Limits on tensor coupling from neutron β decay

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Limits on the tensor couplings generating a Fierz interference term b in mixed Gamow-Teller Fermi decays can be derived by combining data from measurements of angular correlation parameters in neutron decay, the neutron lifetime, and $G_V = G_F V_{ud}$ as extracted from measurements of the $\mathcal{F}t$ values from the $0^+ \rightarrow 0^+$ superallowed decay data set. These limits are derived by comparing the neutron β -decay rate as predicted in the standard model with the measured decay rate while allowing for the existence of beyond the standard model (BSM) couplings. We analyze limits derived from the electron-neutrino asymmetry a, or the beta asymmetry A, finding that the most stringent limits for C_T/C_A under the assumption of no right-handed neutrinos is $-0.0026 < C_T/C_A < 0.0024$ (95% C.L.) for the two most recent values of A. The derived limits on scalar and tensor couplings have the useful property that they are independent of BSM extensions with vector or axial-vector symmetry to first order.

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where \mathcal{O}_i corresponds to the standard Dirac operators

 $(1, \gamma_{\mu}, \gamma_{\mu}\gamma_{5}, \gamma_{5}, \sigma_{\lambda\mu}), \ \sigma_{\lambda\mu} = -i/2(\gamma_{\lambda}\gamma_{\mu} - \gamma_{\mu}\gamma_{\lambda}) \ \text{and} \ p, \ n,$

e, and v represent the hadronic and leptonic fields [7]. The

strength of each type of interaction in the lepton current is given

by a coupling constant C_i and C'_i where $i \in \{V, A, S, T, P\}$ are

In the following we present an analysis of a "lifetimeconsistency test" for neutron β decay, from which we derive relevant limits for beyond the standard model physics, in particular for new scalar and tensor couplings. Our analysis utilizes high precision data from $0^+ \rightarrow 0^+$ decays and neutron decay and does not supplant a more general fitting procedure to obtain limits from all β -decay data [1–3]. We note, however, that our limits are comparable to those obtained from fits to the entire β -decay set when similar assumptions are made (no right-handed neutrinos). This brief report was inspired by comments in Bhattacharya et al. [4] and begun as a part of thesis research [5]; however, we note that additional details have subsequently been published by Ivanov, Pitschmann, and Troitskaya [6].

The general method compares the measured value of the neutron lifetime, whose current particle data group (PDG) value is $\tau_n = 880.1 \pm 1.1$ s, to a prediction of the neutron lifetime using the measured weak coupling strength from $0^+ \rightarrow 0^+$ and the value of the axial-vector coupling constant, $\lambda \equiv g_A/g_V$, extracted from angular correlations measurements. Because this comparison requires the interpretation of specific angular correlation measurements to consistently extract limits, we analyze some specific cases of interest associated with the electron-neutrino correlation a, and the β asymmetry A. As discussed in more detail later in the text, these limits are independent of beyond standard model extensions with vector or axial-vector symmetry to first order. We understand that this treatment is not exhaustive, nor should it supplant direct limits on the Fierz term in the neutron system, but it is intended to indicate the utility of these limits.

Derivation of impact of the Fierz term on the neutron decay *rate*. β decay can be represented, using all possible Lorentzinvariant couplings, by the Hamiltonian density,

$$\mathcal{H} = \sum_{i} (\bar{p}\mathcal{O}_{i}n)(\bar{e}\mathcal{O}_{i}(C_{i} + C_{i}^{'}\gamma_{5})\nu) + \text{H.c.}, \qquad (1)$$

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the vector, axial-vector, scalar, tensor, and pseudoscalar interactions, respectively. In the scenario where $|C_i| = |C_i|$, parity is maximally violated, and in the standard model $|C_V| = |C'_V|$ and $|C_A| = |C'_A|$ and $C_S = C'_S = C_T = C'_T = C_P = C'_P = 0$. These restrictions are experimentally determined, leaving the possibility for deviations below the current experimental precision. Limits on tensor couplings can be derived by noting that the decay rate for neutron β decay can be written as (ignoring, at present, the possibility of a Fierz term)

$$\frac{1}{\tau_n} = \frac{G_V^2}{2\pi^3\hbar} (1+3\lambda^2) f_n (1+\Delta_{\rm RC}),$$
 (2)

where, under the conserved vector current hypothesis, $G_{\rm V} =$ $G_{\rm F}|V_{ud}|, f_n$ is the statistical rate function for the neutron defined as

$$f_n = I_0(x_0)(1 + \Delta_f) = 1.6887,$$
 (3)

$$I_k(x_0) = \int_1^{x_0} x^{1-k} (x_0 - x)^2 \sqrt{x^2 - 1} \, dx, \qquad (4)$$

and where x and x_0 are the electron total energy and endpoint energy in terms of the electron rest mass, and Δ_f is the Coulomb and recoil correction for the phase-space integral $I_0(x_0) = 1.6299$. The standard model electroweak radiative corrections are denoted by $\Delta_{\rm RC} = 3.90(8) \times 10^{-2}$ [8]. G_F is the Fermi coupling constant as extracted from muon decay [9], and V_{ud} is the first element of the Cabibbo-Kobyashi-Maskawa (CKM) quark mixing matrix. One can also predict the neutron decay rate from $0^+ \rightarrow 0^+$ decays, by using the extracted value of \tilde{G}_V from the average $\mathcal{F}t_{0^+ \to 0^+}$ and λ from neutron angular correlation measurements, where if the Fierz

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TABLE I. The statistical weighting and ratio of the phase-space factors is presented for each of the 13 isotopes used in the $0^+ \rightarrow 0^+$ superallowed data set to calculate the average $\mathcal{F}t$ value. $I_k(\tilde{x}_0)$ are the statistical rate functions defined in Eq. (4) and calculated by Towner and Hardy [10,11].

Isotope	$\mathcal{F}t$	$I_0(ilde{x}_0)$	$I_1(\tilde{x}_0)$	$I_1(\tilde{x}_0)/I_0(\tilde{x}_0)$	
^o C 3067.7(4.6)		2.3004(12)	1.42401(74)	0.6190(5)	
¹⁴ O	3071.5(3.3)	42.772(23)	18.743(10)	0.4382(3)	
²² Mg	3078.0(7.4)	418.39(17)	128.948(52)	0.3082(2)	
³⁴ Ar	3069.6(8.5)	3414.5(1.5)	724.56(32)	0.2122(1)	
$^{26}Al^m$	3072.4(1.4)	478.237(38)	143.662(11)	0.3004(1)	
³⁴ Cl	3070.6(2.1)	1995.96(47)	466.26(11)	0.2336(1)	
38 K ^m	3072.5(2.4)	3297.88(34)	701.459(69)	0.2127(1)	
⁴² Sc	3072.4(2.7)	4472.24(1.15)	895.34(23)	0.2002(1)	
⁴⁶ V	3073.3(2.7)	7209.47(90)	1317.17(16)	0.1827(1)	
⁵⁰ Mn	3070.9(2.8)	10745.97(57)	1816.07(10)	0.1690(1)	
⁵⁴ Co	3069.9(3.3)	15766.6(2.9)	2470.63(45)	0.1567(1)	
⁶² Ga	3071.5(7.2)	26400.2(8.3)	3719.7(1.2)	0.1409(1)	
⁷⁴ Rb	3078.0(13.0)	47300.0(110)	5884.1(1.4)	0.1244(4)	
Average	3072.08(79)		、 /	0.2579(1)	

term is zero $\tilde{G}_{\rm V}^2 = G_{\rm V}^2$,

$$\frac{1}{\tau_{0^+}} = \frac{\tilde{G}_{\rm V}^2}{2\pi^3\hbar} (1+3\lambda^2) f_n (1+\Delta_{\rm RC}).$$
(5)

A nonzero Fierz term will alter the neutron decay rate τ_n via a $\langle m_e/E \rangle$ term in the phase-space integral and modify the value of G_V extracted from superallowed Fermi decays to

$$\tilde{G}_{\rm V}^2 = G_{\rm V}^2 \langle 1 + b_{\rm F} \gamma I_1(\tilde{x}_0) / I_0(\tilde{x}_0) \rangle, \tag{6}$$

where $\gamma = \sqrt{1 - (Z\alpha)^2}$, Z is the atomic number, α is the fine structure constant, and \tilde{x}_0 is the end-point energy for the superallowed Fermi decay isotopes, and $I_1(\tilde{x}_0)/I_0(\tilde{x}_0)$ corresponds to the ratio of phase-space integrals over the superallowed decay used in the determination of V_{ud} and $b_F = 2 \operatorname{Re}(C_S/C_V)$ [10]. For the moment we will ignore the changes in λ induced by *b*; this will be addressed in detail after presenting an outline of our approach. In Table I, the 13 isotopes included in the determination of the average $\mathcal{F}t$ are listed with the absolute uncertainty on the measurement and the statistical rate function and the ratio $I_1(\tilde{x}_0)/I_0(\tilde{x}_0)$ [10]. The reported values include both recoil and Coulomb corrections. Writing Eqs. (2) and (5) in terms of G_V , b_F and *b*, we have

$$\frac{1}{\tau_n} = \frac{G_V^2}{2\pi^3\hbar} (1+3\lambda^2) f_n (1+\Delta_{\rm RC})(1+\kappa b),$$
(7)

and

$$\frac{1}{\tau_{0^+}} = \frac{G_V^2}{2\pi^3\hbar} (1+3\lambda^2) f_n (1+\Delta_{\rm RC}) (1+\zeta b_{\rm F}), \qquad (8)$$

where $\kappa = I_1(x_0)/I_0(x_0)$, $\tilde{\kappa} = I_1(\tilde{x}_0)/I_0(\tilde{x}_0)$, and $\zeta = \langle \gamma \tilde{\kappa} \rangle \sim 0.2560$. In Eq. (7), the term $(1 + \kappa b)$ arises from the neutron phase-space integral when $b \neq 0$, and the $(1 + \zeta b_F)$ term in Eq. (8) is from substitution of measured G_V using Eq. (6). Taking the difference between the measured neutron decay rate and the decay rate predicted from $0^+ \rightarrow 0^+$ decays in

terms of measured quantities gives

$$\tau_n K(1+3\lambda^2) = \frac{1+\zeta b_{\rm F}}{1+\kappa b},\tag{9}$$

where all the constants have been combined into $K = \tilde{G}_V^2 f_n (1 + \Delta_{\rm RC})/(2\pi^3\hbar) = 1.934(2) \times 10^{-4} \, {\rm s}^{-1}$, we express Eq. (9). Critically, leading order differences in the predicted versus measured decay rates must come from scalar and tensor-induced couplings in the Fierz term, and any new physics which adjusts the value of G_V and λ affects both rates uniformly (such as right-handed currents). Additionally, the impact of the scalar coupling determined in the superallowed decays is suppressed by ζ because of the much higher end-point energy of these decays relative to neutron β decay.

Under the assumptions of this analysis the Fierz inference term can be approximated in terms of the scalar C_S/C_V and tensor C_T/C_A couplings [1]:

$$b = \frac{2\sqrt{1-\alpha^2}}{1+3\lambda^2} \left[\operatorname{Re}\left(\frac{C_{\rm S}}{C_{\rm V}}\right) + 3\lambda^2 \operatorname{Re}\left(\frac{C_{\rm T}}{C_{\rm A}}\right) \right].$$
(10)

At this point, we already have a reasonably strong constraint on new physics by using Eqs. (9) and (10), and the definition of $b_{\rm F}$,

$$\frac{C_{\rm T}}{C_{\rm A}}(6\lambda^2\gamma_n) = \frac{\delta b}{\tau_n K\kappa} - 2\gamma_n \frac{C_{\rm S}}{C_{\rm V}} - (1+3\lambda^2)\kappa^{-1} \quad (11)$$

where $\delta b = \langle 1 + 2\gamma (C_S/C_V)\tilde{\kappa} \rangle$ and $\gamma_n = \sqrt{1 - \alpha^2}$. Using the PDG values for $\lambda = -1.2701(25)$, $\tau_n = 880.1(1.1)$ s, and $V_{ud} = 0.97425(40)$ [12] and limits on scalar couplings, $C_S/C_V = 0.0011(13)$, from the superallowed data set [10], one can place a limit on the tensor coupling. This results in $2-\sigma$ (95% C.L.) limits of $-0.0009 < C_T/C_A < 0.0125$. Note that if only the Perkeo II result for $\lambda = -1.2739(19)$ [13] is used, then limits shift to $-0.0012 < C_T/C_A < 0.0065$.

b dependence of λ . The limits obtained from Eq. (11) have ignored the fact that λ is determined experimentally by measuring correlation coefficients, which typically are

TABLE II. Experimental results for λ from measurements of A using a single parameter fit to energy dependence of the asymmetry are summarized. For each measurement the reported analysis window and the phase-space integral ratios over that range are listed. The last column estimates the change in the asymmetry where b = 0.001.

Experiment	Α	λ	Energy range	$I_0(x_1, x_2)/I_0(x_0)$	$I_1(x_1, x_2)/I_0(x_0)$	$\Delta A \ (\%)$
Perkeo [19]	-0.1146(19)	-1.262(5)	>200 keV	0.801	0.581	0.06
Perkeo II [13]	-0.11951(50)	-1.2755(13)	325–675 keV	0.843	0.534	0.05
III TPC [20]	-0.1160(9)(12)	-1.266(4)	200–700 keV	0.807	0.583	0.06
UCNA [18]	-0.11952(110)	-1.2756(30)	275–625 keV	0.828	0.557	0.06
Yerozolimsky [21]	-0.1135(14)	-1.2594(38)	250–780 keV	0.824	0.561	0.06

modified by the existence of a Fierz term as in

$$X_m(E_e) = \frac{X_0(E_e)}{1 + b \, m_e/E_e},\tag{12}$$

where $X_m(E_e) \in \{a, A, B, ...\}$ is the measured value of the coefficient as a function of electron energy. For this analysis we will focus on *a* and *A* because closed form expressions can be obtained for limits on tensor couplings and they have the highest sensitivity to λ . The method used to extract the correlation coefficient and the energy range of the analysis will impact the sensitivity to *b*, as will be shown explicitly in the case of *a*. Note that we are also explicitly ignoring imaginary couplings and the affect of tensor and scalar couplings on the angular correlations *a* and *A*, which are second order in C_S and C_T .

λ derived from the measured β-asymmetry parameter A_m . Single parameter fits to the energy dependence of the β asymmetry will modify the measured quantity by $A_0/(1 + b\langle I_1(x_1, x_2)/I_0(x_1, x_2)\rangle)$, where x_1 and x_2 are the limits of the energy range used in analysis. Table II shows the ratio of the phase-space integrals over the reported analysis energy range for several of the experiments that measure the asymmetry and would be subject to the type of dilution analyzed here. The leading order expression of A_o (where one has already corrected the measured asymmetry for small radiative and recoil order corrections [7]) in terms of λ is

$$A_0 = 2|\lambda| \frac{1-|\lambda|}{1+3\lambda^2},\tag{13}$$

which, combined with Eqs. (10)–(12), gives

$$3\lambda^{2} = \left[\frac{h}{C_{\mathrm{T}}/C_{\mathrm{A}}\gamma_{n}\kappa+1}\right]$$
$$= 3\left(\frac{-1-\sqrt{1-A_{m}^{2}(3\tilde{C}_{\mathrm{T}}+2/A_{m})\tilde{C}_{\mathrm{S}}}}{3A_{m}\tilde{C}_{\mathrm{T}}+2}\right)^{2}.$$
 (14)

In Eq. (14), we have made the following substitutions: $\tilde{C}_x = C_x \gamma_n + 1$ and $h = (\delta b / \tau_n K) - \gamma_n \kappa (C_S / C_V) - 1$. We assume here that the BSM scalar and tensor couplings make negligible contributions to the radiative and recoil-order corrections. For BSM couplings at the ≈ 0.01 level, this should certainly be true, as can be seen by inspecting radiative corrections, which are precisely defined in Refs. [14–16]. Equation (14) can be

solved in closed form, producing three roots:

$$C_{\rm T}/C_{\rm A} = \begin{cases} -s/(3A_m\gamma_n), \\ [h(A_m\{s+3A_m/\kappa\}\tilde{C}_{\rm S}-2) \\ -A_m(sh^2+3\tilde{C}_{\rm S}^2/\kappa) \\ \pm 2h\gamma_n\sqrt{1-A_m(s-3A_m/\kappa)\tilde{h}}] \\ \times (3A_m^2\gamma_n\tilde{h}^2)^{-1}, \end{cases}$$
(15)

where $\tilde{h} = (1 + h + C_s \gamma)$ and $s = 2 + 3A_m$, two of which predict large values of C_T/C_A and are ruled out by current experimental limits,

$$\frac{C_{\rm T}}{C_{\rm A}} = \begin{cases} 4.67 \pm 0.05, & \text{(first root)} \\ 25.8 \pm 0.9, & \text{(negative root)} \end{cases}.$$
 (16)

The remaining solution gives a 2- σ limit on the tensor coupling of $-0.0015 < C_T/C_A < 0.0079$, using the PDG values for the measured parameters.

The PDG value of λ includes the result of Mostovoi *et al.* [17], which is determined by simultaneous measurement of the β asymmetry A, and the neutrino asymmetry B, and would therefore require careful analysis to determine the impact a nonzero Fierz term would have on the extracted λ . Using the results from Perkeo II [13] and UCNA [18], $A_m =$ -0.11931(46), we then obtain $-0.0026 < C_T/C_A < 0.0024$ (95% C.L.). Note that the 30% reduction in the limit is in large part because of the increased error bar on A_0 used by the PDG to account for the variations in the current measurements; including this factor would increase the limit to $2\sigma = 0.0039$. Future prospects for reducing this limit via increasing the precision of A_m are shown in Fig. 1, where we see that next generation experiments will reduce the uncertainty on A_m to the point where $\delta \tau_n$ becomes the leading contribution to this limit.

 λ derived from the electron-neutrino correlation parameter a_0 . Measurements of the electron-neutrino correlation parameter a_0 are being proposed and carried out at several cold neutron facilities worldwide with the aim of significantly improving the current precision of 3.9% to < 0.1% [22–24]. Determining the *a* coefficient can be performed by directly measuring the angular distribution of emitted electron and proton in coincidence, in which case the $a_0/(1 + bm_e/E_e)$ scaling can apply (directional method), or via a measurement of the proton energy spectrum. Directly fitting the proton spectrum or using discrete points from the spectrum (spectral fit method) as in Stratowa *et al.* [25] will result in $a_m = a_0 + x_f b$, where $x_f \sim 0.09$ as determined by Monte Carlo. An alternate method of analyzing proton spectral data is an integral analysis, which

has the advantage of being much less sensitive to the presence of a Fierz term, as presented in [26]. In such an analysis one compares the integral rate over a fixed energy range to the total decay rate. This results in a linear scaling:

$$a_m = a_0 + x_I b = \frac{1 - \lambda^2}{1 + 3\lambda^2} + x_I b, \tag{17}$$

where $a_m = -0.103(4)$ is the measured value of the correlation coefficient and $x_I \sim 0.008$ from [26], which was confirmed by this analysis using Monte Carlo. Using Eqs. (10) and (11), and the two expressions for the measured coefficient including a Fierz term, we can find a solution for C_T/C_A in terms of measurement quantities,

$$\frac{C_{\mathrm{T}}}{C_{\mathrm{A}}} = \begin{cases} \frac{h\left(a+\frac{1}{3}\right) - \left(1-a_{m}+x_{I}\gamma_{n}\frac{c_{\mathrm{S}}}{c_{\mathrm{V}}}\right)/\kappa}{\gamma_{n}\left(1-a_{m}+x_{I}\gamma_{n}\frac{c_{\mathrm{S}}}{c_{\mathrm{V}}}\right) - x\gamma_{n}h} & \text{(Linear),} \\ \frac{h\left(1+\frac{1}{3a_{m}}\right) - \left(\frac{1}{a_{m}}-1-\gamma_{n}\kappa\frac{c_{\mathrm{S}}}{c_{\mathrm{V}}}\right)/\kappa}{\gamma_{n}\left(\frac{1}{a_{m}}-1-\kappa\gamma_{n}\frac{c_{\mathrm{S}}}{c_{\mathrm{V}}}-h\kappa\right)} & \text{(Inverse),} \end{cases}$$
(18)

where we have made use of the substitutions defined previously. This approach is more straightforward because of the fact that, unlike *A*, there are no terms which are linear in λ in Eq. (17). Using the current PDG value for $a_0 = -0.1030(40)$ (extracted using the spectral fit method, $x_f = 0.09$) we find limits of $-0.0134 < C_T/C_A < 0.0324$ (95% C.L.).

While this limit is not currently competitive with those obtained through measurements of the β asymmetry, Fig. 1 shows that determining *a* from angular distributions is more sensitive than the spectral fit method to tensor couplings, and the next generation experiments should improve upon current limits set by A_0 . (For example, a measurement such as Nab [22], aiming for a precision of $\Delta a/a \simeq 10^{-3}$, could set constraints of $|C_T/C_A| < 0.0015$ with the current uncertainty on τ_n .)

We also note that the differing sensitivities to the Fierz term afforded by the integral proton spectrum analysis and directional distribution measurements afford an alternate method to extract limits on the Fierz term.

Conclusion. In this analysis we have presented a selfconsistent derivation of limits to tensor couplings in the weak interaction, using experimentally measured quantities from neutron β decay and $0^+ \rightarrow 0^+$ superallowed Fermi decays. By calculating the difference of the measured and predicted neutron β -decay rate, we are able to derive a limit to the tensor coupling $-0.0026 < C_T/C_A < 0.0024$, under the assumption of maximal parity violation and no right-handed neutrinos, where $|C_X| = |C'_X|$. Noting that a nonzero Fierz term would



FIG. 1. (Color online) The 2- σ limits on C_T/C_A are shown for the methods of extracting *a* discussed in the text, along with those from fitting the energy dependence of the β asymmetry *A*. Current limits on *a* are taken from the PDG2012 (Beringer [12]), and future limits denote the proposed sensitivity of experiments such as Nab (Počanić [22]) (directional method), aCORN (Wietfeldt [24]) (directional method), and aSPECT (Baeßler [23]) (fit method). For this analysis we consider the β asymmetry from UCNA (Mendenhall [18]) and Perkeo II (Mund [13]), and the proposed limits are from the PERC experiment (Dubbers [27]).

modify the experimentally reported value of λ , we have shown that the measured correlation coefficients a_m and A_m can be used to set limits on the tensor couplings that are competitive with those obtained from global fits to the available data [1–3]. If the precision of A or a reaches the 0.1% level, then the accuracy of the neutron lifetime becomes the leading contribution to the derived limits.

These results can be used to set constraints on the effective couplings from Bhattacharya *et al.* [4], where the tensor coupling is given as $C_T/C_A = -4(g_T\epsilon_T/g_A)/(1 + \epsilon_L - \epsilon_R))$, where $\epsilon_{R(L)}$ represent the effective right- and left-handed couplings and both are zero in the SM. In general, this directly leads to a limit of $-5.8 \times 10^{-4} < g_T\epsilon_T/(g_A\{1 + \epsilon_L - \epsilon_R\}) < 6.4 \times 10^{-4}$. However, under the assumption that BSM physics arises from tensor couplings then $\epsilon_R = \epsilon_L = 0$, and this simplifies to $g_T\epsilon_T/g_A$.

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