Derivation of reaction cross sections from experimental elastic backscattering probabilities

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The relationship between the backward elastic scattering probabilities and the reaction cross sections is derived. This is a very simple and useful method to extract reaction cross sections for heavy-ion systems. We compare the results of our method with those that use the traditional full elastic scattering angular distributions for several systems at energies near and above the Coulomb barrier. From the calculated reaction and capture cross sections that use the present method, we derive the cross sections of other mechanisms for weak nearly spherical systems.

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I. INTRODUCTION

For a long time, measurements of elastic scattering angular distributions that cover full angular ranges and optical model analysis have been used for the determination of reaction cross sections. The traditional method consists of deriving the parameters of the complex optical potentials which fit the experimental elastic scattering angular distributions and then of deriving the reaction cross sections predicted by these potentials. This can be performed because there is a well-known and clear relationship between the reaction and the elastic scattering processes due to the conservation of the total reaction flux. Any loss from the elastic scattering channel directly contributes to the reaction channel and vice versa. The direct measurement of the reaction cross section is a very difficult task since it would require the measurement of individual cross sections of all reaction channels, and most of them could be reached only by specific experiments. This would require different experimental setups not always available at the same laboratory and, consequently, such direct measurements would demand a large amount of beam time and would take probability some years to be reached. On the other hand, the measurement of elastic scattering angular distributions is much simpler than that. Even so, both the experimental part and the analysis of this latter method are not so simple. In the present paper, as an extension of previous papers of our group [1,2], we present a much simpler method to determine reaction cross sections than the one that uses full elastic scattering angular distribution data. It consists of measuring only elastic scattering at one backward angle, and from that, the extraction of the reaction cross sections can easily be performed.

The paper is organized in the following way. In Sec. II, we derive the formula for the extraction of the reaction cross sections by employing the experimental elastic scattering excitation function at a backward angle. In Sec. III, we use this formula to extract the reaction cross sections for several systems, and then we compare the results with those extracted from the experimental elastic scattering angular distributions for the same systems (^{4}He + ^{92}Mo, ^{4}He + ^{110,116}Cd, ^{4}He + ^{112,120}Sn, ^{6,7}Li + ^{64}Zn, and ^{16}O + ^{208}Pb). In this section, we also show the comparison of the calculated and experimental capture cross sections for the ^{6,7}Li + ^{64}Zn systems, and we predict the approximate cross sections for transfer and inelastic processes for those systems. In Sec. IV, the paper is summarized.

II. RELATIONSHIP BETWEEN THE REACTION CROSS SECTIONS AND THE ELASTIC SCATTERING EXCITATION FUNCTION AT A BACKWARD ANGLE

Quasielastic scattering is defined as the sum of elastic scattering, inelastic excitations, and a few nucleon transfer reactions. So, one defines the quasielastic scattering probability as

$$P_{qe}(E_{\mathrm{c.m.}}, J) = P_{\mathrm{el}}(E_{\mathrm{c.m.}}, J) + P_{\mathrm{in}}(E_{\mathrm{c.m.}}, J) + P_{\mathrm{tr}}(E_{\mathrm{c.m.}}, J), \quad (1)$$

where P_{el}, P_{in}, and P_{tr} are the elastic scattering, inelastic, and transfer probabilities, respectively. The total reaction probability may be written as

$$P_R(E_{\mathrm{c.m.}}, J) = P_{\mathrm{in}}(E_{\mathrm{c.m.}}, J) + P_{\mathrm{tr}}(E_{\mathrm{c.m.}}, J) + P_{\mathrm{cap}}(E_{\mathrm{c.m.}}, J) + P_{\mathrm{BU}}(E_{\mathrm{c.m.}}, J) + P_{\mathrm{DIC}}(E_{\mathrm{c.m.}}, J), \quad (2)$$

where P_R refers to the nonelastic reaction channel probability, P_{cap} is the capture probability (sum of evaporation-residue formation, fusion-fission, and quasifission probabilities or sum of fusion and quasifission probabilities), P_{DIC} is the deep inelastic collision probability, and P_{BU} is the breakup probability, important particularly when weakly bound nuclei are involved in the reaction [3]. Note that the deep inelastic collision process is only important at high energies above the Coulomb barrier. The deep inelastic collision process one can neglect because we are concerned with the low-energy region.

From the conservation of the total reaction flux, one can write [1,3] the expression,

$$P_{\mathrm{el}}(E_{\mathrm{c.m.}}, J) + P_R(E_{\mathrm{c.m.}}, J) = 1, \quad (3)$$

or

$$P_{qe}(E_{\mathrm{c.m.}}, J) + P_{\mathrm{cap}}(E_{\mathrm{c.m.}}, J) + P_{\mathrm{BU}}(E_{\mathrm{c.m.}}, J) = 1. \quad (4)$$

Here and in the following part of this paper, we neglect the deep inelastic collision since we are concerned with low energies. Thus, one can extract the reaction probability $P_R(E_{\mathrm{c.m.}}, J = 0)$

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at $J = 0$ from the experimental elastic scattering probability $P_{\rm el}(E_{\rm c.m.}, J = 0)$ at $J = 0$,

$$P_R(E_{\rm c.m.}, J = 0) = 1 - P_{\rm el}(E_{\rm c.m.}, J = 0) = 1 - d\sigma_{\rm el}(E_{\rm c.m.})/d\sigma_{\rm Ru}(E_{\rm c.m.}). \quad (5)$$

Here, the elastic scattering probability [3–7],

$$P_{\rm el}(E_{\rm c.m.}, J = 0) = d\sigma_{\rm el}/d\sigma_{\rm Ru} \quad (6)$$

for angular momentum $J = 0$ is given by the ratio of the elastic scattering differential cross section and the Rutherford differential cross section at 180°. Furthermore, one can approximate the J dependence of the reaction probability $P_R(E_{\rm c.m.}, J)$ at a given energy $E_{\rm c.m.}$ by shifting the energy [8],

$$P_R(E_{\rm c.m.}, J) \approx P_R\left(E_{\rm c.m.} - \frac{\hbar^2\Lambda}{2\mu R_b^2} - \frac{\hbar^4\Lambda^2}{2\mu^3\omega_b^2 R_b^6}, J = 0\right), \quad (7)$$

where $\Lambda = J(J+1)$, $R_b = R_b(J = 0)$ is the position of the Coulomb barrier at $J = 0$, $\mu = m_0 A_1 A_2/(A_1 + A_2)$ is the reduced mass (m_0 is the nucleon mass), and ω_b is the curvature of the s-wave potential barrier. By employing Eqs. (5) and (7), to convert the sum over the partial waves J into an integral and to express J by the variable $E = E_{\rm c.m.} - \frac{\hbar^2\Lambda}{2\mu R_b^2}$, we obtain the following simple expression:

$$\sigma_R(E_{\rm c.m.}) = \frac{\pi R_b^2}{E_{\rm c.m.}} \int_0^{E_{\rm c.m.}} dE[1 - d\sigma_{\rm el}(E)/d\sigma_{\rm Ru}(E)] \times \left[1 - \frac{4(E_{\rm c.m.} - E)}{\mu\omega_b^2 R_b^2}\right]. \quad (8)$$

The formula (8) relates the reaction cross section with the elastic scattering excitation function at a backward angle. By using the experimental elastic scattering probabilities $P_{\rm el}(E_{\rm c.m.}, J = 0)$ and Eq. (8), one can obtain the reaction cross sections.

It is important to mention that since the generalized form of the optical theorem connects the reaction cross section and forward elastic scattering amplitude [3] from our method, we show that the forward and backward elastic scattering amplitudes are related to each other.

III. RESULTS OF CALCULATIONS

A. Reaction cross sections

In the following, we show the results of our method to extract the reaction cross section by using Eq. (8). To calculate the position R_b and frequency ω_b of the Coulomb barrier, we use the nucleus-nucleus interaction potential $V(R, J)$ of Ref. [9]. For the nuclear part of the nucleus-nucleus potential, the double-folding formalism with the Skyrme-type density-dependent effective nucleon-nucleon interaction is employed [9].

To confirm the validity of our method of the extraction of σ_R, first, we compare the obtained reaction cross sections with those extracted from the traditional experimental elastic scattering angular distributions plus the optical potential method. The results from our method are shown as solid (red) and dashed (blue) lines in all figures from Figs. 1 to 6,

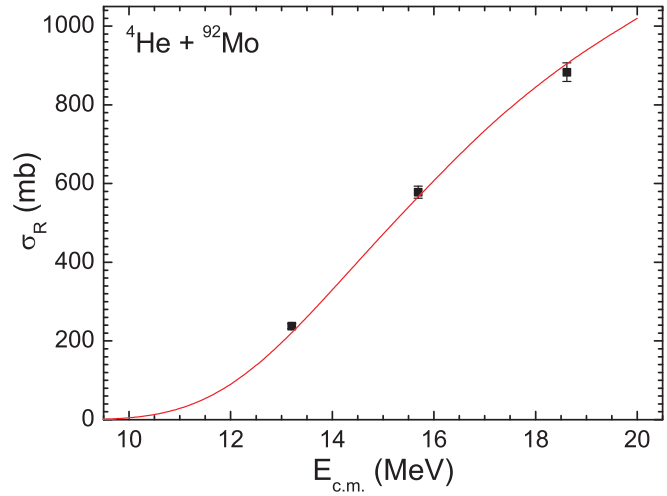

FIG. 1. (Color online) The extracted reaction cross sections that employ Eq. (8) (solid line) for the ^{4}He + ^{92}Mo reaction. The used experimental elastic scattering probabilities at the backward angle are from Ref. [10]. The reaction cross sections extracted from the experimental elastic scattering angular distribution with optical potential are presented by squares [10].

whereas, the results obtained from the traditional full elastic scattering angular distribution data are shown by solid squares. As the backscattering elastic data were not taken at 180° but rather at backward angles in the range from 150° to 170°, the corresponding center-of-mass energies were corrected by the centrifugal potential at the experimental angle as suggested by Timmers *et al.* [4]. In Figs. 7 and 8, we also show results of our calculations for the capture cross sections, and other curves are shown.

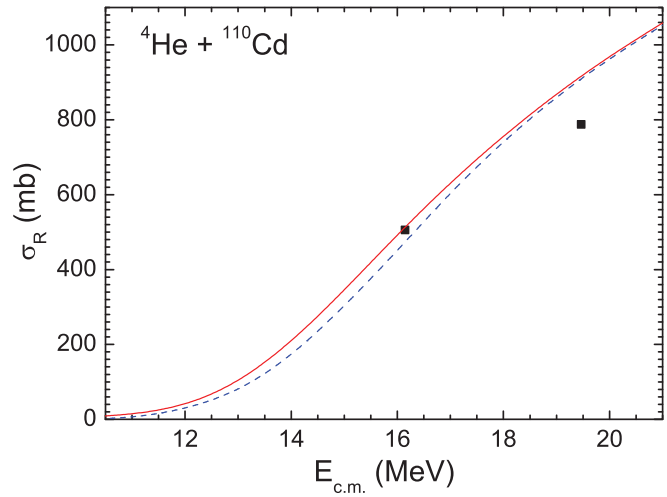

FIG. 2. (Color online) The extracted reaction cross sections that employ Eq. (8) (lines) for the ^{4}He + ^{110}Cd reaction. The used experimental elastic scattering probabilities at the backward angle are from Refs. [12,13] (solid line) and Ref. [14] (dashed lines). The reaction cross sections extracted from the experimental elastic scattering angular distribution with optical potential are presented by squares [10].

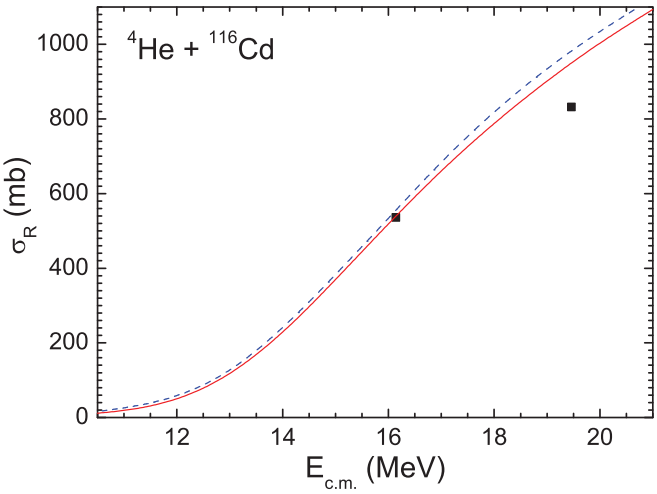

FIG. 3. (Color online) The same as in Fig. 2 but for the ^{4}He + ^{116}Cd reaction.

As can be observed in Figs. 1–8, there is good agreement between the reaction cross sections extracted from experimental elastic scattering at backward angle and from the experimental elastic scattering angular distributions with an optical potential for the reactions ^{4}He + ^{92}Mo, ^{4}He + ^{110,116}Cd, ^{4}He + ^{112,120}Sn, ^{16}O + ^{208}Pb, and ^{6,7}Li + ^{64}Zn at energies near and above the Coulomb barrier. One can see that the used formula (8) is suitable not only for almost spherical nuclei, but also for the reactions with slightly deformed target nuclei. The deformation effect is effectively contained in the experimental P_{el}. For very deformed nuclei, it is not possible experimentally to separate elastic events from the low-lying inelastic excitations. In our calculations, to obtain better agreement for the reactions ^{16}O + ^{208}Pb and ^{6}Li + ^{64}Zn, the extracted reaction cross sections were shifted in energy by 0.3 MeV to higher energies and 0.4 MeV to lower energies with respect to the measured experimental data, respectively. There is no clear physical justification for the energy shift. The most probable reason might be related to the uncertainty associated with the elastic scattering data.

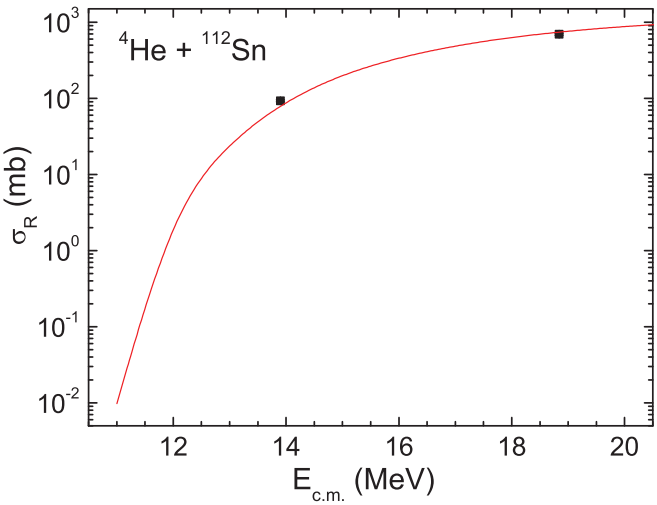

FIG. 4. (Color online) The extracted reaction cross sections that employ Eq. (8) (solid line) for the ^{4}He + ^{112}Sn reaction. The used experimental elastic scattering probabilities at the backward angle are from Ref. [10]. The reaction cross sections extracted from the experimental elastic scattering angular distribution with optical potential are presented by squares [10].

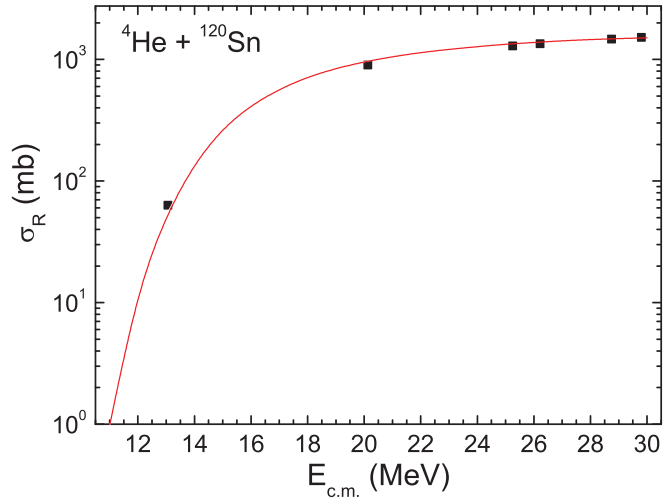

FIG. 5. (Color online) The same as in Fig. 4 but for the ^{4}He + ^{120}Sn reaction.

B. Capture and transfer plus breakup plus inelastic cross sections

By using a similar formalism as the one presented in Sec. II and Eq. (4), the capture cross section can be written, if one

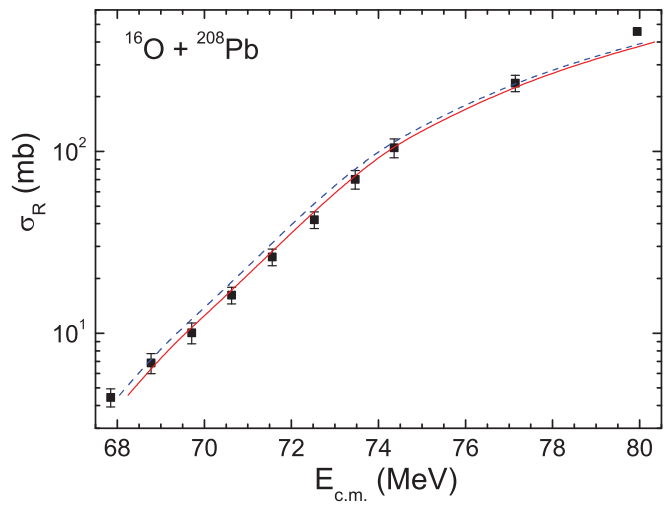

FIG. 6. (Color online) The extracted reaction cross sections that employ Eq. (8) (solid line) for the ^{16}O + ^{208}Pb reaction. The used experimental elastic scattering probabilities at the backward angle are from Ref. [11]. The reaction cross sections extracted from the experimental elastic scattering angular distribution with optical potential are presented by squares [11].

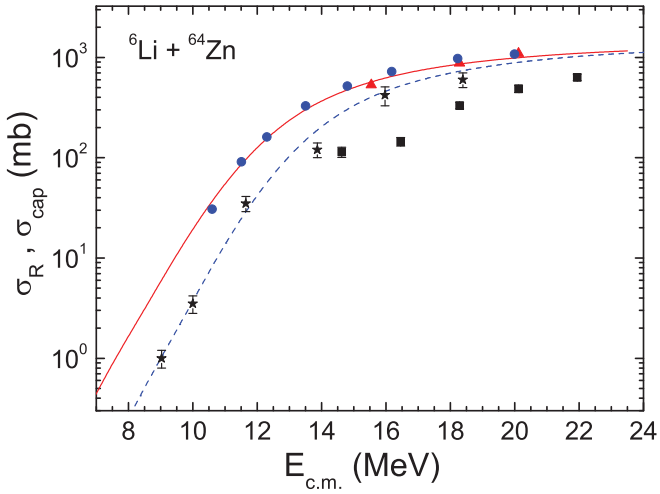

FIG. 7. (Color online) The extracted reaction (solid line) and capture (dashed line) cross sections that employ Eqs. (8) and (9) for the ^{6}Li + ^{64}Zn reaction. The used experimental elastic and quasielastic scattering probabilities at the backward angle are from Refs. [15,16]. The reaction cross sections extracted from the experimental elastic scattering angular distribution with optical potential and capture (fusion) cross sections are presented by circles [15,16], triangles [17,18], squares [17,18], and stars [15,16], respectively.

assumes that $P_{\rm BU} = 0$ since it is much smaller than P_{qe}, as [2]

$$\sigma_{\rm cap}(E_{\rm c.m.}) = \frac{\pi R_b^2}{E_{\rm c.m.}} \times \int_{E_{\rm c.m.}-(\hbar^2\Lambda_{\rm cr}/2\mu R_b^2)}^{E_{\rm c.m.}} dE[1 - d\sigma_{qe}(E)/d\sigma_{\rm Ru}(E)] \times \left[1 - \frac{4(E_{\rm c.m.} - E)}{\mu\omega_b^2 R_b^2}\right], \quad (9)$$

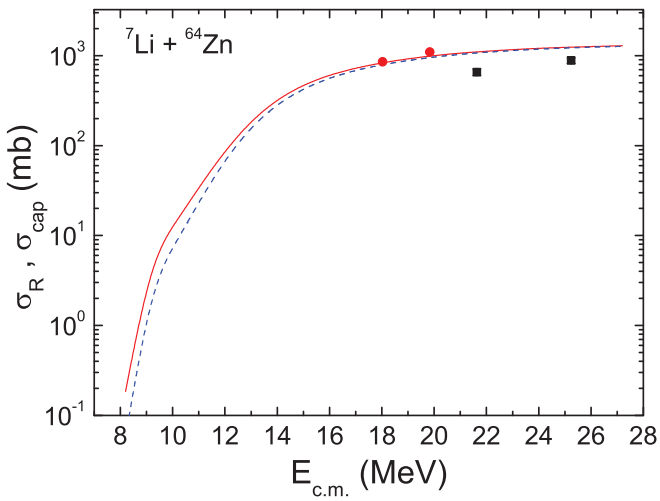

FIG. 8. (Color online) The same as in Fig. 7, but for the ^{7}Li + ^{64}Zn reaction. The reaction cross sections extracted from the experimental elastic scattering angular distribution with optical potential and capture (fusion) cross sections are presented by circles [17,18] and squares [17,18], respectively.

where in $\Lambda_{\rm cr} = J_{\rm cr}(J_{\rm cr} + 1)$, $J_{\rm cr}$ is the critical angular momentum at which the potential pocket in the nucleus-nucleus interaction potential $V(R, J)$ vanishes and capture does not occur. So, the capture cross sections can be extracted from the experimental quasielastic scattering probabilities $P_{qe}(E_{\rm c.m.}, J = 0) = d\sigma_{qe}/d\sigma_{\rm Ru}$ as already demonstrated in Ref. [2].

In Figs. 7 and 8, we also show the results of our calculations for capture cross sections of the ^{6,7}Li + ^{64}Zn systems for which the fusion process can be considered to exhaust the capture cross section. Figure 7 shows that the extracted and experimental capture cross sections are in good agreement for the ^{6}Li + ^{64}Zn reaction at energies near and above the Coulomb barrier for the data taken in Refs. [15,16]. Note that the extracted capture excitation function is shifted in energy by 0.7 MeV to higher energies with respect to the experimental data. This could be the result of different energy calibrations in the experiments on the capture measurement and quasielastic scattering. The data taken in Refs. [17,18] are below our predictions. This fact was already observed and was commented on in Ref. [19], and the reason given for the low fusion cross sections was owing to experimental problems with the high electronic threshold of the events when the data were taken. Figure 8 shows that the capture cross section for the ^{7}Li + ^{64}Zn system, obtained in the same works of Refs. [17,18], is also below our predictions. The same reason for this behavior as for the ^{6}Li + ^{64}Zn system was given in the same Ref. [19] since the ^{6}Li and ^{7}Li data were taken at the same experiment.

The extraction of reaction (capture) cross sections from the experimental elastic (quasielastic) backscattering probabilities leads to uncertainties on the order of 10% at energies above the Coulomb barrier. At energies below the barrier, the uncertainties are larger because a deviation in the elastic (quasielastic) backscattering cross section from the Rutherford cross section is comparable with the experimental uncertainties. Those overall uncertainties are comparable with the ones obtained from the traditional method that uses full elastic scattering angular distributions.

For the ^{7}Li + ^{64}Zn reaction, the Q value of the one neutron stripping transfer is positive, and this process should have a reasonable high probability to occur, whereas, for the ^{6}Li + ^{64}Zn reaction, Q values of neutron transfers are negative. Therefore, one might expect that transfer cross sections for ^{7}Li + ^{64}Zn are larger than for ^{6}Li + ^{64}Zn. With concern for breakup, since ^{6}Li has a smaller threshold energy for breakup than ^{7}Li, one might expect that breakup cross sections for ^{6}Li + ^{64}Zn are larger than for ^{7}Li + ^{64}Zn. Actually, in Fig. 9, one can observe that our calculations show that $\sigma(^{7}\text{Li} + ^{64}\text{Zn}) > \sigma(^{6}\text{Li} + ^{64}\text{Zn})$, where $\sigma = \sigma_R - \sigma_{\rm cap} \approx \sigma_{\rm tr} + \sigma_{\rm in}$ since $\sigma_{\rm tr} + \sigma_{\rm in} \gg \sigma_{\rm BU}$ for these light systems at energies close and below the Coulomb barrier ($\sigma_{\rm tr}$, $\sigma_{\rm in}$, and $\sigma_{\rm BU}$ are the transfer, inelastic scattering, and breakup cross sections, respectively). So, our present method of extracting reaction and capture cross sections from backward elastic scattering data allows the approximate determination of the sum of transfer and inelastic scattering cross sections or $\sigma_{\rm tr} + \sigma_{\rm in} + \sigma_{\rm BU}$ in systems where $P_{\rm BU}$ cannot be neglected. For both systems investigated, the values of these cross sections

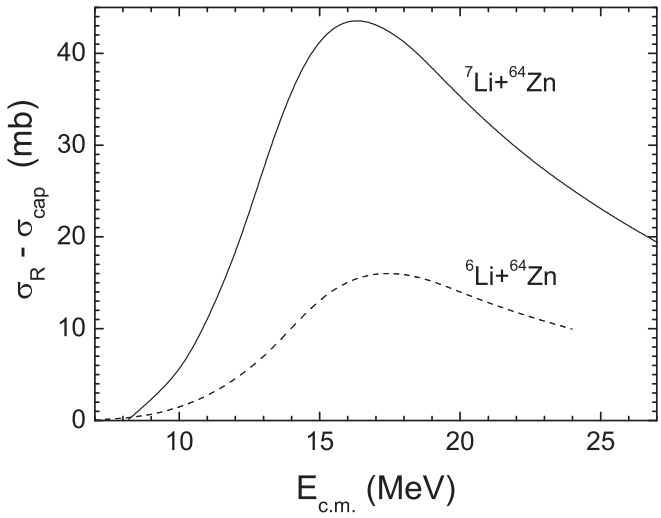

FIG. 9. The extracted $\sigma_R - \sigma_{cap}$ for the reactions $^6\text{Li} + ^{64}\text{Zn}$ (dashed line) and $^7\text{Li} + ^{64}\text{Zn}$ (solid line).

are shown to increase with $E_{c.m.}$, reach a maximum slightly above the Coulomb barrier energy, and after, decrease. The difference between the two curves in Fig. 9 may be considered approximately as the difference of σ_{tr} between the two systems since σ_{in} should be similar for both systems with the same target, apart from the excitation of the bound excited state of ^{7}Li. Because $\sigma_{tr}(^7\text{Li} + ^{64}\text{Zn}) \gg \sigma_{tr}(^6\text{Li} + ^{64}\text{Zn})$, one can find $\sigma_{tr}(^7\text{Li} + ^{64}\text{Zn}) \approx \sigma(^7\text{Li} + ^{64}\text{Zn}) - \sigma(^6\text{Li} + ^{64}\text{Zn})$. The maximum absolute value of the transfer cross section σ_{tr} at energies near the Coulomb barrier is about 30 mb. Figure 9 also shows that the difference between transfer cross sections for ^{7}Li and ^{6}Li are much more important than the possible larger σ_{BU} for ^{6}Li than for ^{7}Li.

IV. SUMMARY

We propose a new and very simple way to determine reaction cross sections, through a relation (8) between the elastic scattering excitation function at backward angle and the reaction cross section. We show, for several systems, that this method works well and that the elastic backscattering technique could be used as an important and simple tool in the study of the reaction cross sections. The extraction of reaction (capture) cross sections from the elastic (quasielastic) scattering at the backward angle is possible with reasonable uncertainties as long as the deviation between the elastic (quasielastic) scattering cross section and the Rutherford cross section exceeds the experimental uncertainties significantly. The behavior of the transfer and inelastic excitation function extracted from the experimental probabilities of the elastic and quasielastic scatterings at the backward angle also was shown.

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