

Compound nucleus aspect of sub-barrier fusion: A new energy scaling behavior

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It has been shown on selected data that heavy ion sub-barrier fusion is of compound nucleus nature. Data subjected to a simple energy scaling demonstrate either the lack of or greatly reduced fusion enhancement. Within the proposed approach, the sub-barrier fusion cross-section could be easily predictable.

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Fusion of two atomic nuclei occurs when the interacting bodies can overcome the barrier formed by the sum of the attractive nuclear and the repulsive Coulomb and centrifugal potentials. If the center of mass (c.m.) of kinetic energy is below the barrier height, classically forbidden fusion can occur as an instantaneous act of barrier quantum tunneling. The tunneling probability depends on the barrier parameters. However, the fusion cross-sections of some nuclei, despite being isotopes of the same element and thus having almost the same barrier, very often differ dramatically. This effect, known as sub-barrier fusion enhancement, is hitherto explained as being caused by the intrinsic properties of individual nuclei, their susceptibility to collective excitations, and nucleon or cluster transfers [1,2]. These direct reactions could modify the barrier through a coupling mechanism of the reaction channels and would lead either to fusion enhancement or to the hindrance of fusion, depending on the specific situation. We intend to show that the opportunity for sub-barrier fusion is determined by the compound nucleus characteristics and that its likelihood is decided mainly by general properties of the participating colliding nuclei. The heavy ion sub-barrier fusion cross-section could be predictable without involving any structural effects of the colliding nuclei.

A vast body of high quality data for sub-barrier fusion cross-sections has become available in recent years. Precise fusion cross-section measurements down to a level of 1 mb and below, which have been obtained at Canberra, Legnaro, Beijing, Argonne, and other facilities, allow comparative studies. The effect of static deformation on fusion probability has been evidenced by the early measurements of the $^{16}\text{O} + ^A\text{Sm}$ fusion reactions [3] (^ASm denotes different samarium isotopes). A large enhancement of the sub-barrier fusion of deformed ^{154}Sm was observed, compared with the spherical ^{144}Sm . The data points obtained for ^{148}Sm were found to lie between the limits for the spherical and statically deformed nuclei. Late experimental results for these systems [4], completed with the $^{17}\text{O} + ^{144}\text{Sm}$ data [5], are shown in Fig. 1.

An example of particularly large sub-barrier enhancement effects can be seen in Fig. 2, which shows the experimental fusion cross-sections for four pairs of Ca and Zr isotopes [6–8]. The data seem difficult to comprehend. All these nuclei are

rather spherical ones. Therefore, their static deformations could not be responsible for the observed effects.

Moreover, the cross-section for ^{48}Ca on ^{90}Zr is higher than for ^{40}Ca on the same target, while the opposite is true for the ^{96}Zr target. This means that not only the properties of individual nuclei, but also their mutually dependent excitations should be taken into account in a complicated way in attempts to describe the data. The experimental results presented in Fig. 2 have been compared with the calculations of the improved quantum molecular dynamics model [8]. The authors show in a juxtaposition that the dynamical effects play an important role in the sub-barrier fusion reactions. In addition, a significant effect of neutron transfer with positive Q values has been pointed out [7,8].

A simple inspection of the data shown in Figs. 1 and 2 reveals an apparent correlation. The system of the largest Q value has the largest sub-barrier fusion probability and vice versa. Q is the energy gain in fusion defined by $M_c = m_1 + m_2 + Q$, where M_c , m_1 , and m_2 are the masses of the compound nucleus and the two colliding partners, respectively. We believe this dependence is a general one. The few exceptions that could be found among the available data systematics deserve special consideration and a re-examination of the relevant experimental data. The greater the difference in Q values for similar systems, the bigger the observed sub-barrier enhancement is. This means that the phase-space available in fusion is an important factor that governs this process in the sub-barrier energy region. To our knowledge, the Q -value impact on sub-barrier fusion has been ignored in all barrier penetration models, which is equivalent to a tacit assumption of $Q = 0$.

There is a physical reason to incorporate the Q -value dependence in sub-barrier fusion models. This process is a rather slow one. For colliding nuclei with energy below the Coulomb barrier, their relative velocity decreases to its minimum at the distance of closest approach. Estimations have been made for the cases of $^4\text{He} + ^{208}\text{Pb}$ and $^{40}\text{Ca} + ^{40}\text{Ca}$ scattering occurring with asymptotic energy equal to 0.75 of the respective Coulomb barrier heights. The relative motions of the considered nuclei appear to be much lower than the typical nucleon velocity inherent to a nucleus. This means that during the contact time between two nuclei, there is enough opportunity for interaction between their nucleons. Thus, the situation arising at the sub-barrier approach resembles that of the double-nuclear adiabatic system formed at higher energies [9]. The system undergoes mutual excitations and

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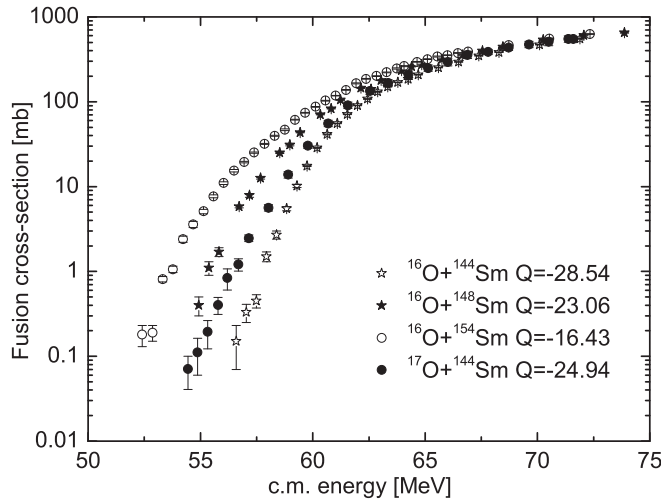


FIG. 1. Fusion excitation functions for $^{16}\text{O} + ^{144,148,154}\text{Sm}$ [4] and $^{17}\text{O} + ^{144}\text{Sm}$ [5] systems, Q stands for energy gain in fusion.

equilibration of collective degrees of freedom. Eventually, the double-nuclear system evolves towards the most probable deep-inelastic channels and towards fusion. According to that concept, fusion is a long-lasting process and its probability is determined by a phase-space availability, which is the trait for the compound nucleus mechanism. If some kind of equilibration could take place in the sub-barrier fusion, then the appropriate experimental data should also exhibit phase-space sensitivity.

The fusion cross-section σ_F has to obey two obvious asymptotic boundary conditions: for $E_{c.m.} \rightarrow -Q$, $\sigma_F \rightarrow 0$ and for $E_{c.m.} > V_C$, $\sigma_F = \pi R^2(1 - V_C/E_{c.m.})$, where $E_{c.m.}$, V_C , and R are the center of mass energy, Coulomb barrier height, and interaction radius, respectively. At an energy below the low energy boundary condition, in fact below an energy threshold, i.e., when $E_{c.m.} < -Q$, the energy conservation makes $\sigma_F = 0$, although for negative Q , which is a very frequent case, the barrier-penetration models yield finite values for σ_F . The higher energy boundary condition $E_{c.m.} > V_C$ gives a geometrical limit for fusion, which holds at the absence of both the direct processes and the upper limit for angular momentum in the fused system.

To compare the cross-sections for various fusion systems, because of the lack of a suitable model for sub-barrier fusion, a simple energy scaling is introduced. The experimental energy $E_{c.m.}$ has to be replaced by a reduced energy parameter E_r , which is given by

$$E_r = \frac{E_{c.m.} + Q}{V_C + Q}. \quad (1)$$

In the above parametrization, the only free parameter is the barrier height V_C . Among the many model approximations for V_C existing in the literature, the simplest one is taken here as $V_C = e^2 Z_1 Z_2 / R$, and the interaction radius R is given by $R = r_0(A_1^{1/3} + A_2^{1/3})$. Here, e , Z_1 , Z_2 , A_1 , and A_2 are the elementary charge, and the atomic and mass numbers of the fusing nuclei, respectively; r_0 is the reduced radius.

An example of the suggested data reduction application can be seen in Fig. 3, where 12 data sets are plotted against the

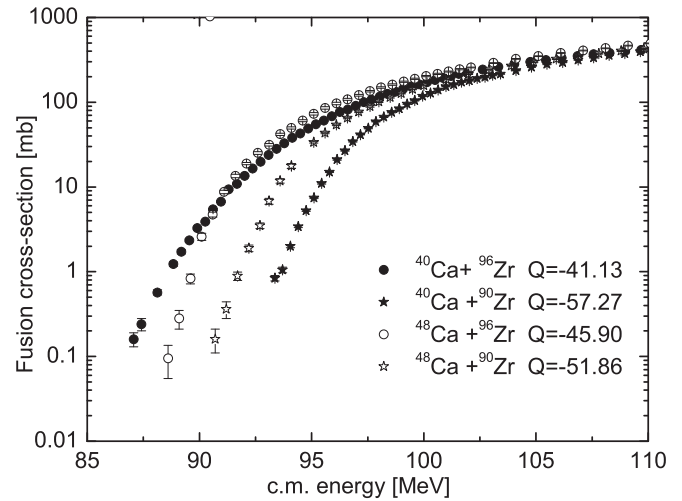


FIG. 2. Experimental fusion data for the combination of $^{40,48}\text{Ca} + ^{90,96}\text{Zr}$ nuclei [6–8], Q stands for fusion Q values.

reduced energy of Eq. (1). The data from Figs. 1 and 2 are completed here by the fusion data for $^{36,32}\text{S} + ^{110}\text{Pd}$ [10], $^{40}\text{Ca} + ^{124}\text{Sn}$ [11], and $^{48}\text{Ca} + ^{124}\text{Sn}$ [12]. All data points shown in Fig. 3 as open symbols are for the data of Figs. 1 and 2, semi-closed symbols are for the other data points. The reduced radius parameter r_0 has been varied within 3.2%, from 0.95×1.44 fm for the $^{40}\text{Ca} + ^{90}\text{Zr}$ system to 0.92×1.44 fm for the $^{48}\text{Ca} + ^{96}\text{Zr}$ one. Actually, it should not be expected that the applied parametrization for R with unvarying r_0 would be adequate for the large neutron excess of the last system. The data points in Fig. 3 scatter within a band with a width corresponding to a factor of 2–3 for the fusion cross-section. This could be either due to the omission here of other fusion data reduction procedures and the neglect of structural effects, or to the limitations of the used scaling prescription, and because of experimental uncertainties. By analyzing a variety

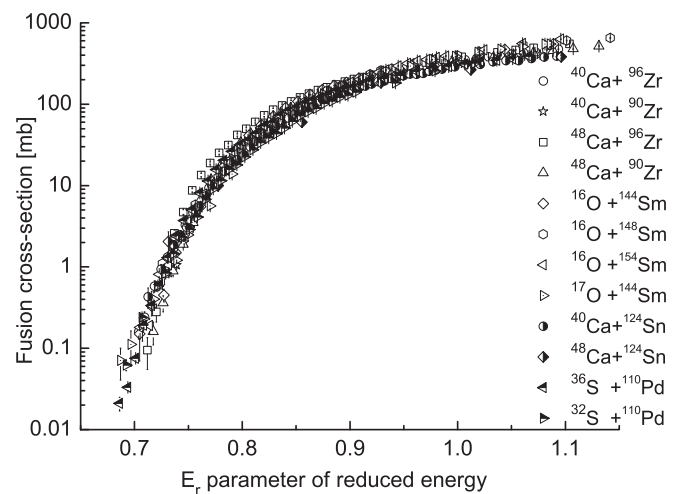


FIG. 3. Experimental fusion data presented in Figs. 1 and 2 and data for $^{36,32}\text{S} + ^{110}\text{Pd}$ [10], $^{40}\text{Ca} + ^{124}\text{Sn}$ [11], and $^{48}\text{Ca} + ^{124}\text{Sn}$ [12] systems in terms of the reduced c.m. energy. The open symbols show data of Figs. 1 and 2, whereas the semi-closed ones are for all other data, see text.

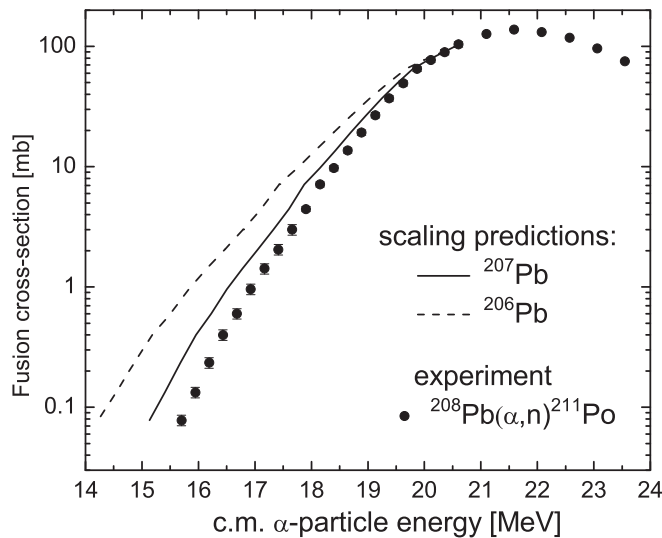


FIG. 4. Sub-barrier fusion cross-section predictions for $\alpha + {}^{206}\text{Pb}$ and $\alpha + {}^{207}\text{Pb}$ systems from the energy scaling of the $\alpha + {}^{208}\text{Pb}$ data [13].

of fusion data, we found that the scaling, Eq. (1), fails to satisfactorily reduce the data if fusing systems differ very much with respect to the total mass and Q value, in terms of V_c . Definitely, a more refined approach to the definition of the Coulomb barrier V_c is desirable, in addition to a scrutiny of the possible fusion probability dependence on the total mass, mass asymmetry, and/or the value of $Z_1 Z_2$ product. Nevertheless, even with these limitations, it remains evident that the long discussed fusion cross-section enhancement is reduced greatly for the selected data set, as seen in Fig. 3, and in some instances by two orders of magnitude. We can conclude that the observed enhancement is predominantly of compound nucleus nature with little room left for other mechanisms that could contribute to the observed cross-section variations.

A straightforward implication of the proposed scaling is its predictive power. The sub-barrier fusion cross-section for a pair of nuclei could be predicted easily if the cross-section data measured for a neighboring system are known. Let us consider the fusion of ${}^4\text{He}$ nuclei with different lead isotopes. The enhancement for the $\alpha + {}^{206,207}\text{Pb}$ fusion cross-sections with respect to the $\alpha + {}^{208}\text{Pb}$ fusion is expected owing to larger Q values for the first two.

Such estimations made with a barrier height equal to 20.5 MeV for all Pb nuclei are shown in Fig. 4, together with the ${}^{208}\text{Pb}(\alpha, n){}^{211}\text{Po}$ reaction data [13]. The cross-section for this reaction at sub-barrier energies almost completely exhausts the total fusion cross-section for that system.

On the grounds of a Q -value criterion suggested here, an even bigger effect on the sub-barrier fusion cross-section should be observed for ${}^3\text{He}$ ions, as well as for weakly bound neutron-rich light projectiles, such as ${}^6\text{He}$ and ${}^8\text{He}$, hitting Pb and Bi target nuclei because of the much larger fusion Q values. It would be interesting to compare yields of these highly exothermic sub-barrier fusion reactions induced by light nuclei of entirely different structures. Some data

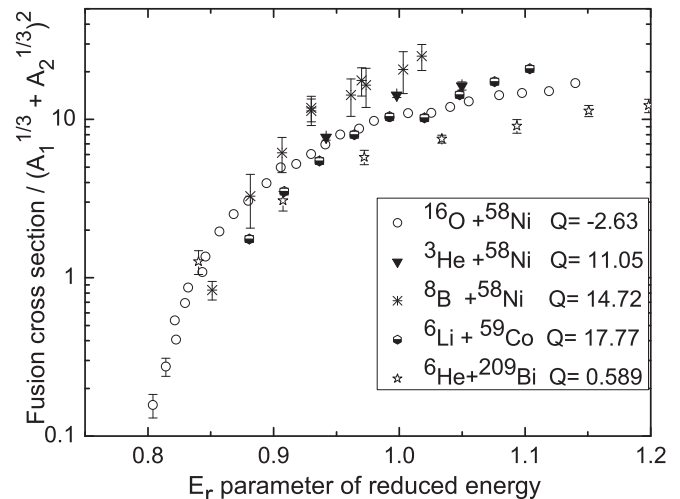


FIG. 5. Reduced cross-section of fusion data for ${}^3\text{He} + {}^{58}\text{Ni}$ [14], ${}^8\text{B} + {}^{58}\text{Ni}$ [15], ${}^{16}\text{O} + {}^{58}\text{Ni}$ [16], ${}^6\text{Li} + {}^{59}\text{Co}$ [17], and ${}^6\text{He} + {}^{209}\text{Bi}$ [18] systems vs the reduced c.m. energy parameter, Eq. (1). Q stands for fusion Q values.

on exotic ion fusion do show the enhancement, whereas an anticipation concerning ${}^3\text{He}$ is waiting an experimental verification.

The proposed treatment of the fusion experimental data seems to be appropriate for sub-barrier fusion measurements. It can be seen in Fig. 2, for example, that some data only below 10 mb obey our Q -value rule. The suggested scaling is not the unique one which takes into account the phase-space available in fusion. More involved algorithms for the energy scaling could be developed and applied. However, even at this stage, Eq. (1) can be useful for comparing fusion data measured at energies close to the barrier. To demonstrate this we took the fusion data for ${}^3\text{He} + {}^{58}\text{Ni}$ [14], ${}^8\text{B} + {}^{58}\text{Ni}$ [15], ${}^{16}\text{O} + {}^{58}\text{Ni}$ [16], ${}^6\text{Li} + {}^{59}\text{Co}$ [17], and ${}^6\text{He} + {}^{209}\text{Bi}$ [18] systems. The first three sets of the data are presented in Fig. 4 of [14] in terms of the reduced fusion cross-section $\sigma_F / (A_1^{1/3} + A_2^{1/3})^2$ against the energies scaled by a factor of $Z_1 Z_2 / (A_1^{1/3} + A_2^{1/3})$. We show in Fig. 5 the reduced fusion cross-sections for these five different systems versus our reduced energy scaling parameter E_r .

The reduced radius r_0 , needed for the barrier height estimations, was taken equal to 0.96×1.44 fm for all five analyzed data sets. Although our data reduction presented in Fig. 5 is not perfect and could be improved, we do not see in Fig. 5 distinct differences between the exotic systems ${}^3\text{He}$, ${}^8\text{B} + {}^{58}\text{Ni}$ fusion data and the data for a reference ${}^{16}\text{O} + {}^{58}\text{Ni}$ system. We are confident that the Q -value effect should be taken into account when comparing various fusion data, otherwise the comparison may lead to unjustified physical conclusions.

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