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Spectroscopy of proton-rich ^{66}Se up to $J^\pi = 6^+$: Isospin-breaking effect in the $A = 66$ isobaric triplet

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Candidates for three excited states in the ^{66}Se have been identified using the recoil-β tagging method together with a veto detector for charged-particle evaporation channels. These results allow a comparison of mirror and triplet energy differences between analog states across the $A = 66$ triplet as a function of angular momentum. The extracted triplet energy differences follow the negative trend observed in the $f_{7/2}$ shell. Shell-model calculations indicate that the strength of the Coulomb isotensor part alone is not sufficient to account for this trend in the case of the $A = 66$ triplet.

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The building blocks of the nucleus, i.e., the protons and the neutrons, are conventionally regarded as two different particle species differing in charge and slightly in mass. However, because these particles are affected similarly by the strong nuclear force, they can be viewed as two different quantum states of a generic particle, the nucleon. This approach leads to the concept of isospin, in which nucleons are distinguished by a z projection T_z of the isospin quantum number T. The isospin representation simplifies the treatment of the two-body nucleon-nucleon interaction and the classification of nuclear states. Isospin symmetry implies that for mirror nuclei, which have the same mass, but where the number of protons and neutrons is interchanged, the resulting analog states with the same T are degenerate. However, this degeneracy is lifted by isospin nonconserving (INC) forces, which lead to the mirror energy differences (MEDs) [1] evaluated as

$$\mathrm{MED}_{J,T} = E^*_{J,T,T_z=-1} - E^*_{J,T,T_z=+1}. \qquad (1)$$

The MEDs relate to isovector energy differences; if the nuclear interaction were charge-symmetric in the absence of the Coulomb force, then the MED ought to be zero. In practice, it is found that the MEDs vary as a function of angular momentum on an energy scale of around ~100 keV. Even on the assumption of perfect symmetry of the wave functions for isobaric analog states, calculating the MED for a specific case can be complex. In addition to the expected two-body Coulomb effects, contributions to the MED are found from monopole effects such as single-particle Coulomb shifts, the electromagnetic spin-orbit interaction, and changes in radius or shape as a function of spin. In cases of weak binding, the breakdown of symmetry can also lead to further effects such as Thomas-Ehrman shifts (TESs) [2,3]. Where mirror states are well bound, there has been considerable success in calculating the MED and a good correspondence is found with experiment for nuclei in the $f_{7/2}$ shell [1].

Analog states in pairs of mirror nuclei are subsets of complete isobaric multiplets, i.e., sequences of isobars where states are characterized by the same T. A simple case is that of $T = 1$ triplets, in nuclei with $T_z = (N - Z)/2 = 0, \pm 1$ where, in addition to the MEDs, the triplet energy differences (TEDs) [1] may be evaluated:

$$\mathrm{TED}_{J,T} = E^*_{J,T,T_z=-1} + E^*_{J,T,T_z=+1} - 2E^*_{J,T,T_z=0}. \qquad (2)$$

The TEDs are isotensor energy differences and probe a different aspect of the two-body interaction. They are sensitive to charge-dependent effects because they reflect the difference between the average of the proton-proton (pp) and neutron-neutron (nn) interactions and the neutron-proton (np) interaction. The TEDs have a special property that make them particularly attractive to study. That is, the TEDs are not expected to be strongly influenced by the single-particle contributions described earlier, but are instead especially sensitive to the details of the isotensor (multipole) interactions. At a fundamental level, these interactions may have one or two possible origins—a Coulomb interaction and/or a nuclear INC interaction—thus, the TEDs have the capability to shed light on the balance between these terms.

Extensive information on the MEDs and TEDs exists for the sd shell, where the relevant nuclei lie close to or on the line of stability (for most recent example, see Ref. [4]). Over the past 15 years, information on low-lying excited states has been gathered in the $f_{7/2}$ shell, allowing the MEDs and TEDs to be studied for the $A = 46$ [5] and $A = 54$ [6] triplets. In the upper fp shell, however, the experimental information is extremely limited for odd-odd $N = Z$ nuclei between ^{56}Ni and ^{100}Sn and almost nonexistent for $T_z = -1$ nuclei. This is undoubtedly attributable to the low production cross sections for such nuclei because they lie very far from the line of stability. Here the nuclear structure is expected to become significantly more

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complex with more orbitals involved. In addition, there is evidence of a sudden structural change when going towards the mass $A = 70$–80 region, and shape coexistence, driven by the increasing occupancy of the $g_{9/2}$ orbital [7], and references therein]. Aside from the pure nuclear structure interest, a deeper understanding of Coulomb and other INC effects across medium-mass $T = 1$ triplets may impact on related areas of physics, including standard model tests [8] and nuclear astrophysics [9]. For these reasons, it would be of high interest to pursue the TED and MED investigations beyond ^{56}Ni. Recently, Obertelli *et al.* [10] identified the 2^+ state in ^{66}Se in a study of two-nucleon removal from a secondary beam of ^{68}Se at Michigan State University (MSU). This constitutes the only definite identification of an excited $2^+, T = 1$ state in the upper fp shell. In this paper, we present the observation of the 2^+, 4^+, and 6^+ states in ^{66}Se, allowing the only TED study to date above the $f_{7/2}$ shell.

In recent years the study of exotic nuclei has been driven by advances in experimental sensitivity concomitant with advances in detection technology. An example of a technique which can extract the signal of an exotic nucleus with exquisite sensitivity is recoil-decay tagging (RDT) [11,12]. This technique exploits the characteristic decay properties of the nucleus of interest to identify it at the focal plane of a recoil separator and then tag the associated γ rays. Recently, RDT has been developed from its initial focus on α-decaying nuclei to be more broadly applicable. A challenging extension has been to β-decaying nuclei because, in general, β decay does not provide a unique tag owing to the three-body nature of the decay. In some special cases, however, β decay can be used, where the decay is Fermi superallowed. Here, the short half-lives ($\sim$100 ms) and high end-point energies ($\sim$10 MeV) differ considerably from neighboring nuclei and provide defining characteristics, which can be exploited by correlating positrons with recoils implanted at the focal plane of a recoil separator. This technique, entitled recoil-β tagging (RBT), is suitable for studying exotic proton-rich nuclei and was first demonstrated for ^{74}Rb [13] at the University of Jyväskylä (JYFL). This initial work has been extended at JYFL to the previously unknown case of ^{78}Y [14] and recently provided additional information on excited states in ^{66}As [15]. To reach the most exotic nuclei on the proton-rich side of the $N = Z$ line in fusion-evaporation reactions, the experimental sensitivity needs to be increased further. These nuclei are also associated with pure neutron emission amid a dominant background of charged-particle evaporation channels. In the present work, a charged-particle veto detector has been developed to suppress the reaction channels associated with proton and α evaporation. The effectiveness of this methodology is demonstrated with the important case of ^{66}Se.

The experiment was performed at JYFL utilizing the K-130 cyclotron, which provided a ^{28}Si beam at an energy of 75 MeV. The beam bombarded a $^{\mathrm{nat}}$Ca target, rolled to a thickness of 0.65 mg/cm^2, with an average intensity of 3 pnA for 36 hr. γ rays were detected at the target position by the JUROGAMII array consisting of 24 clover [16] and 10 tapered [17,18] Compton-suppressed germanium detectors with a total efficiency of 5.5% at 1.33 MeV. A new veto device, UoYtube (University of York tube), consisting of 96 CsI(Tl) crystals read out by photodiodes, was installed at the target position [19]. Fusion recoils were separated from the beam by the gas-filled separator RITU (Recoil Ion Transport Unit) [20,21]. Further identification of the recoils was performed in the GREAT (Gamma Recoil Electron Alpha Tagging) [22] spectrometer, located at RITU's focal plane, where the recoils were finally implanted in a pair of adjacent 700-μm-thick double-sided silicon strip detectors (DSSDs). The GREAT spectrometer also included a large segmented clover-, two JUROGAMII clover-, and planar germanium detectors, which were mounted around the DSSD to observe delayed γ rays. In addition, the planar detector in combination with the DSSD served as a ΔE-E telescope for β particles. Data were collected with the triggerless total data readout (TDR) [23] acquisition system and analyzed with the GRAIN [24] software.

The identification of ^{66}Se γ rays is facilitated by its Fermi superallowed β-decay nature and by the fact that ^{66}Se is produced via two-neutron evaporation, while the other products involve emission of at least one charged particle. With these features in mind, a stepwise procedure was followed to search for γ rays originating from ^{66}Se. In the first instance, the RBT method was applied by correlating 0.5–10-MeV β particles to recoils within a correlation time of 106 ms ($\approx 3 \times t_{1/2}(^{66}\mathrm{Se})$ [25]). Figure 1(a) shows the observed γ rays when these tagging conditions are applied. As expected, transitions from ^{66}As are identified along with contaminants such as ^{65}Ga and ^{65}Ge, corresponding to $3p$ and $2pn$ channels, respectively. Next, charged-particle veto was applied and the success of this approach is demonstrated in Fig. 1(b). This leaves five peaks at 191, 841, 929, 1135, and 1456 keV,

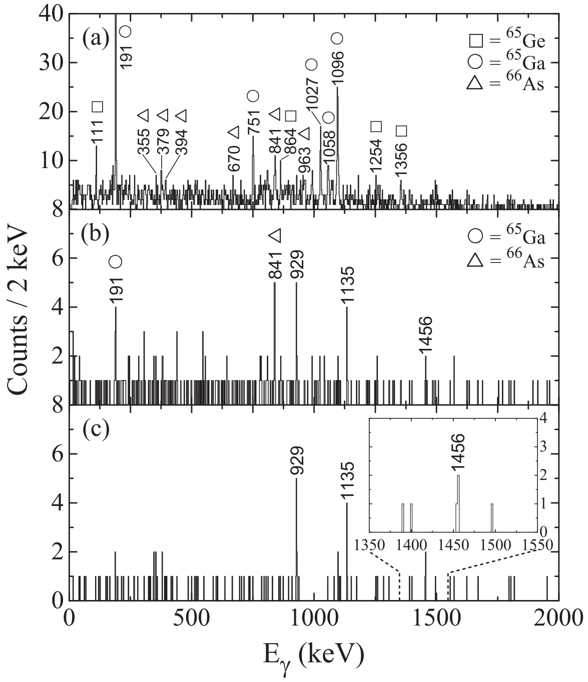

FIG. 1. (a) Recoil-β tagged JUROGAMII singles γ-ray spectrum with a 0.5–10-MeV β gate. (b) Same as (a) but with charged-particle suppression. (c) Same as (b) but with an additional delayed γ-ray veto condition (see text for further details). The inset in panel (c) illustrates the low background region around the 1456-keV line.

where the first two can be associated with ^{65}Ga and ^{66}As, respectively. The γ rays detected in the focal plane germanium detectors in delayed coincidence with a recoil implantation or in prompt coincidence with β decay can be utilized as an additional veto, in a similar manner to the charged particles. The β decay of ^{65}Ga feeds excited states in ^{65}Zn, which are deexcited by various γ rays, as are the isomeric structures in ^{66}As [15]. After the final focal plane veto only three γ rays remain, as can be seen in Fig. 1(c). The γ ray at 929(2) keV [$I_{929\,\mathrm{keV}} = 100(40)$] in Fig. 1(c) deexcites the 2^+ state in ^{66}Se, because it is consistent with the transition energy of 929(7) keV reported in Ref. [10]. The other two peaks at 1135(2) keV [$I_{1135\,\mathrm{keV}} = 70(40)$] and 1456(2) keV [$I_{1456\,\mathrm{keV}} = 50(30)$] are tentatively assigned to deexcite the 4^+ and 6^+ levels, respectively, because the observed pattern represents a typical spectrum of the strongest yrast transitions in an even-even nucleus. The 6^+ assignment over 5^- for the state at 3520 keV is supported by the fact that the energy of 1456 keV is closer to the energy of $6^+ \rightarrow 4^+$ transition than $5^- \rightarrow 4^+$ transition in ^{66}Ge [26,27]. Secondly, the systematics of Se isotopes imply that the 5^- state should remain above the 6^+ state in excitation energy, making the population of the 6^+ state more favorable as it is yrast. Thirdly, no additional γ rays are observed around energies of 500 and 900 keV as the feeding and depopulation of the 5^- state in ^{66}Ge would indicate.

In the case of low statistics, which is especially true for the 1456-keV peak residing in the area of almost zero background [see the inset in Fig. 1(c)], the decay times of the recoils associated with the prompt γ rays can be investigated. The standard deviation ($\sigma_{\Theta_{\mathrm{exp}}} = 1.07$) of the logarithmic β-decay-time distribution, which is obtained by gating on the 929-, 1135-, and 1456-keV lines, meets the recommended limits ($\sigma^{\mathrm{lower}}_{\Theta_{\mathrm{exp}}} = 0.77$, $\sigma^{\mathrm{upper}}_{\Theta_{\mathrm{exp}}} = 1.75$) for 16 events, which indicates that the observed activity originates from the decay of one radioactive species [28]. The derived β-decay half-life of 38^{+13}_{-8} ms is also in agreement with Ref. [25]. This result, together with the arguments above, indicate that the observed γ rays originate from ^{66}Se.

The TED and MED data for $A = 66$ are plotted in Figs. 2(a) and 2(b), respectively, along with the data for nuclei in the $f_{7/2}$ shell. The TEDs follow a negative trend within each triplet while the MEDs vary from case to case. The significant variation of the MEDs reflects the fact they depend strongly on Coulomb multipole effects associated with recoupling the angular momenta of pairs of particles as a function of spin. The sign of the MED depends on whether it is protons or neutrons that are active in a particular member of the mirror pair. In addition, monopole effects will also contribute and will vary in sign from case to case. However, the TEDs are remarkably consistent in sign and, to a large extent, magnitude. This is partly associated with the fact that multipole effects will dominate the TEDs. Indeed, under the assumption of identical wave functions across the triplet, the monopole contributions discussed earlier effectively cancel in the calculation of the TED. Identical wave functions is a reasonable assumption for well-bound states, although in heavier systems, there are predictions of different shape-driving effects that will destroy this symmetry [38].

The fact that the TEDs are negative can be explained in a simple picture because they are directly dependent on the isotensor part of the two-body interaction, i.e., $V_{pp} + V_{nn} - 2V_{np}$ [see Eq. (2)]. The TED decreasing with spin has its origin in two separate effects. Firstly, the number of $T = 1$ np pairs, for a given analog state, is always larger in the odd-odd $N = Z$ nucleus than in the two even-even nuclei. This has been demonstrated both analytically [39] and with shell-model

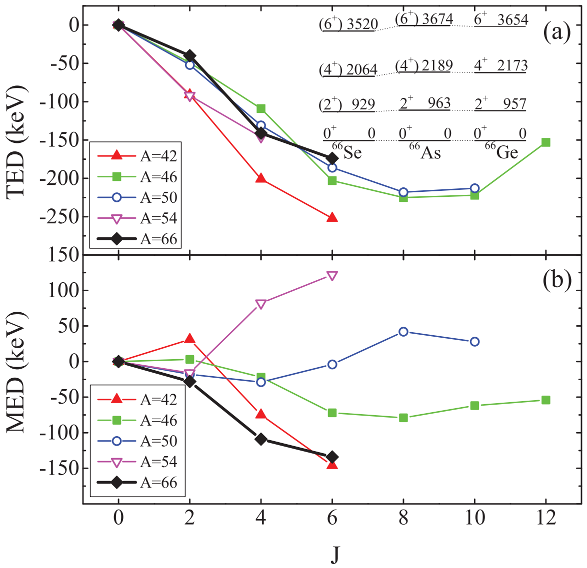

FIG. 2. (Color online) (a) The TEDs for the nuclei in the $A = 42$–66 region. (b) Same as (a) but for the MEDs. Data are taken from the present work and from Refs. [5,6,15,26,29–37]. The inset in panel (a) shows spins, parities, and excitation energies (in keV) of IAS in $A = 66$ nuclei.

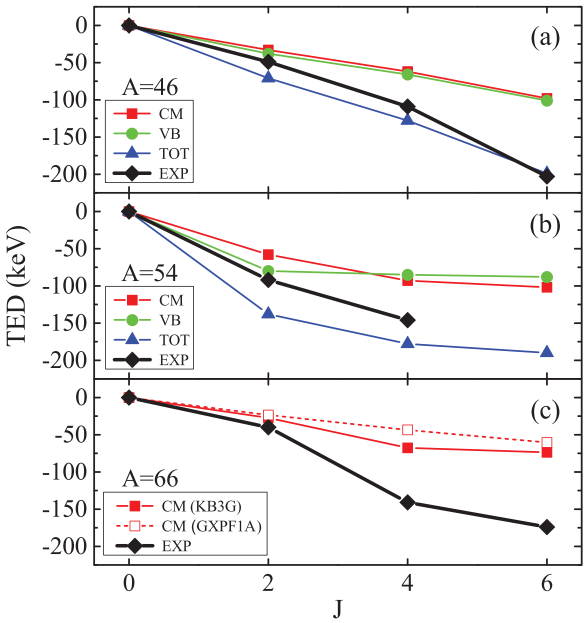

FIG. 3. (Color online) The experimental and shell-model predicted TEDs for (a) $A = 46$, (b) $A = 54$, and (c) $A = 66$ triplets (see text for details).

calculations in the $f_{7/2}$ shell [40]. Secondly, the Coulomb isotensor interaction is positive, but reduces relative to the ground state for increasing angular momentum coupling. The combination of these two effects leads to the negative TED in all cases studied so far. However, in the $f_{7/2}$ shell, it was found that the Coulomb isotensor interaction (CM) alone was not sufficient to account for the TED magnitude [1,6,41]. An additional nuclear isotensor component (VB) of $+100$ keV for $J = 0$ couplings of $f_{7/2}$ particles was identified based on the empirical TED of the $A = 42$ triplet [41]. These results, illustrated in Figs. 3(a) and 3(b) for the $A = 46$, 54 triplets, have been reproduced in the current study for completeness and comparison. It should be emphasized that the fundamental origin of this additional INC term has not been explained and, furthermore, there are no predictions for its requirement in the upper fp shell.

It is obvious that for the $A = 66$ triplet studied here the negative TED behavior continues, as observed in the $f_{7/2}$ shell. In addition, the CM component alone will not account for the observed TED in this heavier case either, which is a new and very interesting observation. This is illustrated in Fig. 3(c), which shows a prediction of the TED for $A = 66$ assuming only a Coulomb isotensor interaction. The calculation was performed using ANTOINE in the fp space with KB3G and GXPF1A interactions, allowing at most five excitations beyond the $f_{7/2}$ and $p_{3/2}$ orbitals. This should be viewed as a simplistic calculation, because it does not include the $g_{9/2}$ orbit. The VB component has not been included because, unlike in the $f_{7/2}$ shell, we have no empirical estimate of the strength. For the $A = 66$ triplet one would need to add the VB component into at least three different orbitals, namely into the $f_{5/2}$, $p_{3/2}$, and $p_{1/2}$. Furthermore, there are no solid grounds, which would dictate how the VB strength should be distributed between the above-mentioned orbitals to reproduce the experimental data. Nevertheless, this simple calculation shows that the Coulomb part alone is insufficient to explain the experimental TED magnitude. It should be noted that the missing $(g_{9/2})^2$ components in the wave functions would only change the prediction for the TED by virtue of the different spin-dependent changes of the Coulomb energy for $g_{9/2}$ wave functions compared with the fp orbitals. It seems unlikely that this would be sufficient to account for the large TEDs seen at high spins.

In a recent theoretical study by Kaneko *et al.* [42], a shell-model analysis of Coulomb displacement energies was performed to study the effect of INC nuclear forces on the triplet displacement energies of the ground states. In general they found that the agreement with the data was much improved in the $f_{7/2}$ shell, when the additional isotensor interaction of $+165$ keV for the $J = 0$, $T = 1$ coupling was introduced. It is interesting to note that the isotensor interaction used is larger than previously considered in this region. In addition, it was found in Ref. [42] that the INC forces were less important for nuclei in the upper fp shell, which stands in contrast with the shell-model results presented in the current study.

Other factors such as deformation effects and TESs, which could, in principle, have an effect on the observed experimental TEDs, should be also considered. However, it can be shown that the Z dependency is removed from the difference in Coulomb energy terms [43] when the TEDs are computed; hence, the shape-changing contribution to the TEDs nearly cancels and is, at most, around 1–2 keV. The TESs [2,3] are known to be strong in the case of weakly or unbound s orbital protons. Even though the mass of the ^{66}Se has not been measured, according to the recently calculated values of one- and two-proton separation energies [42], the observed 4^+ state should be still reasonably well bound. In addition, in ^{66}Se the single-particle configurations of the excited 4^+ and 6^+ states are unlikely to be dominated by low l orbitals; thus, the TES contribution ought to be small. Clearly, a calculation to estimate the effect of the TESs in terms of energy should be carried out, but this is beyond the scope of the current article.

In conclusion, excited states in the proton-rich nucleus ^{66}Se have been identified using the recoil-β tagging method in conjunction with a charged-particle veto device. These data allow the TEDs across the full $A = 66$ triplet to be examined for the first time, providing valuable data for further theoretical studies. The observed TED mirrors the negative trend of the triplets in the $f_{7/2}$ shell. Shell-model calculations in the present work reveal that the Coulomb isotensor component alone is insufficient to account for the experimental TEDs, pointing to a need for an additional nuclear INC interaction, whose origin is not clear. A need for an additional INC interaction has been previously demonstrated for the triplets in the $f_{7/2}$ shell. The current study necessitates that further experimental and especially theoretical studies are undertaken, which could clarify the origin of the missing TED magnitude.

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