Muon capture rate on hydrogen and the values of g_A and $g_{\pi NN}$

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Motivated by the recent developments in the determination of the experimental values of the nucleon axial-vector coupling constant g_A and the pion-nucleon coupling constant $g_{\pi NN}$, we carry out a heavy-baryon chiral perturbation calculation of the hyperfine-singlet μp capture rate Γ_0 to next-to-next-to-leading order (N^{2}LO), with the use of the latest values of g_A and $g_{\pi NN}$. The calculated N^{2}LO value is $\Gamma_0^{\rm theor}(\mu^- p \to \nu_\mu n) = 718 \pm 7\,{\rm sec}^{-1}$, where the estimated next-to-next-to-next-to-leading order contribution dominates the error. This value is in excellent agreement with the experimental value reported by the MuCap Collaboration.

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Muon capture on the proton has been the subject of intensive experimental and theoretical investigations; for reviews, see Refs. [1,2]. Recently, the MuCap Collaboration succeeded in measuring, to 1% precision, the rate Γ_0 of muon capture from the hyperfine-singlet state of a μp atom [3]. The reported experimental value is

$$\Gamma_0^{\rm expt}(\mu^- p \to \nu_\mu n) = 714.9 \pm 5.4({\rm stat}) \pm 5.1({\rm syst})\,{\rm sec}^{-1}\,. \tag{1}$$

Heavy-baryon chiral perturbation theory (HBChPT) provides a systematic framework for calculating $\Gamma_0^{\rm theor}$, and a number of HBChPT-based calculations have been reported [4–6]. HBChPT [7–9] involves two perturbative expansions, one in terms of the expansion parameter $Q/\Lambda_\chi \ll 1$ and the other in terms of $Q/m_N \ll 1$. Here Q is a typical four-momentum transfer involved in the reaction, m_N is the nucleon mass, and $\Lambda_\chi \simeq 4\pi f_\pi \simeq 1\,{\rm GeV}$ is the chiral scale. In order for the theory to match the experimental precision of 1%, one needs to incorporate higher order terms in the expansion in Q/Λ_χ and Q/m_N. In Ref. [6] (to be referred to as RMK), Raha *et al.* evaluated $\Gamma_0^{\rm theor}$ including correction terms up to next-to-next-to-leading order (N^{2}LO). They reported $\Gamma_0^{\rm theor} = 710 \times (1 \pm 0.007)\,{\rm sec}^{-1}$ which at N^{2}LO includes radiative corrections and finite proton size effect. The evaluation of $\Gamma_0^{\rm theor}$ in HBChPT at N^{2}LO involves several low-energy constants (LECs), and the accuracy of the calculated value of $\Gamma_0^{\rm theor}$ at this order depends on the precision with which these LECs are known. Additional uncertainties are due to the truncation at N^{2}LO of a HBChPT expansion. The rate of convergence estimated from the leading order (LO), the next-to-leading order (NLO), and the N^{2}LO contributions to $\Gamma_0^{\rm theor}$ found in Refs. [4–6], indicates that next-to-next-to-next-to-leading order (N^{3}LO) corrections would contribute at most $\sim 1\%$ [6]. In the following we shall primarily concentrate on the uncertainties associated with the N^{2}LO evaluation of $\Gamma_0^{\rm theor}$. As emphasized in RMK, the above 0.7% theoretical error is dominated by the possible variations in the experimental values of the nucleon axial-vector coupling constant g_A and the pion-nucleon coupling constant $g_{\pi NN}$. This situation motivates us to pay particular attention to recent highly noteworthy developments regarding the experimental values of g_A [10,11] and $g_{\pi NN}$ [12], and to reexamine the value of $\Gamma_0^{\rm theor}$, taking into account these developments. The purpose of the present Brief Report is to report on such a study.

We first briefly summarize the treatment of the LECs in RMK. An N^{2}LO calculation of $\Gamma_0^{\rm theor}$ involves four LECs: g_A, $\tilde{B}_2$, $\tilde{B}_3$, and $\tilde{B}_{10}$. $\tilde{B}_2$ is determined from the Goldberger-Treiman (GT) discrepancy

$$\Delta_{GT} \equiv \frac{2m_\pi^2}{(4\pi f_\pi)^2 g_A}\,\tilde{B}_2 = \frac{g_A\, m_N}{g_{\pi NN}\ f_\pi} - 1\,,$$

while Refs. [7,13] relate $\tilde{B}_3$ and $\tilde{B}_{10}$ to the nucleon mean squared axial radius $\langle r_A^2\rangle$ and the nucleon isovector mean squared charge radius $\langle r_V^2\rangle$, respectively, via

$$\tilde{B}_3 = \frac{g_A}{2}(4\pi f_\pi)^2\frac{\langle r_A^2\rangle}{3}\,,$$
$$\frac{1}{6}\langle r_V^2\rangle = -\frac{2\tilde{B}_{10}(\Lambda_\chi)}{(4\pi f_\pi)^2} - \frac{1+7g_A^2}{6(4\pi f_\pi)^2} - \frac{1+5g_A^2}{3(4\pi f_\pi)^2}\ln\left(\frac{m_\pi}{\Lambda_\chi}\right).$$

Since the term associated with $\tilde{B}_{10}$ gives only $\sim$0.1% contribution to $\Gamma_0^{\rm theor}$, and since $\langle r_V^2\rangle$ is relatively well known [14,15], variations in $\Gamma_0^{\rm theor}$ due to the uncertainty in $\langle r_V^2\rangle$ can be safely ignored; RMK used a fixed value, $\langle r_V^2\rangle^{1/2} = 0.765$ fm [16]. The terms associated with $\tilde{B}_2$ and $\tilde{B}_3$ give $\sim$0.7% and $\sim$1.9% contribution to $\Gamma_0^{\rm theor}$, respectively, implying a more pronounced sensitivity of $\Gamma_0^{\rm theor}$ to variations in the input parameters entering $\tilde{B}_2$ and $\tilde{B}_3$. As for the $\tilde{B}_3$ contribution, RMK found that $\sim$10% variation in $\langle r_A^2\rangle^{1/2}$ (or equivalently, in the axial mass parameter m_A) causes $\sim$0.3% changes in $\Gamma_0^{\rm theor}$, which are not totally negligible; it is to be noted that the 10% variation is a rather ample allowance for the uncertainty in $\langle r_A^2\rangle^{1/2}$. The value of $g_{\pi NN}$, which affects $\tilde{B}_2$ via Δ_{GT}, was extracted from nucleon-nucleon scattering and pion-nucleon scattering [17–20], but the resulting values show significant scatter. As an estimated range of variation in $g_{\pi NN}$, RMK adopted $g_{\pi NN} = 13.044$ to 13.40, the smaller value taken from Ref. [17] and the larger value from Ref. [18]. Variations in $g_{\pi NN}$ within this range lead to $\sim$0.2% changes in $\Gamma_0^{\rm theor}$. For g_A, RMK employed as an estimate of its uncertainty the difference between the Particle Data Group (PDG) 2002 value and the PDG 2012 value [21–23]. Variations in g_A within this range cause $\sim$0.6% changes in $\Gamma_0^{\rm theor}$; these changes arise primarily from the overall multiplicative factor $(1+3g_A^2)$ that enters the expression for $\Gamma_0^{\rm theor}$, and also from the contribution

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of the $\tilde{B}_2$ term. The estimated theoretical uncertainty of 0.7% in $\Gamma_0^{\rm theor}$ was obtained by taking the quadratic sum of the above-mentioned individual errors. It is noteworthy that the radiative corrections, which contribute about 2% to $\Gamma_0^{\rm theor}$ [24], are well under control and do not affect the uncertainty in $\Gamma_0^{\rm theor}$; see Ref. [6] for details.

We now turn our attention to the latest experimental developments regarding g_A and $g_{\pi NN}$. Historically, the value of g_A recommended by the PDG has been steadily increasing, and the 2012 PDG value is $g_A = 1.2701 \pm 0.0025$ [21]. Very recently, however, two groups [10,11] reported the value $g_A \simeq 1.276$, extracted from the measurement of the asymmetry parameter A in neutron β decay. This new value is significantly larger than the 2012 PDG value. It is noteworthy that this new value of g_A is consistent with the recently revised value of the neutron mean lifetime, $\tau = 880.1 \pm 1.1$ sec ($S = 1.8$) [21,25], as discussed in Ref. [10]. Furthermore, Ivanov *et al*. [26] pointed out the possibility that these new values of g_A and τ resolve the "antineutrino flux anomaly," a lingering problem in the nuclear reactor neutrino-oscillation experiments. Regarding the value of $g_{\pi NN}$, in a recent notable study [12], Baru *et al.* improved the Goldberger-Miyazawa-Oehme sum rule analysis of Ericson *et al*. [19], and deduced the value, $g_{\pi NN} = 13.116 \pm 0.092$. It is worth emphasizing that Baru *et al*. [12] used the most recent value for the πN scattering length a^+, which had been determined from the high-precision πd atom data [27]. These important developments motivate us to re-evaluate $\Gamma_0^{\rm theor}$ at N^{2}LO with the use of the value of g_A obtained in Refs. [10,11], and the value of $g_{\pi NN}$ deduced in Ref. [12]. As will be discussed in the concluding paragraph, it is assumed here that the electromagnetic effects have been removed from these two experimentally determined hadronic constants.

In calculating $\Gamma_0^{\rm theor}$, we use exactly the same formalism and the input parameters as employed in RMK, *except* the values of g_A and $g_{\pi NN}$; as explained above, we adopt here $g_A = 1.2758 \pm 0.0016$ [10,11] and $g_{\pi NN} = 13.116 \pm 0.092$ [12]. To assess to what extent the uncertainties in g_A and $g_{\pi NN}$ affect the precision in $\Gamma_0^{\rm theor}$, we calculate $\Gamma_0^{\rm theor}$ for four cases. In the first and second cases, $g_{\pi NN}$ is fixed at its central value $g_{\pi NN} = 13.116$, while g_A is taken to be at the lower or upper end of the range within the experimental error. In the third and fourth cases, g_A is fixed at its central value, $g_A = 1.2758$, while $g_{\pi NN}$ is assumed to be at the lower or upper end of the range within the experimental error. Table I shows the values of $\Gamma_0^{\rm theor}$ along with Δ_{GT} calculated for these four cases. We emphasize that the results in this table comprise the radiative corrections and the finite proton-size effects, as estimated in RMK. Table I indicates that the uncertainty in g_A causes $\sim$0.2% variation in $\Gamma_0^{\rm theor}$, and that the uncertainty in $g_{\pi NN}$ leads to $\sim$0.1% variation. To deduce the total uncertainty in $\Gamma_0^{\rm theor}$, we recall that, according to RMK, if one assigns a 10% error to $\langle r_A^2\rangle^{1/2}$ (which is considered to be a rather generous error estimate), it causes about 0.3% variations in $\Gamma_0^{\rm theor}$ at N^{2}LO. By taking the squared sum of the errors that arise from g_A, $g_{\pi NN}$, and $\langle r_A^2\rangle^{1/2}$, we arrive at

$$\Gamma_0^{\rm theor}({\rm N^2LO}) = 718 \times (1 \pm 0.003)\,{\rm sec}^{-1}. \tag{2}$$

It is noteworthy that the new larger value for g_A [10,11] increases the central value of $\Gamma_0^{\rm theor}$ by about 0.8%, as compared with the result in RMK; this change arises primarily from the overall factor $(1+3g_A^2)$ contained in the expression for $\Gamma_0^{\rm theor}$. It is also to be noted that the adoption of the new input for g_A and $g_{\pi NN}$ significantly reduces the uncertainties in $\Gamma_0^{\rm theor}$ obtained in an N^{2}LO calculation. Corrections entering at N^{3}LO are reasonably expected to produce at most a $\sim$1% contribution to $\Gamma_0^{\rm theor}$, uncertainties that are within the present experimental precision. Since the 0.3% uncertainty that arises within an N^{2}LO calculation is much smaller than that due to the possible N^{3}LO contributions, it is reasonable to adopt the central value of $\Gamma_0^{\rm theor}$ in Eq. (2) and attach $\sim$1% error to it: $\Gamma_0^{\rm theor} = 718 \times (1 \pm 0.01)\,{\rm sec}^{-1}$.

To summarize, we have updated the HBChPT calculation of the hyperfine-singlet μp capture rate $\Gamma_0^{\rm theor}$ to N^{2}LO carried out in Ref. [6], using the recently reported values of g_A and $g_{\pi NN}$. We have assumed in this work that the coupling constants, g_A and $g_{\pi NN}$, are pure hadronic constants. The electromagnetic corrections to, e.g., the asymmetry parameter A in polarized neutron β decay which is used by Refs. [10,11] to determine g_A, are known to be very small; e.g., Ref. [28] finds radiative corrections to g_A determined from A to be 0.12%. As to the value of $g_{\pi NN}$ the subtraction constant in the sum rule has been extracted from pionic deuterium where, e.g., isospin violating effects are considered as well as QED effects. The hadronic cross sections entering the dispersion integrals are also assumed to have been corrected for the possible electromagnetic effects; see discussions in Ref. [12] and references therein. However, as shown in a highly illuminating paper by Gasser *et al*. [29], it is virtually impossible to extract pure hadronic values for, e.g., g_A and $g_{\pi NN}$, from experimental data. With the use of $g_A = 1.2758 \pm 0.0016$ [10,11] and $g_{\pi NN} = 13.116 \pm 0.092$ [12], where we assume that the errors quoted include residual electromagnetic effects, the theory favors a larger central value for $\Gamma_0^{\rm theor}$ compared to the previous result [6]. In particular, our calculation that includes radiative and proton finite-size corrections is

$$\Gamma_0^{\rm theor}(\mu^- p \to \nu_\mu n) = 718 \pm 7\ {\rm sec}^{-1}\,, \tag{3}$$

where the error is dominated by the estimated N^{3}LO contributions. This new central value for $\Gamma_0^{\rm theor}$ is still in excellent agreement with the experimental value, Eq. (1), reported by the MuCap Collaboration.

TABLE I. Capture rate $\Gamma_0^{\rm theor}$ and Goldberger-Treiman discrepancy Δ_{GT} calculated with $g_A = 1.2758 \pm 0.0016$ [10,11] and $g_{\pi NN} = 13.116 \pm 0.092$ [12]. $\Gamma_0^{\rm theor}$ is evaluated to N^{2}LO, including radiative and proton finite-size corrections as discussed in Ref. [6].

g_A	$g_{\pi NN}$	Δ_{GT}	$\Gamma_0^{\rm theor}$ (sec^{-1})
1.2774	13.116	−0.011	719.7
1.2742	13.116	−0.013	716.9
1.2758	13.208	−0.019	717.4
1.2758	13.024	−0.005	719.2

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[1] T. Gorringe and H. W. Fearing, Rev. Mod. Phys. **76**, 31 (2003).
[2] P. Kammel and K. Kubodera, Annu. Rev. Nucl. Part. Sci. **60**, 327 (2010).
[3] V. A. Andreev *et al*. (MuCap Collaboration), Phys. Rev. Lett. **99**, 032002 (2007); **110**, 012504 (2013).
[4] S.-i. Ando, F. Myhrer, and K. Kubodera, Phys. Rev. C **63**, 015203 (2000).
[5] V. Bernard, T. R. Hemmert, and U.-G. Meißner, Nucl. Phys. A **686**, 290 (2001).
[6] U. Raha, F. Myhrer, and K. Kubodera, Phys. Rev. C **87**, 055501 (2013).
[7] V. Bernard, N. Kaiser, and U.-G. Meißner, Int. J. Mod. Phys. E **4**, 193 (1995).
[8] V. Bernard, Prog. Nucl. Part. Phys. **60**, 82 (2008).
[9] S. Scherer, Prog. Nucl. Part. Phys. **64**, 1 (2010).
[10] D. Mund *et al*., Phys. Rev. Lett. **110**, 172502 (2013).
[11] M. P. Mendenhall *et al*. (UCNA Collaboration), Phys. Rev. C **87**, 032501 (2013).
[12] V. Baru, C. Hanhart, M. Hoferichter, B. Kubis, A. Nogga, and D. R. Phillips, Phys. Lett. B **694**, 473 (2011); Nucl. Phys. A **872**, 69 (2011).
[13] V. Bernard, H. W. Fearing, T. R. Hemmert, and U.-G. Meissner, Nucl. Phys. A **635**, 121 (1998).
[14] G. G Simon, Ch. Schmitt, F. Borkowski, and V. H. Walter, Nucl. Phys. A **333**, 381 (1980).
[15] I. T. Lorenz, H.-W. Hammer, and U.-G. Meißner, Eur. Phys. J. A **48**, 151 (2012); R. Pohl, R. Gilman, G. A. Miller, and K. Pachucki, Annu. Rev. Nucl. Part. Sci. **63**, 175 (2013).
[16] P. Mergell *et al*., Nucl. Phys. A **596**, 367 (1996).
[17] J. J. de Swart, M. C. M. Rentmeester, and R. G. E. Timmermans, πN Newslett. **13**, 96 (1997); R. A. Arndt, W. J. Briscoe, I. I. Strakovsky, and R. L. Workman, Phys. Rev. C **74**, 045205 (2006).
[18] G. Höhler, in *Landolt-Börnstein*: *Numerical Data and Functional Relationships in Science and Technology*, Group I, Vol. 9 Subvol. b2, edited by H. Schopper (Springer, Berlin, 1983).
[19] T. E. O. Ericson, B. Loiseau, and A. W. Thomas, Phys. Rev. C **66**, 014005 (2002).
[20] D. V. Bugg, Eur. Phys. J. C **33**, 505 (2004).
[21] J. Beringer *et al*. (Particle Data Group), Phys. Rev. D **86**, 010001 (2012).
[22] K. Nakamura *et al*. (Particle Data Group), J. Phys G: Nucl. Part. Phys. **37**, 075021 (2010), recommend $g_A = 1.2694 \pm 0.0028$ ($S = 2.0$).
[23] K. Higawara *et al*. (Particle Data Group), Phys. Rev. D **66**, 010001 (2002), recommend $g_A = 1.2679 \pm 0.0030$; R. M. Barnett *et al*. (Particle Data Group), *ibid*. **54**, 1 (1996), recommend $g_A = 1.2601 \pm 0.0025$ ($S = 1.1$).
[24] A. Czarnecki, W. J. Marciano, and A. Sirlin, Phys. Rev. Lett. **99**, 032003 (2007).
[25] F. E. Wietfeldt and G. L. Greene, Rev. Mod. Phys. **83**, 1173 (2011).
[26] A. N. Ivanov, R. Hollwieser, N. I. Troitskaya, M. Wellenzohn, O. M. Zherebtsov, and A. P. Serebrov, arXiv:1306.1995 [hep-ph].
[27] T. Strauch *et al*., Phys. Rev. Lett. **104**, 142503 (2010); Eur. Phys. J. A **47**, 88 (2011).
[28] M. Fukugita and T. Kubota, Phys. Lett. B **598**, 67 (2004).
[29] J. Gasser, A. Rusetsky, and I. Scimeni, Eur. Phys. J. C **32**, 97 (2003).