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Ambiguities in the rate of oxygen formation during stellar helium burning in the $^{12}C(\alpha,\gamma)$ reaction

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The rate of oxygen formation determines the C/O ratio during stellar helium burning. It is the single most important nuclear input in stellar evolution theory, including the evolution of type II and type Ia supernova. However, the low-energy cross section of the fusion of $^4He + ^{12}C$, denoted as the $^{12}C(\alpha,\gamma)^{16}O$ reaction, still remains uncertain. I analyze and critically review the most recent measurements of complete angular distributions of the outgoing γ rays at very low energies ($E_{c.m.} \geqslant 1.0$ MeV). My analysis of the angular distributions measured with the EUROGAM/GANDI arrays leads to considerably larger error bars than have been published, which excludes them from the current sample of "world data." I show that the current sample of "world data" of the measured $E2$ cross-section factors below 1.7 MeV cluster into two distinct groups that lead to two distinct extrapolations: $S_{E2}(300) \approx 60$ or $S_{E2}(300) \approx 154$ keVb. There is a discrepancy between the measured $E1$-$E2$ phase difference (ϕ_{12}) and unitarity as required by the Watson theorem, which suggests systematic problem(s) in some of the measured γ-ray angular distributions. The ambiguity of the extrapolated $S_{E2}(300)$ together with the previously observed ambiguity of $S_{E1}(300)$ (approximately 80 or 10 keVb) must be resolved by future measurements of complete and detailed angular distributions of the $^{12}C(\alpha,\gamma)$ reaction at very low energies ($E_{c.m.} \leqslant 1.0$ MeV).

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Stellar helium burning, which follows hydrogen burning, is an important stage in the evolution of stars. During this stage the elements carbon and oxygen are formed, and as such it is one of the most vivid examples of the anthropic principle [1]. During this stage carbon is synthesized by the "triple-α process," but at the same time carbon is also destroyed by fusing with an additional α particle to form ^{16}O in the $^{12}C(\alpha,\gamma)^{16}O$ reaction. Hence the formation of oxygen in stellar helium burning determines the C/O ratio, an essential parameter in stellar evolution theory [1].

The importance of the C/O ratio for the evolution of massive stars ($M > 8M_\odot$) that evolve to core collapse (type II) supernova has been discussed extensively [2]. More recently it was shown that the C/O ratio is also important for understanding the ^{56}Ni mass fraction produced by lower mass stars ($M \approx 1.4M_\odot$) that evolve into type Ia supernova (SNeIa) [3]. Thus the C/O ratio is also important for understanding the light curve of SNeIa. Such SNeIa are used as cosmological "standard candles" with which the accelerated expansion of the universe and dark energy were recently discovered [4].

Stellar evolution theory requires knowledge of the C/O ratio with an uncertainty of 5%. This requires accurate measurements at low energies and extrapolation of the measured astrophysical cross-section factors to the Gamow window at 300 keV [1]. Since mainly two ($\ell = 1$ and $\ell = 2$) partial waves contribute to the reaction, accurate angular distribution data are needed at low energies to determine with high accuracy the astrophysical cross-section factors $S_{E1}(300)$ and $S_{E2}(300)$ defined in Ref. [1].

Recently some of the most impressive γ-ray measurements of the $^{12}C(\alpha,\gamma)^{16}O$ reaction were published [5–9], including measurements of complete angular distribution at center-of-mass (c.m.) energies approaching 1.0 MeV. These measurements employ large luminosities of the order of 10^{35} cm^{-2} s^{-1} with integrated luminosities close to one inverse fb [6–9] and a large (fraction of 4π) array of γ-ray detectors (but some of the arrays employ low-efficiency HpGe detectors, which led in some cases to insufficient counting statistics). Such unprecedented data led researchers to expect a resolution of the debate on the value of the low-energy cross section of the $^{12}C(\alpha,\gamma)$ reaction. While these data did not resolve the outstanding questions, they do provide the first possible detailed study of the cross section of the $^{12}C(\alpha,\gamma)$ reaction at low energies approaching 1.0 MeV.

In this paper I analyze and critically review the new measurements of angular distributions of γ rays from the $^{12}C(\alpha,\gamma)$ reaction [5–9]. I focus on angular distribution data in order to reveal trends in the cross-section factors measured at the current lowest energies. Specifically I study the $E2$ cross-section factors (S_{E2}) measured at energies ($E_{c.m.}$) below 1.7 MeV in order to avoid the energy region where higher lying (1^- and 2^+) states dominate and to be most sensitive to the bound 2^+ state at 6.917 MeV in ^{16}O that governs the $E2$ cross section at stellar burning energies. I show that the "world data" on S_{E2} below 1.7 MeV cluster into two groups that differ by an average factor of 2.6, and consequently these data extrapolate to two distinct solutions of $S_{E2}(300) \approx 60$ or $S_{E2}(300) \approx 154$ keVb. The ambiguity in the value of the extrapolated $S_{E2}(300)$ resembles the previously observed ambiguity in the value of the extrapolated $S_{E1}(300)$ where the small value solution of the $E1$ cross-section factor [$S_{E1}(300) \approx 10$ keVb] cannot be ruled out [10,11]. I point out a disagreement of the measured $E1$-$E2$ relative phase angle (ϕ_{12}) with unitarity, which together with the disagreement on the value of the "cascade cross section" [12] defines the major challenges facing future measurements in this field.

I propose stringent requirements needed in future studies (see, for example, Refs. [13,14]) in order to resolve these ambiguities. The exact values and energy dependence of S_{E2} and S_{E1} are essential for extrapolating the proposed

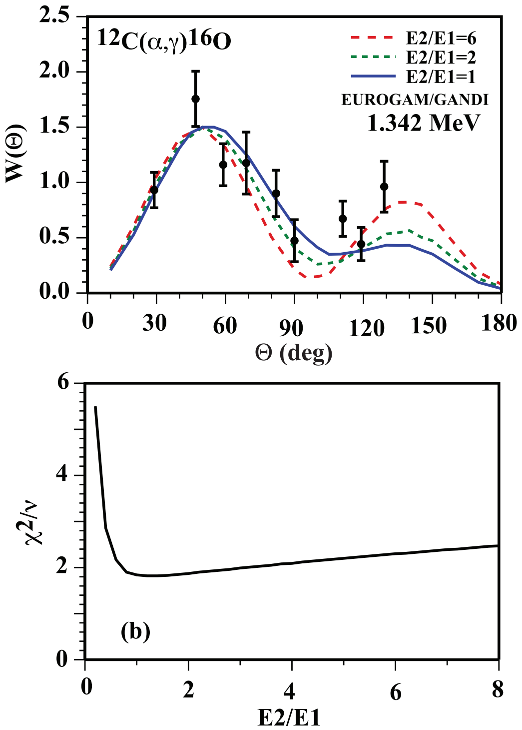

FIG. 1. (Color online) (a) The measured angular distribution of the ^{12}C(α,γ) reaction [8] together with the $E1 + E2$ fits for three values of the $E2/E1$ ratio as discussed in the text. (b) The reduced χ^2/ν obtained for different $E2/E1$ ratios.

measurements of the total reaction cross section to 300 keV (see, for example, Ref. [15]).

I analyzed all the published angular distributions measured at low energy ($E_{\mathrm{c.m.}} < 1.5$ MeV) with the GANDI/EUROGAM array at Stuttgart [6,8] and employed the standard Legendre polynomial expansion as shown, for example, in Eq. (4.3) of Ref. [8] and the published angular attenuation coefficients. The angular distributions measured at 891 and 903 keV, shown in Fig. 4 of Ref. [6], were not included in this analysis since the data points were measured with error bars of nearly 100% (or larger). In order to simplify the analysis I fixed the relative angle (ϕ_{12}) at the value predicted by Eq. (1), discussed below, and varied only one parameter (S_{E2}/S_{E1}) apart from an overall normalization.

As shown in Fig. 1 the $E2/E1$ ratio at 1.342 MeV can be varied by a factor as large as 6 and still yield a similar quality of fit, with only a slight increase in χ^2/ν from 1.8 to 2.4. The same figure demonstrates that the data points measured at backward angles (larger than 90°) provide the largest sensitivity to the $E2/E1$ ratio, but these few (three) data points are measured with poor precision, considerably worst than 10%. It is clear from Fig. 1 that precise data (5–10% statistics) measured with small angular bins (10° or smaller) at large backward angles (90–160°) are essential for an accurate determination of the $E1$ and $E2$ cross-section factors.

The obtained χ^2 values shown in Fig. 1(b) yield $\frac{S_{E2}}{S_{E1}}(1.342) = 1.4^{+1.6}_{-0.6}$ for a fixed value of the relative angle of $\phi_{12} = 54^\circ$ predicted by Eq. (1) and discussed below. The S_{E2}/S_{E1} ratios obtained for all other published angular distributions measured at $E_{\mathrm{c.m.}} < 1.5$ MeV [6,8] are shown in Fig. 2. The large and asymmetric error bars deduced in this analysis are considerably different than those published in Refs. [6–8]. I conclude that the S_{E2}/S_{E1} ratios measured with the EUROGAM/GANDI arrays are not determined with sufficient accuracy, less than 50%, to define the cross-section factors at energies below 1.5 MeV. Thus I do not include these data in the sample of current "world data."

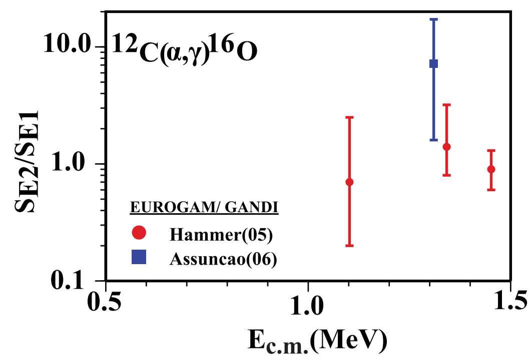

FIG. 2. (Color online) The $E2/E1$ ratios deduced in the current analysis of the data obtained using the EUROGAM/GANDI arrays [6,8].

Excluding the results of the Stuttgart Collaboration [6,8] from the sample of "world data" is in agreement with the finding of Brune and Sayre [16] but is in conflict with Schuermann *et al.* [17] that included the data of the Stuttgart Collaboration [6,8] in their sample of the "world data". In contrast, Schuermann *et al.* [17] removed the data of Redder *et al.* [18] and Ouellet *et al.* [19] from their sample of the "world data". Their selection criteria together with the critical review discussed here and in Ref. [16] would leave only the recent data of Kunz *et al.* [5] and Plag *et al.* [9] in the current sample of "world data" of measured angular distributions at energies below 1.7 MeV. This is a less than a satisfactory situation for such an important cross section.

In Fig. 3 I show the published "world data" of S_{E2} values deduced from angular distributions measured at low energies ($E_{\mathrm{c.m.}} < 1.7$ MeV). I show the new measurements [5,9]

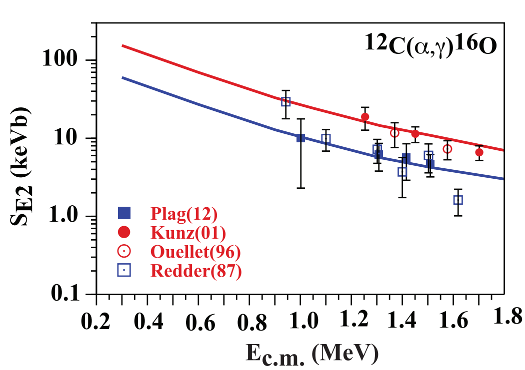

FIG. 3. (Color online) The measured S_{E2} values [5,9,18,19] and the corresponding R-matrix fits. The two distinct groups of data extrapolate to 60 ± 12 and 154 ± 31 keVb. The S_{E2} values measured using the GANDI [6] and EUROGAM [8] arrays are excluded, as discussed in the text.

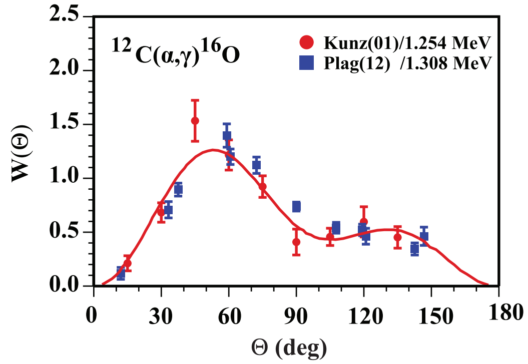

FIG. 4. (Color online) The angular distributions measured by Plag *et al.* [9] superimposed on the published data and fit curve of Kunz *et al.* [5].

together with the previous measurements [18,19] that are not excluded here. My analysis of the angular distributions published in Refs. [5,9,18,19] confirms the published S_{E2} cross-section factors and error bars; hence they are shown in Fig. 3 as published [5,9,18,19]. However, a few data points published with relative error bars of nearly 100% (or larger) are not included in Fig. 3. The results obtained at Stuttgart [6,8] are also not included in this sample, as discussed above.

The data shown in Fig. 3 aggregate into two distinct groups. On the one hand the R-matrix fit of Plag. *et al.* [9] shown in Fig. 3 yields a reasonable fit ($\chi^2/N = 2.0$, $N = 10$) to the data measured by Plag *et al.* [9] and Redder *et al.* [18], but it yields a poor fit ($\chi^2/N = 6.0$, $N = 5$) to the data of both Kunz *et al.* [5] and Oulellet *et al.* [19]. This fit extrapolates to $S_{E2}(300) = 60 \pm 12$ keVb.

On the other hand the R-matrix fit curve of Plag *et al.* [9], when multiplied by 2.57, yields a good fit ($\chi^2/N = 0.75$) to the data of Kunz *et al.* [5] and Oulellet *et al.* [19], but the renormalized curve yields a poor fit ($\chi^2/N = 26.7$) to the data of both Plag *et al.* [9] and Redder *et al.* [18]. This fit extrapolates to $S_{E2}(300) = 154 \pm 31$ keVb. The multiplicative factor of 2.57 may indeed reflect our lack of knowledge of, for example, the α width (spectroscopic factor) of the bound 2^+ state at 6.917 MeV in ^{16}O.

It is worth noting the subtle difference between the angular distribution published by Kunz *et al.* [5] at $E_{c.m.} = 1.254$ MeV with $E2/E1 = 1.28(32)$ as compared to the recent angular distribution published by Plag *et al.* [9] at the nearby energy of $E_{c.m.} = 1.308$ MeV with $E2/E1 = 0.44(20)$. The subtle differences in the data around 90° lead to an $E2$ cross section that is different by a factor of almost 3. Such a large difference is clearly not expected due to the 54-keV difference in energy where these two angular distribution were measured and indicates a major systematic problem. Clearly, these recent angular distributions [5,9] are the most accurate data available today on the $^{12}C(\alpha,\gamma)$ reaction at low energy, but the two different extrapolated values do not allow us to determine $S_{E2}(300)$ with the required accuracy of 10% or better.

I conclude that current "world data" on S_{E2} extracted from angular distributions measured at energies below 1.7 MeV cluster in two distinct groups, leading to two different extrapolations of $S_{E2}(300)$: ≈60 or ≈154 keVb. Neither one of these solutions can be favored or ruled out by the current "world data" of measured angular distributions. In order to resolve this ambiguity in the value of $S_{E2}(300)$ one needs to measure complete and very detailed γ-ray angular distributions for the $^{12}C(\alpha,\gamma)$ reaction with high accuracy (with binning of 10° or less) at very low energies (below 1.5 MeV). As shown in Fig. 1 the data at large backward angles are most sensitive to the $E2/E1$ ratio, but such measurements with γ-ray detectors are challenged by the finite size of the γ-ray detector and the presence of the beam pipe.

The ambiguity in the value of the extrapolated $S_{E2}(300)$ reported in this paper resembles the ambiguity in the value of the extrapolated $S_{E1}(300)$ value where even the data on the β decay of ^{16}N shown in Fig. 18 (and Fig. 16) of Ref. [20] reveal two minima with identical χ^2_β values at $S_{E1}(300) \approx 10$ keVb and $S_{E1}(300) \approx 80$ keVb. The small value of the extrapolated $S_{E1}(300) \approx 10$ keVb has been discussed by many authors [10,11,18,21,22] and cannot be resolved by the modern data as shown in Fig. 5 of Ref. [6]. In order to resolve this ambiguity in the value of $S_{E1}(300)$ the newly proposed experiments [13,14] must measure complete γ-ray angular distributions of the $^{12}C(\alpha,\gamma)$ reaction with high accuracy at low energies (below 1.0 MeV).

The Legendre-polynomial fit of the angular distribution data discussed above also includes an $E1$-$E2$ interference term with a (ϕ_{12}) relative phase angle. This phase angle can be written as [23]

$$\phi_{12} = \delta_2 - \delta_1 + \arctan(\eta/2), \quad (1)$$

where δ_2 and δ_1 are the measured elastic phase shifts for $\ell = 2$ and $\ell = 1$ respectively, and η is the Sommerfeld parameter. Since this relationship was first derived in (multilevel) R-matrix theory [23] it is generally assumed to be a prediction of the R-matrix theory, but the broader validity of Eq. (1) was discussed in Ref. [24] and was previously shown to be a consequence [25] of the Watson theorem [26], which is routed in unitarity. Hence we conclude that Eq. (1) is required by unitarity.

The recently measured angular distributions were analyzed by either fixing the value of the $E1$-$E2$ mixing angle (ϕ_{12}) at the value predicted by Eq. (1) [5,8] or by considering the phase angle (ϕ_{12}) as a fit parameter [8]. The $E1$-$E2$ relative phases (ϕ_{12}) extracted as fit parameters [8] are in strong disagreement with the prediction of Eq. (1), as shown in Fig. 11 of Ref. [8]. Hence we conclude that the relative phase angles measured in the Stuttgart experiment [8] violate unitarity. Such strong deviations from Eq. (1) are observed on resonance around $E_{c.m.} = 2.4$ MeV where the cross sections are large. They indicate poorly understood systematic problems in the measured angular distributions [8], as also concluded in Ref. [16]. Clearly this violation of unitarity must be resolved by future measurements of complete angular distribution measured in the vicinity of the 1^- resonance state of ^{16}O ($E_{c.m.} \approx 2.4$ MeV).

To conclude I analyzed and reviewed new modern measurements of complete angular distributions of γ rays from the $^{12}C(\alpha,\gamma)$ reaction measured at very low energies approaching $E_{c.m.} \approx 1.0$ MeV. While these measurements represent a major improvement of the "world data" and our knowledge of the low-energy cross section of the $^{12}C(\alpha,\gamma)^{16}O$ reaction, I demonstrate that the measured S_{E2} values bifurcate into two groups extrapolating to $S_{E2}(300) \approx 60$ keVb or $S_{E2}(300) \approx 154$ keVb. This ambiguity in the extrapolated $S_{E2}(300)$ value resembles the ambiguity in the extrapolated $S_{E1}(300)$ value where the small $S_{E1}(300) \approx 10$ keVb solution cannot be ruled out in favor of the large ≈ 80 keVb solution. These ambiguities in the extrapolated $S_{E2}(300)$ and $S_{E1}(300)$ values must be considered by practitioners in the field of stellar evolution theory and they must be resolved by the experiments now in progress [13] or in the planning stage [14]. A violation of unitarity of the measured $E1$-$E2$ relative phases (ϕ_{12}) must be resolved as well.

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