PHYSICAL REVIEW C **88**, 059801 (2013)

Comment on "Nonidentical protons"

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(Received 8 March 2013; published 19 November 2013)

We comment on various issues in a recent paper on nonidentical protons as a resolution to the proton radius puzzle.

DOI: 10.1103/PhysRevC.88.059801 PACS number(s): 26.60.−c, 21.65.−f, 13.40.Gp, 14.20.Dh

A recent paper by Mart and Sulaksono [1] suggests the proton radius puzzle [2,3] can be resolved by considering nonidentical protons. This idea is implemented by fitting the proton electric form-factor data with a sum of dipole form factors over a range of radii, which leads to an average proton radius of 0.8333 ± 0.0004 fm, consistent with the muonic hydrogen value. (The article does not address the atomic hydrogen data, which agree with the *ep* scattering result [4].) In this Comment, we criticize their fit, the validity of the extracted proton radius, and comment on some of the discussion in the article.

(i) A significant problem with the fit can be seen from the values of χ^2/N shown in Fig. 4 of Ref. [1]. At the minimum, $\chi^2/N \approx 4$. The reduced χ^2 indicates that the fit does not describe the data well. Thus, one should not attribute much significance to any extracted parameters, e.g., the radius. Figure 5 of Ref. [1] confirms the unsatisfactory fit to the data at the smallest Q^2 values with the most sensitivity to the radius. Although there is some scatter in and poor overlap of the data, the dipole fit is high relative to the average of the data. About 2/3 of the data points are below the fit, some significantly, whereas, about 1/6 of the data points are above the fit, only one by as much as 1σ. Thus, the visual appearance of Fig. 5 confirms the numerical result for χ^2/N—the fit is inadequate—and indicates that the actual radius is larger than the extracted value.

(ii) Modern form-factor fits generally have an uncertainty on the proton radius of ≈0.01 fm. The uncertainty from Ref. [1] is ≈25 times smaller! We believe that this is unrealistically small and results from two issues. First, the large χ^2/N and small radius uncertainty both result from the dipole being an insufficiently flexible parametrization that is unable to agree with the data. This causes χ^2 to grow rapidly as the fit radius is varied. Second, certain uncertainties are ignored. In particular, there is no reason to believe the actual form factor looks like any simple parametrization, and thus, one should try multiple parametrizations to investigate *model* uncertainties. The most thorough study of this type, performed in Ref. [5], includes dipole forms, which were ultimately rejected for use in determining the radius as they were unable to fit the data adequately. The model uncertainty apparently is ignored in Ref. [1].

(iii) Another uncertainty not discussed in Ref. [1] is referred to as truncation error. A straight line to data representing a curved function yields parameters that depend on the region over which the data are fit. For the proton radius, which is extracted from the slope of the form factor at $Q^2 = 0$, the form factor is nonlinear, and a radius obtained from a linear fit will be offset by varying amounts depending on the high-Q^2 cutoff of the data. The offset between the fit and reality is the truncation error. By using a function which more closely approximates the data, one reduces, but does not eliminate, truncation error.

We generated pseudodata from the Kelly form-factor parametrization [6] with a comparable number of cross-section points to the Mainz data set [5] in the low-Q^2 region. The pseudodata were scattered by an estimated form-factor uncertainty of 0.5%. We then studied the extracted radius by using both linear and dipole fits over a variety of ranges with upper limits that varied from 0.01 to 0.6 GeV^2. Figure 1 shows that the dipole fit, although much better than the linear fit, even when restricted to only the lowest-Q^2 data, and thereby attaining the best possible extracted radius, gives an extracted value that is a few tenths of a percent different from the real radius with very large uncertainty due to the few available data points within that range. This effect was not studied in Ref. [1], but it needs to be if one truncates the data since there can be a significant truncation error.

Our opinion is that the fit performed in Ref. [1] does not help resolve the proton radius puzzle; more work needs to be done for a reliable radius extraction. Better estimates of the proton radius from electron scattering can be found in Refs. [5,7–9]. A fit and discussions of issues in radius extraction can be found in Refs. [10,11]. The most reliable fit that extracts a small proton radius consistent with muonic hydrogen is in Ref. [12]. A review of the radius puzzle is given in Ref. [13].

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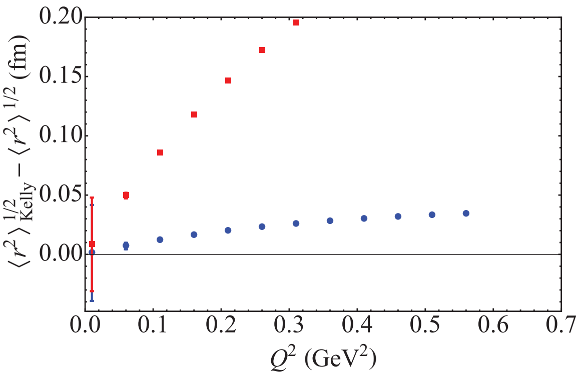

FIG. 1. (Color online) The difference between the radius of the Kelly form-factor fit and that extracted from the Kelly form factor based pseudodata as a function of the upper limit of the fitted data range. The red points result from a linear-fit-based extraction, and the blue points result from a dipole parametrization. It can be seen that, as the fitting range decreases, the extracted radius approaches the "real" radius, but the errors blow up substantially. Even limiting the fit to the lowest given range, the difference between the real radius and that extracted by using the dipole fit is on the order of tenths of a percent—far larger than the error given in Ref. [1].

We conclude with some comments on the discussion in Ref. [1]. We agree with the basic idea that the proton is a dynamical system, and if one could take a snapshot of the system's size, it would vary with time. It also seems clear that the proton size cannot be ignored if one is to understand nuclear matter. But we have disagreements with aspects of the discussion of proton structure that might be misleading. The dipole form factor has been known to only work at the ≈10% level since well before experiments began at Jefferson Laboratory and at the Mainz Microtron [14]. The nonrelativistic limit is inappropriate for serious consideration as the proton is a relativistic system, and one should consider the transverse charge distribution rather than the usual Fourier transform [15]; as a result, one should be aware that there is not a well-defined proton radius in three dimensions—but there is a slope of the form factor at $Q^2 = 0$. Also, as a result, the dipole form has no theoretical significance, and deviations from it should be considered unsurprising rather than necessitating a rigorous physical concept. To push data to very low Q^2 does not improve the quality of the extracted radius without improved uncertainties as indicated above, in the discussion of Fig. 1. Independent of the idea of a dynamically varying proton size, the form factor is the observable, not the charge distribution, and the slope at $Q^2 = 0$ determines the "radius" of the charge distribution. We comment concerning the analysis of Ref. [16] vs the data of Ref. [5]. In Ref. [16], a number of independent older and often flawed experiments with small data sets were consistently analyzed. In Ref. [5], there was an order of magnitude more data taken with numerous systematic checks to minimize uncertainties so that precise form factors could be determined. It is the best cross-sectional data available, even if there are some issues in the analysis [17]. There are experiments planned at Jefferson Laboratory and the Paul Scherrer Institut [18,19] which will measure low-Q^2 electromagnetic scattering. Finally, we note, as a matter of simple logic, that, even if the theoretical argument that leads to fitting the form factor as a sum of dipoles were correct, the converse does not necessarily hold.

This work has been funded, in part, by the US National Science Foundation, the US DOE (Grant No. DE-FG02-99ER4110-21967), the GW Research Enhancement Fund, and the GW Centers and Institute Facilitating Fund.

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