\mu_p S_{1,2}}{2M_p} + \frac{\mathbf{L}\cdot\mathbf{S}}{\mu_{np}}\right), \quad (7)$$

where μ_n and μ_p are the neutron and proton magnetic moments, M_n is the neutron mass, M_p is the proton one, and $\mathbf{L}\cdot\mathbf{S}$ is the spin-orbit operator. The EM terms in the proton-proton channel include one- and two-photon exchange, vacuum polarization, and magnetic moment,

$$V_{\mathrm{EM,pp}}(r) = V_{\mathrm{C1}}(r) + V_{\mathrm{C2}}(r) + V_{\mathrm{VP}}(r) + V_{\mathrm{MM,pp}}(r), \quad (8)$$

where

$$V_{\mathrm{C1}}(r) = \frac{\alpha'}{r}, \quad (9)$$
$$V_{\mathrm{C2}}(r) = -\frac{\alpha\alpha'}{M_p r^2}, \quad (10)$$
$$V_{\mathrm{VP}}(r) = \frac{2\alpha\alpha'}{3\pi r}\int_1^\infty e^{-2m_e rx}\left(1 + \frac{1}{2x^2}\right)\frac{\sqrt{x^2-1}}{x^2}dx, \quad (11)$$
$$V_{\mathrm{MM,pp}}(r) = -\frac{\alpha}{4M_p^2 r^3}\left[\mu_p^2 S_{1,2} + 2(4\mu_p - 1)\mathbf{L}\cdot\mathbf{S}\right]. \quad (12)$$

Note that these potentials are *only* used above $r_c = 3$ fm and thus form factors accounting for the finite size of the nucleon can be set to 1. Energy dependence is present through the parameter

$$\alpha' = \alpha\frac{1 + 2k^2/M_p^2}{\sqrt{1 + k^2/M_p^2}}, \quad (13)$$

where k is the center-of-mass momentum and α is the fine-structure constant. Table I lists the values used for the fundamental constants in this work's calculations.

Even though the contribution of all non-Coulomb electromagnetic terms to the noncentral partial wave phase shifts is rather small when compared to the V_{C1} and V_{OPE} ones, their inclusion is crucial to accurately describe the scattering amplitude. Also the vacuum polarization contribution is needed for the proper calculation of low-energy observables. For these reasons, in this work the potential in the $^1S_0$ partial wave includes all EM terms listed previously, while the rest have only the V_{C1} one. Still, the electromagnetic scattering amplitude is constructed with all terms explicitly, as is shown below.

The Coulomb effects in the short-range part of the interaction are included by a coarse-grained representation, instead of simply extending V_{C1} below r_c, in order to keep the advantage of having only a few interaction radii r_i in that region. This coarse graining is obtained by looking for a δ-shell representation of the interaction, i.e., $\bar{V}_{\mathrm{C1}}(r) = \sum_n V_i^C \Delta r_i \delta(r - r_i) + \theta(r - r_c)V_{\mathrm{C1}}(r)$, where the V_i^C are determined by reproducing the Coulomb scattering amplitude to high precision and are not changed in the fitting process. The first line of Table VI shows the corresponding δ-shell parameters V_i^C.

TABLE I. Values of fundamental constants used.

Constant	Value	Units
$\hbar c$	197.327053	MeV fm
m_{π^0}	134.9739	Mev/c^2
$m_{\pi^\pm}$	139.5675	Mev/c^2
M_p	938.27231	Mev/c^2
M_n	939.56563	Mev/c^2
m_e	0.510999	Mev/c^2
α^{-1}	137.035989	
f^2	0.075	
μ_p	2.7928474	μ_0
μ_n	−1.9130427	μ_0

III. FITTING PROCEDURE

The determination of the $V_{i,n}$ coefficients in Eq. (2) is made through a partial wave decomposition of the potential to calculate observables and reproduce experimental data. Our fitting procedure consists of using the $(\lambda_i)_{l,l'}^{JS}$ parameters to calculate partial wave phase shifts, summing those phase shifts to obtain the scattering amplitude M, extracting observables from M, comparing observables with experimental data by a merit function and minimizing such function with respect of the fitting parameters. In theory such a procedure requires a sum of an infinite number of partial waves in the complete scattering amplitude, but all high angular momentum partial waves in the potential can be written as linear combinations of the low angular momentum ones by means of the relation between the latter and the operator basis (see Appendix A). This allows us to use only a few $(\lambda_i)_{l,l'}^{JS}$ coefficients as independent fitting parameters. Also, the phase shifts of very high angular momentum partial waves are mostly determined by the long-range part of the interaction and their contribution to the scattering amplitude is limited by the centrifugal barrier, therefore in practice a limited number of partial waves is needed and summing up to $J_{\max} = 20$ proofs is sufficiently accurate to compute the strong scattering amplitude below pion production threshold.

Phase shifts are calculated by integrating Schrödinger's equation; the details of such calculation with the δ-shell potential are given in Appendix B. The next subsection reproduces and outlines the expressions necessary to calculate the nuclear and electromagnetic scattering amplitudes. Reference [18] has an exhaustive list of observables that can be extracted from different parametrizations of M. The calculation of the merit function χ^2 is explained in Sec. III B; special attention is

given to the treatment of systematic uncertainties from the experimental data.

In any fit we have *always* constrained the potential to reproduce the deuteron binding energy to its experimental value $B = 2.224\,575(9)$ MeV as well as the np $^1S_0$ scattering length to $\alpha_{^1S_0} = -23.74(2)$ fm.

A. Scattering amplitude

The on-shell scattering amplitude M can be expressed in terms of five complex quantities; several parametrizations exist for this purpose and for definiteness we choose the Wolfenstein parameters where

$$M(\mathbf{k}_f, \mathbf{k}_i) = a + m(\sigma_1, \mathbf{n})(\sigma_2, \mathbf{n}) + (g-h)(\sigma_1, \mathbf{m})(\sigma_2, \mathbf{m}) + (g+h)(\sigma_1, \mathbf{l})(\sigma_2, \mathbf{l}) + c(\sigma_1 + \sigma_2, \mathbf{n}), \quad (14)$$

where $\mathbf{l}$, $\mathbf{m}$, $\mathbf{n}$ are three unitary orthogonal vectors along the directions of $\mathbf{k}_f + \mathbf{k}_i$, $\mathbf{k}_f - \mathbf{k}_i$ and $\mathbf{k}_i \wedge \mathbf{k}_f$ and $\mathbf{k}_f$, $\mathbf{k}_i$ are the final and initial relative nucleon momenta respectively. The parameters a, m, g, h, c depend on the scattering angle θ and k, also any scattering observable in our database can be written in terms of them [18,19].

The partial wave decomposition of the $M^s_{m'_s,m_s}$ matrix elements due to a certain interaction is

$$M^s_{m'_s,m_s}(\theta) = \frac{1}{2ik}\sum_{J,l',l}\sqrt{4\pi(2l+1)}Y^{l'}_{m'_s-m_s}(\theta,0) \times C^{l',s,J}_{m_s-m'_s,m'_s,m_s} i^{l-l'}\left(S^{J,s}_{l,l'} - \delta_{l',l}\right)C^{l,s,J}_{0,m_s,m_s}, \quad (15)$$

where $C^{l,s,J}_{m_l,m_s,m_J}$ is a Clebsch-Gordan coefficient, $Y^l_m(\theta,\phi)$ the spherical harmonic, $\delta_{l,l'}$ a Kronecker δ, and $S^{J,s}_{l,l'}$ are the S matrix elements with the corresponding phase shifts of such interaction. Denoting the phase shifts as $\delta^{J,s}_{l,l'}$, for the singlet ($s=0$, $l=l'=J$) and triplet uncoupled ($s=1$, $l=l'=J$) channels the S matrix is simply $e^{2i\delta^{J,s}_{l,l}}$, in the triplet coupled channel ($s=1$, $l=J\pm1$, $l'=J\pm1$) it reads

$$S^J = \begin{pmatrix} e^{2i\delta^{J,1}_{J-1}}\cos 2\varepsilon_J, & ie^{i(\delta^{J,1}_{J-1}+\delta^{J,1}_{J+1})}\sin 2\varepsilon_J \\ ie^{i(\delta^{J,1}_{J-1}+\delta^{J,1}_{J+1})}\sin 2\varepsilon_J, & e^{2i\delta^{J,1}_{J+1}}\cos 2\varepsilon_J \end{pmatrix}, \quad (16)$$

with ε_J the mixing angle. The scattering amplitude has a contribution for every term considered in the potential; this allows us to separate M in one part due to the nuclear interaction and another coming from the EM terms,

$$M = M_{\rm EM} + M_{\rm N}. \quad (17)$$

The pp and np electromagnetic amplitudes read

$$M_{\rm EM,pp} = M_{\rm C1} + M_{\rm C2} + M_{\rm VP} + M_{\rm MM,pp}, \quad (18)$$

$$M_{\rm EM,np} = M_{\rm MM,np}. \quad (19)$$

Given the finite range nature of the nuclear interaction, $M_{\rm N}$ has a fast convergence when summing over partial waves and allows a rapid calculation every time the fitting parameters are varied during the fitting procedure. Meanwhile, the $M_{\rm EM}$ part of a pp scattering has a slow convergence due to the interplay among different long-range contributions. Actually, $M_{\rm C2}$ and $M_{\rm MM,pp}$ require summations up to $l = 1000$. Fortunately, since $M_{\rm EM}$ does not depend on the fitting parameters it only has to be calculated once and stored.

The expressions to calculate every part of the pp electromagnetic scattering amplitude are well known [4,20,21] and we reproduce them here for completeness. The Coulomb scattering amplitude is given by

$$f_{{\rm C1},k}(\theta) = \frac{1}{2ik}\sum_l (2l+1)[e^{2i(\sigma_l-\sigma_0)}-1]P_l(\theta) = -\frac{\eta}{k}\frac{e^{-i\eta\ln(1/2)(1-\cos\theta)}}{1-\cos\theta}, \quad (20)$$

where $P_l(\theta)$ are the Legendre polynomials, $\eta = \alpha' M_p/(2k)$, and the Coulomb phase shifts are calculated with $\sigma_l = \arg\Gamma(l+1+i\eta)$.

Since the two-photon exchange potential $V_{\rm C2}$ has a $1/r^2$ dependence it can be absorbed into the centrifugal barrier of the radial Schrödinger's equation and the latter is solved analytically using Coulomb wave functions of noninteger l. This procedure leads to the $V_{\rm C2}$ phase shifts

$$\rho_l = \sigma_\lambda - \sigma_l + \frac{(l-\lambda)\pi}{2}, \quad (21)$$

where λ is obtained by solving $\lambda(\lambda-1) = l(l+1) - \alpha\alpha'$. Now the amplitude can be calculated with

$$f_{{\rm C2},k}(\theta) = \frac{1}{2ik}\sum_l (2l+1)e^{2i(\sigma_l-\sigma 0)}[e^{2i\rho_l}-1]P_l(\theta). \quad (22)$$

A similar expression to the one in Eq. (22) describes the vacuum polarization (VP) scattering amplitude replacing the ρ_l phase shifts for the VP ones, which are usually denoted by τ_l. Since the values for τ_l are rather small, even for large values of k and l, the approximation

$$f_{{\rm VP},k}(\theta) = \frac{1}{k}\sum_l (2l+1)e^{2i(\sigma_l-\sigma 0)}\tau_l P_l(\theta) \quad (23)$$

is a good starting point to calculate $f_{{\rm VP},k}$ as it can be expanded by a series in powers of η where $f_{\rm VP} = f^{(0)}_{\rm VP} + f^{(1)}_{\rm VP} + f^{(2)}_{\rm VP} + \cdots$. The leading order is obtained using the first Born approximation and is expressed as

$$f^{(0)}_{{\rm VP},k}(\theta) = -\frac{\alpha}{3k\pi}\eta\frac{F(k,\theta)}{1-\cos\theta}, \quad (24)$$

where

$$F(k,\theta) = -\frac{5}{3} + X + \sqrt{1+X}\left(1-\frac{X}{2}\right) \times \ln\left[\frac{(1+X)^{1/2}+1}{(1+X)^{1/2}-1}\right], \quad (25)$$

with $X = 2m_e^2/[k^2(1-\cos\theta)]$. The real part of the subleading-order term can be computed with

$$\mathrm{Re}\left[f^{(1)}_{{\rm VP},k}(\theta)\right] = \frac{4\alpha}{3k\pi(1-\cos\theta)}\eta^2\left(\frac{1-\cos\theta}{1+\cos\theta}\right)^{1/2} \times \left[\tan^{-1}\left(\frac{1+\cos\theta}{1-\cos\theta}\right)^{1/2} - \tan^{-1}\left(\frac{m_e^2}{k^2}\frac{1+\cos\theta}{1-\cos\theta}\right)^{1/2}\right] \quad (26)$$

and a good approximation for the corresponding imaginary part is

$$\mathrm{Im}\left[f^{(1)}_{\mathrm{VP},k}(\theta)\right] \approx \frac{\alpha}{3k\pi(1-\cos\theta)}\eta^2\left[\ln\left(\frac{1}{X}\right)\right] \times\left[\ln\left(\frac{k}{m_e}\right)-\frac{3}{2}\ln\left(\frac{2}{1-\cos\theta}\right)\right]. \tag{27}$$

An expansion up to this order has been found to be accurate enough to describe $f_{\mathrm{VP},k}$ for the energy range discussed in this work.

The treatment of identical particles in a pp scattering reaction requires the antisymmetrization of the $M^s_{m'_s,m_s}$ matrix elements; this is easily done by

$$M^s_{\mathrm{X}m'_s,m_s} = [f_{\mathrm{X},k}(\theta)+(-)^s f_{\mathrm{X},k}(\pi-\theta)]\delta_{m'_s,m_s}, \tag{28}$$

where $X = C1, C2, \mathrm{VP}$.

For the magnetic-moment pp amplitude it is necessary to calculate the partial-wave K matrix which is defined by $S-1 = 2iK(1-iK)^{-1}$. Since $V_{\mathrm{MM,pp}}$ is proportional to the spin-orbit and tensor operator there is no contribution to the spin singlet channel and $(K_{\mathrm{MM,pp}})^{J,0}_{l,l} = 0$, the spin triplet channel elements are given by

$$\begin{aligned}
(K_{\mathrm{MM,pp}})^{J=l,1}_{l,l} &= -M_p k(2f_T - f_{LS})I_{l,l},\\
(K_{\mathrm{MM,pp}})^{J=l+1,1}_{l,l} &= -M_p k\left(-\frac{2l}{2l+3}f_T - l f_{LS}\right)I_{l,l},\\
(K_{\mathrm{MM,pp}})^{J=l+1,1}_{l+2,l+2} &= -M_p k\left(-\frac{2l+6}{2l+3}f_T - (l+3)f_{LS}\right)\\
&\quad\times I_{l+2,l+2},\\
(K_{\mathrm{MM,pp}})^{J=l+1,1}_{l,l+2} &= -M_p k\left(6\frac{\sqrt{(l+1)(l+2)}}{2l+3}f_T\right)I_{l,l+2},
\end{aligned} \tag{29}$$

where f_T and f_{LS} are the coefficients of the tensor and spin-orbit operators in the potential, i.e.,

$$f_T = -\frac{\alpha\mu_p^2}{4M_p^2}, \quad f_{LS} = -\frac{\alpha(4\mu_p-1)}{2M_p^2}, \tag{30}$$

the $I_{l,l'}$ terms are integrals of the $1/r^3$ dependence with Coulomb wave functions and are given by

$$\begin{aligned}
I_{l,l} &= \frac{1}{2l(l+1)}\\
&\quad+\frac{1-\pi\eta+\pi\eta\coth(\pi\eta)-2\eta^2\sum_{n=0}^{l}(n^2+\eta^2)^{-1}}{2l(l+1)(2l+1)},\\
I_{l,l+2} &= \tfrac{1}{6}\,|l+1+i\eta|^{-1}\,|l+2+i\eta|^{-1}\,.
\end{aligned} \tag{31}$$

In principle the set of equations in (29) allows us to calculate $S_{\mathrm{MM,pp}}$ and use the latter to obtain the $M_{\mathrm{MM,pp}}$ matrix elements via the partial-wave decomposition of Eq. (15); unfortunately the numerical effort to reach convergence by summing over J, l and l' is too big to be practical. Using the approximation $S_{\mathrm{MM,pp}} - 1 \approx 2iK_{\mathrm{MM,pp}}$ gives rise to a contribution

$$Z_{LS} = -\frac{M_p}{\sqrt{2}}f_{LS}\sum_{\text{odd } l} e^{2i(\sigma_l-\sigma_0)}\frac{2l+1}{l(l+1)}P_l(\theta) \tag{32}$$

to the $M^1_{1,0}$ matrix element and the same with a minus sign to the $M^1_{0,1}$ one. Fortunately, this series can be calculated analytically with

$$Z_{LS} = -\frac{M_p f_{LS}}{\sin\theta\sqrt{2}}(e^{-i\eta\ln(1/2)(1-\cos\theta)} + e^{-i\eta\ln(1/2)(1+\cos\theta)} - 1). \tag{33}$$

The use of this result significantly improves the convergence rate of $M_{\mathrm{MM,pp}}$.

The neutron-proton EM amplitude can be expressed in terms of the Wolfenstein-like parameters a, b, c, d, e [21–23], usually known as the Saclay parameters, and we reproduce this result for completeness as well:

$$\begin{aligned}
a_{\mathrm{EM,np}}(s,t) &= \frac{\alpha}{t\sqrt{s}}\left\{\left(F_1^nF_1^p + tF_2^nF_2^p\right)\left[s-M_n^2-M_p^2+\frac{t}{8sk^2}\{[s-(M_n+M_p)^2][3s-(M_n-M_p)^2]\right.\right.\\
&\quad\left.+2[s-(M_n-M_p)^2](\sqrt{s}-M_n-M_p)^2\} + \frac{t^2}{16sk^4}[s-(M_n-M_p)^2](\sqrt{s}-M_n-M_p)^2\right]\\
&\quad\left.+\left(F_1^nF_2^p+F_2^nF_1^p\right)t\left[2\sqrt{s}-M_n-M_p+\frac{t}{2k^2}(\sqrt{s}-M_n-M_p)\right]\right\},\\
b_{\mathrm{EM,np}}(s,t) &= \frac{\alpha}{t\sqrt{s}}\left[\left(F_1^nF_1^p - tF_2^nF_2^p\right)\left\{s-M_n^2-M_p^2+\frac{t}{8sk^2}[s+(M_n-M_p)^2][s-(M_n+M_p)^2]\right\}\right.\\
&\quad\left.+\left(F_1^nF_2^p - F_2^nF_1^p\right)t\left(M_m-M_p\right)\right],\\
c_{\mathrm{EM,np}}(s,t) &= \frac{\alpha}{2\sqrt{s}}\left(F_1^n+2M_nF_2^n\right)\left(F_1^p+2M_pF_2^p\right),\\
d_{\mathrm{EM,np}}(s,t) &= -c(s,t),\\
e_{\mathrm{EM,np}}(s,t) &= -i\frac{\alpha\sin\theta}{t\sqrt{s}}\left[\left(F_1^nF_1^p+tF_2^nF_2^p\right)\left\{s-M_n^2-M_p^2-\frac{M_n+M_p}{2\sqrt{s}}[s+(M_n-M_p)^2]+\frac{\sqrt{s}-M_n-M_p}{\sqrt{s}+M_n+M_p}\frac{t}{2}\right\}\right.\\
&\quad\left.+\left(F_1^nF_2^p+F_2^nF_1^p\right)[2k^2\sqrt{s}+t(\sqrt{s}-M_m-M_p)]\right],
\end{aligned} \tag{34}$$

where s and t are the Mandelstam invariants [24] and can be calculated by $k^2 = [s - (M_n + M_p)^2][s - (M_n - M_p)^2]/4s$ and $t = -2k^2(1 - \cos\theta)$. F_1 and F_2 are the Dirac and Pauli form factors, which on the point-particle approximation read

$$F_1^p = 1, \quad F_1^n = 0, \quad F_2^p = \frac{\mu_p - 1}{2M_p}, \quad F_2^n = \frac{\mu_n}{2M_n}. \tag{35}$$

The transformation between the Wolfenstein and Saclay parametrizations can be found in [18].

One important remark has to be made about the S matrix and the phase shifts that describe it. The nuclear phase shifts presented in this work are extracted with respect to the EM wave functions, as this is also the case for the many other phase-shift analyses and potentials in the literature [4,6–9,11]. For this reason, in the pp channel of the S_N matrix in Eq. (15) one has to make the replacement

$$S_N - 1 \to e^{i(\sigma_l+\rho_l+\tau_l)}(S_{MM,pp})^{1/2}(S_N - 1) \times (S_{MM,pp})^{1/2} e^{i(\tau_l+\rho_l+\sigma_l)}. \tag{36}$$

The np channel has a similar correction due to the magnetic-moment potential but its contribution is rather small and given the larger uncertainties of the data the effect is not statistically significant, therefore we do not include it in our calculations. Also, it should be noted that the ρ_l and τ_l phase shifts, as well as the K matrix elements of Eq. (29), are calculated with respect to the Coulomb wave functions, which gives rise to the $e^{2i(\sigma_l-\sigma_0)}$ term in Eqs. (22), (23), and (32).

B. Calculation and minimization of the merit function χ^2

With the scattering amplitude described by the Wolfenstein parameters it is possible to calculate observables for any scattering angle θ and center-of-mass momentum k, calculate a chi square merit function χ^2 to asses the ability of the δ-shell potential to reproduce the experimental data in our database, and adjust the $(\lambda)^{JS}_{l,l'}$ parameters by a least-squares fitting to minimize χ^2. The data are grouped by experiments and most of those measure an observable at a single laboratory frame energy E_{LAB} at different scattering angles; only a few of them are measured at a fixed angle for different values of E_{LAB} and the total cross-section experiments include measurements also at different laboratory energies since there is no scattering angle involved.

Every experimental dataset can be subject to a known and common systematic uncertainty (normalized data), an arbitrarily large systematic uncertainty (floated data), or no systematic uncertainty at all (absolute data); this is recorded by the experimentalist every time a set of measurements is made. In all three cases the merit function χ^2_t of a single dataset is given by

$$\chi^2_t = \sum_{i=1}^{n} \frac{[o_i(k_i,\theta_i)/Z - t_i(k_i,\theta_i,(\lambda)^{JS}_{l,l'})]^2}{[\delta o_i(k_i,\theta_i)/Z]^2} + \frac{(1-1/Z)^2}{(\delta_{sys}/Z)^2}, \tag{37}$$

where o_i and δo_i are the experimental value of and observable and the corresponding statistical uncertainty at point i, t_i is the theoretical value, δ_{sys} is the systematic uncertainty of the experiment, and Z is a scaling factor. The last term in Eq. (37) is usually denoted as χ^2_{sys}. Absolute data have $\delta_{sys} = 0$ and are not scaled ($Z = 1$). The optimal value of Z for normalized and floated data is obtained by minimizing χ^2_t with respect to Z; this leads to

$$Z = \left(\sum_{i=1}^{n} \frac{o_i t_i}{\delta o_i^2} + \frac{1}{\delta^2_{sys}}\right) \Big/ \left(\sum_{i=1}^{n} \frac{t_i^2}{\delta o_i^2} + \frac{1}{\delta^2_{sys}}\right), \tag{38}$$

where the k, θ, and λ dependence has been omitted. Since floated data have an arbitrarily large and common systematic uncertainty, this type of data use Eq. (38) with $\delta_{sys} = \infty$ so $\chi^2_{sys} = 0$. For normalized data the value of δ_{sys} is given by experimentalists, in most cases $Z \neq 1$, therefore $\chi^2_{sys} \neq 0$ and the normalization is counted as an extra data point. In some normalized datasets the systematic uncertainty can give a rather large contribution to χ^2_t, probably due to an underestimation of δ_{sys}. To correct for this underestimation, if a dataset has $\chi^2_{sys} > 9$ we float these data and no extra normalization data are counted; this is in line with the 3σ criterion which will be explained below. Finally, the total χ^2 is simply the sum of χ^2_t of every np and pp dataset.

To minimize the merit function χ^2 with respect to the fitting parameters $(\lambda)^{JS}_{l,l'}$ we use the Levenberg-Marquardt method. This method requires the calculation of derivatives of every calculated observable t_i with respect to all the fitting parameters to construct an approximation to the Hessian matrix,

$$H_{n,m} \approx 2\alpha_{n,m} = 2\sum_{i=1}^{N} \frac{1}{\delta o_i^2} \frac{\partial t_i\left(k,\theta,(\lambda)^{JS}_{l,l'}\right)}{\partial \lambda_n} \frac{\partial t_i\left(k,\theta,(\lambda)^{JS}_{l,l'}\right)}{\partial \lambda_m}. \tag{39}$$

This matrix is used to calculate the optimal change in parameters to reduce the number of steps needed to minimize χ^2. Once a minimum has been found, inversion of the α matrix gives the usual covariance matrix of the fitting parameters. An advantage of the δ-shell potential is that the derivatives in Eq. (39) can be computed analytically and simultaneously with the corresponding observable. This approach greatly reduces the numerical effort needed to minimize χ^2. Moreover, it also sidesteps the large inaccuracies triggered by a determination of the covariance matrix using a numerical evaluation of second derivatives and crossed derivatives at the minimum for a function with 46 parameters (we easily found nonpositive covariance matrices in this way). Details of the numerical algorithm can be found in [25].

IV. SELECTION OF DATA

Our database contains a total of 2972 pp and 4737 np published scattering data up to the year 2013 with $E_{LAB} \leqslant 350$ MeV. Unfortunately there seems to be mutually incompatible data, most likely due to under- and overestimations of statistical and systematic uncertainties from the experimental side. A clear example of these inconsistencies, at backward angles in this case, are the two datasets of np differential cross section at 162 MeV [26,27] plotted in the bottom right panel

of Fig. 1 for illustration purposes. Note that the bottom-right panel compares the unscaled data with the three fits. To deal with these inconsistencies we improve the 3σ criterion introduced in this context by the Nijmegen group in their 1993 partial-wave analysis [4] which became an essential tenet of their success and the subsequent high quality fits following thereafter [5–8]. This criterion discards mutually incompatible data, but has the unwanted side effect of eventually preventing a fraction of the data to contribute positively to the final fit. This is so because no distinction is made between mutually incompatible datasets in similar kinematical conditions and which of them, if any, are actually incompatible with the remaining data in different kinematical conditions as encoded in the phenomenological parametrization which intertwines all kinematical regions below pion production threshold. We propose below an extended self-consistent 3σ criterion which actually differentiates both situations.

First, let us explain the traditional 3σ criterion used so far in the literature claiming a final $\chi^2/\mathrm{DOF} \lesssim 1$. For a set of n measurements with a Gaussian distribution, the quantity $z \equiv \chi^2/n$ will satisfy the normalized probability distribution

$$\mathcal{P}_n(z) = \frac{n(nz/2)^{1/2-1}}{2\Gamma(n/2)} e^{-nz/2}. \tag{40}$$

According to the 3σ criterion a dataset is considered inconsistent with the rest of the database (more specifically, a phenomenological model representing such database is meant in practice), if z has a probability smaller than 0.27%. In most cases a dataset will have a highly improbable z value if the statistical errors are either underestimated (z will be very high) or overestimated (z will be very low); then for every n there is an interval between $z_{\min}$ and $z_{\max}$ of allowed values of z. Such end points are defined by

$$\begin{aligned} 0.0027 &= \int_0^{z_{\min}(n)} \mathcal{P}_n(z)dz = 1 - \frac{\Gamma(n/2, nz_{\min}/2)}{\Gamma(n/2)}, \\ 0.0027 &= \int_{z_{\max}(n)}^{\infty} \mathcal{P}_n(z)dz = \frac{\Gamma(n/2, nz_{\max}/2)}{\Gamma(n/2)}. \end{aligned} \tag{41}$$

The decision to float data with $\chi^2_{\mathrm{sys}} > 9$ is a consequence of applying the 3σ criterion with $n = 1$ to the normalization data.

Our selection process aims to obtain the largest possible database that contains only consistent data with each other. To be able to compare a single dataset with the rest it is necessary to have a model describing all of the available data as accurately as possible. Therefore, we start by fitting our δ-shell potential to the complete database with $N = 2972|_{pp,\mathrm{exp}} + 159|_{pp,\mathrm{norm}} + 4737|_{np,\mathrm{exp}} + 259|_{np,\mathrm{norm}} = 3131|_{pp} + 4993|_{np}$ and obtain $\chi^2 = 3543.74|_{pp} + 8390.27|_{np}$ which yields $\chi^2/\mathrm{DOF} = 1.48$. This larger-than-1 value was expected since we know that mutually incompatible experimental data are present. A comparison of the two np differential cross-section experiments at 162 MeV with this initial fit is shown on the top left panel in Fig. 1. Note that at angles where the error bars do not overlap, the model gives an "in the middle" solution where the sum of both contributions to χ^2 is minimized. Now every dataset can be tested using the 3σ criterion and

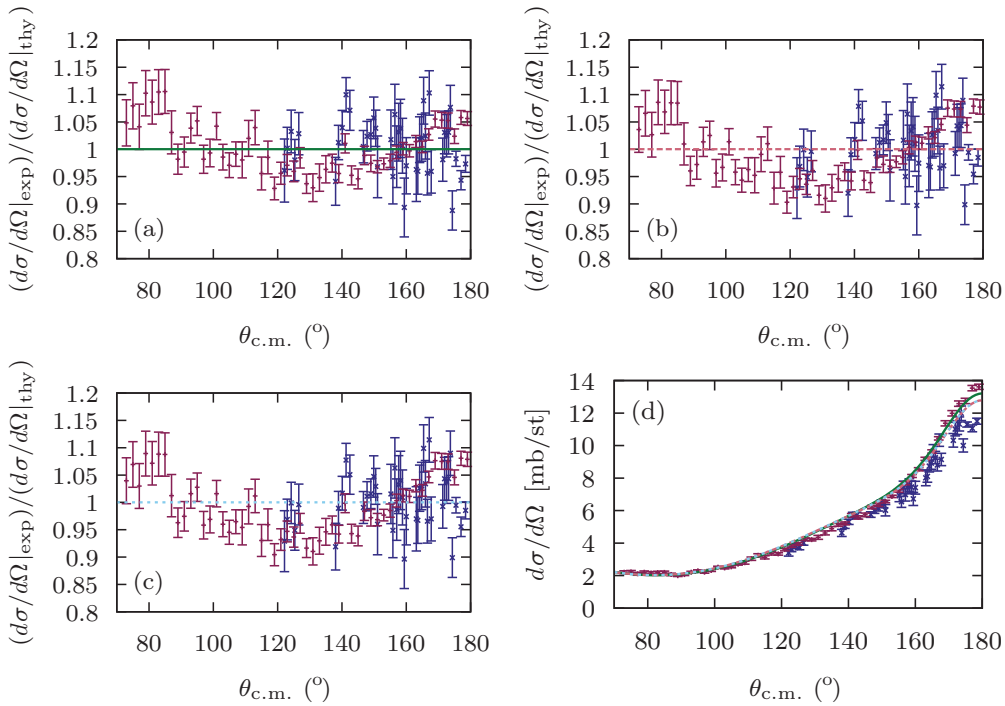

FIG. 1. (Color online) np differential cross section at 162 MeV. The top left, top right, and bottom left panels compare the experimental datasets of BO78 [26] (blue crosses with error bars) and RA98 [27] (red lines with error bars) to a fit to the complete database (green solid line), to the initial and final consistent databases (light-red dashed and light-blue dotted lines) respectively as discussed in the text. Every data point is scaled to the corresponding fit. The bottom right panel compares the unscaled data with the three fits.

compared with the rest of the database via the first fit (in this case both BO78 [26] and RA98 [27] have $z > z_{\max}$). Every dataset failing to satisfy $z_{\min} \leqslant z \leqslant z_{\max}$ is excluded and the remaining $N = 3008|_{pp} + 3438|_{np}$ data make what we call the initial and mutually consistent database. By construction, this is a very close approximation to the minimal mutually consistent database. A second fit is then performed, this time to these initial and mutually compatible data only, and $\chi^2 = 3061.97|_{pp} + 3634.34$ is obtained, which yields $\chi^2/\mathrm{DOF} = 1.05$. At this point the standard 3σ criterion stops. However, looking at the top right panel of Fig. 1 where the same experimental data are compared to the second fit, one can notice that the theoretical model is now closer to the BO78 [26] values even though the latter played no role in the determination of the fitting parameters. It is fair then to ask if the discarded BO78 [26] data, or any other of the initially rejected sets, are compatible with the initial and mutually consistent data base.

To analyze this point, we apply anew the 3σ criterion to all of the datasets using the second fit. We find that this time $z_{\mathrm{RA98}} > z_{\max}$, while $z_{\mathrm{BO78}} < z_{\max}$ instead; this initially discarded BO78 [26] dataset is now recovered along with a total of 269 data and the parameters can be refitted again. This particular example shows the potential good features of the Gross and Stadler proposal [8].

Therefore, we apply this improved 3σ criterion systematically to the full database in a self-consistent manner. Namely, the process can be repeated iteratively until no more data are recovered or rejected. The bulk of recovered data is obtained the second time the 3σ criterion is applied; for successive steps only one or two datasets are recovered or rejected. These statistical fluctuations can be regarded as a marginal effect provided the range of variation in the fitting parameters is substantially smaller than their final quoted uncertainty. Our final fit meets this requirement.

A final and mutually consistent database with $N = 2996|_{pp} + 3717|_{np}$ data is obtained and the last refitting of the parameters is carried out, yielding $\chi^2 = 3051.64|_{pp} + 3958.08|_{np}$ while the value of $\chi^2/\mathrm{DOF} = 1.05$ is conserved. Finally the bottom left panel in Fig. 1 compares both experiments with the last fit, but the differences with the top right panel are very small since the fitting parameters for the second fit turn out to be very similar to the final ones. Tables II and III list all the pp and np data included in the final and mutually consistent database to which the parameters $(\lambda)^{J,S}_{l,l'}$ are fitted. Tables IV and V show the pp and np rejected data.

We note that our final database includes *both* pp and np. However, if we restrict to the np channel as done in Ref. [8] we find that those data close to the boundary of *their* acceptance/rejection interval are also close to the boundary of *our* acceptance/rejection interval, as the corresponding dataset chi square, χ^2_t, are rather similar. The inclusion or rejection in our case is supported by the pp observables.

V. RESULTS

With a consistent database it is possible to properly estimate statistical errors in the potential parameters. In Ref. [11] we give the corresponding partial-wave independent fitting parameters which, as explained above, proved more convenient as primary quantities to carry out the fit due to smaller statistical correlations. Here, Table VI shows the strength coefficients $V_{i,n}$ and their statistical uncertainties propagated from the experimental data via the usual covariance matrix and applying Appendix A to the results of Ref. [11]. With these parameters and the covariance matrix it is possible to also estimate and propagate statistical error bars for calculations made with the δ-shell potential. For example Tables VII–IX show pp isovector, np isovector, and np isoscalar phase shifts respectively with statistical errors extracted from experimental data for a few partial waves at different kinetic laboratory frame energies. Comparing our results with Tables IV and V in [4] we find that for low angular momentum partial waves our estimates for the errors tend to be smaller than the ones obtained by the Nijmegen group in their 1993 PWA, while for higher values of l the Nijmegen errors are smaller.

Deuteron wave functions, static properties, and form factors can be calculated with the parameters of the $^3S_1$-$^3D_1$ coupled channel for the bound state and the errors have been propagated. The results have already been shown in our previous work [11] and will not be discussed further here.

VI. CONCLUSIONS AND OUTLOOK

The main problem in analyzing the existing np and pp scattering data lies in the large number of experiments in different kinematical regions measuring a variety of observables with different accuracies and a largely heterogeneous abundance in the $(E_{\mathrm{LAB}}, \theta)$ plane. On the other hand, for a given LAB energy and scattering angle just five complex quantities suffice to describe the scattering amplitude. Therefore the measurement of ten different observables at a given point in this plane would reconstruct the full amplitude up to a global phase. In terms of a partial wave decomposition at a fixed energy one needs about as many angles as partial waves are needed for the scattering amplitude to converge. For the energies listed in the table this gives a total number of points in the $(E_{\mathrm{LAB}}, \theta)$ plane much smaller than the number of available data. Unfortunately, the experiments were not designed from this point of view, and a rather different nonhomogeneous distribution is at our disposal. In addition, there are mutually inconsistent experiments and a proper selection of data must be carried out. We have explored with success an interesting criterion suggested by Gross and Stadler [8] in their covariant spectator theory analysis of np scattering data. This criterion improves with respect to the more customary 3σ criterion applied since the 1993 benchmarking fit.

The best known way of smoothly interpolating energy values between the experimentally measured ones is to assume a phenomenological potential. Of course, there exist conditions regarding the analytical behavior of the scattering amplitude and its partial wave contributions which become relevant at sufficiently low energies and depend only on the long-distance behavior of the potential and for the case of OPE corresponds to the appearance of a left hand cut at $p^2_{\mathrm{CM}} = -m^2_\pi/4$ in the partial-wave amplitudes. In this paper we

TABLE II. The *pp* scattering datasets used in the fit. The first column gives the energy, or energy range, of the experiment in MeV; the notation for the reference and the observable type is adopted from the SAID group [28]. n_1 represents the number of total measurements in the dataset, while n_2 is the final number of measurements which can be different from n_1 due to rejection of outliers an the inclusion of normalization data. See also main text.

E_{LAB}	Ref.	Type	θ	n_1	n_2	Sys	χ^2_{sys}	Z	χ^2_t	χ^2_t/n_2
0.3–0.4	BR64 [29]	DSG	91.2	5	5		float	0.735	8.84	1.77
0.4	TH78 [30]	DSG	58.4–95.0	4	4		float	1.003	0.76	0.19
0.4	TH78 [30]	DSG	26.0–110.0	17	18	0.2%	0.024	1.000	32.72	1.82
0.4	TH78 [30]	DSG	58.4–95.0	4	4		float	0.999	10.94	2.74
0.4	TH78 [30]	DSG	58.4–95.0	4	4		float	1.000	5.62	1.40
0.4	TH78 [30]	DSG	58.4–98.0	6	6		float	0.999	16.39	2.73
0.4	TH78 [30]	DSG	58.4–95.0	4	4		float	0.999	13.01	3.25
0.4	TH78 [30]	DSG	40.0–46.0	3	4	0.2%	0.094	0.999	0.97	0.24
0.4	TH78 [30]	DSG	58.4–95.0	4	4		float	1.002	6.43	1.61
0.4	TH78 [30]	DSG	58.4–98.0	6	6		float	1.000	10.55	1.76
0.5	TH78 [30]	DSG	24.0–110.0	39	40	0.2%	0.689	1.002	43.40	1.08
0.7	TH78 [30]	DSG	24.0–110.0	26	27	0.2%	1.257	1.002	22.83	0.85
1.0	TH78 [30]	DSG	24.0–110.0	31	32	0.2%	0.938	1.002	30.43	0.95
1.4	KN66 [31]	DSG	12.0–70.0	11	11		float	0.995	6.25	0.57
1.9	KN66 [31]	DSG	12.0–90.0	13	12		float	0.994	5.19	0.43
2.4	KN66 [31]	DSG	12.0–100.0	14	14		float	0.994	7.37	0.53
3.0	KN66 [31]	DSG	12.0–90.0	13	13		float	0.993	14.26	1.10
5.0	IM75 [32]	DSG	16.0–90.1	17	18	0.4%	1.444	0.995	20.22	1.12
5.0	BA82 [33]	P	19.9–90.3	11	12	1.0%	0.070	1.003	6.09	0.51
6.1	SL68 [34]	DSG	12.0–100.1	17	17		float	1.003	14.62	0.86
7.0	IM75 [32]	DSG	16.0–90.1	17	18	0.4%	3.228	0.993	19.27	1.07
8.0	IM75 [32]	DSG	16.0–90.1	17	18	0.4%	3.251	0.993	14.87	0.83
8.1	SL68 [34]	DSG	12.0–90.1	16	16		float	1.003	18.20	1.14
9.6	OB80 [35]	AYY	90.0	1	1		no systematic error		0.22	0.22
9.6	SL67 [36]	P	36.1–52.1	5	5		no systematic error		2.17	0.43
9.7	JO59 [37]	DSG	90.0	1	1		no systematic error		0.58	0.58
9.7	JO59 [37]	DSG	10.0–89.8	26	26		float	1.014	14.81	0.57
9.7	JA70 [38]	DSG	26.1–60.1	5	5		float	1.016	2.06	0.41
9.8	BA82 [33]	P	15.4–90.4	15	16	1.0%	0.821	1.009	20.56	1.28
9.9	JA70 [38]	DSG	25.1–100.2	10	10		float	1.002	7.07	0.71
10.0	HU75 [39]	P	20.0–80.0	7	7		no systematic error		9.26	1.32
11.4–26.5	CA67 [40]	AYY	90.0	4	4		float	0.990	1.06	0.27
11.4–26.5	CA67 [40]	AXX	90.0	4	4		float	0.991	3.71	0.93
13.6	JA70 [38]	DSG	20.1–110.2	11	12	0.3%	1.212	0.997	8.46	0.71
15.6	SL67 [36]	P	40.2–90.4	4	4		no systematic error		1.67	0.42
16.2	BL59 [41]	P	50.2	1	1		no systematic error		0.76	0.76
17.7	BO58 [42]	P	60.3	1	1		no systematic error		0.33	0.33
18.2	YN54 [43]	DSG	30.0–90.0	8	9	1.5%	0.037	1.003	4.99	0.55
19.7	JA76 [44]	DSG	20.1–90.3	13	14	0.4%	0.019	1.001	7.34	0.52
19.8	BU59 [45]	DSG	14.0–90.0	15	16	5.0%	0.000	1.000	22.91	1.43
19.8	RO59 [46]	DSG	18.0–35.1	7	8	5.0%	0.117	1.017	5.87	0.73
20.0	AB62 [47]	AYY	75.0	1	1		no systematic error		1.12	1.12
25.6	JE60 [48]	DSG	10.1–89.6	23	23		float	1.016	12.34	0.54
25.7	KR94 [49]	D	25.2–60.3	8	9	1.3%	2.779	1.022	12.32	1.37
25.7	KR94 [49]	R	25.2–60.3	6	7	1.3%	0.197	1.006	3.93	0.56
25.7	KR94 [49]	A	40.3–60.3	2	3	1.3%	0.045	1.003	2.52	0.84
27.0	JA67 [50]	AYY	90.0	1	1		no systematic error		0.33	0.33
27.4	CH63 [51]	P	45.0	1	1		no systematic error		0.10	0.10
27.6	AS65 [52]	R	23.2–54.6	2	3	3.0%	0.049	0.993	0.57	0.19
27.6	AS65 [52]	A	23.2–54.6	3	4	3.0%	0.018	0.996	4.04	1.01
28.2	JO59 [37]	DSG	90.0	1	1		no systematic error		0.73	0.73
30.0	BA63 [53]	P	45.0	1	2	4.0%	0.037	0.992	4.83	2.41
31.1	JO59 [37]	DSG	90.0	1	1		no systematic error		0.003	0.003
34.2	JO59 [37]	DSG	90.0	1	1		no systematic error		0.93	0.93
36.8	CH63 [51]	P	60.0–70.0	2	2		no systematic error		0.21	0.10

TABLE II. (*Continued*.)

E_{LAB}	Ref.	Type	θ	n_1	n_2	Sys	χ^2_{sys}	Z	χ^2_t	χ^2_t/n_2
36.9	JO59 [37]	DSG	90.0	1	1		no systematic error		0.26	0.26
37.2	GA70 [54]	AYY	90.0	1	1		no systematic error		0.18	0.18
37.2	GA70 [54]	AXX	90.0	1	1		no systematic error		5.74	5.74
38.3	CH63 [51]	P	45.0	1	1		no systematic error		2.34	2.34
39.4	JO58 [55]	DSG	8.1–89.4	27	28	0.9%	0.386	0.994	28.33	1.01
39.6	JO59 [37]	DSG	90.0	1	1		no systematic error		0.30	0.30
41.0	KR56 [56]	DSG	90.0	1	1		no systematic error		0.86	0.86
44.7	JO59 [37]	DSG	90.0	1	1		no systematic error		0.88	0.88
46.0	PA58 [57]	P	45.5	1	1		no systematic error		1.52	1.52
46.9	GA71 [58]	AYY	90.0	1	1		no systematic error		4.60	4.60
47.5	AS65 [59]	A	23.5–87.1	5	5	5.0%	0.088	0.985	1.43	0.29
47.5	NI69 [60]	AYY	90.0	1	2	10.0%	0.003	1.005	0.008	0.004
47.5	NI69 [60]	AXX	90.0	1	2	10.0%	3.710	0.807	4.17	2.08
47.8	AS65 [52]	R	23.5–87.1	5	6	5.0%	0.008	1.004	4.98	0.83
47.8	AS65 [52]	A	23.5–87.1	5	6	5.0%	0.156	1.020	2.60	0.43
49.4	BA67 [61]	DSG	13.0–90.6	28	29	0.3%	0.186	1.001	34.15	1.18
49.7	CH63 [51]	P	45.0	1	1		no systematic error		4.76	4.76
49.9	BA63 [53]	P	45.0	1	1		no systematic error		0.02	0.02
50.0	GR63 [62]	D	70.0	1	1		no systematic error		1.32	1.32
50.0	SM89 [63]	P	16.2–78.7	10	11	0.4%	3.366	1.007	10.25	0.93
50.1	BE86 [64]	DSG	14.0–90.0	24	25	1.6%	1.747	1.021	11.35	0.45
50.2	JO59 [37]	DSG	90.0	1	1		no systematic error		0.25	0.25
51.5	KR56 [56]	DSG	90.0	1	1		no systematic error		0.95	0.95
51.5	NI61 [65]	DSG	16.2–35.5	9	10	4.5%	1.675	0.942	7.07	0.71
51.7	CH63 [51]	P	60.0	1	1		no systematic error		0.54	0.54
51.7	CH63 [51]	P	60.0	1	1		no systematic error		1.28	1.28
51.8	NI61 [65]	DSG	35.5–90.8	9	10	2.5%	4.039	0.950	15.13	1.51
52.0	NI63 [66]	AYY	90.0	1	1		no systematic error		2.31	2.31
52.0	NI63 [66]	CKP	90.0	1	1		no systematic error		3.12	3.12
52.3	SA68 [67]	DSG	14.2–90.8	29	26	0.5%	0.034	0.999	20.29	0.78
52.3	TA78 [68]	P	30.4–101.8	12	13	2.0%	0.537	1.015	7.29	0.56
53.2	CH63 [51]	P	75.0	1	1		no systematic error		1.82	1.82
53.2	CH63 [51]	P	75.0	1	1		no systematic error		1.19	1.19
56.0	PA58 [57]	P	45.6	1	1		no systematic error		0.05	0.05
56.1	JO59 [37]	DSG	90.0	1	1		no systematic error		0.09	0.09
58.5	CH63 [51]	P	45.0	1	1		no systematic error		0.63	0.63
58.5	CH63 [51]	P	45.0	1	1		no systematic error		0.07	0.07
61.9	JO59 [37]	DSG	90.0	1	1		no systematic error		0.006	0.006
66.0	PA58 [57]	P	20.4–71.0	11	12	2.8%	1.147	1.030	10.88	0.91
68.2	TA78 [68]	P	31.5–101.0	12	13	2.0%	1.177	1.022	6.98	0.54
68.3	YO60 [69]	DSG	10.2–89.0	26	26	1.1%	0.601	1.009	24.56	0.94
68.4	JO59 [37]	DSG	90.0	1	1		no systematic error		0.04	0.04
69.5	KR56 [56]	DSG	90.0	1	1		no systematic error		0.03	0.03
70.0	CH63 [51]	P	45.0	1	1		no systematic error		0.06	0.06
70.0	KR56 [56]	DSG	30.0–80.0	4	4		float	1.023	9.65	2.41
70.0	CH63 [51]	P	45.0	1	1		no systematic error		2.06	2.06
71.0	PA58 [57]	P	45.8	1	1		no systematic error		0.44	0.44
73.5	JA68 [70]	AYY	90.0	1	1		no systematic error		0.39	0.39
78.0	PA58 [57]	P	45.8	1	1		no systematic error		0.87	0.87
78.5	KR56 [56]	DSG	90.0	1	1		no systematic error		0.003	0.003
86.0	PA58 [57]	P	45.9	1	1		no systematic error		0.001	0.001
95.0	KR56 [56]	DSG	40.0–90.0	6	6		float	0.990	0.68	0.11
95.0	KR56 [56]	DSG	25.0–90.0	6	6		float	1.006	1.99	0.33
95.0	KR56 [56]	DSG	90.0	1	1		no systematic error		0.002	0.002
95.0	PA58 [57]	P	20.6–86.4	14	15	2.8%	0.000	1.000	16.84	1.12
97.0	CH63 [51]	P	45.0	1	1		no systematic error		2.43	2.43
97.0	CH63 [51]	P	45.0	1	1		no systematic error		1.18	1.18
97.7	WI68 [71]	P	16.4–88.6	13	14	0.8%	0.148	0.997	12.60	0.90

TABLE II. (*Continued*.)

E_{LAB}	Ref.	Type	θ	n_1	n_2	Sys	χ^2_{sys}	Z	χ^2_t	χ^2_t/n_2
98.0	TA60 [72]	P	10.2–81.4	14	15	2.0%	0.982	0.980	15.51	1.03
98.0	TH60 [73]	D	20.5–61.3	5	5		no systematic error		7.51	1.50
98.0	JA65 [74]	R	31.3–72.0	5	5		no systematic error		6.54	1.31
98.0	JA68 [70]	AYY	90.0	1	1		no systematic error		0.02	0.02
98.0	JA65 [74]	RP	31.6–62.6	4	4		no systematic error		0.55	0.14
98.0	TH60 [73]	P	20.5–61.3	5	6	3.0%	0.033	0.995	4.44	0.74
98.0	TH60 [73]	P	41.0–61.3	3	4	3.0%	1.055	0.969	5.40	1.35
98.8	WI68 [71]	DSG	22.5–88.6	19	20	1.0%	0.007	1.001	15.57	0.78
102.0	PA58 [57]	DSG	30.8–66.4	3	3		float	0.983	7.35	2.45
102.0	PA58 [57]	P	30.8–66.4	3	4	2.8%	1.030	1.028	3.13	0.78
107.0	PA58 [57]	DSG	30.8–66.5	3	3		float	0.927	0.56	0.19
107.0	PA58 [57]	P	30.8–66.5	3	4	2.8%	0.108	0.991	6.95	1.74
118.0	PA58 [57]	DSG	20.6–88.2	15	14		float	0.952	14.23	1.02
118.0	PA58 [57]	P	20.6–96.7	16	17	2.8%	1.905	0.961	29.49	1.73
127.0	PA58 [57]	DSG	31.0–66.7	3	3		float	0.952	0.52	0.17
127.0	PA58 [57]	P	31.0–66.7	3	4	2.8%	0.149	0.989	8.36	2.09
130.0	BA57 [75]	P	20.6–81.9	4	5	3.3%	0.243	0.984	2.78	0.56
137.0	PA58 [57]	P	31.1–66.9	3	4	2.8%	0.411	0.982	3.10	0.78
137.5	HE63 [76]	RP	43.0–82.1	5	5		no systematic error		0.64	0.13
138.0	CA63 [77]	D	31.0–82.0	4	4		no systematic error		3.75	0.94
138.0	CA63 [77]	P	20.7–88.0	15	16	5.0%	0.264	1.026	23.49	1.47
139.0	HE63 [76]	A	31.1–82.1	6	7	4.0%	0.878	0.963	3.23	0.46
140.0	TH60 [78]	R	31.1–82.1	6	6		no systematic error		4.84	0.81
140.4	JA64 [79]	RP	31.4–82.2	6	5		no systematic error		0.71	0.14
140.7	CO67 [80]	P	16.6–87.9	20	20	0.8%	0.708	1.007	17.33	0.87
142.0	TA60 [72]	P	5.2–82.1	27	26		float	1.063	22.65	0.87
142.0	HW60 [81]	D	12.5–82.1	8	7		no systematic error		11.51	1.64
142.0	BI63 [82]	R	24.0–90.0	8	8		no systematic error		10.62	1.33
142.0	HW60 [81]	P	12.4–82.1	8	9	3.0%	0.001	1.001	7.07	0.79
143.0	BI61 [83]	D	31.1–92.2	7	7		no systematic error		8.36	1.19
143.0	JA63 [84]	A	32.2–84.8	6	6		no systematic error		4.67	0.78
143.0	JA68 [70]	AYY	60.0–90.0	2	2		no systematic error		0.14	0.07
144.0	JA71 [85]	DSG	3.6–41.4	27	26	0.6%	0.916	0.994	42.21	1.62
144.1	CO67 [80]	DSG	16.6–36.2	6	7	0.9%	0.308	0.995	2.64	0.38
144.1	CO67 [80]	DSG	41.4–87.9	15	15	0.6%	0.024	1.001	14.97	1.00
147.0	PA58 [57]	P	6.2–87.8	28	29	2.8%	0.148	1.011	32.33	1.11
147.0	PA58 [57]	DSG	31.1–67.0	3	3		float	0.991	1.47	0.49
147.0	PA58 [57]	DSG	6.2–31.1	11	11		float	1.158	22.00	2.00
147.0	PA58 [57]	DSG	41.4–112.0	15	15		float	1.064	9.98	0.67
147.0	PA58 [57]	DSG	20.7–72.0	11	11		float	1.066	9.02	0.82
147.0	CA56 [86]	DSG	90.0	1	1		no systematic error		0.70	0.70
155.0	MI67 [87]	DSG	10.4–89.8	23	22	4.0%	0.381	0.975	18.48	0.84
170.0	BA57 [75]	P	31.3–82.5	7	8	3.3%	0.209	0.985	2.71	0.34
174.0	FI55 [88]	P	20.8–72.4	5	6	6.6%	0.845	0.939	4.11	0.69
183.1	PR91 [89]	P	18.1	1	1		no systematic error		1.97	1.97
185.4	PI92 [90]	P	5.5–21.4	20	21	1.4%	0.102	0.996	11.92	0.57
197.4	RT98 [91]	P	7.4–89.9	41	42	1.3%	0.032	1.002	62.09	1.48
197.4	RT98 [91]	AXX	7.4–89.9	41	42	2.5%	0.372	1.015	42.93	1.02
197.4	RT98 [91]	AYY	7.4–89.9	41	42	2.5%	0.245	1.012	51.70	1.23
197.4	RT98 [91]	AZX	7.4–89.9	41	42	2.5%	0.983	1.025	36.45	0.87
197.4	LO00 [92]	AZZ	11.6–89.9	39	40	2.0%	2.717	1.033	30.66	0.77
197.8	HA97 [93]	P	9.5–36.7	14	15	1.3%	0.016	1.002	22.19	1.48
197.8	HA97 [93]	AXX	9.5–36.7	14	15	2.4%	0.448	1.016	11.66	0.78
197.8	HA97 [93]	AYY	9.5–36.7	14	15	2.4%	0.284	1.013	6.69	0.45
197.8	HA97 [93]	AZX	9.5–36.7	14	15	2.4%	0.991	1.024	16.99	1.13
197.8	WI99 [94]	D	10.5–78.4	10	11	0.5%	0.120	1.002	12.48	1.13
197.8	WI99 [94]	AP	9.9–31.9	5	6	2.0%	0.051	1.005	0.74	0.12
197.8	WI99 [94]	A	9.9–31.9	5	6	2.0%	1.228	1.022	9.99	1.66

TABLE II. (*Continued*.)

$E_{\rm LAB}$	Ref.	Type	θ	n_1	n_2	Sys	$\chi^2_{\rm sys}$	Z	χ^2_t	χ^2_t/n_2
197.8	WI99 [94]	RP	9.9–31.9	5	6	2.0%	0.556	0.985	2.14	0.36
197.8	WI99 [94]	R	9.9–31.9	5	6	2.0%	0.110	1.007	2.35	0.39
204.0	RY71 [95]	DSG	52.7–85.3	3	4	5.4%	0.271	0.972	11.49	2.87
205.0	GR79 [96]	P	50.3	1	2	3.0%	0.536	0.978	1.40	0.70
209.1	BU78 [97]	P	18.9–50.2	3	4	2.8%	0.050	1.006	2.52	0.63
209.1	BU78 [97]	D	18.9–50.2	3	4	2.8%	0.186	1.012	3.55	0.89
209.1	BU78 [97]	R	18.9–50.2	3	4	2.8%	0.000	1.000	0.60	0.15
209.1	BU78 [97]	RP	31.4	1	2	2.8%	0.367	0.983	1.55	0.78
210.0	AB58 [98]	P	30.0	2	2	no systematic error			3.12	1.56
210.0	BA57 [75]	P	13.7–83.0	8	9	3.3%	0.001	1.001	8.61	0.96
210.0	TI61 [99]	P	30.0–90.0	7	8	3.6%	0.002	0.999	1.90	0.24
213.0	MA66 [100]	DSG	8.9–38.7	13	13	1.3%	0.950	0.987	19.05	1.47
213.0	MA66 [100]	P	8.9–38.7	13	14	3.1%	0.034	0.994	18.40	1.31
213.0	GO62 [101]	D	30.0 - 90.0	7	7	no systematic error			7.24	1.03
213.0	EN61 [102]	R	30.0–90.0	7	7	no systematic error			3.58	0.51
213.0	EN61 [102]	A	80.0–90.0	2	2	no systematic error			1.80	0.90
213.0	GO64 [103]	RP	30.0–90.0	5	5	no systematic error			5.29	1.06
213.0	EN61 [102]	A	30.0–70.0	5	6	2.0%	0.646	1.016	3.56	0.59
213.0	GO62 [101]	P	30.0–80.0	6	7	5.0%	0.013	0.994	14.60	2.09
213.0	GO62 [101]	P	30.0–80.0	6	7	5.0%	0.045	0.989	12.32	1.76
213.0	GO62 [101]	P	30.0–80.0	6	7	5.0%	0.004	1.003	8.69	1.24
217.0	TI61 [99]	P	60.0–120.0	7	8	12.0%	0.411	0.923	8.10	1.01
217.0	TI61 [99]	P	60.0–110.0	6	7	12.0%	0.403	0.924	2.11	0.30
225.0	GR79 [96]	P	35.9	1	2	1.0%	0.094	0.997	0.17	0.08
241.0	ON89 [104]	P	60.0–88.0	8	9	5.0%	0.043	1.010	18.75	2.08
241.0	ON89 [104]	D	60.0–88.0	8	9	5.0%	3.013	1.087	19.01	2.11
241.0	ON89 [104]	R	60.0–88.0	8	9	5.0%	2.642	1.081	5.85	0.65
241.0	ON89 [104]	A	60.0–88.0	8	9	5.0%	0.123	1.018	18.17	2.02
241.0	ON89 [104]	MSSN	60.0–88.0	8	9	5.0%	0.029	0.991	9.17	1.02
241.0	ON89 [104]	MSKN	60.0–88.0	8	9	5.0%	0.026	1.008	5.25	0.58
250.0	PR98 [105]	P	7.4–89.4	41	42	1.3%	0.446	0.991	32.44	0.77
250.0	PR98 [105]	AXX	7.4–89.4	41	42	2.4%	0.522	0.983	34.06	0.81
250.0	PR98 [105]	AYY	7.4–89.4	41	42	2.4%	0.203	0.989	50.32	1.20
250.0	PR98 [105]	AZX	7.4–89.4	41	42	2.4%	0.001	0.999	38.36	0.91
260.0	CH56 [106]	DSG	17.0–63.3	5	6	5.2%	0.018	1.007	7.68	1.28
266.0	AM78 [107]	P	50.9	1	1	no systematic error			2.11	2.11
276.0	CH57 [108]	P	19.3–76.8	6	7	7.5%	2.197	0.889	9.65	1.38
280.0	PR98 [105]	P	7.5–89.0	41	42	1.3%	0.197	1.006	47.26	1.13
280.0	PR98 [105]	AXX	7.5–89.0	41	42	2.4%	0.646	1.019	26.05	0.62
280.0	PR98 [105]	AYY	7.5–89.0	41	42	2.4%	1.155	1.026	46.63	1.11
280.0	PR98 [105]	AZX	7.5–89.0	41	42	1.3%	0.192	1.006	44.11	1.05
285.0	AE76 [109]	DSG	4.6–15.8	22	22	float		0.960	32.71	1.49
294.4	PR98 [105]	P	7.5–89.2	40	41	1.3%	0.156	1.005	33.40	0.81
294.4	PR98 [105]	AXX	7.5–89.2	40	41	2.4%	0.009	1.002	39.97	0.97
294.4	PR98 [105]	AYY	7.5–89.2	40	41	2.4%	0.011	1.003	55.55	1.35
294.4	PR98 [105]	AZX	7.5–89.2	40	41	2.4%	0.005	1.002	39.14	0.95
300.0	OT84 [110]	DSG	90.0	1	2	1.8%	4.102	1.036	4.41	2.21
302.9	AU84 [111]	AZZ	80.0–98.0	10	11	2.1%	0.688	0.983	15.82	1.44
305.0	BE68 [112]	AYY	59.6–103.9	14	15	8.0%	2.646	0.870	16.83	1.12
307.0	CH67 [113]	P	33.6–79.4	6	6	float		0.981	0.57	0.09
310.0	CH54 [114]	DSG	6.5–21.7	7	6	float		1.099	14.84	2.47
310.0	CH54 [114]	P	6.5–21.7	7	8	4.0%	0.102	0.987	4.53	0.57
310.0	CH57 [108]	D	23.1–80.5	6	6	no systematic error			5.47	0.91
310.0	CH57 [108]	R	22.4–80.1	6	6	no systematic error			8.69	1.45
310.0	PR98 [105]	P	7.5–89.4	40	41	1.3%	0.108	0.996	41.99	1.02
310.0	PR98 [105]	AXX	7.5–89.4	40	41	2.4%	0.602	0.981	29.69	0.72
310.0	PR98 [105]	AYY	7.5–89.4	40	41	2.4%	1.438	0.971	54.16	1.32
310.0	PR98 [105]	AZX	7.5–89.4	40	41	2.4%	0.116	0.992	40.44	0.99

TABLE II. (*Continued*.)

E_{LAB}	Ref.	Type	θ	n_1	n_2	Sys	χ^2_{sys}	Z	χ^2_t	χ^2_t/n_2
312.0	BE80 [115]	P	4.3–30.7	13	14	0.6%	0.320	1.003	19.67	1.40
312.0	BE80 [115]	D	4.3–30.7	13	14	1.2%	0.023	1.002	7.21	0.51
312.0	BE80 [115]	R	4.3–30.7	13	14	1.2%	0.008	0.999	18.96	1.35
312.0	BE80 [115]	A	4.3–30.7	13	14	1.2%	0.005	1.001	18.69	1.34
314.0	ON89 [104]	P	60.0–88.0	8	9	5.0%	1.544	1.062	13.16	1.46
314.0	ON89 [104]	R	60.0–88.0	8	9	5.0%	3.319	1.091	9.41	1.05
314.0	ON89 [104]	A	60.0–88.0	8	9	5.0%	5.818	0.879	9.49	1.05
314.0	ON89 [104]	MSSN	60.0–88.0	8	9	5.0%	0.015	1.006	3.93	0.44
314.0	ON89 [104]	MSKN	60.0–88.0	8	9	5.0%	0.029	0.991	3.16	0.35
315.0	CH57 [108]	DSG	21.6–89.4	7	7	float		1.007	5.40	0.77
315.0	CH57 [108]	P	21.6–76.2	6	7	4.0%	2.833	0.933	6.01	0.86
315.0	VA64 [116]	AYY	90.0	1	1	no systematic error			0.30	0.30
315.0	KA65 [117]	AYY	45.0	1	1	no systematic error			0.03	0.03
315.0	KA65 [117]	CKP	45.0	1	1	no systematic error			0.0003	0.0003
316.0	SI56 [118]	A	25.4–76.3	3	3	no systematic error			2.81	0.94
318.0	MC81 [119]	P	36.6	1	2	1.0%	0.106	1.003	0.25	0.13
320.0	AL61 [120]	AYY	90.0	1	1	no systematic error			0.82	0.82
320.0	AM78 [107]	P	51.4	1	1	no systematic error			0.15	0.15
324.0	BU78 [97]	P	19.4–51.4	3	4	1.5%	0.084	1.004	2.17	0.54
324.0	BU78 [97]	P	19.4–51.4	3	4	1.5%	0.035	1.003	4.18	1.04
324.0	BU78 [97]	D	19.4–51.4	3	4	1.5%	0.554	1.011	10.79	2.70
324.0	BU78 [97]	R	19.4–51.4	3	4	1.5%	0.289	0.992	12.28	3.07
324.0	BU78 [97]	RP	32.2	1	2	1.5%	0.081	0.996	1.07	0.53
325.0	SH82 [121]	DSG	22.3–88.6	19	20	1.0%	0.142	1.004	26.01	1.30
327.8	BU78 [97]	P	55.8	1	2	1.5%	0.146	1.006	0.21	0.10
328.0	RY71 [95]	DSG	40.9–83.9	5	6	2.9%	5.950	1.071	15.86	2.64
328.0	BE66 [122]	P	49.1–88.9	13	14	6.2%	0.224	0.971	6.04	0.43
330.0	FI54 [123]	DSG	5.3–29.7	17	15	float		0.978	14.59	0.97
330.0	BE68 [112]	AYY	59.7–99.9	13	14	8.0%	1.163	0.914	5.15	0.37
332.0	AB75 [124]	DSG	51.3–85.7	12	12	float		0.935	19.03	1.59
334.5	AL70 [125]	DSG	43.1–89.4	10	10	float		1.014	4.35	0.44
334.5	AL70 [125]	P	43.1–89.4	11	12	6.0%	1.237	0.933	11.45	0.95
341.0	ON89 [104]	P	38.0–98.0	15	16	5.0%	0.019	0.993	27.81	1.74
341.0	ON89 [104]	D	38.0–98.0	15	16	5.0%	0.391	1.031	25.11	1.57
341.0	ON89 [104]	R	38.0–98.0	15	16	5.0%	1.638	1.064	22.59	1.41
345.0	CH51 [126]	DSG	15.2–53.2	10	9	float		0.960	6.77	0.75
345.0	CH51 [126]	DSG	35.6–89.2	17	16	5.0%	2.865	1.085	17.98	1.12
348.0	AE76 [109]	DSG	4.6–16.0	22	22	float		1.054	23.47	1.07
350.0	PR98 [105]	P	7.6–89.9	40	41	1.3%	0.723	1.011	52.60	1.28
350.0	PR98 [105]	AXX	7.6–89.9	40	41	2.4%	0.122	1.008	56.35	1.37
350.0	PR98 [105]	AYY	7.6–89.9	40	41	2.4%	1.386	0.972	42.79	1.04
350.0	PR98 [105]	AZX	7.6–89.9	40	41	2.4%	0.350	0.986	35.93	0.88

have used a rather simple phenomenological potential which incorporates indisputable physical effects in their certified domain of applicability and a unknown contribution which summarizes our ignorance of the intermediate and short-range parts. Our analysis is consistent with the venerable and CD-OPE contribution above a distance of 3 fm. The simplicity of the potential should not be confused as unrealistic, as by definition our ignorance can be parametrized according to anyone's prejudices. We choose to implement the appealing idea of coarse graining of the interaction, since the shortest distance which can be probed below pion production threshold corresponds to a NN relative de Broglie wavelength resolution of $\Delta r \sim 1/\sqrt{m_\pi M_N} \sim 0.6$ fm. This simple scale consideration along with the role of the centrifugal barrier allows us to foresee the number of needed fitting parameters for the intermediate-range part [10,11,15] which turns out to be on the order of the actual number of parameters.

A successful fit to pp and np scattering data below the pion production threshold was achieved using a δ-shell representation for the short- and intermediate-range part of the nucleon-nucleon interaction; the long-range part was described by charge dependent OPE and EM interactions.

A partial-wave decomposition of the potential allowed us to incorporate charge dependence in the $^1S_0$ parameters while

TABLE III. Same as Table II for *np* scattering data.

E_{LAB}	Ref.	Type	θ	n_1	n_2	Sys	χ^2_{sys}	Z	χ^2_t	χ^2_t/n_2
0.0	LO74 [127]	SGT		1	1	no systematic error			2.95	2.95
0.0	DI75 [128]	SGT		1	1	no systematic error			4.23	4.23
0.0	HO71 [129]	SGT		1	1	no systematic error			0.37	0.37
0.0	KO90 [130]	SGT		1	1	no systematic error			8.04	8.04
0.0	FU76 [131]	SGT		1	1	no systematic error			4.27	4.27
0.1–0.6	AL55 [132]	SGT		5	5	no systematic error			2.26	0.45
0.2–0.8	DA13 [133]	SGT		65	65	no systematic error			90.36	1.39
0.5–3.2	EN63 [134]	SGT		2	2	no systematic error			3.97	1.99
0.5–24.6	CL72 [135]	SGT		114	114	float		0.995	136.87	1.20
0.8–20.0	CL69 [136]	SGT		17	15	no systematic error			14.08	0.94
1.3	ST54 [137]	SGT		1	1	no systematic error			1.14	1.14
1.5–27.5	DA71 [138]	SGT		27	28	0.1%	0.582	0.999	24.98	0.89
2.7	HR69 [139]	DSG	130.0–150.0	2	2	no systematic error			0.64	0.32
3.0	HR69 [139]	DSG	130.0–150.0	2	2	no systematic error			3.00	1.50
3.3	HR69 [139]	DSG	130.0–150.0	3	3	no systematic error			1.64	0.55
3.6–11.6	WI95 [140]	SGTT		9	9	no systematic error			12.52	1.39
3.7	HR69 [139]	DSG	130.0–150.0	4	4	no systematic error			2.10	0.52
4.0	HR69 [139]	DSG	130.0–150.0	4	4	no systematic error			0.72	0.18
4.3	HR69 [139]	DSG	130.0–150.0	4	4	no systematic error			2.85	0.71
4.7	HR69 [139]	DSG	130.0–150.0	4	4	no systematic error			3.25	0.81
4.7	HA53 [141]	SGT		1	1	no systematic error			3.78	3.78
4.9	HR69 [139]	DSG	130.0–150.0	4	4	no systematic error			2.78	0.69
5.0–19.7	WA01 [142]	SGTL		6	6	no systematic error			9.85	1.64
5.1	HR69 [139]	DSG	130.0–150.0	4	4	no systematic error			6.69	1.67
5.2	HR69 [139]	DSG	130.0–150.0	4	4	no systematic error			9.67	2.42
7.2–14.0	BR58 [143]	SGT		6	6	no systematic error			14.99	2.50
7.6	WE92 [144]	P	65.8–124.8	4	5	3.0%	0.170	0.988	10.40	2.08
10.0	BO01 [145]	DSG	59.9–180.0	6	7	0.8%	0.062	0.998	1.87	0.27
10.0	HO88 [146]	P	44.5–165.3	12	13	4.0%	1.177	1.043	13.41	1.03
10.7–17.1	WA01 [142]	SGTT		3	3	no systematic error			5.60	1.87
11.0	MU71 [147]	P	90.0	1	1	no systematic error			0.09	0.09
12.0	WE92 [144]	P	46.0–125.2	8	9	3.0%	1.506	0.963	14.12	1.57
13.5	TO77 [148]	P	90.0	1	1	no systematic error			0.37	0.37
13.7	SC88 [149]	AYY	90.0	1	1	no systematic error			6.33	6.33
14.0	AR70 [150]	DSG	80.0–100.0	3	4	1.6%	0.210	0.993	0.89	0.22
14.0	SC88 [149]	AYY	90.0	1	1	no systematic error			1.92	1.92
14.1	SE55 [151]	DSG	70.0–173.0	6	7	4.0%	1.035	0.959	1.91	0.27
14.1	BR81 [152]	P	50.6–156.6	10	11	3.0%	0.038	1.006	4.36	0.40
14.1	AL53 [153]	DSG	48.0–154.5	8	8	float		0.942	1.30	0.16
14.1	GR65 [154]	DSG	90.0–170.0	5	5	float		1.828	2.32	0.46
14.1	NA60 [155]	DSG	89.0–165.0	4	5	0.7%	1.053	1.007	2.12	0.42
14.1	WE92 [144]	P	45.9–125.2	5	6	3.0%	0.470	0.979	4.32	0.72
14.1	SH74 [156]	DSG	52.5–172.0	8	8	no systematic error			3.66	0.46
14.1	PO52 [157]	SGT		1	1	no systematic error			0.04	0.04
14.1	BU97 [158]	DSG	89.7–155.7	6	7	7.1%	0.005	0.995	5.14	0.73
14.5	FI77 [159]	P	40.0–120.0	8	9	5.0%	0.014	0.994	8.83	0.98
14.8	TO77 [148]	P	90.0	1	1	no systematic error			0.08	0.08
15.7	MO67 [160]	DSG	56.6–161.8	16	16	float		1.021	11.06	0.69
15.8	CL98 [161]	DT	132.4	1	1	no systematic error			2.80	2.80
15.8–110.0	BO61 [162]	SGT		34	35	2.0%	1.543	0.975	39.04	1.12
16.0	TO77 [148]	P	90.0	1	1	no systematic error			0.30	0.30
16.0	WE92 [144]	P	46.0–125.2	5	6	3.0%	0.783	1.027	8.17	1.36
16.2	GA72 [163]	P	70.0–130.0	3	3	no systematic error			0.56	0.19
16.2	BR96 [164]	SGTT		1	1	no systematic error			0.02	0.02
16.2	BR97 [165]	SGTL		1	1	no systematic error			0.07	0.07
16.4	BE62 [166]	P	100.0–140.0	3	4	9.3%	0.001	1.003	2.84	0.71
16.4	JO74 [167]	P	90.0–150.0	4	4	no systematic error			4.36	1.09
16.8	MU71 [147]	P	90.0	1	1	no systematic error			0.09	0.09

TABLE III. (*Continued*.)

E_{LAB}	Ref.	Type	θ	n_1	n_2	Sys	χ^2_{sys}	Z	χ^2_t	χ^2_t/n_2
16.9	MO74 [168]	P	40.0–140.0	4	5	6.0%	0.887	0.944	3.53	0.71
16.9	TO88 [169]	P	51.0–143.7	11	12	2.0%	0.059	0.995	15.87	1.32
16.9	TO88 [169]	P	136.5–166.5	4	5	1.0%	0.000	1.000	0.35	0.07
17.0	WI84 [170]	P	33.1–122.9	6	7	2.0%	0.001	1.001	4.10	0.59
17.4	OC91 [171]	DT	132.9	1	2	5.5%	0.198	0.976	1.44	0.72
17.8–29.0	PE60 [172]	SGT		5	5	no systematic error			6.94	1.39
17.9	GA55 [173]	DSG	80.0–175.0	11	12	1.9%	3.879	0.963	16.44	1.37
18.5	WE92 [144]	P	65.6–125.0	4	5	3.0%	0.224	1.014	3.02	0.60
19.0	WI84 [170]	P	33.1–122.9	6	7	3.0%	0.000	1.001	4.00	0.57
19.6–28.0	GR66 [174]	SGT		3	4	0.1%	0.003	1.000	3.17	0.79
19.7	DA59 [175]	SGT		1	1	no systematic error			0.32	0.32
20.5	LA65 [176]	P	21.5–100.5	9	10	18.8%	0.483	1.131	7.17	0.72
21.1	MO74 [168]	P	40.0–140.0	6	7	3.0%	0.140	0.989	4.31	0.62
21.6	JO74 [167]	P	50.0–170.0	7	7	float		1.303	3.35	0.48
21.6	SI89 [177]	P	77.5–150.0	5	6	4.0%	1.039	1.041	7.44	1.24
22.0	WI84 [170]	P	33.1–151.4	8	9	3.1%	1.767	1.041	17.36	1.93
22.2	FI90 [178]	DSG	104.6–164.9	5	6	10.0%	0.000	0.998	1.65	0.28
22.5	FL62 [179]	DSG	65.0–175.0	12	12	float		0.979	5.94	0.49
22.5	SC63 [180]	DSG	7.0–51.0	6	7	3.3%	0.012	1.004	3.09	0.44
22.5	FL62 [179]	SGT		1	2	2.0%	0.137	0.993	0.89	0.44
23.1	MA66 [181]	AYY	130.0–174.0	4	4	no systematic error			0.39	0.10
23.1	PE63 [182]	P	50.0–150.0	6	7	4.0%	0.039	1.008	4.10	0.59
23.1	MU71 [147]	P	140.0–150.0	2	3	20.0%	0.075	0.945	0.31	0.10
23.1	MA66 [181]	P	140.0–150.0	2	3	12.2%	0.023	0.982	0.28	0.09
23.7	BE62 [166]	P	80.0–140.0	4	5	10.9%	0.127	0.961	1.30	0.26
24.0	RO70 [183]	DSG	89.0–164.7	4	5	0.5%	3.312	0.991	11.50	2.30
24.0	BU73 [184]	DSG	71.3–157.9	4	4	float		0.996	1.41	0.35
24.0	MA72 [185]	DSG	39.3–50.5	2	2	no systematic error			0.39	0.20
24.6–59.4	BR70 [186]	SGT		8	9	3.0%	0.002	0.999	8.45	0.94
25.0	WI84 [170]	P	33.1–151.4	8	9	2.9%	1.895	1.040	8.63	0.96
25.0	FI90 [178]	DSG	104.6–164.9	5	6	10.0%	0.009	0.990	4.28	0.71
25.0	SR86 [187]	P	50.7–148.4	11	12	2.5%	1.833	0.966	14.76	1.23
25.0	SR86 [187]	P	128.7–164.6	5	6	2.5%	0.029	0.996	5.12	0.85
25.3	DR79 [188]	DSG	180.0	1	1	no systematic error			0.07	0.07
25.5	OC91 [189]	DT	131.1	1	1	no systematic error			0.007	0.007
25.8	MO77 [190]	DSG	20.1–90.5	8	9	3.0%	0.170	0.988	4.90	0.54
25.8	MO77 [190]	DSG	89.5–178.0	8	9	3.0%	0.023	0.995	3.73	0.41
26.9–72.5	BO85 [191]	SGT		5	5	no systematic error			6.13	1.23
27.2	BU73 [184]	DSG	71.3–157.8	5	5	float		1.001	0.85	0.17
27.4	FI90 [178]	DSG	104.6–164.9	5	6	10.0%	0.005	0.993	5.45	0.91
27.5	SC63 [180]	DSG	7.0–72.0	8	9	3.0%	0.606	1.023	1.53	0.17
27.5	SC63 [180]	DSG	159.0–173.0	3	3	float		0.868	2.36	0.79
27.5	WI84 [170]	P	33.1–151.4	8	8	3.0%	1.339	1.035	10.12	1.26
29.0	BE97 [192]	DSG	40.0–140.0	6	7	5.0%	0.041	0.990	16.66	2.38
29.6	MU71 [147]	P	60.0–120.0	3	4	10.0%	0.232	1.048	1.48	0.37
29.6	EL75 [193]	P	50.0–150.0	11	11	no systematic error			2.51	0.23
29.9	FI90 [178]	DSG	104.6–164.9	5	6	10.0%	0.005	0.993	15.26	2.54
30.0	LA65 [176]	P	21.5–158.5	12	13	8.3%	0.329	0.952	13.65	1.05
30.0	WI84 [170]	P	33.1–151.4	8	9	2.9%	0.175	1.012	5.54	0.62
31.1	DR79 [188]	DSG	180.0	1	1	no systematic error			0.27	0.27
31.6	RY72 [194]	P	60.5–100.6	2	2	no systematic error			2.46	1.23
32.5	SC63 [180]	DSG	7.0–82.0	9	10	2.1%	4.982	1.047	15.32	1.53
32.5	SC63 [180]	DSG	129.0–173.0	6	7	4.0%	2.770	0.933	7.49	1.07
32.5	RY72 [194]	P	80.5	1	1	no systematic error			0.90	0.90
32.9	FI90 [178]	DSG	89.4–164.8	6	7	10.0%	0.001	0.997	13.77	1.97
33.0	WI84 [170]	P	33.1–151.4	8	9	2.9%	2.732	1.048	8.70	0.97
33.0–350.0	LI82 [195]	SGT		72	73	1.1%	0.279	0.994	72.89	1.00
34.5	BE97 [192]	DSG	40.0–140.0	6	7	5.0%	0.005	0.996	8.09	1.16

TABLE III. (*Continued*.)

E_{LAB}	Ref.	Type	θ	n_1	n_2	Sys	χ^2_{sys}	Z	χ^2_t	χ^2_t/n_2
35.8	FI90 [178]	DSG	89.4–164.8	6	7	10.0%	0.022	1.015	15.76	2.25
36.0	WI84 [170]	P	33.1–151.4	8	9	2.9%	2.726	1.048	10.27	1.14
37.5	SC63 [180]	DSG	7.0–92.0	10	11	2.0%	1.055	1.021	7.13	0.65
37.5	SC63 [180]	DSG	118.0–173.0	7	8	4.0%	4.353	0.917	8.73	1.09
38.0	TA53 [196]	SGT		1	2	2.6%	0.398	0.984	1.09	0.55
39.7	FI90 [178]	DSG	89.4–164.8	6	7	10.0%	0.032	1.018	19.77	2.82
40.0	LA65 [176]	P	21.5–158.5	15	16	10.6%	0.601	1.082	11.32	0.71
40.0	WI84 [170]	P	33.1–151.4	8	9	2.9%	0.972	1.029	14.81	1.65
40.0	BL85 [197]	DSG	90.0–178.5	3	3	no systematic error			1.44	0.48
42.5	SC63 [180]	DSG	7.0–102.0	11	12	2.0%	0.789	1.018	15.98	1.33
42.5	SC63 [180]	DSG	78.0–173.0	11	12	4.0%	2.946	0.931	17.62	1.47
45.0	BL85 [197]	DSG	90.0–178.5	3	3	no systematic error			1.58	0.53
45.0	BE97 [192]	DSG	40.0–140.0	6	7	5.0%	0.013	0.994	8.95	1.28
47.5	SC63 [180]	DSG	7.0–102.0	11	12	2.0%	1.609	1.025	22.74	1.90
47.5	SC63 [180]	DSG	78.0–173.0	11	12	4.0%	1.900	0.945	19.05	1.59
49.0	BE97 [192]	DSG	40.0–140.0	6	7	5.0%	0.135	0.982	8.05	1.15
50.0	WO85 [198]	DT	179.0	1	1	no systematic error			0.03	0.03
50.0	BL85 [197]	DSG	90.0–178.5	3	3	no systematic error			0.82	0.27
50.0	LA65 [176]	P	21.5–158.5	15	16	4.7%	0.228	1.022	8.03	0.50
50.0	MO77 [190]	DSG	20.3–90.8	8	9	3.0%	0.055	1.007	4.70	0.52
50.0	MO77 [190]	DSG	69.2–173.3	12	13	3.0%	0.510	0.979	10.46	0.80
50.0	JO77 [199]	AYY	109.0–174.0	4	5	25.0%	1.102	0.738	2.51	0.50
50.0	RO78 [200]	P	69.3–149.6	9	10	3.6%	1.285	0.959	8.24	0.82
50.0	GA80 [201]	P	60.6–120.6	7	7	4.0%	0.050	1.009	4.04	0.58
50.0	FI80 [202]	AYY	108.0–174.0	4	5	7.8%	0.869	0.927	3.73	0.75
50.0	WI84 [170]	P	33.1–151.4	8	9	3.4%	3.460	1.063	17.37	1.93
50.0	FI90 [178]	DSG	89.4–164.8	6	7	10.0%	0.051	0.977	11.06	1.58
50.0	FI80 [202]	P	108.0–174.0	4	5	2.0%	0.001	0.999	0.88	0.18
50.0	FI80 [202]	P	108.0–174.0	4	5	2.0%	0.000	1.000	0.33	0.07
52.5	SC63 [180]	DSG	7.0–112.0	12	13	1.7%	2.594	1.027	25.06	1.93
52.5	SC63 [180]	DSG	78.0–173.0	11	12	3.8%	0.484	0.974	11.88	0.99
53.0	BE97 [192]	DSG	40.0–140.0	6	7	5.0%	0.303	0.972	4.98	0.71
55.1	BL85 [197]	DSG	90.0–178.5	3	3	no systematic error			0.73	0.24
57.5	SC63 [180]	DSG	7.0–112.0	12	13	2.0%	2.041	1.029	17.01	1.31
57.5	SC63 [180]	DSG	78.0–173.0	11	12	4.0%	1.671	0.948	19.43	1.62
58.8	BE76 [203]	DSG	11.8–42.3	9	10	10.0%	0.097	0.969	5.84	0.58
60.0	LA65 [176]	P	21.5–158.5	16	17	3.9%	4.552	0.917	22.86	1.34
61.0	BL85 [197]	DSG	90.0–166.0	2	2	no systematic error			0.41	0.20
62.2	BL85 [197]	DSG	90.0–178.5	3	3	no systematic error			3.91	1.30
62.5	SC63 [180]	DSG	7.0–112.0	12	13	2.0%	0.001	0.999	30.84	2.37
62.5	SC63 [180]	DSG	78.0–173.0	11	12	4.0%	5.308	0.908	21.47	1.79
62.7	BE97 [192]	DSG	40.0–140.0	6	7	5.0%	0.089	0.985	15.56	2.22
63.1	KI80 [204]	DSG	39.4–165.8	19	17	3.0%	1.086	0.969	29.49	1.73
65.0	BL85 [197]	DSG	90.0–178.5	3	3	no systematic error			0.34	0.11
66.0	HA92 [205]	SGTL		1	1	no systematic error			1.34	1.34
67.5	BR92 [206]	P	38.6–103.1	12	13	4.0%	0.200	1.018	9.14	0.70
67.5	BR92 [206]	P	82.0–155.2	19	20	4.0%	0.958	0.961	24.64	1.23
67.5	BE76 [203]	DSG	11.9–42.4	9	10	10.0%	0.152	0.961	9.00	0.90
67.5	HA91 [207]	AZZ	104.8–168.1	20	21	6.0%	0.623	0.953	15.39	0.73
70.0	BL85 [197]	DSG	90.0–178.5	3	3	no systematic error			2.29	0.76
70.0	SC63 [180]	DSG	7.0–122.0	12	13	2.0%	0.039	1.004	31.49	2.42
70.0	SC63 [180]	DSG	78.0–173.0	11	12	4.0%	8.589	0.883	18.76	1.56
70.0	LA65 [176]	P	21.5–158.5	16	17	3.9%	6.693	0.899	23.68	1.39
72.8	BE97 [192]	DSG	40.0–140.0	6	7	5.0%	0.111	0.983	2.72	0.39
76.2	BL85 [197]	DSG	90.0–166.0	2	2	no systematic error			3.81	1.91
76.7	BE76 [203]	DSG	11.9–49.6	11	11	10.0%	0.901	0.905	8.85	0.80
80.0	SC63 [180]	DSG	7.0–112.0	12	13	2.0%	0.649	0.984	16.37	1.26
80.0	SC63 [180]	DSG	78.0–173.0	11	12	4.0%	3.202	0.928	14.02	1.17

TABLE III. (*Continued*.)

E_{LAB}	Ref.	Type	θ	n_1	n_2	Sys	χ^2_{sys}	Z	χ^2_t	χ^2_t/n_2
80.0	LA65 [176]	P	21.5–158.5	16	17	4.2%	1.369	0.951	9.80	0.58
86.5	BE76 [203]	DSG	11.9–49.7	11	12	10.0%	1.683	0.870	19.07	1.59
89.5	SC63 [180]	DSG	7.0–122.0	13	14	2.0%	1.714	0.974	15.00	1.07
89.5	SC63 [180]	DSG	78.0–173.0	11	12	4.0%	3.476	0.925	13.73	1.14
90.0	LA65 [176]	P	21.5–158.5	16	17	5.1%	1.075	0.947	10.19	0.60
90.0	CH57 [208]	DSG	9.0–175.0	17	17	float		0.909	24.43	1.44
91.0	SA54 [209]	DSG	59.8–176.6	25	25	float		0.932	21.26	0.85
93.4–106.8	CU55 [210]	SGT		4	4	no systematic error			1.81	0.45
95.0	ME04 [211]	DSG	27.5–150.0	10	11	5.0%	0.700	0.958	7.81	0.71
95.0	ST57 [212]	P	22.5–159.5	15	16	8.0%	0.710	0.933	32.23	2.01
96.0	KL02 [213]	DSG	152.4–175.0	11	12	5.0%	0.307	1.028	8.10	0.67
96.0	BL04 [214]	DSG	80.0–160.0	9	10	3.0%	0.492	0.979	13.04	1.30
96.0	JO05 [215]	DSG	19.9–75.6	12	12	no systematic error			22.67	1.89
96.0	GR58 [216]	DSG	29.3–58.8	4	5	5.0%	0.097	1.016	0.47	0.09
96.8	BE76 [203]	DSG	11.9–49.8	11	12	10.0%	0.565	0.925	16.54	1.38
98.0	HI56 [217]	P	58.6–159.5	9	10	14.3%	0.325	0.919	5.78	0.58
99.0	SC63 [180]	DSG	7.0–122.0	13	14	1.7%	1.554	0.979	20.16	1.44
99.0	SC63 [180]	DSG	78.0–173.0	11	12	3.8%	0.148	0.985	14.45	1.20
100.0	LA65 [176]	P	21.5–158.5	16	17	7.3%	0.178	1.031	7.27	0.43
105.0	TH55 [218]	DSG	6.2–61.4	7	8	8.0%	1.394	1.094	2.45	0.31
107.6	BE76 [203]	DSG	12.0–50.0	11	12	10.0%	0.350	0.941	17.83	1.49
108.5	SC63 [180]	DSG	7.0–122.0	13	14	2.0%	0.630	0.984	15.77	1.13
108.5	SC63 [180]	DSG	78.0–173.0	11	12	4.0%	0.159	0.984	21.44	1.79
110.0	LA65 [176]	P	22.0–158.0	16	17	10.0%	0.184	1.043	18.93	1.11
118.8	BE76 [203]	DSG	12.0–50.1	11	12	10.0%	0.439	0.934	14.83	1.24
120.0	LA65 [176]	P	22.0–158.5	16	17	14.9%	0.041	1.030	8.12	0.48
120.0	CO64 [219]	SGT		1	1	no systematic error			0.003	0.003
125.0–168.0	SH65 [220]	SGT		2	3	12.0%	1.990	0.831	3.69	1.23
125.9–344.5	GR85 [221]	SGT		12	13	1.5%	0.262	0.992	3.63	0.28
126.0	CA64 [222]	P	33.0–81.9	6	7	10.0%	0.235	1.049	4.39	0.63
128.0	HO60 [223]	DSG	78.1–169.7	10	11	2.2%	1.380	0.974	4.43	0.40
128.0	HO60 [223]	P	78.1–169.7	10	11	10.0%	0.029	0.983	11.21	1.02
128.0	PA62 [224]	DT	124.0–160.0	5	5	no systematic error			8.87	1.77
128.0	CO64 [219]	DT	170.0	1	1	no systematic error			0.0001	0.0001
129.0	MS66 [225]	DSG	73.2–176.8	15	16	6.5%	0.693	0.946	11.34	0.71
129.0	HO74 [226]	DSG	32.6–92.0	9	10	16.0%	0.003	1.008	4.94	0.49
129.0	HO74 [226]	DSG	76.2–167.3	16	17	7.0%	0.136	0.974	10.69	0.63
130.0	RA56 [227]	DSG	25.0–155.0	14	15	3.2%	3.637	0.939	15.01	1.00
130.5	BE76 [203]	DSG	11.0–50.2	11	12	10.0%	0.171	0.959	13.69	1.14
135.0	LE63 [228]	A	42.1–83.6	5	6	4.0%	0.003	1.002	3.66	0.61
137.0	TH55 [218]	DSG	6.3–61.8	7	8	5.0%	0.805	0.955	4.76	0.60
137.0	LE63 [228]	R	42.1–83.6	5	5	no systematic error			6.72	1.34
137.0	GR58 [216]	DSG	19.3–58.3	5	6	5.0%	2.139	0.927	3.95	0.66
140.0	ST62 [229]	P	20.7–159.3	14	15	4.4%	1.285	0.950	25.17	1.68
142.8	BE76 [203]	DSG	11.0–50.3	11	12	10.0%	0.085	0.971	3.14	0.26
150.0	MS66 [225]	DSG	63.2–176.8	16	17	6.5%	0.298	0.964	7.38	0.43
155.4	BE76 [203]	DSG	11.1–50.5	11	11	10.0%	0.331	0.942	21.71	1.97
162.0	BO78 [26]	DSG	178.5–122.2	43	43	float		0.916	65.91	1.53
168.5	BE76 [203]	DSG	11.1–50.6	11	11	10.0%	0.410	0.936	12.18	1.11
175.3	DA96 [230]	P	86.6–106.0	20	20	float		0.772	40.88	2.04
177.9	BO78 [26]	DSG	179.2–122.0	44	44	float		0.922	48.81	1.11
180.0	BI91 [231]	SGTT		1	1	no systematic error			0.25	0.25
180.0	BI91 [231]	SGTL		1	1	no systematic error			0.05	0.05
181.0	SO87 [232]	P	57.5–126.1	10	10	4.0%	0.363	0.976	13.08	1.31
181.0	SO87 [232]	AYY	57.5–126.1	10	11	8.0%	0.004	0.995	9.93	0.90
181.8	BE76 [203]	DSG	11.1–50.8	11	12	10.0%	0.859	0.907	12.54	1.04
194.0	SA06 [233]	DSG	92.7–177.0	15	16	1.5%	0.004	0.999	29.23	1.83
195.6	BE76 [203]	DSG	11.2–50.9	11	12	10.0%	0.212	0.954	18.03	1.50

TABLE III. (*Continued*.)

E_{LAB}	Ref.	Type	θ	n_1	n_2	Sys	χ^2_{sys}	Z	χ^2_t	χ^2_t/n_2
197.0	SP67 [234]	DT	147.4–126.9	3	3	no systematic error			2.75	0.92
199.0	TH68 [235]	DSG	76.9–158.1	8	6	float		0.990	11.88	1.98
200.0	KA63 [236]	SGT		1	1	no systematic error			0.04	0.04
203.2	DA96 [230]	P	77.6–101.0	24	25	3.1%	0.163	0.987	27.48	1.10
210.0	BE76 [203]	DSG	11.2–51.1	11	12	10.0%	0.069	0.974	12.24	1.02
211.5	BO78 [26]	DSG	178.0–120.4	43	43	float		0.933	33.12	0.77
212.0	KE82 [237]	DSG	88.6–177.4	39	36	3.2%	0.174	0.987	40.48	1.12
212.0	KE82 [237]	DSG	15.8–72.9	4	5	2.0%	0.258	1.010	1.33	0.27
212.0–319.0	KE82 [237]	SGT		3	3	float		1.047	0.50	0.17
217.2	DA96 [230]	P	77.6–101.0	24	25	3.1%	0.707	0.974	23.56	0.94
220.0	CL80 [238]	P	49.6–162.1	16	17	3.0%	0.279	0.984	22.29	1.31
220.0	CL80 [238]	DT	98.3–152.5	10	11	3.0%	0.000	0.999	8.71	0.79
220.0	AM77 [239]	RT	161.0	1	2	3.0%	0.015	1.004	0.34	0.17
220.0	AM77 [239]	RPT	161.0	1	2	3.0%	0.000	1.000	1.06	0.53
220.0	AX80 [240]	RT	97.6–152.5	7	8	3.0%	1.447	0.964	10.37	1.30
220.0	AX80 [240]	AT	97.6–152.5	7	8	3.0%	0.014	1.004	18.53	2.32
220.0	BA89 [241]	AYY	71.0–144.2	16	17	7.5%	0.762	0.935	9.75	0.57
220.0	BA89 [241]	P	71.0–144.2	17	17	2.5%	0.129	1.009	5.74	0.34
220.0	BA89 [241]	P	71.0–144.2	16	17	5.0%	0.007	0.996	8.02	0.47
224.3	BE76 [203]	DSG	11.2–51.2	11	12	10.0%	0.020	1.014	9.86	0.82
225.0	BI91 [231]	SGTT		1	1	no systematic error			0.08	0.08
225.0	BI91 [231]	SGTL		1	1	no systematic error			0.05	0.05
228.0	BA89 [241]	RT	160.9	1	1	no systematic error			5.73	5.73
229.1	BO78 [26]	DSG	178.3–119.6	49	49	float		0.933	66.91	1.37
239.5	BE76 [203]	DSG	11.3–51.4	11	12	10.0%	0.657	0.919	5.38	0.45
247.2	BO78 [26]	DSG	178.4–118.8	53	53	float		0.944	41.79	0.79
260.0	KE50 [242]	DSG	37.7–180.0	15	16	4.0%	0.221	0.981	26.59	1.66
260.0	AH98 [243]	RT	105.4–159.0	8	9	3.0%	2.920	0.949	23.18	2.58
260.0	AH98 [243]	AT	105.4–159.0	8	9	3.0%	0.053	0.993	12.33	1.37
260.0	AH98 [243]	AT	104.4–118.0	3	4	3.0%	0.012	0.997	1.24	0.31
260.0	AH98 [243]	DT	105.4–159.0	8	9	3.0%	0.001	1.001	3.72	0.41
260.0	AH98 [243]	DT	104.4–118.0	3	4	3.0%	0.001	1.001	7.84	1.96
260.0	AH98 [243]	P	105.4–159.0	8	9	2.0%	0.001	1.001	8.03	0.89
260.0	AH98 [243]	P	104.4–118.0	3	4	2.0%	0.086	1.006	4.73	1.18
260.0	AR00 [244]	P	90.0–118.0	8	9	1.8%	0.264	0.991	8.32	0.92
260.0	AR00 [244]	P	102.0–162.0	16	17	1.8%	0.400	0.989	17.48	1.03
260.0	AR00 [244]	AYY	90.0–118.0	8	9	3.9%	0.538	0.971	6.12	0.68
260.0	AR00 [244]	AYY	102.0–162.0	16	17	3.9%	2.698	0.936	18.64	1.10
260.0	AR00 [244]	AZZ	86.0–118.0	9	10	7.2%	2.561	0.885	9.17	0.92
260.0	AR00 [244]	AZZ	102.0–162.0	16	17	7.2%	2.860	0.878	23.51	1.38
260.0	AN00 [245]	D	88.0–120.0	5	6	2.4%	0.328	0.986	7.93	1.32
260.0	AN00 [245]	D	104.0–160.0	8	8	float		0.853	5.90	0.74
260.0	AN00 [245]	D0SK	104.0–160.0	8	8	float		0.770	2.90	0.36
260.0	AN00 [245]	DT	88.0–120.0	5	6	2.4%	0.000	1.000	5.00	0.83
260.0	AN00 [245]	AT	96.0–120.0	4	5	2.4%	0.029	0.996	10.61	2.12
260.0	AN00 [245]	AT	104.0–160.0	8	9	2.4%	0.079	0.993	13.05	1.45
260.0	AN00 [245]	RT	96.0–120.0	4	5	2.4%	0.000	1.000	3.21	0.64
260.0	AN00 [245]	RT	104.0–160.0	8	9	2.4%	0.047	1.005	6.17	0.69
260.0	AN00 [245]	NNKK	104.0–160.0	8	9	2.4%	0.014	0.997	12.37	1.37
260.0	AN00 [245]	NSKN	96.0–120.0	4	5	2.4%	0.000	1.000	0.78	0.16
260.0	AN00 [245]	NSKN	104.0–160.0	8	9	2.4%	0.006	1.002	2.48	0.28
260.0	AN00 [245]	NSSN	96.0–120.0	4	5	2.4%	0.001	1.001	2.55	0.51
261.0	DA96 [230]	P	68.6–89.0	21	22	2.8%	2.590	1.045	35.91	1.63
265.8	BO78 [26]	DSG	178.8–118.0	63	63	float		0.953	66.07	1.05
267.2	BE76 [203]	DSG	11.4–51.7	11	11	10.0%	0.024	1.015	6.07	0.55
277.0	BI91 [231]	SGTT		1	1	no systematic error			0.06	0.06
277.0	BI91 [231]	SGTL		1	1	no systematic error			0.02	0.02
284.0	DA02 [246]	P	113.0–176.3	14	15	3.0%	0.177	0.987	8.60	0.57

TABLE III. (*Continued*.)

E_{LAB}	Ref.	Type	θ	n_1	n_2	Sys	χ^2_{sys}	Z	χ^2_t	χ^2_t/n_2
284.8	BO78 [26]	DSG	178.7–117.2	73	73	float		0.929	79.81	1.09
295.0–328.0	GR82 [247]	SGT		2	3	0.5%	0.370	1.003	6.51	2.17
295.0–328.0	GR82 [247]	SGTT		2	3	0.5%	0.000	1.000	7.11	2.37
295.0–328.0	GR82 [247]	SGTL		2	3	0.5%	0.000	1.000	0.61	0.20
304.2	BO78 [26]	DSG	178.7–115.7	79	79	float		0.975	73.63	0.93
307.0	CH67 [113]	P	33.1–141.5	8	8	3.0%	0.069	0.992	11.54	1.44
309.6	BE76 [203]	DSG	11.5–52.1	11	12	10.0%	0.711	1.084	12.48	1.04
310.0	CA57 [108]	P	21.6–164.9	19	18	4.0%	0.043	0.992	10.98	0.61
312.0	BA93 [248]	P	50.2–129.4	24	25	4.0%	0.935	1.039	19.31	0.77
312.0	BA94 [249]	AZZ	50.2–89.6	11	12	4.0%	0.188	0.983	19.95	1.66
312.0	FO91 [250]	SGTL		1	1	no systematic error			0.01	0.01
314.0	DA02 [246]	P	113.0–176.3	14	15	3.0%	0.026	0.995	15.49	1.03
315.0	AR00 [244]	P	102.0–162.0	16	17	1.2%	0.651	0.990	24.30	1.43
315.0	AR00 [244]	AYY	78.0–118.0	11	12	3.7%	2.351	0.943	13.56	1.13
315.0	AR00 [244]	AYY	102.0–162.0	16	17	3.7%	4.534	0.921	25.22	1.48
315.0	AR00 [244]	AZZ	102.0–162.0	16	17	7.1%	5.742	0.830	23.95	1.41
315.0	AN00 [245]	D	80.0–120.0	6	7	1.9%	0.729	0.984	2.96	0.42
315.0	AN00 [245]	D	104.0–162.0	8	8	float		0.879	14.51	1.81
315.0	AN00 [245]	D0SK	80.0–120.0	6	6	float		0.773	8.68	1.45
315.0	AN00 [245]	D0SK	104.0–162.0	8	8	float		0.791	5.47	0.68
315.0	AN00 [245]	DT	80.0–120.0	6	7	1.9%	0.000	1.000	13.11	1.87
315.0	AN00 [245]	AT	80.0–120.0	6	7	1.9%	0.059	0.995	3.80	0.54
315.0	AN00 [245]	AT	104.0–162.0	8	9	1.9%	0.232	0.991	8.76	0.97
315.0	AN00 [245]	RT	80.0–120.0	6	7	1.9%	0.000	1.000	2.29	0.33
315.0	AN00 [245]	RT	104.0–162.0	8	9	1.9%	0.050	0.996	6.41	0.71
315.0	AN00 [245]	NSKN	88.0–120.0	5	6	1.9%	0.001	0.999	1.66	0.28
315.0	AN00 [245]	NSKN	104.0–162.0	8	9	1.9%	0.065	0.995	14.64	1.63
315.0	AN00 [245]	NSSN	80.0–120.0	6	7	1.9%	0.005	0.999	7.95	1.14
315.0	AN00 [245]	NSSN	104.0–162.0	8	9	1.9%	0.000	1.000	10.01	1.11
318.0	AH98 [243]	RT	105.1–159.0	8	9	3.0%	0.806	0.973	4.61	0.51
318.0	AH98 [243]	AT	105.1–159.0	8	9	3.0%	0.457	0.980	9.75	1.08
318.0	AH98 [243]	AT	89.2–118.1	5	6	3.0%	0.383	0.981	9.64	1.61
318.0	AH98 [243]	DT	105.1–159.0	8	9	3.0%	0.017	0.996	7.34	0.82
318.0	AH98 [243]	DT	89.2–118.1	5	6	3.0%	0.000	1.000	4.24	0.71
318.0	AH98 [243]	P	105.1–159.0	8	9	2.0%	0.512	0.986	7.34	0.82
318.0	AH98 [243]	P	89.2–118.1	5	6	2.0%	0.078	0.994	9.03	1.51
319.0	KE82 [237]	DSG	11.1–94.5	7	7	float		1.105	7.14	1.02
324.1	BO78 [26]	DSG	178.8–114.9	81	81	float		0.927	89.03	1.10
325.0	AS77 [251]	P	44.9–159.4	42	38	12.0%	0.433	1.079	53.47	1.41
325.0	AM77 [239]	RT	160.5	1	2	3.0%	0.103	0.990	1.58	0.79
325.0	AM77 [239]	RPT	160.5	1	2	3.0%	0.000	1.000	0.14	0.07
325.0	AS77 [251]	DT	87.3–149.0	8	9	3.0%	0.000	1.000	4.54	0.50
325.0	CL80 [238]	P	45.0–159.4	21	18	3.0%	4.280	0.938	31.20	1.73
325.0	CL80 [238]	DT	84.2–152.9	12	13	3.0%	0.096	0.991	11.54	0.89
325.0	AX80 [240]	RT	76.8–153.5	9	10	3.0%	0.072	1.008	13.70	1.37
325.0	AX80 [240]	AT	76.8–144.5	8	9	3.0%	1.133	0.968	8.84	0.98
325.0	BA89 [241]	AYY	61.9–145.9	19	20	6.8%	0.621	1.054	13.73	0.69
325.0	BA89 [241]	P	61.9–145.9	19	20	2.5%	0.008	0.998	9.57	0.48
332.0	BI91 [231]	SGTT		1	1	no systematic error			1.19	1.19
332.0	BI91 [231]	SGTL		1	1	no systematic error			1.53	1.53
337.0	BA89 [241]	RT	160.5	1	1	no systematic error			7.70	7.70
343.0	AM77 [239]	RT	141.7–167.0	4	4	no systematic error			4.55	1.14
343.8	BE76 [203]	DSG	11.6–52.5	11	12	10.0%	0.006	1.008	13.11	1.09
344.0	DA02 [246]	P	113.0–176.3	14	15	3.0%	0.256	1.015	18.68	1.25
344.3	BO78 [26]	DSG	179.1–114.1	80	79	float		0.952	81.56	1.03
350.0	SI56 [252]	P	46.4–158.2	10	9	float		0.992	5.97	0.66
350.0	AS62 [253]	DSG	160.7–173.8	7	7	float		1.019	5.63	0.80

TABLE IV. Same as Table II but for rejected pp data.

$E_{\rm LAB}$	Ref.	Type	θ	n_1	n_2	Sys	$\chi^2_{\rm sys}$	Z	χ^2_t	χ^2_t/n_2
0.3–0.4	DO97 [254]	DSG	90.0	14	15	0.1%	4.083	0.998	318.20	21.21
0.4	TH78 [30]	DSG	58.4–95.0	4	4	float		0.998	18.45	4.61
0.4	TH78 [30]	DSG	24.0–110.0	22	23	0.2%	0.016	1.000	62.88	2.73
14.2	KI60 [255]	DSG	18.1–114.2	17	18	10.0%	0.008	0.991	1.39	0.08
20.2	CA67 [40]	P	34.8–90.3	8	9	11.8%	0.019	0.984	1.20	0.13
46.9	GA71 [58]	AXX	90.0	1	1	no systematic error			15.73	15.73
66.0	PA58 [57]	DSG	25.5–71.0	10	10	float		1.037	1.78	0.18
95.0	PA58 [57]	DSG	25.7–86.4	13	13	float		0.970	2.94	0.23
137.0	PA58 [57]	DSG	31.1–66.9	3	3	float		0.977	0.04	0.01
241.0	ON89 [104]	DT	60.0–88.0	8	9	5.0%	0.000	0.999	1.12	0.12
308.0	GR79 [96]	P	51.3	1	2	3.0%	0.002	0.999	0.002	0.001
314.0	ON89 [104]	DT	60.0–88.0	8	9	5.0%	0.046	0.989	1.43	0.16
327.0	GR79 [96]	P	36.7	1	2	1.0%	0.000	1.000	7.1×10^{-7}	3.6×10^{-7}
341.0	ON89 [104]	A	38.0–98.0	15	16	5.0%	0.681	0.959	44.59	2.79
350.0	OT84 [110]	DSG	90.0	1	1	float		1.074	0.00	0.00

TABLE V. Same as Table II but for rejected np data.

$E_{\rm LAB}$	Ref.	Type	θ	n_1	n_2	Sys	$\chi^2_{\rm sys}$	Z	χ^2_t	χ^2_t/n_2
0.5–2.0	PO82 [256]	SGT		3	3	no systematic error			30.24	10.08
1.0–2.5	FI54 [257]	SGT		2	2	no systematic error			14.51	7.25
2.5	DV71 [258]	SGT		1	1	no systematic error			14.04	14.04
12.0	BR08 [259]	P	32.6–143.8	15	16	1.5%	5.271	0.966	41.63	2.60
14.1	SU67 [260]	DSG	11.9–92.8	16	16	float		0.999	3.19	0.20
31.5	BE97 [192]	DSG	40.0–140.0	6	7	5.0%	0.014	1.006	57.54	8.22
37.5	BE97 [192]	DSG	40.0–140.0	6	7	5.0%	0.105	0.984	25.82	3.69
41.0	BE97 [192]	DSG	40.0–140.0	6	7	5.0%	0.587	0.962	26.81	3.83
58.5	BE97 [192]	DSG	40.0–140.0	6	7	5.0%	1.427	0.940	64.20	9.17
67.5	GO94 [261]	DSG	87.3–172.9	15	16	5.0%	0.058	0.988	121.93	7.62
67.7	BE97 [192]	DSG	40.0–140.0	6	7	5.0%	0.132	0.982	31.29	4.47
88.0–150.9	ME66 [262]	SGT		6	7	0.1%	0.001	1.000	0.33	0.05
143.0	KU61 [263]	P	41.0–118.0	8	8	float		0.835	24.17	3.02
152.0	PA71 [264]	DSG	77.8–169.3	13	13	float		1.048	69.02	5.31
162.0	RA98 [27]	DSG	73.0–179.0	54	55	4.0%	0.042	0.992	423.16	7.69
194.5	BO78 [26]	DSG	179.2–121.2	42	42	float		0.923	79.02	1.88
199.0	TH68 [235]	P	76.9–158.1	8	9	10.0%	0.006	0.992	28.35	3.15
199.9	FR00 [265]	DSG	81.1–179.3	102	103	5.0%	0.025	0.992	283.51	2.75
200.0	KA63 [236]	DSG	6.3–173.8	20	21	2.1%	0.010	1.002	54.12	2.58
219.8	FR00 [265]	DSG	80.8–179.3	104	105	5.0%	0.232	0.976	748.77	7.13
240.2	FR00 [265]	DSG	80.6–179.3	107	108	5.0%	0.599	0.961	552.16	5.11
247.2–344.3	DE73 [266]	SGT		4	5	0.2%	1.530	1.002	53.13	10.63
260.0	AN00 [245]	D0SK	88.0–120.0	5	6	2.4%	3.709	0.954	24.78	4.13
260.0	AN00 [245]	NSSN	104.0–160.0	8	9	2.4%	0.000	1.000	1.34	0.15
261.9	FR00 [265]	DSG	80.3–179.3	108	109	5.0%	1.083	0.948	437.21	4.01
280.0	FR00 [265]	DSG	80.0–179.3	109	110	5.0%	4.221	0.897	1396.64	12.70
300.0	DE54 [267]	DSG	35.0–175.0	15	16	10.0%	0.047	0.978	42.35	2.65
300.2	FR00 [265]	DSG	79.8–179.3	111	111	float		0.874	452.00	4.07
315.0	AR00 [244]	P	78.0–118.0	11	12	1.2%	3.249	0.978	42.91	3.58
315.0	AR00 [244]	AZZ	78.0–118.0	11	12	7.1%	7.601	0.804	39.91	3.33
315.0	AN00 [245]	NNKK	104.0–162.0	8	9	1.9%	0.005	1.001	1.36	0.15
319.0	KE82 [237]	DSG	66.7–177.0	64	65	3.9%	0.032	0.993	274.99	4.23
320.1	FR00 [265]	DSG	79.5–179.2	110	110	float		0.874	577.12	5.25
325.0	BA89 [241]	P	61.9–145.9	19	20	4.3%	0.005	0.997	5.29	0.26
340.0	FR00 [265]	DSG	79.3–179.2	112	112	float		0.877	512.30	4.57
350.0	AS62 [253]	DSG	114.2–165.1	10	10	float		1.023	76.22	7.62

TABLE VI. Delta-shell potential parameters $V_{i,n}$ (in fm^{-1}) with their errors for all operators. These parameters are *not* the fitting parameters and are obtained from the 46 *independent* partial waves $(\lambda_i)^{JS}_{l,l'}$ (see Appendix A). We take $N = 5$ equidistant points with $\Delta r = 0.6$ fm. Rows marked with* indicates that the corresponding strengths coefficients are not independent or zero. In the first line we provide the central component of the δ shells corresponding to the EM effects below $r_c = 3$ fm. These parameters remain fixed within the fitting process.

Operator	$V_{1,x}$ $r_1 = 0.6$ fm	$V_{2,x}$ $r_2 = 1.2$ fm	$V_{3,x}$ $r_3 = 1.8$ fm	$V_{4,x}$ $r_4 = 2.4$ fm	$V_{5,x}$ $r_5 = 3.0$ fm
$V_C[pp]_{\rm EM}$	0.0073308	0.0063680	0.0033378	0.0036882	0.0009250
c	0.195(3)	−0.059(2)	−0.0125(4)	−0.0059(2)	−0.00215(7)
τ	−0.085(3)	−0.021(1)	−0.0035(2)	0.0025(1)	−0.00072(2)
σ	−0.045(2)	0.0029(8)	0.0120(3)	−0.00405(9)	0.00122(3)
$\sigma\tau$	−0.0649(9)	0.0563(6)	0.0033(1)	0.00292(7)	0.00041(1)
t	0.0	−0.026(2)	0.0067(5)	−0.0026(2)	0.00047(8)
$t\tau$	0.0	0.0591(6)	0.0168(4)	0.0083(2)	0.00112(7)
ls	0.0	−0.198(2)	0.0169(4)	−0.0053(2)	0.00080(5)
$ls\tau$	0.0	−0.0837(8)	0.0112(3)	−0.0029(1)	0.00041(4)
$l2$	−0.0325(5)	0.075(2)	−0.0099(3)	0.0030(1)	−0.00046(3)
$l2\tau$	0.0141(5)	0.0156(7)	−0.0041(2)	0.00084(6)	−0.00007(1)
$l2\sigma$	0.0075(4)	0.0207(7)	−0.0032(1)	0.00094(5)	−0.00020(1)
$l2\sigma\tau$	0.0108(2)	−0.0009(3)	−0.00091(8)	0.00034(3)	−0.000039(6)
$ls2$	0.0	−0.066(3)	0.0134(4)	−0.0037(2)	0.00068(5)
$ls2\tau$	0.0	−0.007(1)	0.0047(3)	−0.00134(9)	0.00013(3)
T	0.0022(9)	0.0007(3)	−0.0005(1)	0.0	0.00005(2)
σT^*	−0.0022(9)	−0.0007(3)	0.0005(1)	0.0	−0.00005(2)
tT^*	0.0	0.0	0.0	0.0	0.0
τz^*	0.0	0.0	0.0	0.0	0.0
$\sigma\tau z^*$	0.0	0.0	0.0	0.0	0.0
$l2T^*$	−0.0004(2)	−0.00012(4)	0.00009(2)	0.0	−0.000008(3)
$l2\sigma T^*$	0.0004(2)	0.00012(4)	−0.00009(2)	0.0	0.000008(3)

TABLE VII. *pp* isovector phase shifts.

$E_{\rm LAB}$	$^1S_0$	$^1D_2$	$^1G_4$	$^3P_0$	$^3P_1$	$^3F_3$	$^3P_2$	ε_2	$^3F_2$	$^3F_4$	ε_4	$^3H_4$
1	32.651	0.001	0.000	0.133	−0.080	−0.000	0.014	−0.001	0.000	0.000	−0.000	0.000
	±0.003	±0.000	±0.000	±0.000	±0.000	±0.000	±0.000	±0.000	±0.000	±0.000	±0.000	±0.000
5	54.841	0.042	0.000	1.582	−0.892	−0.004	0.213	−0.052	0.002	0.000	−0.000	0.000
	±0.006	±0.000	±0.000	±0.003	±0.001	±0.000	±0.001	±0.000	±0.000	±0.000	±0.000	±0.000
10	55.256	0.164	0.003	3.737	−2.037	−0.031	0.648	−0.201	0.013	0.001	−0.004	0.000
	±0.010	±0.000	±0.000	±0.008	±0.002	±0.000	±0.002	±0.000	±0.000	±0.000	±0.000	±0.000
25	48.789	0.694	0.040	8.607	−4.862	−0.230	2.489	−0.811	0.105	0.019	−0.049	0.004
	±0.013	±0.001	±0.000	±0.020	±0.006	±0.000	±0.005	±0.001	±0.000	±0.000	±0.000	±0.000
50	39.185	1.709	0.152	11.524	−8.208	−0.682	5.854	−1.708	0.341	0.102	−0.197	0.026
	±0.017	±0.004	±0.000	±0.032	±0.012	±0.002	±0.009	±0.004	±0.001	±0.001	±0.000	±0.000
100	25.444	3.781	0.418	9.497	−13.259	−1.473	10.980	−2.643	0.832	0.461	−0.554	0.110
	±0.033	±0.008	±0.002	±0.056	±0.019	±0.009	±0.018	±0.008	±0.008	±0.005	±0.001	±0.000
150	15.254	5.618	0.703	4.816	−17.620	−2.061	14.008	−2.913	1.192	1.013	−0.877	0.220
	±0.046	±0.014	±0.006	±0.066	±0.025	±0.019	±0.019	±0.012	±0.017	±0.009	±0.003	±0.002
200	7.041	7.192	1.006	−0.227	−21.506	−2.505	15.784	−2.901	1.327	1.634	−1.129	0.341
	±0.057	±0.021	±0.012	±0.062	±0.036	±0.033	±0.025	±0.016	±0.022	±0.016	±0.005	±0.006
250	0.318	8.538	1.307	−5.121	−24.903	−2.718	16.715	−2.703	1.223	2.200	−1.308	0.464
	±0.074	±0.024	±0.017	±0.063	±0.051	±0.046	±0.032	±0.022	±0.027	±0.022	±0.006	±0.012
300	−5.098	9.577	1.586	−9.663	−27.811	−2.547	16.951	−2.312	0.921	2.644	−1.434	0.572
	±0.100	±0.031	±0.019	±0.089	±0.064	±0.054	±0.033	±0.030	±0.039	±0.027	±0.007	±0.019
350	−9.342	10.183	1.842	−13.677	−30.221	−1.922	16.600	−1.730	0.461	2.970	−1.541	0.650
	±0.134	±0.052	±0.026	±0.141	±0.076	±0.068	±0.030	±0.039	±0.055	±0.042	±0.010	±0.026

TABLE VIII. *np* isovector phase shifts.

$E_{\rm LAB}$	$^1S_0$	$^1D_2$	$^1G_4$	$^3P_0$	$^3P_1$	$^3F_3$	$^3P_2$	ε_2	$^3F_2$	$^3F_4$	ε_4	$^3H_4$
1	62.077	0.001	0.000	0.181	−0.107	−0.000	0.022	−0.001	0.000	0.000	−0.000	0.000
	±0.018	±0.000	±0.000	±0.000	±0.000	±0.000	±0.000	±0.000	±0.000	±0.000	±0.000	±0.000
5	63.660	0.041	0.000	1.657	−0.933	−0.004	0.258	−0.048	0.002	0.000	−0.000	0.000
	±0.044	±0.000	±0.000	±0.003	±0.001	±0.000	±0.001	±0.000	±0.000	±0.000	±0.000	±0.000
10	60.020	0.155	0.002	3.754	−2.057	−0.026	0.727	−0.184	0.011	0.001	−0.003	0.000
	±0.063	±0.000	±0.000	±0.008	±0.002	±0.000	±0.002	±0.000	±0.000	±0.000	±0.000	±0.000
25	51.071	0.674	0.032	8.479	−4.877	−0.198	2.627	−0.763	0.091	0.016	−0.039	0.003
	±0.099	±0.001	±0.000	±0.021	±0.006	±0.000	±0.005	±0.001	±0.000	±0.000	±0.000	±0.000
50	40.885	1.702	0.132	11.300	−8.318	−0.611	6.011	−1.661	0.308	0.092	−0.170	0.021
	±0.147	±0.004	±0.001	±0.033	±0.013	±0.002	±0.009	±0.004	±0.002	±0.001	±0.000	±0.000
100	27.210	3.775	0.370	9.140	−13.571	−1.381	11.062	−2.665	0.775	0.437	−0.506	0.093
	±0.230	±0.008	±0.009	±0.057	±0.019	±0.009	±0.018	±0.008	±0.009	±0.005	±0.001	±0.000
150	17.100	5.571	0.606	4.336	−18.034	−2.004	14.004	−2.982	1.110	0.971	−0.829	0.194
	±0.266	±0.014	±0.030	±0.066	±0.025	±0.020	±0.020	±0.012	±0.017	±0.010	±0.003	±0.002
200	8.830	7.106	0.849	−0.796	−21.958	−2.509	15.697	−2.971	1.212	1.563	−1.095	0.309
	±0.267	±0.022	±0.059	±0.062	±0.037	±0.033	±0.026	±0.017	±0.022	±0.016	±0.005	±0.006
250	1.971	8.424	1.119	−5.749	−25.375	−2.781	16.542	−2.748	1.074	2.096	−1.295	0.423
	±0.281	±0.024	±0.081	±0.064	±0.051	±0.046	±0.032	±0.022	±0.028	±0.022	±0.006	±0.012
300	−3.618	9.432	1.438	−10.325	−28.308	−2.649	16.697	−2.328	0.744	2.513	−1.444	0.518
	±0.344	±0.032	±0.085	±0.091	±0.065	±0.054	±0.033	±0.030	±0.040	±0.027	±0.007	±0.019
350	−8.044	9.994	1.829	−14.349	−30.747	−2.040	16.280	−1.727	0.260	2.822	−1.571	0.578
	±0.453	±0.054	±0.083	±0.145	±0.076	±0.069	±0.030	±0.040	±0.056	±0.043	±0.010	±0.026

the rest of the parameters are charge independent. A comprehensive and ready-to-use review of the necessary theory to compute the full on-shell electromagnetic scattering amplitude was made. This includes every single EM contribution, as well as the energy dependent parameter α' and, as already found out in the previous high quality analysis, turns out to be crucial for the correct description of pp and np scattering observables. The 3σ criterion was used iteratively to obtain a fully consistent database with $N_{\rm data} = 6713$ including normalization data. The complete data selection process described here allowed us

TABLE IX. *np* isoscalar phase shifts.

$E_{\rm LAB}$	$^1P_1$	$^1F_3$	$^3D_2$	$^3G_4$	$^3S_1$	ε_1	$^3D_1$	$^3D_3$	ε_3	$^3G_3$
1	−0.186	−0.000	0.006	0.000	147.647	0.104	−0.005	0.000	0.000	−0.000
	±0.000	±0.000	±0.000	±0.000	±0.010	±0.001	±0.000	±0.000	±0.000	±0.000
5	−1.494	−0.010	0.219	0.001	117.954	0.657	−0.180	0.002	0.012	−0.000
	±0.004	±0.000	±0.000	±0.000	±0.022	±0.004	±0.000	±0.000	±0.000	±0.000
10	−3.062	−0.064	0.847	0.012	102.292	1.125	−0.671	0.005	0.080	−0.003
	±0.010	±0.000	±0.001	±0.000	±0.031	±0.009	±0.002	±0.000	±0.000	±0.000
25	−6.357	−0.421	3.729	0.170	80.136	1.734	−2.788	0.043	0.553	−0.053
	±0.033	±0.000	±0.008	±0.000	±0.044	±0.020	±0.009	±0.001	±0.000	±0.000
50	−9.663	−1.142	9.057	0.727	62.160	2.057	−6.423	0.315	1.615	−0.265
	±0.069	±0.003	±0.027	±0.001	±0.053	±0.037	±0.026	±0.004	±0.003	±0.001
100	−14.207	−2.304	17.387	2.248	42.712	2.473	−12.198	1.444	3.498	−0.996
	±0.106	±0.022	±0.049	±0.010	±0.053	±0.068	±0.048	±0.016	±0.016	±0.009
150	−17.977	−3.178	21.962	3.892	30.392	2.938	−16.407	2.643	4.825	−1.919
	±0.119	±0.055	±0.066	±0.033	±0.058	±0.095	±0.058	±0.033	±0.028	±0.030
200	−21.235	−3.953	24.017	5.484	20.936	3.448	−19.682	3.485	5.749	−2.879
	±0.144	±0.086	±0.085	±0.057	±0.072	±0.112	±0.071	±0.051	±0.034	±0.053
250	−24.030	−4.672	24.846	6.885	12.945	4.036	−22.324	3.931	6.427	−3.799
	±0.185	±0.102	±0.096	±0.070	±0.087	±0.111	±0.080	±0.071	±0.043	±0.069
300	−26.418	−5.283	25.324	7.986	5.837	4.753	−24.369	4.099	6.938	−4.669
	±0.226	±0.104	±0.114	±0.078	±0.098	±0.113	±0.108	±0.095	±0.053	±0.084
350	−28.418	−5.704	25.950	8.729	−0.644	5.610	−25.699	4.121	7.294	−5.517
	±0.261	±0.125	±0.189	±0.104	±0.105	±0.169	±0.191	±0.121	±0.064	±0.118

to recover a total of 300 data that a single application of the 3σ criterion would instead have rejected. The fitting parameters, their uncertainties, and correlations were determined from the final and mutually consistent database. Thus, errors in the potential can be propagated to any other quantities obtained using the δ-shell representation. Phase shifts were extracted from the data and are presented here with the corresponding statistical error bars. With a consistent database it is also possible to test the statistical significance of the inclusion of other potentials where the intermediate-range contributions are explicitly modelled.

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APPENDIX A: TRANSFORMATIONS BETWEEN PARTIAL WAVE AND OPERATOR BASIS

As mentioned in the main text we use an operator basis extending the AV18 potentials in coordinate space. We use the following definitions:

$$S_{12} = 3\sigma_1 \cdot \hat{\mathbf{r}}\sigma_2 \cdot \hat{\mathbf{r}} - \sigma_1 \cdot \sigma_2, \tag{A1}$$

$$T_{12} = 3\tau_{z1}\tau_{z2} - \tau_1 \cdot \tau_2. \tag{A2}$$

Thus, the total potential in the operator basis reads

$$V = \sum_{n=1,21} V_n(r) O^n. \tag{A3}$$

Here, the first 14 operators are the same charge-independent ones used in the Argonne v_{14} potential and are given by

$$\begin{aligned} O^{n=1,14} = {} & 1,\ \tau_1\cdot\tau_2,\ \sigma_1\cdot\sigma_2,\ (\sigma_1\cdot\sigma_2)(\tau_1\cdot\tau_2),\ S_{12},\ S_{12}(\tau_1\cdot\tau_2), \\ & \mathbf{L}\cdot\mathbf{S},\ \mathbf{L}\cdot\mathbf{S}(\tau_1\cdot\tau_2),\ L^2,\ L^2(\tau_1\cdot\tau_2),\ L^2(\sigma_1\cdot\sigma_2), \\ & L^2(\sigma_1\cdot\sigma_2)(\tau_1\cdot\tau_2),\ (\mathbf{L}\cdot\mathbf{S})^2,\ (\mathbf{L}\cdot\mathbf{S})^2(\tau_1\cdot\tau_2). \end{aligned} \tag{A4}$$

These 14 components are denoted by the abbreviations c, τ, σ, $\sigma\tau$, t, $t\tau$, ls, $ls\tau$, $l2$, $l2\tau$, $l2\sigma$, $l2\sigma\tau$, $ls2$, and $ls2\tau$. The remaining terms are

$$\begin{aligned} O^{n=15,21} = {} & T_{12},\ (\sigma_1\cdot\sigma_2)T_{12}\ ,\ S_{12}T_{12},\ (\tau_{z1}+\tau_{z2}), \\ & (\sigma_1\cdot\sigma_2)(\tau_{z1}+\tau_{z2})\ ,\ L^2T_{12},\ L^2(\sigma_1\cdot\sigma_2)T_{12}. \end{aligned} \tag{A5}$$

These terms are charge dependent and are labeled as T, σT, tT, τz, $\sigma\tau z$, $l2T$, and $l2\sigma T$.

Since we use the low angular momentum partial-wave strength coefficients as fitting parameters we need a transformation to construct the complete potential and extract the high angular momentum partial-wave parameters from the latter. The high angular momentum partial waves up to $J = 20$ are important to calculate the strong on-shell scattering amplitude. The transformation from the operator basis to partial waves is direct and is given by

$$\begin{aligned} (\lambda_i)^{J,S}_{l,l'} = {} & \mu_{\alpha\beta}\Delta r[1-(-1)^{l+S+T}]\Big[\delta_{l,l'}\Big(V_{i,c}+(4T-3)V_{i,\tau}+(4S-3)V_{i,\sigma}+(4S-3)(4T-3)V_{i,\sigma\tau} \\ & +l(l+1)[V_{i,l2}+(4T-3)V_{i,l2\tau}+(4S-3)V_{i,l2\sigma}+(4S-3)(4T-3)V_{i,l2\sigma\tau}] \\ & +\frac{1}{2}[J(J+1)-l(l+1)-S(S+1)][V_{i,ls}+(4T-3)V_{i,ls\tau}] \\ & +\frac{1}{4}[J(J+1)-l(l+1)-S(S+1)]^2[V_{i,ls2}+(4T-3)V_{i,ls2\tau}] \\ & +\delta_{1,T}[3\tau_{zi}\tau_{zj}-(4T-3)][V_{i,T}+(4S-3)V_{i,\sigma T}+l(l+1)(V_{i,l2T}+(4S-3)V_{i,l2\sigma T})]\Big) \\ & +\delta_{1,S}\Big(2\delta_{J,l}\delta_{J,l'}-\frac{2(J-1)\delta_{J-1,l}\delta_{J-1,l'}}{2J+1}-\frac{2(J+2)\delta_{J+1,l}\delta_{J+1,l'}}{2J+1}+\frac{6\sqrt{J(J+1)}(\delta_{J+1,l}\delta_{J-1,l'}+\delta_{J-1,l}\delta_{J+1,l'})}{2J+1}\Big) \\ & \times[V_{i,t}+(4T-3)V_{i,t\tau}]\Big], \end{aligned} \tag{A6}$$

where the tT, τz, and $\sigma\tau z$ are not included since in our analysis the contribution of these operators is set to zero.

Evaluation of Eq. (A6) at the ^1S_0np, ^1S_0pp, $^3P_0$, $^1P_1$, $^3P_1$, $^3S_1$, ε_1, $^3D_1$, $^1D_2$, $^3D_2$, $^3P_2$, ε_2, $^3F_2$, $^1F_3$, $^3D_3$ results in a set of 15 equations. By imposing that the only charge dependent parameters are on the $^1S_0$ partial wave the condition $V_{i,T} = -V_{i,\sigma T} = -6V_{i,l2T} = 6V_{i,l2\sigma T}$ must be fulfilled and the number of independent variables is reduced to 15. The solution of this system of equations is given by

$$V_{i,O} = \sum_{JS,l\leqslant l'} \frac{1}{M_p\Delta r} c_{O,JSll'}(\lambda_i)^{JS}_{ll'}, \tag{A7}$$

where O runs over the labels of the operator basis. The numerical values of the $c_{O,JSll'}$ coefficients are displayed in Table X where $JSll'$ are expressed in the spectroscopic

TABLE X. Transformation coefficients $c_{O,JSll'}$ between the 15 low angular momentum partial waves used to parametrize the δ-shell potential and the 15 independent operators of the complete potential.

Operator	^1S_0np	^1S_0pp	$^3P_0$	$^1P_1$	$^3P_1$	$^3S_1$	ε_1	$^3D_1$	$^1D_2$	$^3D_2$	$^3P_2$	ε_2	$^3F_2$	$^1F_3$	$^3D_3$
c	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{40}$	0	$\frac{3}{16}$	0	0	0	0	$\frac{3}{10}$	$\frac{\sqrt{6}}{5}$	$-\frac{9}{80}$	$-\frac{1}{80}$	0
τ	$\frac{1}{48}$	$\frac{1}{24}$	$\frac{1}{8}$	$-\frac{3}{40}$	0	$-\frac{3}{16}$	0	0	0	0	$\frac{1}{10}$	$\frac{\sqrt{6}}{15}$	$-\frac{3}{80}$	$\frac{1}{80}$	0
σ	$-\frac{1}{16}$	$-\frac{1}{8}$	$\frac{1}{8}$	$-\frac{3}{40}$	0	$\frac{1}{16}$	0	0	0	0	$\frac{1}{10}$	$\frac{\sqrt{6}}{15}$	$-\frac{3}{80}$	$\frac{1}{80}$	0
$\sigma\tau$	$-\frac{1}{48}$	$-\frac{1}{24}$	$\frac{1}{24}$	$\frac{3}{40}$	0	$-\frac{1}{16}$	0	0	0	0	$\frac{1}{30}$	$\frac{\sqrt{6}}{45}$	$-\frac{1}{80}$	$-\frac{1}{80}$	0
t	0	0	0	0	0	0	$\frac{\sqrt{2}}{16}$	0	0	0	0	$\frac{5\sqrt{6}}{48}$	0	0	0
$t\tau$	0	0	0	0	0	0	$-\frac{\sqrt{2}}{16}$	0	0	0	0	$\frac{5\sqrt{6}}{144}$	0	0	0
ls	0	0	0	0	$-\frac{3}{8}$	0	$\frac{\sqrt{2}}{56}$	$-\frac{1}{40}$	0	$-\frac{1}{24}$	$\frac{3}{8}$	$\frac{\sqrt{6}}{8}$	0	0	$\frac{1}{15}$
$ls\tau$	0	0	0	0	$-\frac{1}{8}$	0	$-\frac{\sqrt{2}}{56}$	$\frac{1}{40}$	0	$\frac{1}{24}$	$\frac{1}{8}$	$\frac{\sqrt{6}}{24}$	0	0	$-\frac{1}{15}$
$l2$	$-\frac{1}{96}$	$-\frac{1}{48}$	$-\frac{9}{32}$	$-\frac{1}{160}$	$\frac{9}{32}$	$-\frac{1}{32}$	$-\frac{\sqrt{2}}{56}$	$-\frac{1}{160}$	$\frac{1}{32}$	$\frac{1}{32}$	$-\frac{9}{160}$	$-\frac{9\sqrt{6}}{40}$	$\frac{9}{160}$	$\frac{1}{160}$	$\frac{1}{160}$
$l2\sigma$	$\frac{1}{96}$	$\frac{1}{48}$	$-\frac{3}{32}$	$\frac{1}{160}$	$\frac{3}{32}$	$-\frac{1}{96}$	$-\frac{\sqrt{2}}{168}$	$-\frac{1}{480}$	$-\frac{1}{32}$	$\frac{1}{96}$	$-\frac{3}{160}$	$-\frac{3\sqrt{6}}{40}$	$\frac{3}{160}$	$-\frac{1}{160}$	$\frac{1}{480}$
$l2\tau$	$-\frac{1}{288}$	$-\frac{1}{144}$	$-\frac{3}{32}$	$\frac{1}{160}$	$\frac{3}{32}$	$\frac{1}{32}$	$\frac{\sqrt{2}}{56}$	$\frac{1}{160}$	$\frac{1}{96}$	$-\frac{1}{32}$	$-\frac{3}{160}$	$-\frac{3\sqrt{6}}{40}$	$\frac{3}{160}$	$-\frac{1}{160}$	$-\frac{1}{160}$
$l2\sigma\tau$	$\frac{1}{288}$	$\frac{1}{144}$	$-\frac{1}{32}$	$-\frac{1}{160}$	$\frac{1}{32}$	$\frac{1}{96}$	$\frac{\sqrt{2}}{168}$	$\frac{1}{480}$	$-\frac{1}{96}$	$-\frac{1}{96}$	$-\frac{1}{160}$	$-\frac{\sqrt{6}}{40}$	$\frac{1}{160}$	$\frac{1}{160}$	$-\frac{1}{480}$
$ls2$	0	0	$\frac{1}{4}$	0	$-\frac{3}{8}$	0	$\frac{\sqrt{2}}{28}$	$\frac{1}{40}$	0	$-\frac{1}{24}$	$\frac{1}{8}$	$\frac{\sqrt{6}}{4}$	0	0	$\frac{1}{60}$
$ls2\tau$	0	0	$\frac{1}{12}$	0	$-\frac{1}{8}$	0	$-\frac{\sqrt{2}}{28}$	$-\frac{1}{40}$	0	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{\sqrt{6}}{12}$	0	0	$-\frac{1}{60}$
T	$-\frac{1}{24}$	$\frac{1}{24}$	0	0	0	0	0	0	0	0	0	0	0	0	0

notation $^{2S+1}l_J$ with $l = 0, 1, 2, 3, 4, 5, \ldots$ conventionally named S, P, D, F, G, H, ... for the diagonal matrix elements. The nondiagonal terms corresponding total angular momentum J are labeled as ε_J.

APPENDIX B: CALCULATION OF PHASE SHIFTS FROM δ-SHELL POTENTIAL

1. Uncoupled partial waves

The two-body scattering amplitude M can be constructed directly using the phase shift for every partial wave. To calculate such phase shifts it is necessary to integrate the corresponding Schrödinger equation. In the case of uncoupled partial waves ($l = l' = J$) the potential is central and the reduced equation reads

$$-\frac{d^2}{dr^2}u_l(k,r) + \left[U_{l,l}^{l,S}(r) + \frac{l(l+1)}{r^2}\right]u_l(k,r) = k^2 u_l(k,r), \tag{B1}$$

where $U_{l,l}^{J,S}(r) = 2\mu_{\alpha,\beta}V_{l,l}^{J,S}(r)$ is the reduced potential. To solve this differential equation we use the fact that inside the region $r_i < r < r_{i+1}$ the δ-shell potential is zero and $u_l(k,r) = u_{l,i}(k,r)$, with $u_{l,i}$ the solution of the free particle equation

$$-\frac{d^2}{dr^2}u_{l,i}(k,r) + \frac{l(l+1)}{r^2}u_{l,i}(k,r) = k^2 u_{l,i}(k,r). \tag{B2}$$

Therefore in the neighborhood of r_i we can take a solution of the type

$$u_l(k,r) = [1-\theta(r-r_i)]u_{l,i-1}(k,r) + \theta(r-r_i)u_{l,i}(k,r). \tag{B3}$$

Substituting this solution in Eq. (B1) and considering the linear independence of $\delta(r-r_i)$ and $\delta'(r-r_i)$, two conditions are found to hold at every concentration radius:

$$\begin{aligned} u_{l,i}(k,r_i) &= u_{l,i-1}(k,r_i) \equiv u(k,r_i), \\ \left[\frac{d}{dr}u_{l,i}(k,r) - \frac{d}{dr}u_{l,i-1}(k,r)\right]_{r=r_i} &= \lambda_i u(k,r_i), \end{aligned} \tag{B4}$$

where the l, S, J labels of the λ_i coefficients have been omitted. The first equation implies the continuity of the wave function, while the second expresses the relation between the strength coefficients λ_i and the change in the log derivative of wave function at the interaction points r_i.

Since $u_{l,i}(k,r)$ is the solution of the free particle equation, it can be written as a linear combination of the reduced spherical Bessel functions,

$$\hat{j}_l(x) = x j_l(x), \quad \hat{y}_l(x) = x y_l(x). \tag{B5}$$

For convenience we choose a linear combination with the form

$$u_{l,i}(k,r) = A_i[\hat{j}_l(kr) - \tan\delta_i(k)\hat{y}_l(kr)], \tag{B6}$$

where A_i and δ_i are constants to be determined. This representation allows us to take a wave function with the asymptotic form

$$u_{l,N}(k,r) = \hat{j}_l(kr) - \tan\delta_N \hat{y}_l(kr) \tag{B7}$$

for distances greater than the last concentration radius r_N. In this case δ_N is the phase shift resulting from the scattering process. The condition of regularity at the origin is met with

$$u_{l,0}(k,r) = A_0\hat{j}_l(kr), \tag{B8}$$

which imposes the condition $\tan\delta_0 = 0$.

Now, only an equation for expressing $\tan\delta_i(k)$ in terms of $\tan\delta_{i-1}(k)$ is needed to calculate $\delta_N(k)$. For simplicity we define

$$\varphi_{l,i}(k,r) = \hat{j}_l(kr) - \tan\delta_i(k)\hat{y}_l(kr), \tag{B9}$$

which allows us to write

$$u_l(k,r_i) = A_i\varphi_{l,i}(k,r_i) = A_{i-1}\varphi_{l,i-1}(k,r_i). \quad \text{(B10)}$$

Multiplying the second equation in (B4) by $u_l(k,r)$ and using (B10) conveniently the term $A_{i-1}A_i$ appears on both sides of the equation and we get

$$\varphi_{l,i-1}(k,r_i)\left.\frac{d}{dr}\varphi_{l,i}(k,r)\right|_{r=r_i} - \varphi_{l,i}(k,r_i)\left.\frac{d}{dr}\varphi_{l,i-1}(k,r)\right|_{r=r_i} = \lambda_i\varphi_{l,i-1}(k,r_i)\varphi_{l,i}(k,r_i). \quad \text{(B11)}$$

Inserting the definition (B9) and considering the Wronskian relation

$$\hat{y}_l(kr)\frac{d}{dr}\hat{j}_l(kr) - \hat{j}_l(kr)\frac{d}{dr}\hat{y}_l(kr) = k \quad \text{(B12)}$$

we get

$$\tan\delta_i(k) - \tan\delta_{i-1}(k) = -\frac{\lambda_i}{k}\varphi_{l,i-1}(k,r_i)\varphi_{l,i}(k,r_i). \quad \text{(B13)}$$

This expression can be considered a discrete version of the variable phase equation, since in the limit of a continuous interaction the form of the latter is recovered. Finally, solving for $\tan\delta_i(k)$ we get

$$\tan\delta_i(k) = \frac{\tan\delta_{i-1}(k) - \frac{\lambda_i}{k}\hat{j}_l(kr_i)\varphi_{l,i-1}(k,r_i)}{1 - \frac{\lambda_i}{k}\hat{y}_l(kr_i)\varphi_{l,i-1}(k,r_i)} \quad \text{(B14)}$$

and using $\tan\delta_0 = 0$ as a boundary condition we are able to calculate δ_N.

For completeness we show how to get the entire solution to (B1). Equation (B7) implies the condition $A_N = 1$; this allows us to calculate all the A_i constants using Eq. (B10) as

$$A_{i-1} = A_i\varphi_{l,i}(k,r_i)\varphi^{-1}_{l,i-1}(k,r_i). \quad \text{(B15)}$$

2. Coupled partial waves

The tensor part of the NN interaction couples the triplet partial waves with $l = l' = J \pm 1$. In this case a set of two differential equations must be solved simultaneously to extract the corresponding phase shifts. The coupled reduced Schrödinger equation reads

$$\begin{aligned} &-\frac{d^2}{dr^2}v(k,r) + \left[U^{J,1}_{J-1,J-1}(r) + \frac{(J-1)J}{r^2}\right]v(k,r) \\ &\quad + U^{J,1}_{J-1,J+1}(r)w(k,r) = k^2 v(k,r), \\ &-\frac{d^2}{dr^2}w(k,r) + \left[U^{J,1}_{J+1,J+1}(r) + \frac{(J+1)(J+2)}{r^2}\right]w(k,r) \\ &\quad + U^{J,1}_{J+1,J-1}(r)v(k,r) = k^2 w(k,r). \end{aligned} \quad \text{(B16)}$$

This equation has two linearly independent solutions that we can label α and β, and its asymptotic behavior can be written as

$$\begin{aligned} v^{(\alpha)}(k,r) &\to \cot\delta^1_{J-1}\hat{j}_{J-1}(kr) - \hat{y}_{J-1}(kr), \\ w^{(\alpha)}(k,r) &\to \tan\varepsilon_J\left[\cot\delta^1_{J-1}\hat{j}_{J+1}(kr) - \hat{y}_{J+1}(kr)\right], \\ v^{(\beta)}(k,r) &\to -\tan\varepsilon_J\left[\cot\delta^1_{J+1}\hat{j}_{J-1}(kr) - \hat{y}_{J-1}(kr)\right], \\ w^{(\beta)}(k,r) &\to \cot\delta^1_{J+1}\hat{j}_{J+1}(kr) - \hat{y}_{J+1}(kr), \end{aligned} \quad \text{(B17)}$$

where δ_{J-1}, δ_{J+1} and ε_J are the phase shifts in the BB or eigenphase parametrization [268]. For a given value of J we use the following notation for the δ-shell reduced potential matrix elements:

$$\begin{aligned} U^{J,1}_{J-1,J-1} &= \sum_{i=1}^{N}\lambda_i^{J-1}\delta(r-r_i), \\ U^{J,1}_{J+1,J+1} &= \sum_{i=1}^{N}\lambda_i^{J+1}\delta(r-r_i), \\ U^{J,1}_{J-1,J+1} = U^{J,1}_{J+1,J-1} &= \sum_{i=1}^{N}\tilde{\lambda}_i\delta(r-r_i). \end{aligned} \quad \text{(B18)}$$

Similarly as we did for the central potential case, we consider the solution inside the interval $r_i < r < r_{i+1}$ to be the solution of the free particle equations

$$\begin{aligned} -\frac{d^2}{dr^2}v_i(k,r) + \frac{(J-1)J}{r^2}v_i(k,r) &= k^2 v_i(k,r), \\ -\frac{d^2}{dr^2}w_i(k,r) + \frac{(J+1)(J+2)}{r^2}w_i(k,r) &= k^2 w_i(k,r). \end{aligned} \quad \text{(B19)}$$

Again, we construct the solution in the neighborhood of r_i as

$$\begin{aligned} v(k,r) &= [1-\theta(r-r_i)]v_{i-1}(k,r) + \theta(r-r_i)v_i(k,r), \\ w(k,r) &= [1-\theta(r-r_i)]w_{i-1}(k,r) + \theta(r-r_i)w_i(k,r). \end{aligned} \quad \text{(B20)}$$

Substituting these solutions in Eq. (B16) with the reduced δ-shell potential it is possible to get boundary conditions similar to the ones in (B4),

$$\begin{aligned} v_i(k,r_i) = v_{i-1}(k,r_i) &\equiv v(k,r_i), \\ w_i(k,r_i) = w_{i-1}(k,r_i) &\equiv w(k,r_i), \\ \left[\frac{d}{dr}v_i(k,r) - \frac{d}{dr}v_{i-1}(k,r)\right]_{r=r_i} &= \lambda_i^{J-1}v(k,r_i) + \tilde{\lambda}_i w(k,r_i), \\ \left[\frac{d}{dr}w_i(k,r) - \frac{d}{dr}w_{i-1}(k,r)\right]_{r=r_i} &= \lambda_i^{J+1}w(k,r_i) + \tilde{\lambda}_i v(k,r_i). \end{aligned} \quad \text{(B21)}$$

To integrate the coupled Schrödinger equation we consider two linearly independent solutions (v_1, w_1) and (v_2, w_2) that in the region $0 \leqslant r < r_1$ are

$$\begin{aligned} v_{1,0}(k,r) &= \hat{j}_{J-1}(kr), \quad w_{1,0}(k,r) = 0, \\ v_{2,0}(k,r) &= 0, \quad w_{2,0}(k,r) = \hat{j}_{J+1}(kr). \end{aligned} \quad \text{(B22)}$$

Some explanation should be given to the indices: the first one is used to differentiate between the two linearly independent solutions while the second indicates a free particle solution inside the interval $r_i < r < r_{i+1}$. For consistency in notation

we consider $r_0 = 0$ but it should be noted that no concentration radius at $r = 0$ is present in the δ-shell potentials of Eq. (B18).

Also it should be pointed out that the differential equations in (B19) are uncoupled and therefore the solutions can be expressed again as a linear combination of the reduced spherical Bessel functions. With this in mind we write

$$\begin{aligned} v_{1,i}(k,r) &= A_{1,i}\hat{j}_{J-1}(kr) + B_{1,i}\hat{y}_{J-1}(kr), \\ w_{1,i}(k,r) &= C_{1,i}\hat{j}_{J+1}(kr) + D_{1,i}\hat{y}_{J+1}(kr), \\ v_{2,i}(k,r) &= A_{2,i}\hat{j}_{J-1}(kr) + B_{2,i}\hat{y}_{J-1}(kr), \\ w_{2,i}(k,r) &= C_{2,i}\hat{j}_{J+1}(kr) + D_{2,i}\hat{y}_{J+1}(kr). \end{aligned} \tag{B23}$$

The α and β asymptotic wave functions can be formed as linear combinations of both solutions for distances greater than the last concentration radius r_N, i.e.,

$$\begin{aligned} v^{\alpha}(k,r) &= \alpha_1 v_{1,N}(k,r) + \alpha_2 v_{2,N}(k,r), \\ w^{\alpha}(k,r) &= \alpha_1 w_{1,N}(k,r) + \alpha_2 w_{2,N}(k,r), \\ v^{\beta}(k,r) &= \beta_1 v_{1,N}(k,r) + \beta_2 v_{2,N}(k,r), \\ w^{\beta}(k,r) &= \beta_1 w_{1,N}(k,r) + \beta_2 w_{2,N}(k,r). \end{aligned} \tag{B24}$$

Matching these linear combinations in the form of Eq. (B23) with the asymptotic solutions of Eq. (B17) it is possible to get the expressions

$$\begin{aligned} \frac{A_{1,N} + \alpha A_{2_N}}{B_{1,N} + \alpha B_{2,N}} &= \frac{C_{1,N} + \alpha C_{2,N}}{D_{1,N} + \alpha D_{2,N}} = \cot\delta^1_{J-1(J+1)}, \\ \frac{D_{1,N} + \alpha D_{2,N}}{B_{1,N} + \alpha B_{2,N}} &= \tan\varepsilon_J, \end{aligned} \tag{B25}$$

where we have defined $\alpha \equiv \alpha_2/\alpha_1$. The first equation has two solutions on α: one corresponds to the α-state eigenphase shift and the other to the β one. The second equation can be used unambiguously to obtain the mixing angle ε_J.

Matching the equations of Eq. (B22) with the ones in (B23) we can get

$$\begin{aligned} A_{1,0} &= 1, \quad & B_{1,0} &= 0, \\ C_{1,0} &= 0, \quad & D_{1,0} &= 0, \\ A_{2,0} &= 0, \quad & B_{2,0} &= 0, \\ C_{2,0} &= 1, \quad & D_{2,0} &= 0. \end{aligned} \tag{B26}$$

Like in the previous subsection, we need an expression for the $v_{1(2),i}$, $w_{1(2),i}$ wave functions in terms of the $v_{1(2),i-1}$, $w_{1(2),i-1}$ ones. Combining the boundary conditions for (B21) with the linear combinations of (B23) and using properly the Wronskian relation in Eq. (B12) we were able to get the following expressions for the constants $\{A, B, C, D\}_{1(2),i}$ in terms of the constants $\{A, B, C, D\}_{1(2),i-1}$:

$$\begin{aligned} B_i &= B_{i-1} + \frac{1}{k}\hat{j}_{J-1}(kr_i)\left\{\lambda_i^{J-1}[A_{i-1}\hat{j}_{J-1}(kr_i) + B_{i-1}\hat{y}_{J-1}(kr_i)] + \tilde{\lambda}_i[C_{i-1}\hat{j}_{J+1}(kr_i) + D_{i-1}\hat{y}_{J+1}(kr_i)]\right\}, \\ A_i &= [\hat{j}_{J-1}(kr_i)]^{-1}[A_{i-1}\hat{j}_{J-1}(kr_i) + (B_{i-1} - B_i)\hat{y}_{J-1}(kr_i)], \\ D_i &= D_{i-1} + \frac{1}{k}\hat{j}_{J+1}(kr_i)\left\{\lambda_i^{J+1}[C_{i-1}\hat{j}_{J+1}(kr_i) + D_{i-1}\hat{y}_{J+1}(kr_i)] + \tilde{\lambda}_i[A_{i-1}\hat{j}_{J-1}(kr_i) + B_{i-1}\hat{y}_{J-1}(kr_i)]\right\}, \\ C_i &= [\hat{j}_{J+1}(kr_i)]^{-1}[C_{i-1}\hat{j}_{J+1}(kr_i) + (D_{i-1} - D_i)\hat{y}_{J+1}(kr_i)], \end{aligned} \tag{B27}$$

where the 1 and 2 indices have been suppressed for simplicity.

Finally, the relations in Eq. (B27) can be used with the boundary conditions of (B26) to calculate the constants $\{A, B, C, D\}_{1(2),N}$ and ultimately use them together with (B25) to get the eigenphase shifts for the coupled channels with orbital angular momentum $l = J \pm 1$.

APPENDIX C: NUMERICAL DETAILS

As already noted in our previous works, the δ-shell representation is just a method to solve Schrödinger's equation. Thus it can also be used to integrate the long-range part of the interaction such as OPE. The only difference is that for pp the free particle-wave functions are replaced by both regular and irregular Coulomb wave functions. Some caution is needed handling the boundary $r_i = r_c$ since Coulomb wave functions from the $r > r_c$ region must match the free particle-wave functions for $r < r_c$. In this case the δ-shell coefficients are *fixed* to the long-range potential value, i.e., $V_i \equiv V(r_i)$, and are no longer fitting parameters. Therefore, the accuracy of the integration depends on the number of sampling points for $r > r_c$. According to our discussion in Refs. [10,11,15] the Nyquist optimal sampling theorem suggests keeping $\Delta r = 0.6$ fm throughout. For OPE it is sufficient to take $N \sim 20$. As a final remark, note that the question of accuracy in the unknown region never arises, i.e., there is no point in improving *beyond* the $\Delta r \sim 1/p_{\max}$ resolution for a maximum CM momentum $p_{\max}$ which in our case is given by the pion production threshold.

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