

Fractal structure of near-threshold quarkonium production off cold nuclear matter

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We investigate near-threshold production of quarkonium resonances in cold nuclear matter through a phenomenologically motivated scaling theory with two exponents, which are fixed by existing data on near-threshold J/ψ production in proton-nucleus collisions. Interestingly, it seems possible to extend one of the exponents to the production of other mesons in cold nuclear matter. The scaling theory can be tested and refined in experiments at the upcoming high-intensity Facility for Antiproton and Ion Research (FAIR) accelerator complex at GSI.

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Quantum chromodynamics (QCD) is a very well established theory for strong interactions, but it is hard to generate predictions from first principles using it. As a result, a large body of data remains to be integrated into the theory. One exception has been the near-threshold production of light mesons. An effective field theory, called chiral perturbation theory, has been very successfully applied to this set of problems, and at the same time also connected to lattice QCD. In the last two decades there has been enormous progress in the theory and increased clarity in observations of the production of light mesons in inelastic pp collisions below the threshold for double meson production. This has had repercussions in related fields such as the exploration of CP violations in decays of hadrons. In this paper we suggest that there are interesting open questions in the near-threshold production of charmonia, which may similarly benefit from the coming experiments at the Facility for Antiproton and Ion Research (FAIR).

The near-threshold production of J/ψ and other charmonia is of great interest today. Since the mass of the proton, $M_p = 0.938$ GeV, and the mass of the J/ψ , $M_H = 3.097$ GeV, the threshold energy for production of J/ψ in pp collisions, in the center of mass of the colliding particles, $\sqrt{S_0} = 4.973$ GeV. Unfortunately this is not small enough to apply any of the usual hadron effective theories, since there are too many hadron states with masses below the J/ψ , and all of them would have to be accounted for in an appropriate effective theory. On the other hand, $\sqrt{S_0}$ is too small for quantitatively accurate use of perturbative QCD along with nonrelativistic QCD (NRQCD) or other models, a procedure which has seen some success at higher energy [1–4].

In this paper we explore the problem using a tool which has been useful for the discovery of effective field theories in the past: the investigation of power laws in data. If power law scaling works as the description of some data then the nonperturbative content is codified into a very small set of parameters. This in itself is a gain. Moreover, in the past, the exponents, called either fractal dimensions or anomalous dimensions, according to the context, have suggested description through

effective field theories. The reason is that such exponents are eigenvalues of renormalization group transformations [5]. In the Appendix we briefly review the arguments which connect power laws with scaling symmetries.

We are interested in the production of the J/ψ , in pp and pA collisions near the threshold energy $\sqrt{S_0}$. We follow the convention of writing the center-of-mass (CM) energy, \sqrt{S} , in the equivalent pp system; for a fixed target configuration this means $S = 2M_p(E_b + M_p)$, where E_b is the beam energy. The total inclusive cross section, σ , can only be a function of \sqrt{S} , M_H , M_p , and the nuclear mass M_A . Then, a dimensional argument allows us to write

$$S_0\sigma = f(Y, A, h), \quad \text{where} \quad Y = \frac{1}{2} \ln \left(\frac{S}{S_0} \right),$$

$$A = \frac{M_A}{M_p}, \quad h = \frac{M_H}{M_p}, \quad (1)$$

and f is a dimensionless function. In this definition of A , we neglected the effects of nuclear binding, which are expected to be less than 1%, and isospin effects, which could be slightly larger. We take masses and branching ratios from [6]. Since we discuss only the J/ψ , we will lighten the notation by dropping h from the list of arguments of f .

In this paper we report a scaling analysis of J/ψ cross sections in a dilepton channel in pp and pA collisions from the lowest up to ISR energies [7–22], but not beyond. Within this data corpus, corrections for kinematic acceptance limitations of each experiment needed for global analyses are discussed in [3,23]. We decided to examine $B\sigma$ rather than σ , where B is the branching ratio in the dielectron or dimuon channel. The reason is that over the years the value of B has moved by more than its error bar. When the inclusive cross section in one of these dilepton channels is measured, this uncertainty does not affect the result. Some experiments correct their data for nuclear effects according to a formula A^α , with α obtained from their data. To start with, we undid this correction, since this power is part of our global analysis.

Extremely close to threshold the variable Y is small and close to zero. In proton-nucleus scattering, we expect some Fermi motion: even though the CM of the nucleus is at rest, individual nucleons may be moving. The typical energy of

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this movement is of the order of the binding energy per nucleon [24], and hence comparable to other effects which we have neglected. Clearly, Fermi motion can be detected with experiments close to threshold since the cross section vanishes otherwise. However, for $Y > 0.1$, the effect can be neglected.

Notice that even at the smallest Y for which data is available there are many inelastic channels open. In the hadronic language, the J/ψ may be produced through intermediate ψ' , or χ_c states, or through virtual $\bar{D}D$ states, or along with multiple light mesons. In the partonic language, there may be colored intermediate states which exchange soft gluons with the rest of the hadronic system, especially near threshold, thereby spoiling QCD factorization. In any case, these different channels may give rise to different dependence in transverse and longitudinal momentum of the J/ψ . As always, the analysis of total inclusive cross sections, which involve summing over all intermediate states, is much simpler and more robust than the analysis of each channel separately.

The form of the function $f(Y, A, h)$ eventually has to be computed from QCD, and it may be arbitrarily complicated. For example, one expects to be able to compute it using perturbative QCD when h is large and Y small [25] or h is small and Y large [1–4]. An important prerequisite for this is the factorization of the initial state into parton distribution functions, which are nonperturbative matrix elements in QCD, which have always been extracted from experiments, and are only now becoming amenable to lattice computations. If the factorization theorems were valid then certain kinematic scalings could be expected to hold [26] which are seen to fail for $Y \approx 1$ [22]. So, in the near-threshold region for J/ψ perturbative QCD inspired models for quarkonium production is not viable. Alternatively, in the region where both Y and A are large, there is now some possibility of computing the function using the color glass condensate picture [27]. Given the complexity of the full problem, we do not seek a model description which might be valid for all Y and A , but restrict our attention to $O(10^{-2}) \leq Y \leq O(10^0)$.

Very close to threshold, where all the particles are moving nonrelativistically, $Y \propto v^2$, where v is the velocity of the lightest particle. In this region, one expects to be able to perform a partial-wave expansion of the amplitude, and hence write f as a Taylor expansion in v , and hence \sqrt{Y} . For $Y > 0.1$ such a Taylor expansion may not be exactly valid, but it might still be interesting to investigate a power law in Y . In pp collisions, with $A = 1$, we find that the data admits a simple power-law fit with

$$Bf(Y, 1) = Bf_1 Y^\beta,$$

$$\text{with } Bf_1 = (2.0 \pm 0.4)S_0 \text{ nb}, \quad \beta = 3.20 \pm 0.26, \quad (2)$$

where the covariance of the parameters is -0.934 . The quality of the description is shown in Fig. 1. In [16] a different phenomenological parametrization was suggested, using intuition based on perturbative QCD and widely used forms of parton distributions. Fitting this form to the pp data, we find

$$Bf(Y, 1) = K(1 - \exp(-Y))^\nu,$$

$$\text{where } K = (59 \pm 5)S_0 \text{ nb}, \quad \nu = 6.5 \pm 0.6, \quad (3)$$

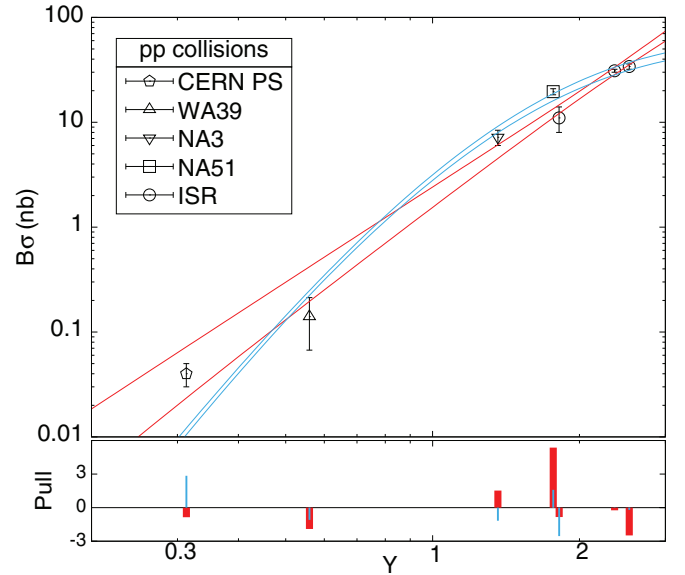


FIG. 1. (Color online) Cross section for the production of J/ψ in pp collisions. The band in red encloses the 68% confidence limits of the two-parameter fit of Eq. (2), and that in blue of Eq. (3). We also show the pull from the data for each fit color-coded similarly. For the former the largest contributions to χ^2 come from $Y > 1$; for the latter, from $Y < 1$.

with covariance of the parameters being 0.823. Overall, the quality of fit is similar to the power law. It is instructive to examine the pull on these fits from various pieces of data. First, we note that for both fits there is a tension between the NA51 measurement [17] and that at the lowest ISR energy [11]. As a result, future high-precision experiments in the range $1 \leq Y \leq 2$ would be very welcome. We also find that the largest contributions to χ^2 for the form in Eq. (2) come from $Y > 1$, whereas the largest contributions to the fit form in Eq. (3) come from $Y < 1$. This is consistent with the argument for the latter form from the parton language, and supports a preference for the power law for small Y . Precision experiments in the range $Y \leq 0.6$ will certainly help in deciding between these models.

We return to the scaling form and try a power law parametrization for the dependence on A . Such a power law, A^α , has been attempted by all experimental groups. However, both the NA50 [19,20] and NA60 [22] experiments independently seem to show that a parametrization of this kind, with α independent of Y , is inadequate at the two energies where each experiment has taken data. To accommodate this, we can write

$$f(Y, A) = fY^\beta A^{\alpha(Y)}. \quad (4)$$

We explain in the Appendix the origin of such a multifractal scaling. The data corpus supports a particularly simple dependence on Y :

$$\alpha = (0.76 \pm 0.02) + (0.10 \pm 0.01)Y, \quad (5)$$

with covariance -0.984 between the two coefficients. The data from CERN-PS [10] were not used in this fit, because of the large errors in this measurement; its inclusion does not change the central values of the fit significantly but increases the errors.

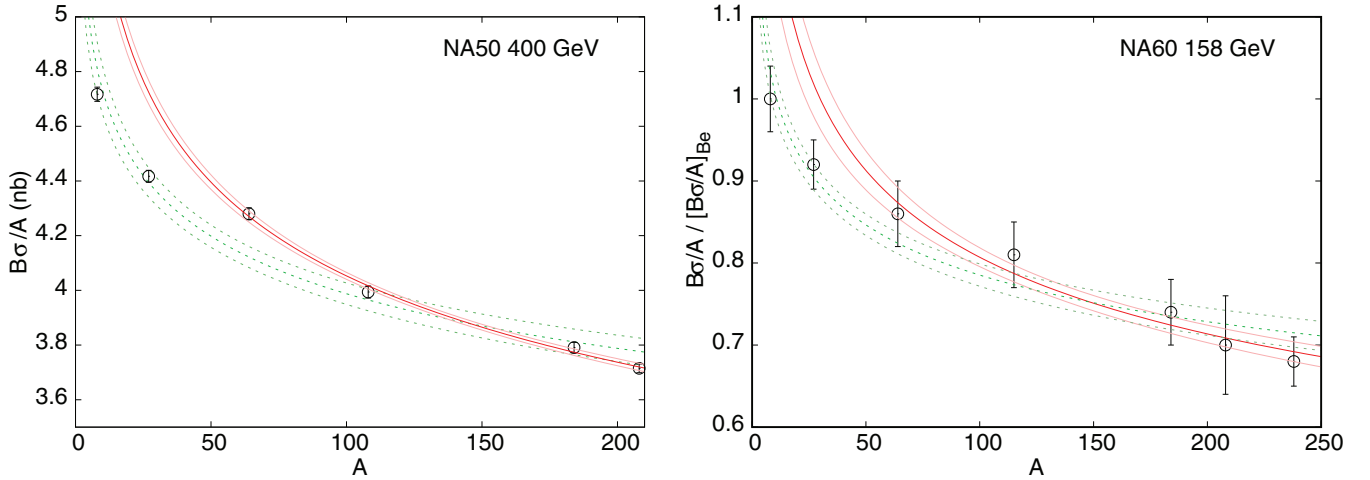


FIG. 2. (Color online) The dependence on A of the inclusive J/ψ production cross section measured in [20] (left panel) and [22] (right panel). Note that the power-law parametrization is an excellent description of the data for $A > 50$ (full line) but is substantially worse when applied to all A (dashed line) in the NA50 data. The errors in the NA60 data are too large for this discrimination, although the trend is similar. The bands enclose the 68% confidence limits of the fits.

It would be interesting to test in future experiments whether this exponent is modified near $Y = 0.1$ as the thresholds for $\psi(2S)$ and the χ s are approached.

However, we would like to examine this parametrization in some more detail. Recall that the scaling theory naturally has

$$f(Y, A) \simeq Y^\beta f(\tilde{A}) \simeq f Y^\beta A^\alpha, \quad (6)$$

where \tilde{A} is a renormalized parameter, and the power laws are asymptotic expressions. We have already argued that the function $f(Y, A)$ may be a complicated function of its arguments when it is fully computed in QCD. However, for this analysis, we are interested in power laws which arise when one or more of these parameters go to zero or infinity. For this reason, we will not be very perturbed if data for all A are not explained by a single power α . It is sufficient for the purposes of a scaling analysis if the power law is valid for $A \gg 1$. The extensive data taken by NA50 [20] are able to test this accurately. Since all the data in this set were taken for fixed Y , we can simply fit $f(Y, A) = f A^\alpha$. For $A > 50$ we find $\alpha = 0.883 \pm 0.005$ and $Bf = (6.9 \pm 0.2)S_0$ nb, with $\chi^2/\text{DOF} = 1.08$ (where DOF denotes degrees of freedom). In contrast, if data for all A are taken into account, then one obtains a substantially worse fit with $\chi^2/\text{DOF} = 20$. This is shown in Fig. 2.

It is of interest to compare the above parametrization with a widely used model of cold nuclear effects in J/ψ production. When the Glauber model is treated in the eikonal approximation, then one obtains the stretched exponential form

$$f(Y, A) = A \exp(-\gamma A^{1/3}) f(Y). \quad (7)$$

In the model, the dimensionless number $\gamma = \rho \sigma_{abs} \lambda$, with ρ being the nuclear density, σ_{abs} having the interpretation of a cross section for absorption of J/ψ in cold nuclear medium, and $\lambda A^{1/3}$ being the path length of the J/ψ in the nucleus (see, for example, [28]). A Glauber model of this kind supposes multiple incoherent collisions off a large number of nuclei

along a classical path followed by the J/ψ . This is unlikely to be correct for all A . If we apply it only to $A > 50$ from the data set of [20], then we find $\chi^2/\text{DOF} = 2.4$ for a fit which gives $\gamma = 0.071 \pm 0.005$. Clearly the goodness of fit is marginally acceptable, but is worse than the power law.¹ Using the values $\rho = 0.16/\text{fm}^3$ and $\lambda = 1.1$ fm, the above fit for γ gives $\sigma_{abs} = (4.0 \pm 0.3)$ mb. This is consistent with the values extracted for the same Y in [22]. Extending this model to all A definitely lowers the quality of the fit, with $\chi^2/\text{DOF} = 5$.

If we wish to test the scaling for large A , then the only experimental data one can use are NA38 [14], NA50 [19,20], HERA-B [21], and NA60 [22]. The HERA-B data are not available in a form suitable for reanalysis. The errors in the NA38 data are substantially larger than the other two remaining sets. The power-law parametrization of the NA60 data shown in Fig. 2 for $A > 50$ gives $\alpha = 0.76 \pm 0.02$. However, NA60 has another data set [22] taken at the same Y as the data in [20]. This data set is compatible with the data at the lower energy in the same experiment at the 95% confidence level, and with the data in [20] at the 68% level. So, the systematic errors in α seem to be such that there is little dependence on Y . In agreement with this, it turns out that the data from [19] yields a best-fit value of $\alpha = 0.79 \pm 0.02$, using only the data for $A > 50$, and this is compatible with α which is independent of Y . A simple fractal power law

$$\alpha \simeq 0.76 \pm 0.02 \quad (8)$$

could be supported by data, provided one takes into account only heavy nuclei with $A > 50$ when modeling the data by Eq. (6). Distinguishing between the behaviors in Eqs. (5) and (8) is something that experiments with $Y < 1$ could also contribute to the study of J/ψ .

¹A three-parameter fit to a stretched exponential $p_1 \exp(-p_2 A^{p_3})$ gives a best fit, with $p_3 \simeq 1/2$, which is indistinguishable from the power law.

Optical models of shadowing in very-low-energy nucleon-nucleus scattering predict $\alpha = 2/3$ for pions, in agreement with measurements [29]. If we want to extend the scaling theory to other mesons, using h as a perturbative parameter, then one could write $\alpha = 2/3 + 0.0317h$, in order to accommodate the result in Eq. (8). Using this formula one finds for the Υ a value $\alpha = 0.98 \pm 0.02$, in agreement with the value $\alpha = 0.962 \pm 0.006 \pm 0.008$ reported by E772 [30]. Coincidentally, the formula is also in agreement with the measured values of α for K , ρ , and ω production at low energy [31]. Since we do not have a theory of nuclear effects near threshold, this fact is, for now, merely an interesting phenomenological observation.

The scaling theory is inadequate to relate the values of $f(\tilde{h})$ in experiments with different initial states: for example pA , $\bar{p}A$, $\pi^\pm A$, etc. If some form of the factorization theorems were valid, then there could be relations such as

$$\frac{f(pA \rightarrow H)}{f(\bar{p}A \rightarrow H)} = \frac{f(\pi A \rightarrow H)}{f(\pi A' \rightarrow H)}, \quad (9)$$

where $f(ab \rightarrow h) = \sigma S_0$ and σ is the total inclusive scattering cross section for the reaction $ab \rightarrow h$ at a fixed Y . Such tests could also be performed with light ions instead of pions. The validity of factorization would be an interesting investigation especially since it is often used in crossed near-threshold reactions to make a connection between hadron decays and the CKM matrix elements.

Proceeding beyond total inclusive cross sections near threshold is hard due to the paucity of data. There is scattered evidence for cold nuclear effects in $\langle p_T^2 \rangle$, from NA38 [14] and HERA-B [21]. However, there is too little data for a systematic study of the effect. The sparse corpus shows a roughly linear rise of $\langle p_T^2 \rangle$ with Y , from a vanishing value at $Y = 0$. Clearly, high statistics studies of p_T and x_F distributions of the J/ψ near threshold would be very welcome.

Since data with beam energy $E_b < 100$ GeV are very sparse [7,10,12], the SIS-100 accelerator at FAIR in GSI presents an opportunity to probe the region of $Y \leq 0.4$ very thoroughly with modern statistics. With a beam luminosity of 1 Hz/nb, fair event rates could be obtained, and these scaling laws can be tested well. This would make the FAIR an ideal test bed for exploring the near-threshold production process for J/ψ as well as cold nuclear effects, including questions about factorization and p_T and x_F distributions. A wish list would contain measurements with pp and a variety of pA collisions to check the scaling of Eq. (5). The pp data could also be used to check the scaling exponent of Eq. (2) and whether it is compatible with the pA result. A range of A can be used to test the region of validity of the power law A^α . A systematic study of p_T and x_F distributions would also be extremely useful.

In summary, we have extracted a power law parametrization of the cross section for J/ψ production in pA collisions for $Y < 2$ and $A > 50$ in the form

$$B\sigma = f A^\alpha Y^\beta \quad \text{with} \quad f = (2.1 \pm 0.1) \text{ nb}, \\ \alpha = 0.76 \pm 0.02, \quad \text{and} \quad \beta = 3.20 \pm 0.26. \quad (10)$$

The exponent α could be a simple fractal dimension as above, or a multifractal dimension as in Eq. (5). This parametrization works at least as well as, and marginally better than, other

parametrizations for the near-threshold production of J/ψ . Our analysis uncovered some gaps in the data which upcoming experiments are in a position to fill. With such data it should be possible to decide between power-law parametrizations of the data and more complex behavior. Such data can also decide between a simple fractal dimension for α or a multifractal dimension. Interestingly, the fractal dimension α can be extended to other mesons such as the π , K , and Υ . Apart from being useful parametrizations of data, such power laws also reveal dynamical symmetries which equate a physical system with one Y and A to another with different Y and A . Such identities should eventually be computable from QCD. Since these scaling laws present fundamental restrictions on QCD, they should be of priority in upcoming low-energy and high-intensity experiments at the SIS-100/300.

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APPENDIX: SCALING, FRACTALS, AND MULTIFRACTALS

Meaningful relations between variables relevant to a physical system can be expressed in terms of dimensionless variables. If they are denoted $\pi_0, \pi_1, \pi_2, \pi_3$, etc., then one can write

$$\pi_0 = f(\pi_1, \pi_2, \pi_3, \dots). \quad (A1)$$

In the limit when $\pi_1 \rightarrow 0$, one may have the simplification that $f(\pi_1, \pi_2, \dots) \rightarrow f(\pi_2, \dots)$. This special case is called the scaling limit. An example is that of structure functions in deep-inelastic scattering, $F(x, \alpha_s)$, where x is the Bjorken variable and α_s the strong coupling [32]. In the limit $\alpha_s \rightarrow 0$ one obtains scaling, i.e., $F(x, \alpha_s) \rightarrow F(x)$. Since π_1 is not uniquely defined, one may have instead chosen to work with $1/\pi_1$. So one may examine asymptotic scaling for either $\pi_1 \rightarrow 0$ or ∞ .

In other cases one may find a behavior known as broken scaling. Then as $\pi_1 \rightarrow 0$, one finds

$$f(\pi_1, \pi_2, \dots) \rightarrow \pi_1^{\mu_1} f(\tilde{\pi}_2, \dots), \quad (A2)$$

where μ_1 is called an anomalous (or fractal) dimension and the $\tilde{\pi}_2 = r(\pi_1, \pi_2, \dots)$, etc. are called renormalized variables. One often encounters multiplicative renormalization in the form $\tilde{\pi}_2 = \pi_2/\pi_1^\nu$, but additive terms such as $\tilde{\pi}_2 = \pi_2/(\Pi_1 + \pi_1)^\nu$ are also known. Multiplicative renormalization implies an invariance under scaling: a change $\pi_2 \rightarrow \lambda\pi_2$ can be absorbed into the multiplicative change $\pi_1 \rightarrow \lambda^{1/\nu}\pi_1$. This is an asymptotic invariance of the function $f(\tilde{\pi}_2, \dots)$. For additive renormalization the invariance consists of a change $\pi_2 \rightarrow \lambda\pi_2$ implying $\pi_1 \rightarrow \lambda^{1/\nu}(\pi_1 - \Pi_1)$.

One may now examine the behavior of the function as $\tilde{\pi}_2 \rightarrow 0$ (or ∞). If there is broken scaling in this variable, then

one has

$$f(\pi_1, \pi_2, \pi_3, \dots) \rightarrow \pi_1^{\mu_1} \tilde{\pi}_2^{\mu_2} f(\tilde{\pi}_3, \dots). \quad (\text{A3})$$

Multiplicative renormalization implies two independent and constant powers, called fractal dimensions. However, for additive renormalization, μ_2 depends on π_1 . This is called multifractal scaling.

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