Excited states of many-body systems in the fermion dynamical symmetry model with random interactions

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In this Brief Report we investigate excited yrast states under random interactions in the framework of the fermion dynamical symmetry model, for the ensemble with spin-0 ground states. Interesting correlations are seen between R_6 and R_4 (where $R_I \equiv E_{I_1^+}/E_{2_1^+}$) by using the Mallmann plot, for cases with both SP(6) symmetry and SO(8) symmetry.

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In 1997, Johnson, Bertsch, and Dean, discovered that the spin-0 ground-state dominance in even-even nuclei can be obtained by using the two-body random ensemble [1]. Since then, many efforts have been devoted to understanding this puzzle and to studying various regularities of many-body systems under random interactions. See Refs. [2–4] for comprehensive reviews.

Among the many regularities under random Hamiltonians, collective motion is of particular interest. In Ref. [5] Bijker and Frank found that for spin-0 ground states of *sd* bosons, collective vibrational and rotational motions are dominant as the number of *sd* bosons is large enough (say, >10). However, in fermion systems the dominance of rotation depends on specific choices of two-body interactions and symmetries of the systems. For example, rotational motion gradually arises when the quadrupole-quadrupole interaction is enhanced in Hamiltonian [6]. Also, in Ref. [7] it was found that the collective features in the two-body random ensemble arise from the quadrupole-quadrupole component in the Hamiltonian. In Ref. [8], statistics of $R_4 = E_{4_1^+}/E_{2_1^+}$ showed that a system with SP(6) symmetry in the fermion dynamical symmetry model [fermion dynamical symmetry model (FDSM) [9,10]; explained below] presents both vibrational and rotational motions, while a system with SO(8) symmetry presents R_4 without any obvious peaks, suggesting that collective motion in the FDSM with random interactions is related to the symmetries of both the Hamiltonian and the truncation scheme. In Ref. [11] Johnson *et al.* studied the features of seniority, vibrations, and rotations in the Mallmann plot [12], namely, a plot of (R_I, R_4) (where R_I is defined by the ratio of the first I^+ state energy to the first 2^+ state energy, $R_I \equiv E_{I_1^+}/E_{2_1^+}$), for both fermion and boson systems. In Ref. [13] Lei *et al.* presented the Mallmann plot for *sd*-boson systems under random interactions and demonstrated remarkable correlations between R_I and R_4, corresponding to the seniority, harmonic vibration and rotation.

This work is a generalization of Refs. [8,11,13], for systems dictated by the symmetries of the FDSM, under random interactions. Here let us introduce the FDSM and its Hamiltonian briefly. The FDSM [9] is one of the SD-pair approximations of the nuclear shell model. It was a further development of the Ginocchio toy model [10], in which both the SD nucleon pairs and the Hamiltonian are constructed in the so-called *k-i* basis [9,10], following either the SO(8) or the SP(6) symmetry. The FDSM Hamiltonian is

$$H = G_0 S^\dagger S + G_2 D^\dagger \cdot D + \sum_{r=1}^{2 \text{ or } 3} B_r P^r \cdot P^r,$$

where $S^\dagger$ and $D^\dagger$ are S and D pairs, P^r is the multipole operator, and r ranges from 1 to 2 for SP(6) symmetry and from 1 to 3 for SO(8) symmetry. G_0, G_2, and B_r are taken to be random values following the Gaussian distribution with average 0 and width 1.

We investigate cases with SO(8) symmetry and pair number $N = 3$–5 and with SP(6) symmetry and $N = 3$–7. We diagonalize 5000 sets of the random Hamiltonians and calculate their yrast states. For these systems, most spin-0 ground-state probabilities, denoted $P(0)$, are around 50%, with exceptions as follows. $P(0)$ values for SO(8) symmetry with $N = 5$ and $P(0)$ for the SP(6) system with $N = 7$ are relatively lower (37% and 44%, respectively); $P(0)$ values for SP(6) symmetry with $N = 3$ and 6 are relatively higher (72% and 67%, respectively).

We now focus on cases with spin-0 ground states and present the distributions of R_4 and the correlation of (R_6, R_4) for these random samples in Fig. 1. We note without further details that the correlation of (R_I, R_4) ($I = 8, 10, \ldots$) is very close to that of (R_6, R_4).

The statistics for the SO(8) symmetry of the FDSM are presented in Figs. 1(a)–1(c) and 1(a′)–1(c′). One sees no obvious statistical peak at $R_4 = 2$ or 3.33 in Figs. 1(a)–1(c) or pronounced statistics at $(R_6, R_4) = (3, 2)$ or (7, 3.33) in Figs. 1(a′)–1(c′). On the other hand, one sees clear peaks in Figs. 1(d)–1(h) and 1(d′)–1(h′) for the SP(6) systems, i.e., similar to patterns observed in *sd* bosons by Bijker and Frank [5]. These results are consistent with those in Ref. [8].

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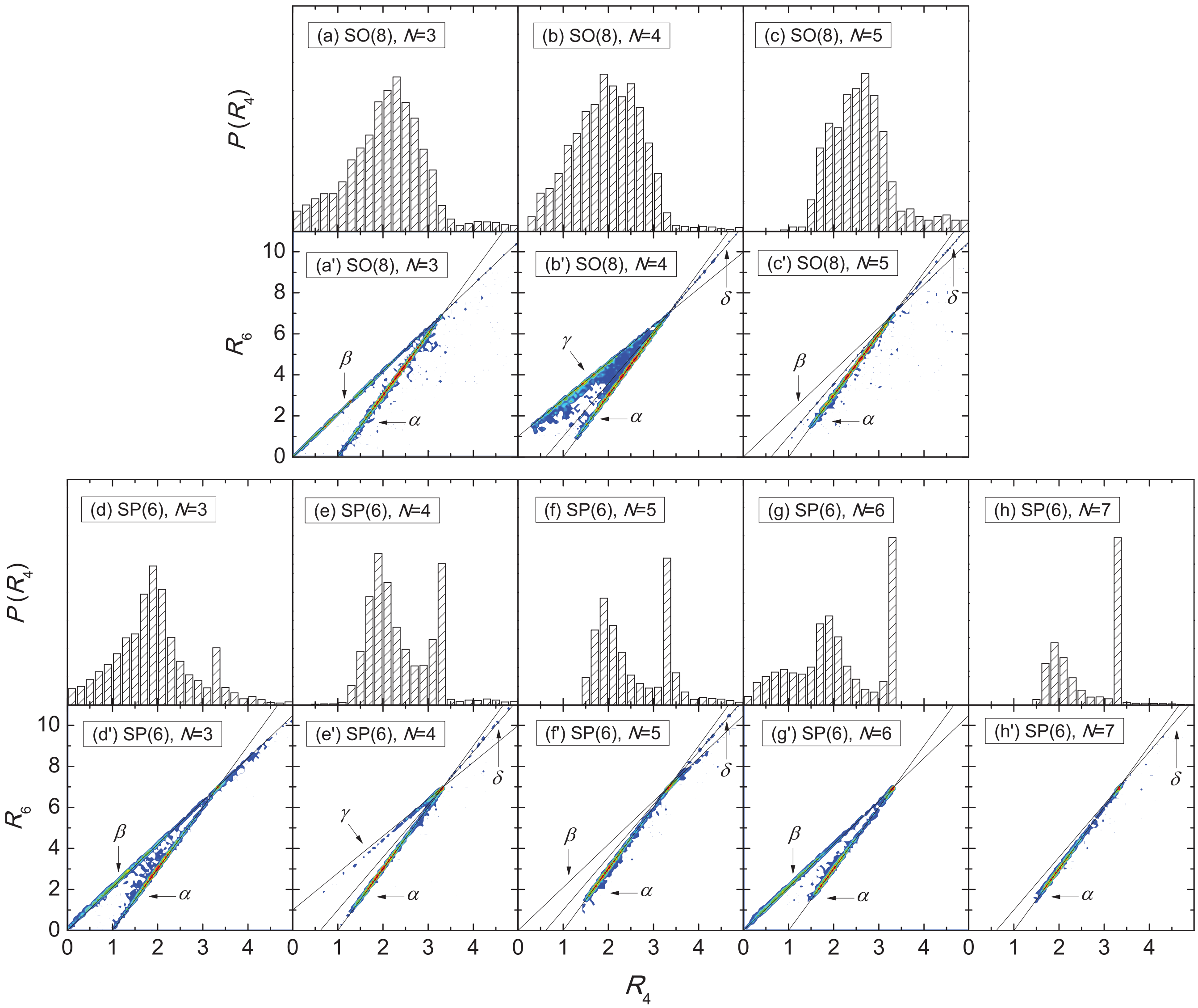

FIG. 1. (Color online) Distribution of R_4 and (R_6, R_4) for FDSM systems under the two-body random ensemble Hamiltonian. No vibrational or rotational peaks, $(\frac{I}{2}, 2)$ or $(\frac{I(I+1)}{6}, 3.33)$, are found in SO(8), but they are in in SP(6) systems. Four linear correlations, denoted α, β, γ, and δ, between R_I and R_4 are noted. See the text for details.

Interestingly, for random samples close to rotational motion in the SP(6) systems, we see that $B_2 < G_2 \simeq G_0$ ($B_1 > B_2$), i.e., the requirement of the SU(3) symmetry holds. In this Brief Report relations between the interaction parameters in brackets mean statistical ones.

Now we look at the patterns exhibited in the Mallmann plot, the correlation between R_6 and R_4, shown in Fig. 1. There are, in total, four types of elegant correlations, denoted α, β, γ, and δ, respectively. We note that the first three correlations have all been pointed out for *sd* bosons in Ref. [13]. The correlation δ is discerned in this Brief Report. Below we discuss these correlations in detail.

The correlation α is a universal correlation in all FDSM systems. The analytical expression of the correlation α is $R_I = \frac{I(I-2)}{8} R_4 - \frac{I(I-4)}{4}$. Thus we have $R_6 = 3R_4 - 3$, and the values of R_4 range from 1 to 3.33. The correlation α is described by a straight line passing through two $(R_6, R_4) = (3, 2)$ and (7, 3.33), corresponding to the vibrational and rotational motions, respectively. For SO(8) symmetry, the two-body interaction parameters in the α correlation follow $G_2 > B_2$, $G_0 > B_2$, $B_3 > B_2$ ($B_1 > B_2$); and for SP(6) symmetry, they well follow $G_2 \simeq B_2 > G_0$, the requirement of the SU(2) subgroup limit in the SP(6) FDSM.

In Ref. [13], the correlation α is presented in terms of the anharmonic vibrator model [14–16], in which

$$E_{I_1^+} = \frac{I}{2} E_{2_1^+} + \frac{I(I-2)}{8} \varepsilon_4,$$

where ε_4 is a parameter with a constant value. Numerical experiments in Ref. [13] did show the constancy of ε_4. Here, however, this is not the case. In Fig. 2 we present the correlation between $E_{4_1^+}$ and $E_{2_1^+}$. One sees that $E_{4_1^+}$ and $E_{2_1^+}$ do not follow a linear correlation with given slope (2.0) and a constant ε_4

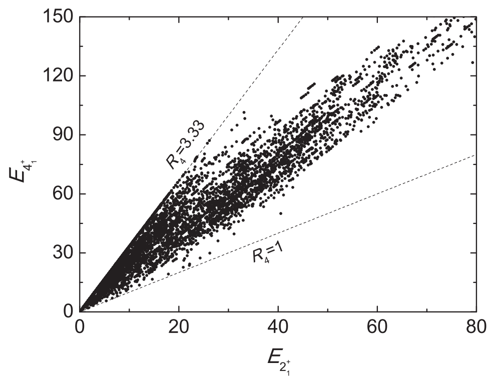

FIG. 2. Correlations between $E_{4_1^+}$ and $E_{2_1^+}$ for random samples corresponding to the correlation α. The unit for both $E_{4_1^+}$ and $E_{2_1^+}$ is arbitrary. Here we include all samples with $|R_6 - 3R_4 + 3| < 0.01$ in Fig. 1. No linear correlation is found, which means that the correlation α cannot be represented by the anharmonic vibrator model.

very closely. Therefore the origin of the correlation α warrants further studies in the future.

The correlation β is given by $R_I = \frac{I(I+1)}{20} R_4$, and thus $R_6 = 2.1 R_4$. It is described by a straight line through $(R_I, R_4) = (0,0)$, and $(\frac{I(I+1)}{6}, 10/3)$. For systems with SO(8) symmetry, the random parameters of the correlation β follow $B_2 > B_3$, $G_2 > B_3$, $B_1 > B_3$; and for systems with SP(6) symmetry, one has $G_0 > B_2 > G_2$. The correlation γ is given by $R_6 = \frac{9}{5} R_4 + 1$; for SO(8) symmetry, $G_0 > G_2$, $B_2 > G_2$, $B_3 > G_2$, and for SP(6) symmetry, $G_0 > G_2 > B_2$ $(B_2 < B_1)$. In Ref. [13], the correlations β and γ for sd-boson systems are found to be represented by the U(5) limit, i.e., d-boson condensation. In the FDSM there are three subgroups, SO(5), SO(7), and SU(2), which are interpreted by anharmonic spherical vibrators. Unfortunately, none of these three dynamical symmetries have a clear correspondence with the β and γ correlation here.

The correlation δ is given by $R_I = \frac{(I-2)(I+3)}{14} R_4 - \frac{(I-4)(I+5)}{14}$, and we have $R_6 = \frac{18}{7} R_4 - \frac{11}{7}$. It is described by a straight line passing through $(R_I, R_4) = (1, 1)$ and $(\frac{I(I+1)}{6}, 3.33)$, the values of the seniority and rotational modes, respectively. For SO(8) symmetry, they follow $B_2 > B_3$, $G_2 > B_3$ $(B_1 > B_3)$; and for SP(6) symmetry, $G_0 > G_2$, $G_0 > B_2$ $(B_1 > B_2)$.

To summarize, in this Brief Report we generalize the studies in Refs. [8,11,13] and study excited states in the yrast states of many-body systems under random two-body interactions, within the framework of the FDSM. The Mallmann plot is found to be very useful in discerning regular patterns among the random ensembles.

As in Ref. [8], collective motions of vibration and rotation do not arise in systems with SO(8) symmetry but arise dominantly in SP(6) systems. In addition, linear correlations between R_6 and R_4 are pointed out. They are called α, β, γ, and δ, respectively. These correlations are very similar to those in sd-boson systems [13], but with the following features.

(i) The correlation α is universal and not represented by the anharmonic vibrator model. Correlations between (R_I, R_4) are very different for different pair numbers N; α and β correlations are dominant when the pair number equals 3 and 6, and α and γ correlations are dominant when $N = 4$.

(ii) Correlations β and γ do not follow the vibrational limit; the probability of seniority mode among the entire ensemble, namely, $(R_I, R_4) = (1, 1)$, is almost 0.

(iii) The linear correlations, β, γ, and δ, hold well in the region of R_4 much larger than 3.33. This is given by the low 2_1^+ state energies.

The random parameters for the rotational peak and the α correlation in SP(6) systems are found, as expected, to follow the requirements of the SU(3) limit and the SU(2) limit, respectively. They correspond to the rotational and vibrational motions. The origin of other correlations remains unknown and thus future studies are warranted.

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