Roles of system size and excitation energy in probing nuclear dissipation with giant dipole resonance γ rays

W. Ye,^{1,*} N. Wang,¹ and X. Chang²

¹Department of Physics, Southeast University, Nanjing 210096, Jiangsu Province, People's Republic of China ²Department of Modern Physics, University of Science and Technology of China, Hefei 230026, Anhui Province, People's Republic of China (Received 2 July 2013; revised manuscript received 16 September 2013; published 7 November 2013)

Using the stochastic Langevin model, we calculate the evolution of postsaddle giant dipole resonance (GDR) γ -ray multiplicity (M_{γ}) as a function of the postsaddle friction strength (β) with excitation energy for ²⁴⁰Cf, ²²⁴Th, and ²⁰⁰Pb systems. It is found that with raising the excitation energy the sensitivity of the postsaddle γ emission to friction is substantially increased. We further find that M_{γ} shows a greater sensitivity to β with increasing size of the fissioning nucleus. The results suggest that on the experimental side, populating those fissioning systems with a large size and a high energy favors better determination of the postsaddle friction strength by measuring the GDR γ multiplicity emitted in the fission process.

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I. INTRODUCTION

Measurements on excitation functions of prescission particles and evaporation residue cross sections deviate significantly from predictions using the standard Bohr-Wheeler statistical models, especially at high energy [1-12]. This is considered as arising from the neglect of friction effects in fission [13-26]. Stochastic approaches to fission have thus been widely applied to address the deviation between measured and theoretical multiplicity. A systematic investigation [27] based on Langevin models suggested that by assuming that the friction is weak inside the barrier but increases with deformation from the saddle to the scission points, one can simultaneously reproduce many observables including particle multiplicities and evaporation residue cross sections for both light and heavy compound nuclei. With the nonequilibrium statistical-operator theory, Aleshin [28] has found a rise of the reduced friction coefficient with deformation. However, it has been pointed out that Langevin calculations [27] using the full one-body dissipation (OBD) strength give a too-large evaporation residue cross section or a too-small fission probability as compared to experimental values. By reducing the strength of the wall formula, a better agreement with the experimental values of prescission neutron multiplicity was obtained by Pomorski et al. [16]. These results were confirmed in recent works [17,25]. Moreover, the same excitation function data of light fissioning systems used in Ref. [27] were also reproduced in Ref. [17], where a modified OBD strength (which is a decreasing function of deformation) was utilized.

While the two types of deformation-dependent friction give a similar presaddle friction strength, the predicted postsaddle friction strength has a great difference. We note that a strong postsaddle friction suggested in Ref. [27] stems from a fit to multiplicity data of heavy systems such as ²⁵¹Es. We also note that calculations with the modified OBD strength [17] or with the chaos-weighted wall formula [25] assuming a low postsaddle friction focused on a reproduction of fission data of light decaying system, but they significantly underestimate multiplicity data of heavy systems (A > 250). Because the shape dependence of friction has been identified as one of the key issues [26] when Langevin models are applied to handle fission of a hot nucleus and because presaddle friction has been well constrained within a narrow range [17,22,25,29], it is clear that the precise knowledge of the magnitude of postsaddle friction becomes very crucial for probing the deformation dependence of friction in nuclear fission. Currently, a number of studies have been carried out in constraining the strength of presaddle friction through various observables, such as evaporation residues [6] and its spin distributions [22], fission probabilities [30,31], and the width of fission-fragment charge distributions [29]. In contrast, fewer efforts have been made to better determine the strength of saddle-to-scission friction.

In contrast to the above-mentioned quantities that depend on presaddle dissipation only, in order to pin down the property of postsaddle dissipation, suitable observables that are sensitive to saddle-to-scission dissipation are critical. Light particles are evaporated along the entire fission path when the fissioning nucleus proceeds from the equilibrium spherical shape to the scission configuration. They thus carry fundamental information on the postsaddle nuclear dissipation. Besides neutrons and light charged particles (LCPs) that have been employed in the studies of postsaddle friction [27,32], giant dipole resonance (GDR) γ rays [5,33–36] have also been considered as important indicators of postsaddle friction. Calculations based on Langevin models [33,37] have reproduced well the experimental data on prescission GDR γ energy spectra and multiplicities of ²²⁴Th systems. Moreover, it has been found that prescission γ multiplicity is a more sensitive signature of nuclear friction than prescission light particles [27].

To date, experimental data on the GDR γ emission in the fission process of a hot nucleus are still scarce. Also, earlier measurements were performed only at several energy points which lie in a limited range of excitation energy. Given that

^{*}Corresponding author: yewei@seu.edu.cn

the prescission GDR γ multiplicity constitutes an important tool of fission dynamics, relevant theoretical analyses may become very necessary once systematic data on GDR γ become available from new experiments. Prescission particles including γ rays are a function of excitation energy [17,38]. In addition, postsaddle particles vary sizably with increasing size of the fissioning nucleus [13,39]. In this context, to guide experimental explorations the present work is devoted to studying under which experimental conditions the sensitivity of the GDR γ emission to postsaddle friction can be enhanced. To this end, we investigate the roles of system size and excitation energy in probing postsaddle friction with GDR γ rays in the framework of Langevin models. The stochastic approach [14,16,17,19,21,22,25,27] has been demonstrated to successfully describe a large volume of experimental data on many observables including multiplicities of prescission light particles and GDR γ rays and evaporation residue cross sections for a lot of compound nuclei over a wide range of angular momentum and fissility.

II. THEORETICAL MODEL

An account of the combination of the dynamical Langevin equation with a statistical decay model (CDSM) is given; for more details, see Refs. [27,37,40]. The dynamic part of the CDSM is described by the Langevin equation that is expressed by entropy. We employ the following one-dimensional overdamped Langevin equation [27] to perform the trajectory calculations:

$$\frac{dq}{dt} = \frac{T}{M\beta} \frac{dS}{dq} + \sqrt{\frac{T}{M\beta}} \Gamma(t).$$
(1)

Here q is the dimensionless fission coordinate and is defined as half the distance between the center of mass of the future fission fragments divided by the radius of the compound nucleus, and β is the dissipation strength. M is the inertia parameter that drops out of the overdamped equation. The temperature in Eq. (1) is denoted by T and $\Gamma(t)$ is a fluctuating force with $\langle \Gamma(t) \rangle = 0$ and $\langle \Gamma(t)\Gamma(t') \rangle = 2\delta(t - t')$. The driving force of the Langevin equation is calculated from the entropy

$$S(q, E^*) = 2\sqrt{a(q)[E^* - V(q)]}$$
 (2)

where E^* is the excitation energy of the system.

The liquid-drop parametrization of Myers and Swiatecki [41,42] for the potential is used. The potential energy is given by

$$V(A, Z, L, q) = a_2 \left[1 - k \left(\frac{N - Z}{A} \right)^2 \right] A^{2/3} [B_s(q) - 1]$$

+ $c_3 \frac{Z^2}{A^{1/3}} [B_c(q) - 1] + c_r L^2 A^{-5/3} B_r(q).$
(3)

Here we have dropped terms which do not depend on the deformation coordinate q. The parameters a_2 , c_3 , k, and c_r in Eq. (3) are not related to q and their values are taken from Ref. [42]. $B_s(q)$, $B_c(q)$, and $B_r(q)$ are the surface, Coulomb,

and rotational energy terms, respectively. In our dynamical calculations we use a $\{c, h, \alpha\}$ [43] parametrization of the compound nucleus shape. Since only symmetrical fission is considered, the parameter describing the asymmetry of the shape is set to $\alpha = 0$. B_r is proportional to the inverse of the rigid body moment of inertia. One can parametrize $B_s(q)$ and $B_c(q)$ as a function of q in the form [44].

$$B_s(q) = \begin{cases} 1 + 2.844(q - 0.375)^2, & \text{if } q < 0.452, \\ 0.983 + 0.439(q - 0.375), & \text{if } q \ge 0.452. \end{cases}$$
(4)

and

$$B_{c}(q) = \begin{cases} 1 + [1 - B_{s}(q) + B_{f}/E_{ssp}]/2X & \text{for } q \ge 0.452, \\ 1 - 1.422(q - 0.375)^{2} & \text{for } q < 0.452. \end{cases}$$
(5)

where E_{ssp} is the surface energy of a spherical nucleus with fissility X. For a sphere $B_s = B_c = B_r = 1$.

A Langevin study [45] with the Lublin-Strasbourg liquiddrop model [46] and the finite-range liquid-drop model [47] demonstrated that prescission particle multiplicities obtained are similar for the two different parametrizations of the potential V(q).

In constructing the entropy, the deformation-dependent level density parameter is used:

$$a(q) = a_1 A + a_2 A^{2/3} B_s(q), \tag{6}$$

where A is the mass number, and $a_1 = 0.073$ and $a_2 = 0.095$ are taken from Ignatyuk *et al.* [48].

In the CDSM, light-particle evaporation is coupled to the fission mode by a Monte Carlo procedure. The emission width of a particle of kind $v(=n, p, \alpha)$ is given by [49]

$$\Gamma_{\nu} = (2s_{\nu} + 1) \frac{m_{\nu}}{\pi^2 \hbar^2 \rho_c(E^*)} \\ \times \int_0^{E^* - B_{\nu}} d\varepsilon_{\nu} \rho_R(E^* - B_{\nu} - \varepsilon_{\nu}) \varepsilon_{\nu} \sigma_{\rm inv}(\varepsilon_{\nu}), \quad (7)$$

where s_{ν} is the spin of the emitted particle ν , and m_{ν} its reduced mass with respect to the residual nucleus. The level densities of the compound and residual nuclei are denoted by $\rho_c(E^*)$ and $\rho_R(E^* - B_{\nu} - \varepsilon_{\nu})$. B_{ν} are the liquid-drop binding energies. ε is the kinetic energy of the emitted particle and $\sigma_{inv}(\varepsilon_{\nu})$ is the inverse cross sections [49].

For the emission of giant dipole γ quanta we take the formula of Lynn [50],

$$\Gamma_{\gamma} = \frac{3}{\rho_c(E^*)} \int_0^{E^*} d\varepsilon \rho_c(E^* - \varepsilon) f(\varepsilon), \qquad (8)$$

with

$$f(\varepsilon) = \frac{4}{3\pi} \frac{1+\kappa}{m_n c^2} \frac{e^2}{\hbar c} \frac{NZ}{A} \frac{\Gamma_G \varepsilon^4}{(\Gamma_G \varepsilon)^2 + (\varepsilon^2 - E_G^2)^2}, \quad (9)$$

with $\kappa = 0.75$. E_G and Γ_G are the position and width of the GDR, and their values are taken from Ref. [40].

A formula suggested in Ref. [27] is used to evaluate the deformation dependence of the charged-particle emission barriers:

$$V_c(q) = V_v \times B_c(q). \tag{10}$$

Here $V_{\nu} = \frac{(Z-Z_{\nu})Z_{\nu}K_{\nu}}{R_{\nu}+1.6}$ with $K_{\nu} = 1.32$ for α , and 1.15 for protons. $R_{\nu} = 1.21[(A - A_{\nu})^{1/3} + A_{\nu}^{1/3}] + (3.4/\varepsilon_{\nu}^{1/2})\delta_{\nu,n}$, where A_{ν} and ε_{ν} is the mass number and the kinetic energy of the emitted particle $\nu = n, p, \alpha$.

Because the mass formula [51] contains the deformationdependent surface energy term and Coulomb energy term, the particle binding energy B_i ($i = n, p, \alpha$) is also a function of q [27,32,52], and it can be written as

$$B_i(q) = M_p(q) - M_d(q) + M_i.$$
 (11)

where M_i $(i = n, p, \alpha)$ is the mass of the emitted particles. $M_p(q)$ and $M_d(q)$ are the masses of the mother and daughter nuclei, respectively.

The CDSM describes the fission process as follows: At early times, the decay of the system is modeled by means of the Langevin equation. After the fission probability flow over the fission barrier attains its quasistationary value, the decay of the compound system is described by a statistical branch. In the statistical branch we calculate the decay widths for particle emission and the fission width and use a standard Monte Carlo cascade procedure with the weights $\Gamma_i / \Gamma_{\text{tot}}$ with $(i = fission, n, p, \alpha, \gamma)$ and $\Gamma_{\text{tot}} = \sum_i \Gamma_i$. This procedure allows for multiple emissions of light particles and higher chance of fission. In case fission is decided there, one switches again to the Langevin equation for computing the evolution from saddle to scission. Prescission various particle multiplicities are calculated by counting the number of corresponding evaporated particle events registered in the dynamic and statistical branch of the CDSM. To accumulate sufficient statistics, 10⁷ Langevin trajectories are simulated.

Because it is not simple to know the initial condition of an experimentally formed compound nucleus, we choose a δ function at the potential bottom as the initial condition for solving Eq. (1). The Langevin equation is started at the position of the ground state of the spherical nucleus. For starting a trajectory an orbital angular momentum value is sampled from the fusion spin distribution, whose form reads

$$\frac{d\sigma(\ell)}{d\ell} = \frac{2\pi}{k^2} \frac{2\ell+1}{1 + \exp[(\ell - \ell_c)/\delta\ell]}.$$
 (12)

The parameters ℓ_c and $\delta \ell$ are the critical angular momenta for fusion and diffuseness, respectively. The final results are weighted over all relevant waves; i.e., the spin distribution is used as the angular momentum weight function.

The choice of initial conditions in the form of δ function and fusion spin distributions given by Eq. (12) means that the present model is restricted to a situation where an equilibrated compound system has been formed in a heavy-ion fusion reaction before the fission process starts. It is unsuitable to describe the fast-fission reactions [27].

III. RESULTS AND DISCUSSION

The overdamped Langevin equation predicts a fission rate that is different from that estimated by the full Langevin equation. In a recent work, Boilley *et al.* [53] discussed the accuracy of the approximation of overdamping in great detail, in particular for describing the saddle-to-scission fission motion. As a consequence, β extracted from the data by using the former equation could be different from the β by using the latter equation. Friction could be model dependent.

Before presenting our results, it is interesting to mention the validity range of the used CDSM. The overdamped Langevin equation [27] is derived from Langevin equation by neglecting the term $\dot{p}/(\beta M)$. Here \dot{p} denotes the momentum of the collective motion conjugate to the coordinate q. It has been shown [54] that when the friction coefficient β is larger than 2ω (which is the angular frequency related to the potential), the motion is not periodical and has an overdamped character. In this case, the collective momentum relaxes much faster than the coordinate. For overdamped Langevin dynamics, i.e., in the overdamped region, the value of the collective kinetic energy is small and thus neglected in the square brackets in Eq. (2) that is employed to evaluate the driving force of the Langevin equation.

A comprehensive analysis for various types of fission data made in Ref. [2] has showed that a strong friction is needed to fit experimental data when the excitation energy of the decaying system is high. The conclusion has been further confirmed in a great of number of recent experimental studies. For example, a very recent experimental analysis of the width of fission-fragment charge distributions by Schmitt *et al.* [29] indicated the importance of introducing friction effects at high energy. A systematic application of CDSM to fusion-fission reactions has indicated that the overdamped Langevin equation can provide a good description of many observables [27].

In addition, it has been shown that the quasistationary fission rates given by the approximation of the overdamping are different from those obtained via a full Langevin approach, and that the deviation becomes smaller with increasing the friction strength. Our present calculations with friction strength larger than 3×10^{21} s⁻¹ satisfy the conditions that use the CDSM, which is based on the approximation of overdamped motion [40].

A. Role of excitation energy in probing β with M_{γ}

In this study the presaddle friction strength is set as $3 \text{ zs}^{-1} (1 \text{ zs}^{-1} = 10^{21} \text{ s}^{-1})$, in accordance with recent theoretical estimates and experimental analyses for those observables [17,22,25,27,29] that are determined by presaddle friction. Light particles including GDR γ rays can be evaporated when the decaying nucleus descends from the saddle to the scission points; they are thus sensitive to the average strength of the saddle-to-scission friction β . Therefore, in this work dynamical calculations are performed considering different values of β , which is equal to (3, 5, 7, 10, 15, 20) zs⁻¹ throughout the whole postsaddle fission process.

We first survey the evolution of the emitted M_{γ} as a function of β with excitation energy. As illustration we show in Fig. 1 the calculation for ²⁴⁰Cf systems at three excitation energies $E^* = 45$, 80, and 180 MeV, which cover a broad excitationenergy domain.

It is observed from the figure that the slope of the curve of M_{γ} vs β , which reflects the sensitivity of the postsaddle GDR γ emission on the postsaddle friction strength, is influenced strongly by excitation energy. Specifically, at low energy of



FIG. 1. (Color online) The emitted postsaddle GDR γ -ray multiplicities (M_{γ}) as a function of the postsaddle friction strength (β) for ²⁴⁰Cf systems at critical angular momentum $\ell_c = 30\hbar$ for three excitation energies $E^* = 45$ MeV (squares), 80 MeV (circles), and 180 MeV (triangles), respectively. The lines are to guide the eye.

45 MeV, M_{γ} changes 2.15 (×10⁻³) as β varies from 3 to 20 zs⁻¹. The change is apparently smaller than that at $E^* =$ 80 MeV, where it reaches 4.76 (×10⁻³), meaning a greater sensitivity to β . Furthermore, the slope becomes much steeper when E^* arrives at 180 MeV. The physical understanding for this phenomenon is as follows: Compared to neutrons and LCPs, GDR γ decays are a relatively weak decay channel that yields a small postsaddle γ multiplicity; accordingly, this lowers its sensitivity to friction.

With an increase in excitation energy, evaporation can compete with fission more effectively, yielding a greater emission of presaddle neutrons and GDR γ rays (Fig. 2).



FIG. 2. (Color online) Presaddle multiplicity of neutrons (a) and GDR γ rays (b) at different β for ²⁴⁰Cf systems at $\ell_c = 30\hbar$ for three excitation energies $E^* = 45$ MeV (squares), 80 MeV (circles), and 180 MeV (triangles), respectively. The lines are to guide the eye.

However, at high energy, even though more presaddle emission including GDR γ rays carries away the excitation energy, a considerable portion of the excitation energy is still left that is available for further particle evaporation during the descend from the saddle to the scission point, which causes an increase in the number of emitted particles. So, with growing energy the multiplicity of various particles including that of GDR γ rises quickly; for instance, at $\beta = 7 \text{ zs}^{-1}$ the value of M_{ν} at $E^* = 80$ and 180 MeV is approximately three and nine times larger than the one at $E^* = 45$ MeV. A large M_{γ} increases its sensitive dependence on β . Therefore, the conclusion deduced from Fig. 1 offers the new experimental indicator of determination of value of the postsaddle friction strength; that is, raising excitation energy can substantially enhance the sensitivity of γ emission to postsaddle dissipation. In other words, creating a condition of high excitation energy in experiment can place a more stringent constraint on the postsaddle friction strength determined from analyzing GDR γ data in fission.

When a nucleus fissions, it will undergo a deformation. The role of deformation in fission dynamics has been stressed [55–57]. In particular, Lestone [52] showed that the opposite influence of deformation on neutrons and LCPs was critical to reproduce the two kinds of prescission multiplicity data simultaneously. In this respect, the deformation parameter is different from other types of parameters such as fission barrier. What the latter primarily affects is the competition between fission channel and all light particle decay channels, and not the competition among the different kinds of light particle decay.

We investigate here the influence of deformation on the results shown in Fig. 1. As an example, we make a calculation at $E^* = 100$ MeV for the cases with and without deformation effects.

Besides Coulomb emission barrier of LCPs, deformation also affects the binding energies of neutrons and LCPs; see Fig. 3. The latter corrections are particularly important for correctly evaluating the change of the neutron and the LCPs multiplicity with deformation, as indicated by Lestone [52]. This is because deformation decreases the binding energy of neutrons but increases that of LCPs. A rise of binding energies of LCPs surpasses the influence stemming from a drop of their Coulomb emission barriers on the LCPs multiplicity.

As a consequence, accounting for the deformation effects can sizably increase postsaddle neutrons and suppress LCP decays (see Table I). Considering the strong competition among different decaying channels, an enhanced neutron emission will decrease the γ multiplicity. The rising speed of a decreasing γ multiplicity in the presence of deformation effects with β is slowed down; i.e., the sensitivity to β is weakened. The expectation is confirmed in Fig. 4, implying the importance of increasing the magnitude of M_{γ} for obtaining a larger sensitivity on friction. Thus, the calculation taking account of the deformation effects further demonstrates that populating a fissioning system with higher energies favors a more accurate determination of the strength of the postsaddle friction with GDR γ emission, a conclusion that reinforces that derived from Fig. 1.



FIG. 3. (Color online) (a) Change in neutron, proton, and α -particle binding energies as a function of deformation coordinate q relative to the spherical binding energies for compound systems ²⁴⁰Cf. (b) Emission barrier (V_c) of protons and α particles of ²⁴⁰Cf as a function of q.

B. Role of system size in probing β with M_{γ}

In addition to the appreciable modification due to a variation in excitation energy to the particle multiplicity, it has been revealed that postsaddle particle numbers are also a function of the mass number of fissioning nuclei [17,27,39]. Apart from that, earlier experimental measurements of GDR γ emission were made for fissioning nuclei spanning a mass region of A =(200–240) [30,33–36]. Under this circumstance, it is necessary to examine the role of system size in probing the postsaddle friction with GDR γ multiplicity. For this purpose, we compare the sensitivity of M_{γ} to β for three decaying systems, i.e., ²⁰⁰Pb, ²²⁴Th, and ²⁴⁰Cf, which have a marked difference in their sizes.

The most prominent feature seen from Fig. 5 is that the M_{γ} of ²⁴⁰Cf shows a greater sensitivity to friction than that of ²²⁴Th, and the latter experiences a more obvious change

TABLE I. The computed multiplicities of postsaddle neutrons (M_n) , protons (M_p) and α particles (M_α) of ²⁴⁰Cf systems at various β for the cases with and without deformation effects at $\ell_c = 30\hbar$ and $E^* = 100$ MeV.

$\frac{\beta}{(10^{21} \text{ s}^{-1})}$	Without deformation effects			With deformation effects		
	M_n	M_p	M_{lpha}	M_n	M_p	M_{lpha}
3	0.896	0.0217	0.0334	1.030	0.0166	0.0215
5	1.211	0.0296	0.0447	1.407	0.0218	0.0272
7	1.479	0.0360	0.0537	1.722	0.0260	0.0317
10	1.816	0.0440	0.0646	2.114	0.0310	0.0368
15	2.268	0.0537	0.0773	2.628	0.0369	0.0430
20	2.623	0.0611	0.0864	3.019	0.0412	0.0473



FIG. 4. (Color online) Deformation effects on the emitted postsaddle GDR γ -ray multiplicities as a function of β for ²⁴⁰Cf at $\ell_c = 30\hbar$ and $E^* = 100$ MeV. Open and solid symbols represent the results without and with deformation effects, respectively.

with β than the case of light ²⁰⁰Pb nucleus, indicating a strong influence of size of a fissioning nucleus on the sensitivity. It is a consequence of an apparent difference in the emitted postsaddle GDR γ multiplicity for the three nuclei. Two reasons contribute to the difference.

First, under the same conditions of excitation energy and angular momentum, presaddle neutrons (a dominant decay channel of an excited nucleus) and GDR γ rays rise with decreasing size of decaying nuclei; see Fig. 6. Moreover, one can notice that for light ²⁰⁰Pb, presaddle GDR multiplicities are far larger than postsaddle ones. A picture like ²⁰⁰Pb is observed for ²²⁴Th. These are in contrast with the heavy ²⁴⁰Cf, which has a greater postsaddle γ multiplicity. A stronger presaddle neutron emission for light fissioning systems carries away more energy, which decreases postsaddle emission, including GDR γ rays.



FIG. 5. (Color online) Comparison of the sensitivity of the emitted postsaddle GDR γ -ray multiplicities on the postsaddle friction strength β for three systems ²⁰⁰Pb (squares), ²²⁴Th (circles), and ²⁴⁰Cf (triangles) at $\ell_c = 30\hbar$ and $E^* = 120$ MeV.



FIG. 6. (Color online) Presaddle multiplicity of neutrons (a) and GDR γ rays (b) at different β for three systems ²⁰⁰Pb (squares), ²²⁴Th (circles), and ²⁴⁰Cf (triangles) at $\ell_c = 30\hbar$ and $E^* = 120$ MeV. The lines are to guide the eye.

Another reason is that a heavier system has a longer saddle-to-scission distance, which provides a longer time for evaporating GDR γ rays. Also, a stronger friction hinders the fission more strongly, leading to a longer saddle-to-scission delay that makes more time available for particle emission. As a result, when the fissioning nucleus becomes heavier, the postsaddle γ multiplicity at a stronger friction becomes greater; that is, M_{γ} displays a larger sensitivity to β for a heavier system.

The calculation above does not contain deformation effects. For a lighter fissioning nucleus, deformation plays a smaller role in particle emission than it does in a heavier fissioning nucleus, as the fission of a light system involves a smaller deformation. We check the influence arising from a variation in the deformation effects on particle evaporation with system size on the conclusion drawn from Fig. 5 and find that it is minor.

Therefore, the comparison of the three systems with different sizes suggests that on the experimental side, producing a compound system with a larger size can increase the sensitivity of GDR γ emission to postsaddle dissipation and, thereby, it can provide a stronger constraint on the strength of postsaddle friction.

We have also carried out the same calculations as those presented in Figs. 1 and 5 but at slightly different presaddle friction strengths and at other angular momenta. The results obtained are analogous to those discussed above and, hence, they are not repeated here.

A discernible variation of presaddle neutrons and GDR γ rays of ²⁰⁰Pb with postsaddle β is seen in Fig. 6, though it is not very prominent. This could be due to backstreaming effects. Nuclear fission is a stochastic process and backstreaming is typical of Brownian motion [26,27]. Because of the influence

of random forces, the fissioning system has a probability to come back to inside the barrier even if it passes over the potential barrier. Since the backstreaming trajectories experience the dissipation outside the saddle, as pointed out in a recent work [58] where the influence of postsaddle friction on evaporation residue cross sections was observed, the postsaddle dissipation strength thus has an influence on the magnitude of the backstreaming effects and, correspondingly, it affects presaddle evaporation multiplicities. Also, excitation energy, angular momentum, and the size of the fissioning system could influence the amplitude of the backstreaming effect and, hence, the presaddle emission.

It has been demonstrated that for nuclei with $A \ge 200$, the fission barrier height predicted from some representative macroscopic models differs very little: see Refs. [26,59] for more details. On the other hand, a slight variation in the height of fission barrier only has an effect on those observables, such as fission probability and evaporation residue cross sections that are directly related to fission barriers, but its effect on prescisson neutrons is small. Furthermore, for the decaying system considered here, neutrons are a dominant decay channel whereas the emission of GDR γ rays is quite weak. It means that even if the number of emitted pressaddle neutrons has a change caused by the small difference in the fission barrier, the influence on postsaddle GDR γ rays is rather small. The three factors mentioned above exhibit that using fission barriers predicted from different macroscopic models such as liquid-drop model [42] and finite-range model [47] in calculation has a minor effect on our results.

C. Influence of level-density parameter on M_{γ} vs β

Level-density parameters are a crucial input quantity both in the statistical calculations of the decay widths and in the Langevin simulations of the decay of the fission nucleus. Two sets of coefficients a_1 and a_2 are often used in the level-density parameter formula [see Eq. (6)]. One set is from Ignatyuk *et al.*'s (Ign) prescription [48]. The other set is taken from predictions from Tõke and Swiatecki (TS) [60], who suggested that $a_1 = 0.0685$ MeV⁻¹ and $a_2 = 0.274$ MeV⁻¹. It was illustrated [61] that the single-particle level densities obtained with the phenomenological Thomas-Fermi method [60] and with the Yukawa folded mean-field approach are similar.

Since it has been demonstrated [17,25,27] that the Ign coefficients can provide a good and systematic fit to fission data, they are used in the calculations of Figs. 1 and 5.

Fission rates turn out to be sensitive to the level-density parameter [25], so the Ign and TS coefficients give different particle multiplicities including GDR γ rays. Figure 7 shows that while the calculated M_{γ} using TS coefficients are different from that using Ign coefficients (Fig. 1), the predicted influence of excitation energy on the sensitivity of GDR γ rays to friction remains the same.

In Fig. 8, we show the results of M_{γ} as a function of β for ²⁰⁰Pb, ²²⁴Th, and ²⁴⁰Cf calculated using TS coefficients. By comparing Fig. 8 with Fig. 5, one can easily see that different parametrizations of level-density parameters do not change the role that system size plays in the sensitivity of M_{γ} to β . Also,



FIG. 7. (Color online) Same as Fig. 1, but the level-density parameter of Tõke *et al.* [60] is used in calculation (open symbols connected by solid curves). The results of using Ignatyuk's level-density parameter [48] are also shown (solid symbols connected by dashed curves).

we note that compared to the results of Ign coefficients, TS coefficients give a larger M_{γ} for heavy ²⁴⁰Cf and a smaller M_{γ} for light ²²⁴Th and ²⁰⁰Pb. This could be due to the significant difference of postsaddle evaporation multiplicities for heavy systems ²⁴⁰Cf and light systems ²²⁴Th and ²⁰⁰Pb [27,39] as well as the effect of different level-density parametrizations on various pre- and postsaddle particle emissions and on the competition among the different types of particle decay channels.

Previous discussions illustrate that our conclusions are robust with respect to a variation in level-density parameter and in fission barrier as well as to an inclusion of deformation effects in calculation. The robustness could stem from the fact that excitation energy and system size are two independent parameters affecting postsaddle GDR γ rays. Consequently, while a variation in other parameters could modify M_{γ} , this does not significantly alter the roles of excitation energy and system size in GDR γ emission as a signature of nuclear dissipation.

Our present calculations show that conditions of high energy and large system size favor better determination of β from GDR γ emission. However, the calculated M_{γ} and hence the extracted value of β has a dependence on the choice of the level-density parameter. Apart from that, the approximation of the overdamping might introduce another ambiguity on the



FIG. 8. (Color online) Same as Fig. 5, but the level-density parameter of Tõke *et al.* [60] is used in calculation (open symbols connected by solid curves). The results of using Ignatyuk's level-density parameter [48] are also shown (solid symbols connected by dashed curves).

value of β . Therefore, more experimental and theoretical researches are still needed for a quantification of β .

IV. CONCLUSIONS

Based on the Langevin equation coupled to a statistical model of particle emission, we have calculated the change of the postsaddle GDR γ multiplicity M_{γ} with the strength of postsaddle friction β at different excitation energies. It has been shown that the sensitivity of M_{γ} to β rises significantly with increasing excitation energy. Moreover, we find that the sensitivity is obviously greater for heavy decaying systems and that the level density parametrization affects the final results. Our findings suggest that experimentally, when employing GDR γ rays to constrain the postsaddle dissipation strength, it is optimal to produce fissioning systems with larger sizes and higher energies.

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