Reply to "Comment on 'Nonidentical protons' "

T. Mart and A. Sulaksono
Departemen Fisika, FMIPA, Universitas Indonesia, Depok 16424, Indonesia
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We reply to the Comment by Downie *et al.*, which concerns the proton radius extracted from elastic electron-proton scattering data by using the assumption of nonidentical protons.

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In the first part of our original paper [1], we have assumed nonidentical protons in our calculation, i.e., the radii of protons vary within $r \pm \Delta r$, where r is the average value. Both r and Δr values are extracted from the latest and most precise elastic electron-proton scattering data from Mainz [2]. In the second part of our paper, we use both r and Δr to estimate the effect of the nucleon radius variation on symmetric nuclear matter (SNM) and neutron star matter (NSM) by considering the so-called excluded volume effect in the calculation. The latter is quite important because, in both SNM and NSM investigations, it is customary to assume a point particle approximation ($r = 0$).

Downie *et al.* [3] criticize the validity of the extracted r and comment on some of the discussions given in the first part of our paper. Whereas, we agree with the conclusion of the Comment that more statistical consideration is needed for a more conclusive result, we believe that the following Reply is important for the sake of clarity of our result given in our original paper [1].

(i) To extract the value of r in Ref. [1], we have only used the experimental data, given in Ref. [4], that consist of 77 data points. These data were obtained from the Rosenbluth separation technique. The reason for using them has been explained in the original paper [1]. The best fit yields a relative cutoff variation of $\Delta = 21.5\%$ and $\chi^2/N = 4.52$. This relatively "large" χ^2/N originates from the fact that the data are extremely precise and fluctuating. This is hard to see in a normal plot as shown in panel (a) of Fig. 1 or in our original paper [1]. However, if we plot the ratio of the $G_{E,p}(Q^2)$ form factor to the standard dipole $G_{\rm dip.}(Q^2) = (1 + Q^2/0.71)^{-2}$ or the deviation of the form factor from the standard dipole as shown in panel (b) or (c) of Fig. 1, this fluctuation becomes apparent. We note that, in the latter [panel (c)], the difference between models and data is more obvious. Furthermore, at the lowest available Q^2, the data do not approach the form-factor normalization $F(Q^2 = 0) = 1$. This is in contrast to the extracted data given in Ref. [5] as well as predictions of other models. Thus, it is obviously difficult to fit these data by only using a smooth function. Note that, despite the good agreement with our result, we did not include the data given in Ref. [5] in our fit.

Although the assumption of a smooth form factor could be considered as a first approximation, other models display the same behavior, e.g., the double dipole and Friedrich-Walcher ones shown in Fig. 1. Moreover, as anticipated in the Introduction of our original paper, this fluctuation could also

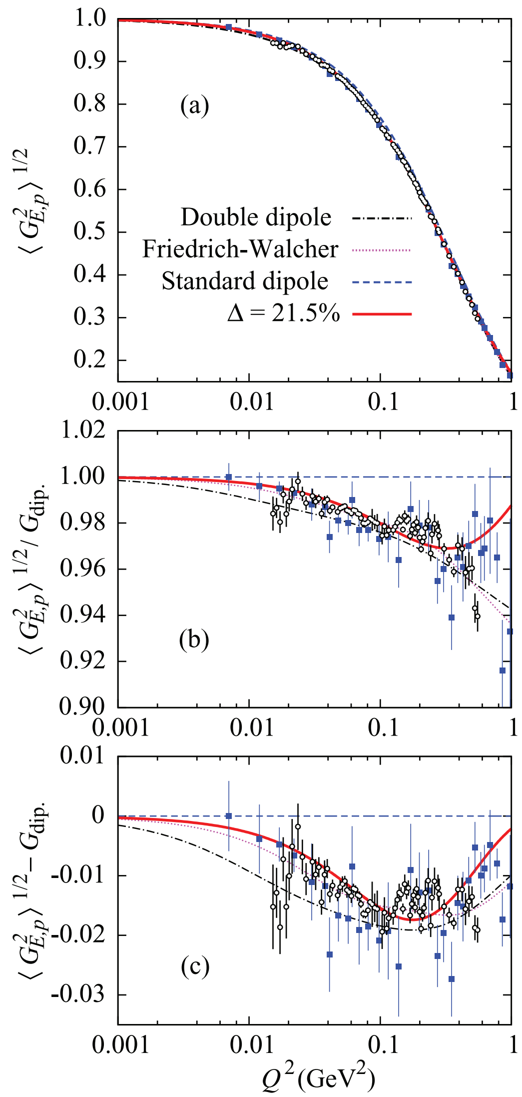

FIG. 1. (Color online) Log-normal representations of Fig. 5 of Ref. [1] in a wider Q^2 range. Panel (a) shows a comparison between the root-mean square of the proton form factor $G_{E,p}(Q^2)$ obtained from different models [1,4] and the experimental data [2,5]. Panel (b) displays the same result as in panel (a) but in terms of the ratio between the form factors and the standard dipole form factor $G_{\rm dip.}(Q^2) = (1 + Q^2/0.71)^{-2}$. The deviation of these form factors and the experimental data from the standard dipole form factor is exhibited in panel (c).

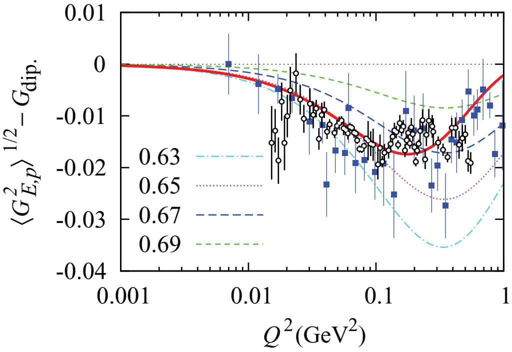

FIG. 2. (Color online) The same as Fig. 1, but for the comparison of experimental data with different parametrizations of the "pure" dipole form factor $(1 + Q^2/\Lambda^2)^{-2}$. The values of Λ^2 for the corresponding lines are shown in the figure. The solid line corresponds to the averaged dipole form factor with $\Delta = 21.5\%$ [1] as in Fig. 1.

originate from some unknown physical process that should be considered in the cross-sectional formula Eq. (4) of Ref. [1]. It is also important to note that the accuracy of the new Mainz data [2] becomes worse in the low-Q^2 region. In general, however, the agreement of our result with all data below $Q^2 = 1$ GeV^2 is better than other models as clearly shown in panel (c) of Fig. 1.

(ii) We agree that other uncertainties that come, e.g., from the model uncertainty and the truncation error, should be considered for a more comprehensive discussion of the error bar of the extracted r. In our original paper, the reported error bar originates only from MINUIT. However, we still believe that the low-Q^2 data will improve the extracted radius. Although the four models displayed in Fig. 1 show a slightly different behavior at $Q^2 \approx 0$, they lead to a significantly different proton charge radius r. Given that both the truncation error and the extracted radius depend strongly on the low-Q^2 experimental data behavior, it is very unfortunate to realize that, at this very decisive region, the Mainz data become less precise. Thus, we believe that future experiments should always focus on the lower Q^2 but with smaller error bars.

(iii) The limitation of a dipole form factor has been discussed in the Comment [3] as well as in our original paper [1]. However, it must be emphasized here that there is a significant difference between the pure dipole form factor and the averaged dipole form factor as shown by the dotted (blue) and solid (red) lines in panel (b) of Fig. 1. We choose the averaged dipole form factor merely because it looks more natural than any other form factor. Of course, in the future, other averaged form factors can be investigated for the sake of comparison with the present result. To show the limitation of a pure dipole form factor, in Fig. 2, we show the variation in this form factor with the variation in the cutoff (Λ^2) with the same convention as in panel (c) of Fig. 1 and compare it with our original result as well as experimental data. Figure 2 obviously indicates that the pure dipole form factors cannot be used to describe the present data, in contrast to the averaged one.

As a conclusion, we believe that the result of our fit of the proton radius given in our original paper is still useful for an exploratory study on the effect of nonidentical protons assumption as well as for use in the NSM matter as discussed in detail in Ref. [1]. Especially, it is important to call attention to the proton structure in the nuclear and neutron star matter sectors. We agree that other uncertainties that come from the model uncertainty and truncation error should be considered for a more precise determination of the proton charge radius. More complex form factors as well as error bars that come from the model uncertainties will be considered in the future.

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