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Potential-model estimate of the mass of ^{11}O(g.s.)

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By using mirror symmetry and information from ^{11}Li, we have used a simple potential model to estimate the energies of the s^2 and p-shell components of ^{11}O(g.s.). We present the predicted ^{11}O mass in terms of the mixing of these two components.

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I. INTRODUCTION

Two estimates of the mass of the ground state (g.s.) of ^{11}O have appeared recently. Charity *et al.* [1] used the energies of ^{11}Li(g.s.), its analog in ^{11}Be, and the double analog in ^{11}B together with the quadratic form of the isobaric multiplet mass equation (IMME) to estimate the mass of ^{11}O(g.s.). Their result was a mass excess of 46.70(84) MeV, which corresponds to $E_{2p} = 3.21(84)$ MeV relative to ^{9}C + $2p$. One of us used a fit [2] to energy differences of several mirror pairs in the region and obtained a value of 5.41(11) MeV [3] for an s^2 fractional occupancy of $P(s^2) = 0.33(6)$ [4]. Even for $P(s^2) = 1.0$, that estimate would have been 4.12 MeV. This large difference prompted us to attempt a potential-model calculation for this case.

A simple potential model has proven quite successful in reproducing mirror energy differences (MEDs) for several light mirror pairs. Whenever the nuclear structure is reliably known, the calculations usually agree with the experimental values to about 30 keV. However, the case of ^{11}Li/^{11}O presents special problems as we see below.

II. CALCULATIONS AND RESULTS

The model assumes mirror symmetry. The procedure is described in detail elsewhere [5]. We treat ^{11}Li [6] as consisting of a ^{9}Li core plus two neutrons in the sd and $1p$ shells and ^{11}O as a ^{9}C core plus two protons in the same configurations. A straightforward calculation would use the energies in ^{10}Li [7] as $^9\text{Li} + n$, plus mirror symmetry, to compute the energies of the mirrors in ^{10}N as $^9\text{C} + p$ and then use these as cores to compute ^{11}O from ^{11}Li. However, two problems exist. All the states of ^{10}Li are unbound, and only one of the mirrors in ^{10}N is known. In the absence of experimental data, we can calculate the energies of the ^{10}N states and then use them. This procedure worked well for the case of ^{19}Mg [8] where we used calculated ^{18}Na energies. In a subsequent experiment [9], measured ^{18}Na energies agreed very well [10] with our calculated values.

However, in ^{10}Li, two of the low-lying states are s wave, and a simple potential model does not support an s-wave neutron resonance. In computing the masses of the lowest states in the $A = 10$ isospin quintet [11], we surmounted this problem by performing calculations for slightly bound states and by extrapolating to the unbound g.s. energy of 26 keV. We estimated that this extrapolation introduced an additional uncertainty of about 35 keV. Here, however, we also need the second s-wave structure, whose energy is $E_n = 0.73$ MeV, and that extrapolation is much less reliable. Rather than attempt that procedure, we have chosen to use the result of a recent fit [12] to mirror energy differences for $2s_{1/2}$ states in several light nuclei. This state is outside the fitted range of the other nuclei, and that produces some additional uncertainty. We estimate the uncertainty associated with the calculation of the energy of the s^2 component in ^{11}O(g.s.) to be about 100 keV.

This problem does not exist for the p-shell core states because the potential model does support p-wave neutron resonances. But, another problem arises: Because the valence and core nucleons are in the same major shell, several core states could contribute, and we plan to only use the first two. For this reason, we estimate the uncertainty in the computed energy of the p-shell component of ^{11}O(g.s.) to also be about 100 keV.

Table I lists the relevant s-wave core states in ^{10}Li [7], the calculated proton energies in ^{10}N, and the resulting $2p$ energies in ^{11}O. It is not known which of the s-wave structures in ^{10}Li is 1^- and which is 2^-, but that ambiguity produces only an 18-keV uncertainty in the s^2 energy. For the p-shell component, we assume the lower, at 0.26 MeV, is 1^+ and the upper, at 0.53 MeV, is 2^+. Again, changing the order has a tiny effect on the p-shell $2p$ energy. Energies of the s^2- and p-shell components of ^{11}O(g.s.) are listed in Table II.

TABLE I. Experimental (^{10}Li) and calculated (^{10}N) energies (MeV) of two lowest s-wave structures and results for ^{11}O.

J^{π} [a]	$E_n(^{10}\text{Li})$ [a]	$E_p(^{10}\text{N})$	$E_{2p}(^{11}\text{O})$ [b]
$(1^-, 2^-)$	0.026	1.81 [c]	3.73
$(2^-, 1^-)$	0.73	2.34 [d]	3.59

[a] Reference [7].
[b] Present paper.
[c] Reference [11].
[d] Reference [12].

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[*] Deceased.

TABLE II. Energies (MeV) of ^{11}O(g.s.) for various components.

Component	E_{2p}
s^2	3.66
p shell	4.86
Mixed[a]	4.46[7(10)][b]

[a] Computed for $P(s^2) = 0.33(6)$.

[b] The first uncertainty is from uncertainty in $P(s^2)$, and the second uncertainty is the estimated uncertainty in the procedure.

The amount of the s^2 component in ^{11}Li(g.s.) is not well established. Estimates have ranged from very small to quite large. The question as to whether a small d^2 component should be included is still unsettled. Consideration of the matter radius of ^{11}Li led us to conclude [4] that the s^2 component was $P(s^2) = 0.33(6)$. However, several quite different values of matter radius have been extracted from interaction and/or reaction cross sections so that a different selection of those results could have produced a value of $P(s^2)$ outside this range. As stated, the amount of d^2 also is not known, but it is known to be small. In any case, the d^2- and p-shell Coulomb energy differences will be about the same, so we need not consider d^2 separately.

Plotted in Fig. 1 is the calculated $2p$ energy of ^{11}O(g.s.) for the entire range of values of $P(s^2)$ with vertical lines at $P(s^2) = 0.33(6)$. The horizontal lines represent the estimate of $E_{2p} = 3.21(84)$ MeV from Charity *et al.* [1]. [Here and elsewhere, E_{2p} is just the negative of the $2p$ separation energy.] We note agreement at the 1σ level for any $P(s^2) \geqslant 0.59$. A better determination of this quantity would be very useful.

III. SUMMARY

We have used a simple potential model to estimate the energies in ^{11}O of the pure s^2 and p-shell components. We have presented our results for the $2p$ energy of ^{11}O(g.s.) as a function of the fraction of the s^2 component. Unlike the previous analysis [3] based on MED alone, the present potential model can find agreement with the IMME analysis of Charity *et al.* [1] if $P(s^2) > 0.59$. The MED result differed from the IMME prediction by 1.1σ even at the maximum value of $P(s^2) = 1$.

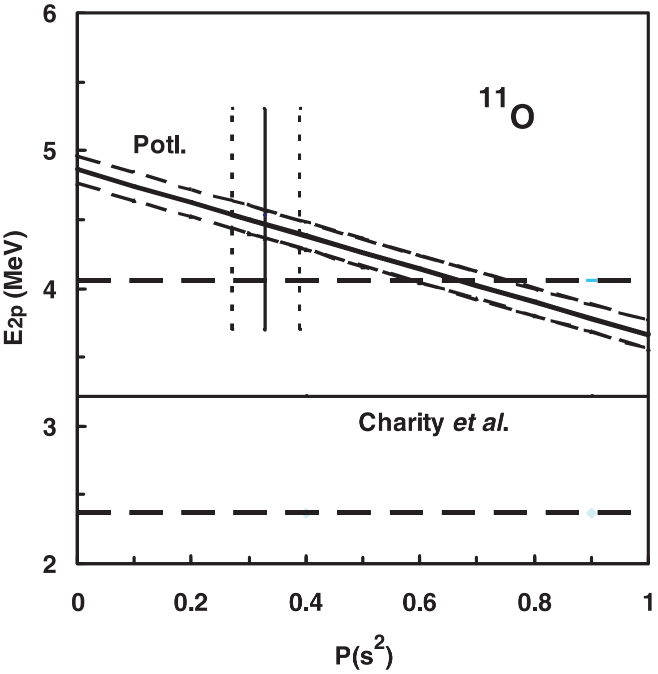

FIG. 1. (Color online) The downward-sloping lines represent our computed estimate (solid lines) and accompanying uncertainty (dashed lines) of the $2p$ energy of ^{11}O(g.s.) as a function of the s^2 fraction $P(s^2)$. The vertical lines are at $P(s^2) = 0.33(6)$. The horizontal lines represent the estimate of Charity *et al.* [1].

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