In-medium nucleons and nucleonic systems: Infinite nuclear matter

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In the present work we discuss the modifications of infinite nuclear matter properties. The modifications are performed in the framework of the in-medium modified Skyrme model. The model is developed to study the properties of in-medium nucleons and nucleonic systems. The mesonic sector of the model contains the nonlinear pion fields propagating in the nuclear medium. The properties of in-medium pions are defined by the pion-nucleus optical potential. The isospin-breaking part of the optical potential and the isospin-breaking effects in the mesonic sector generate the isospin-breaking effects in the baryonic sector. Further, the isospin-breaking effects in the baryonic sector are related to the asymmetric-matter properties. First, we discuss the binding energy per nucleon and the bulk properties of the isospin-symmetric nuclear matter. Then, we include the isospin-breaking effects and discuss the asymmetric-matter properties.

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I. INTRODUCTION

Studies of nucleon properties at finite density and temperature are important for understanding the properties of baryonic matter under extreme conditions. The discovery of the effect by European Muon Collaboration (EMC effect) [1–3], pointing to changes of nucleon properties in nuclear matter (e.g., swelling of the nucleon), initiated further experimental measurements [4–8] and different theoretical approaches [9–16]. The measurements, particularly, have been devoted to studies of electromagnetic (EM) form factors of bound nucleons. For example, the polarization-transfer phenomenon in proton knock-out reactions $^4\mathrm{He}(\vec{e}, e', \vec{p}\,)^3\mathrm{H}$ had prompted continuous experimental interest during the last decade [5–8]. Those experimental measurements showed the quenching effect in the polarization-transfer double ratio $\mathcal{R} = (P'_x/P'_z)_{^4\mathrm{He}}/(P'_x/P'_z)_{^1\mathrm{H}}$. From the theoretical point of view, the quenching effect can be interpreted as an indication of changes in the nucleon's EM form factors in the nuclear medium [9–15] or as an effect of final-state interactions and two-body current contributions [16].

Among the existing approaches related to the hadron properties in nuclear matter, many theoretical models are based on the concept of in-medium nucleons perceiving their individuality. However, properties of nucleons may change in the nuclear medium. Therefore, depending on the energy scale and as is asserted in Refs. [13,15], the changes in EM structures of the bound nucleons may be the actual case observed during the experimental measurements of the polarization-transfer double ratio $\mathcal{R}$. The studies of in-medium EM form factors will help to analyze the difference in behaviors of in-medium protons and neutrons. Furthermore, knowledge of the behavior of in-medium nucleons would be useful for studying the properties of infinite and finite nucleonic systems in a more self-consistent way.

The main focus of present and future works is on modifications of infinite nuclear matter properties, studies of ground-state properties of magic nuclei and nuclei near the shell closure, and descriptions of nuclear reactions. The approach we are going to use is semiphenomenological. To justify this approach, one should satisfy as much as possible phenomenological requirements—the model must be able to reproduce the experimental data and explain the related phenomena. Therefore, we perform our studies step by step. As an initial step I, we concentrate on the analysis of nucleon properties in symmetric and asymmetric nuclear matter. However, we relate those nucleon properties to the bulk properties of infinite nucleonic systems. The nucleon properties in finite nuclei and the relation of those properties to finite nuclear systems will be the subject of the next step II, which will be considered in our future works.

The paper is organized in the following way. In the next section we discuss a topological solitonic approach to nuclear matter and formulate our tasks. Further, in that section we present the generalized Lagrangian (Sec. II A) and discuss the medium functionals describing the influence of the surrounding environment on the in-medium nucleon properties and discuss the peculiarities of our model by comparing with other approaches (Sec. II B). In Sec. III we discuss the classical (Sec. III A) and quantum (Sec. III B) solitons in nuclear matter. Then, in Sec. IV we study the binding energy per nucleon, extracting the contributions of volume (Sec. IV A) and symmetry-energy (Sec. IV B) terms. The results are presented in Sec. V, where we discuss symmetric (Sec. V A) and asymmetric nuclear matter (Sec. V B) properties, separately. In Sec. V C we discuss the small corrections due to the explicit isospin-breaking effects in the mesonic sector. Finally, in Sec. VI we summarize our results and present the outlook for further studies.

II. NUCLEAR MATTER IN A SOLITONIC APPROACH

While the objects of primary interest (constituents of nuclear matter) are assumed to have an extended structure, a chiral solitonic approach may serve as one of the appropriate tools during our studies. Therefore, the theoretical framework in this work will be an in-medium modified solitonic model.

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We will use the in-medium modified Skyrme model [17] where, in addition to the initial studies considering the kinetic and mass term modifications [18], the modification of the Skyrme's quartic term has also been taken into account. However, in the present work, we will generalize the model presented in Ref. [17]; i.e., we take into account the isospin-breaking effects as has been done in Refs. [19,20] in order to study the Nolen-Schiffer anomaly (NSA) in mirror nuclei [21,22].

It is necessary to note that the calculations in Refs. [19,20] are based on the idea of an effective neutron-proton mass difference, which is supposed to change in nuclear matter relatively to its free-space value. During those studies, the electromagnetic part of the in-medium neutron-proton mass difference was calculated by taking into account the changes in EM form factors of nucleons due to the surrounding nuclear environment.

Naturally, the approach developed in Refs. [19,20] can be applied to studies of quenching effect in the polarization-transfer double ratio $\mathcal{R}$ during the proton knock-out $^4\mathrm{He}(\vec{e}, e', \vec{p})^3\mathrm{H}$ reaction. However, before calculating $\mathcal{R}$ in reactions involving finite nuclei, one has to improve the model because the initial approach presented in Ref. [18] and later versions [19,20] have a shortcoming; i.e., the renormalization of the nucleon's mass in nuclear medium was large. It is necessary to note that our framework employs a nonrelativistic approach to the nucleon and nuclear matter properties. Therefore, it is difficult to explain the large renormalization of the nucleon's effective mass in our model, in contrast to the relativistic mean-field approaches. For example, the mass ratio $m_{\mathrm{N}}^*/m_{\mathrm{N}} \approx 0.8$ for the nucleon in $^{16}\mathrm{O}$ led to further difficulties during the calculation of the Nolen-Schiffer anomaly in the framework of the in-medium modified Skyrme model [20].[1] The main conclusion in Ref. [20] was the *stringent* restriction to the possible modifications of the nucleon's effective mass in nuclei.

Technically, the large renormalization of the nucleon's effective mass may be related to the behavior of the Skyrme term in nuclear matter. For example, when the Skyrme term is intact in nuclear matter the renormalization of the nucleon's effective mass is large [18] while the modification of the Skyrme term in nuclear matter diminishes the renormalization effect [17]. From a physical point of view, the first situation may be interpreted as attributable to ignorance of the nucleon's core modifications in nuclear matter. The second case, in contrast, implies the core modifications too. While the original Skyrme term is important for stabilization of the nucleon-soliton, its medium-modified version seems to be important for stabilization of nuclear matter. For example, further developments of the initial approach allowed correct reproduction of the bulk properties of symmetric nuclear matter [17].

Consequently, in the present work we propose a generalized version of the in-medium modified Skyrme model which unifies the ideas of Refs. [17–20]. We formulate the generalized Lagrangian which takes into account the following: the nucleon's "outer-shell" and "inner-core" modifications in a symmetric nuclear medium, the corresponding symmetric-matter properties, the isospin-breaking effects in the mesonic sector and the subsequent effects in the single baryonic sector, changes in nucleon properties due to an isospin-asymmetric nuclear environment, and the corresponding asymmetric-matter properties.

In other words, after formulation of the generalized Lagrangian, first we rediscuss the modification of nucleon properties in symmetric nuclear matter and study the correlations of those properties with the symmetric-matter characteristics (e.g., the volume term in the binding energy formula, the compressibility of symmetric nuclear matter, and the density dependence of pressure in symmetric matter). Then we discuss isospin-symmetry-breaking effects in the mesonic sector, the corresponding isospin-breaking effects in the baryonic sector (e.g., the neutron-proton mass difference), and their relation to asymmetric-matter properties (e.g., the nuclear symmetry energy, the compressibility of asymmetric matter, and asymmetric-matter characteristics near the saturation point). These formulated tasks will be the content of step I.

A. Generalization of the in-medium modified Lagrangian

We start from the in-medium modified Skyrme-model Lagrangian [19,20] and generalize it by taking into account the modifications of the Skyrme's quartic term as has been done in Ref. [17]. The generalized Lagrangian has the following form:

$$\mathcal{L}^* = \mathcal{L}^*_{\mathrm{sym}} + \mathcal{L}^*_{\mathrm{asym}}. \tag{1}$$

It is schematically separated into two parts: the isospin-symmetric part $\mathcal{L}^*_{\mathrm{sym}}$ and the isospin-asymmetric part $\mathcal{L}^*_{\mathrm{asym}}$.

The isospin-symmetric part contains three terms:

$$\mathcal{L}^*_{\mathrm{sym}} = \mathcal{L}^*_2 + \mathcal{L}^*_4 + \mathcal{L}^*_{\chi\mathrm{SB}}. \tag{2}$$

Here $\mathcal{L}^*_2$ is the in-medium modified nonlinear σ-model Lagrangian, $\mathcal{L}^*_4$ is the in-medium modified Skyrme's stabilizing term, and $\mathcal{L}^*_{\chi\mathrm{SB}}$ is the in-medium modified chiral-symmetry-breaking term. Their explicit forms are given as

$$\mathcal{L}^*_2 = \frac{F_\pi^2}{16}\left\{\alpha_s^{02}\,\mathrm{Tr}(\partial_0 U \partial_0 U^\dagger) - \alpha_p^0\,\mathrm{Tr}(\partial_i U \partial_i U^\dagger)\right\}, \tag{3}$$

$$\mathcal{L}^*_4 = -\frac{1}{16e^2\zeta_\tau}\,\mathrm{Tr}\,[U^\dagger\partial_0 U, U^\dagger\partial_i U]^2 + \frac{1}{32e^2\zeta_s}\,\mathrm{Tr}\,[U^\dagger\partial_i U, U^\dagger\partial_j U]^2, \tag{4}$$

$$\mathcal{L}^*_{\chi\mathrm{SB}} = \frac{F_\pi^2 m_\pi^2}{8}\,\alpha_s^{00}\,\mathrm{Tr}\,(U-1), \tag{5}$$

where Einstein's summation convention is always assumed (if not specified otherwise).

The isospin-asymmetric part contains two terms:

$$\mathcal{L}^*_{\mathrm{asym}} = \Delta\mathcal{L}_{\mathrm{mes}} + \Delta\mathcal{L}^*_{\mathrm{env}}. \tag{6}$$

Here $\Delta\mathcal{L}_{\mathrm{mes}}$ is the isospin-breaking term that arises from the explicit symmetry breaking in the mesonic sector, and the term $\Delta\mathcal{L}^*_{\mathrm{env}}$ takes into account the isospin asymmetry of the nucleus.

[1] Hereafter, a superscripted asterisk indicates an explicit medium modification. For an inexplicit medium modified expression we use a symbol without the asterisk.

Their explicit forms are

$$\Delta\mathcal{L}_{\text{mes}} = \frac{F_\pi^2}{16}\sum_{a=1}^{2}\mathcal{M}_-^2\,\text{Tr}(\tau_a U)\text{Tr}(\tau_a U^\dagger), \tag{7}$$

$$\Delta\mathcal{L}_{\text{env}}^* = -\frac{F_\pi^2}{32}\sum_{a,b=1}^{2}\varepsilon_{ab3}\,\frac{\Delta\chi}{m_\pi}\text{Tr}(\tau_a U)\text{Tr}(\tau_b\partial_0 U^\dagger), \tag{8}$$

where $\mathcal{M}_-^2 = (m_{\pi^\pm}^2 - m_\pi^2)/2$.

The chiral SU(2) matrix U has the form $U = \exp(2i\tau_a\pi_a/F_\pi)$, where π_a $(a = 1,2,3)$ are the Cartesian isospin components of the pion field. The pion decay constant and the Skyrme parameter have values $F_\pi = 108.783$ MeV and $e = 4.854$, respectively [20]. The neutral-pion mass is $m_\pi = 134.977$ MeV, which is its Particle Data Group (PDG) value [23]. The model parameters are chosen in such a way that the exact PDG values of nucleon masses in free space, $m_{\text{p}} = 938.27$ MeV and $m_{\text{n}} = 939.56$ MeV [23], are reproduced correctly. Consequently, by ignoring the electromagnetic effects in pion masses, all of these choices of parameters induce the following value for the charged pion mass: $m_{\pi^\pm} = 135.015$ MeV. (For the detailed explanations see Refs. [19,20].)

The medium functionals, α_s^{00}, α_s^{02}, α_p^0 and $\Delta\chi$, depend on nuclear density $\rho(\vec{r})$ and represent the influence of the surrounding environment on the single-nucleon properties. They mostly represent the "outer-shell" changes of the in-medium nucleon.

In the present work, based on the ideas of Ref. [17], the in-medium modification of the Skyrme term is generalized as

$$\mathcal{L}_4 \Rightarrow \mathcal{L}_4^* = \frac{1}{\zeta_\tau}\,\mathcal{L}_4^{(\text{time})} + \frac{1}{\zeta_s}\,\mathcal{L}_4^{(\text{space})}. \tag{9}$$

Here the medium functionals ζ_τ and ζ_s also depend on nuclear density $\rho(\vec{r})$ and represent the "inner-core" modifications of the in-medium nucleon.

Finally, the Lagrangian in Eq. (1) with the specific modified term $\mathcal{L}_4^*$ in Eq. (4) is called the *generalized* in-medium modified Lagrangian and will be used in our calculations after we define the medium functionals. The peculiarities of the generalized Lagrangian are discussed in the next section.

B. Model peculiarities and consistency with other approaches

Here, we briefly outline the basic peculiarities of the generalized Lagrangian in Eq. (1). First of all we note that the effective Lagrangian in the linear approximation reproduces the pion Lagrangian in nuclear matter [19],

$$\mathcal{L}_{\text{low}}^* = \frac{1}{2}\sum_{\lambda=\pm,0}\left\{\partial_\mu\pi^{\lambda\dagger}\partial^\mu\pi^\lambda - \pi^{\lambda\dagger}\left(m_{\pi^\lambda}^2 + \hat{\Pi}^\lambda\right)\pi^\lambda\right\}. \tag{10}$$

Here $\hat{\Pi}^{\pm,0}$ are the self-energies of charged and neutral pions $\pi^{\pm,0}$ and depend on the functionals α_s^{00}, α_s^{02}, α_p^0, and $\Delta\chi$ [24]. In such a way, the pion Lagrangian in the nuclear medium, Eq. (10), is related to the generalized Lagrangian in Eq. (1). Moreover, the medium modifications are consistent with other approaches in Refs. [25–27].

As an example, we consider the $\mathcal{L}_2^*$ term given in Eq. (3), which includes temporal and spatial parts. The factors in Eq. (3) can be presented as $F_{\pi,t}^* = F_\pi\sqrt{\alpha_s^{02}}$ and $F_{\pi,s}^* = F_\pi\sqrt{\alpha_p^0}$, respectively. Then, with the set of parameters defined in Refs. [19,20], $F_{\pi,s}^*$ will decrease in nuclear matter. One can easily check that so does the ratio $F_{\pi,s}^*/F_{\pi,t}^*$. These results are in qualitative agreement with the results obtained in the framework of in-medium chiral perturbation theory [25,26] and the QCD sum rules approach [27].

The pion mass terms $\mathcal{L}_{\chi SB}^* + \Delta\mathcal{L}_{\text{mes}}$ in the generalized Lagrangian in Eq. (1) represent the in-medium modified version of the Lagrangian which was originally proposed by Rathske [28]. Note that the Lagrangian in Ref. [28] describes the isospin-breaking effects in the Skyrme model. The $\mathcal{L}_{\chi SB}^*$ and $\Delta\mathcal{L}_{\text{mes}}$ terms take into account the medium modifications of neutral-pion mass and the isospin-breaking effect in the mesonic sector and generate the subsequent isospin-breaking effects in the baryonic sector [19,20]. Schematically, if one uses $F_{\pi,t}^* = F_\pi\sqrt{\alpha_s^{02}}$ instead of F_π then the effective pion mass in an isospin-symmetric environment can be defined as $m_\pi^* = m_\pi\sqrt{\alpha_s^{00}/\alpha_s^{02}}$. Then, the set of parameters given in Refs. [19,20] reproduces the monotonically increasing function of density $m_\pi^*(\rho)$. This is again consistent with QCD sum rule studies [27].

The term $\Delta\mathcal{L}_{\text{env}}^*$ takes into account the isospin asymmetry of the surrounding environment; it is asymmetric under the interchange of neutron $\rho_{\text{n}}(\vec{r})$ and proton $\rho_{\text{p}}(\vec{r})$ distributions. Consequently, it is the term most responsible for the modification of the neutron-proton mass difference in asymmetric nuclear matter as well as for the self-consistent modifications of asymmetric-matter properties. Moreover, the Lagrangian in Eq. (8) contains the Weinberg-Tomozawa term as the relation

$$\frac{\tilde{b}_1\delta\rho}{4\pi\eta} = -\frac{m_\pi\delta\rho}{8\pi\eta f_{\pi,\text{ph}}^2} = b_1^{\text{l.o.}}\delta\rho \tag{11}$$

is based on the isovector s-wave scattering length in the chiral expansion to the lowest order (see the discussions in Refs. [29,30]). This term naturally appears due to the explicit energy dependence in the pion-nucleus optic potential [24].

Now let us discuss the remaining functionals ζ_τ and ζ_s entering into the in-medium modified Skyrme term $\mathcal{L}_4^*$. It is well known that the Skyrme's quartic stabilizing term may be related to the vector meson dominance model [31], which can be realized in implicit gauge symmetry of the nonlinear sigma model Lagrangian [32]. In this sense the medium modification via the functionals ζ_τ and ζ_s, in general, may be related to the vector-meson properties in nuclear matter. For example, reducing the Skyrme parameter in the nuclear medium may correspond to a decrease of the $g_{\rho\pi\pi}$ coupling. Then one can treat it as a change of the ρ meson width or as a diminishing value of the ρ meson mass in the nuclear medium. There are experimental indications of similar changes in the ρ meson properties [33–35] and some theoretical predictions [36,37].

In the spirit of chiral effective Lagrangians, the Skyrme model Lagrangian could be considered as a truncated version of the general low-energy Lagrangian. Then the total information from the higher order terms accumulate effectively in the Skyrme parameter. Consequently, the in-medium modifications of the higher derivative terms in the general low-energy

Lagrangian can be expressed in terms of the in-medium modified Skyrme parameter. Therefore, the generalized in-medium modified Skyrme model already has all the necessary ingredients and, in principle, could be also relevant (at least qualitatively) to studies of nuclear many-body problems.

For example, due to its stabilizing role, the Skyrme term will also be responsible for the medium modifications, first of all, near the saturation point and at larger densities. Because at high densities the nucleon's core modifications may become important, one should take into account those modifications in order to avoid the collapse of nuclear matter to a singularity [17].

There are already alternate approaches to treat nuclear many-body problems in the framework of the Skyrme model. One of the ways was pioneered by Kutschera *et al.* [38] and later Klebanov [39] discussed a possible formation of skyrmionic matter with a simple cubic crystalline structure due to tensor forces between the neighboring nucleons of the unit cell. There have been a series of studies related to the energetically favorable crystalline structures [40–47] and parallel studies of the Skyrme model on the hypersphere [48–55]. The former are still under discussion and concentrate on the investigations of face-centered cubic (FCC) structure and the possible phase transitions due to baryon charge delocalization from the lattice points to the lattice sides [56–60]. In a quantum mechanical Hartree-Fock-type approach, minimizations of the FCC crystal energy are performed by using the effective in-medium interactions calculated in the framework of the in-medium Skyrme model [61]. One can also mention the studies of exotic many-body systems where the nuclei are considered as topologically nontrivial configurations [62–66].

However, in the above-mentioned alternate approaches, the in-medium skyrmions lose their individuality partially or completely from the beginning and therefore seem to be not relevant for the discussions of nucleon knock-out reactions and disintegration processes during heavy-ion collisions. Here we would like to note that in the framework of the present approach one may always trace the properties of an individual nucleon in dense matter (including finite nuclei), except for an extreme (very high density) case. Nevertheless, that extremal situation can also be naturally explained via the stabilization mechanism in terms of the in-medium energy-momentum tensor form factors [67]. The conclusion is that at very high densities the solitonic picture may no longer be preserved since the skyrmions start to overlap and may lose their individuality.

All of the alternate approaches within the Skyrme model have to deal with modifications of Lagrangian parameters in nuclear matter. As an example, the authors of [62–66] describe light and medium-heavy nuclei properties via the change of Lagrangian parameters (in particular, the Skyrme parameter) from nucleus to nucleus. They call this a "calibration." In this context, our modification via the functionals ζ_τ and ζ_s is an alternate path to those and other discussed approaches.

III. NUCLEONS IN NUCLEAR MATTER

The peculiarities of the quantization procedure are discussed in detail in Refs. [19,20]. The corresponding in-medium modified expressions for the classical soliton mass, for the quantized symmetric rotator's energy eigenvalues, and for the electromagnetic form factors of nucleons and all relevant discussion also can be found there. The difference in the present work is that the Skyrme parameter must change as $e \to e^* = \zeta_\tau^{1/2} e$ or $e \to e^* = \zeta_s^{1/2} e$, depending on the corresponding (time-dependent or time-independent) part of the Lagrangian. Consequently, the simple ζ_τ^{-1} or ζ_s^{-1} factors appear in the expressions related to $\mathcal{L}_4^*$. It is trivial to trace the ζ^{-1} factor due to an infinite nuclear matter approximation considered in the present work.[2] Moreover, one can consider the density as a constant parameter and all density functionals become simply functions of the density parameter. In this case, one can formulate a transparent minimization scheme which allows the simple parametrization of medium functions as discussed below in the present work.

A. Classical solitons

In infinite nuclear matter, using the spherically symmetric hedgehog ansatz for the chiral field

$$U = \exp\{i\vec{\tau}\,\vec{n}F(r)\}, \quad \vec{n} = \vec{r}/r, \tag{12}$$

and the condition[3]

$$a^{*2} = 2\mathcal{M}_-^2\,\frac{\Lambda^*_{\rm mes}}{\Lambda^*} \tag{13}$$

one obtains the following Lagrangian:

$$L^* = -M^*_{\rm NP} + a^*\Lambda^*_{\rm env}, \tag{14}$$

where

$$M^*_{\rm NP} = \sqrt{\frac{\alpha_p^0}{\zeta_s}}\,m^*(\beta), \tag{15}$$

$$m^*(\beta) = \frac{4\pi F_\pi}{e}\int_0^\infty dx\,x^2\left\{\frac{1}{8}\left[F_x^2 + \frac{2\sin^2 F}{x^2}\right] + \frac{\sin^2 F}{x^2}\left[F_x^2 + \frac{\sin^2 F}{2x^2}\right] + \frac{\beta^2}{2}\sin^2\frac{F}{2}\right\}, \tag{16}$$

$$\Lambda^* = \left(\alpha_p^0\zeta_s\right)^{-3/2}\left(\alpha_s^{02}\Lambda_2 + \zeta_\tau^{-1}\alpha_p^0\zeta_s\Lambda_4\right), \tag{17}$$

$$\Lambda^*_{\rm mes} = \left(\alpha_p^0\zeta_s\right)^{-3/2}\Lambda_2, \tag{18}$$

$$\Lambda^*_{\rm env} = \left(\alpha_p^0\zeta_s\right)^{-3/2}\frac{\Delta\chi}{2m_\pi}\Lambda_2, \tag{19}$$

$$\Lambda_2 = \frac{2\pi}{3e^3F_\pi}\int_0^\infty dx\,x^2\sin^2 F, \tag{20}$$

$$\Lambda_4 = \frac{8\pi}{3e^3F_\pi}\int_0^\infty\left(F_x^2 + \frac{\sin^2 F}{x^2}\right)x^2\sin^2 F\,dx. \tag{21}$$

Here we introduced the notation $F_x = \partial F/\partial x$ and the scaled variable $x = (\alpha_p^0\zeta_s)^{1/2} eF_\pi r$ and we defined the density-dependent function

$$\beta^2 = \left(\frac{m_\pi^2}{e^2F_\pi^2}\right)\frac{\alpha_s^{00}}{\left(\alpha_p^0\right)^2\zeta_s}. \tag{22}$$

[2]Some additional changes appear in describing the phenomena related to finite nuclei and we will discuss them in a separate work.

[3]For more details, see Ref. [19].

The corresponding equation of motion is obtained by variation of the Lagrangian in Eq. (14) and has the form

$$(x^2+8\sin^2 F)F_{xx}-x^2\beta^2\sin F(1-2a^*\gamma^2\cos F)$$
$$+2xF_x+\left(4F_x^2-\frac{4\sin^2 F}{x^2}-1\right)\sin 2F=0, \quad (23)$$

where we defined another density function

$$\gamma^2=\frac{\Delta\chi}{12m_\pi^3\alpha_s^{00}}=\frac{\Delta\chi}{24\rho_0\alpha_s^{00}}. \quad (24)$$

Therefore, the classical solution F depends on the two parameters β and γ: $F=F(x;\beta,\gamma)$. The boundary conditions have the forms

$$\begin{array}{ll} F\to\pi-ax, & x\to 0,\\ F\to b(1+\tilde\beta x)x^{-2}\,e^{-\tilde\beta x}, & x\to\infty,\\ \tilde\beta=\beta\sqrt{1-2a^*\gamma^2} & \end{array}$$

and satisfy the baryon number one ($B=1$) solution.

B. Quantum solitons

Considering the time-dependent rotations in isotopic space, one can get the following time-dependent Lagrangian (see Ref. [19]):

$$L^*=-M_{\rm NP}^*-\mathcal{M}_-^2\Lambda_{\rm mes}^*+\frac{\omega_1^2+\omega_2^2}{2}\Lambda^*$$
$$+\frac{(\omega_3+a^*)^2}{2}\Lambda^*+(\omega_3+a^*)\Lambda_{\rm env}^*, \quad (25)$$

where $\vec\omega$ is the angular velocity of the spinning skyrmion. Furthermore, defining the canonical conjugate variables in the body-fixed reference system $T_i=\partial L^*/\partial\omega_i$ and using the canonical quantization method one obtains from the time-dependent Lagrangian in Eq. (25) the following Hamiltonian:

$$\hat H=M_{\rm NP}^*+\mathcal{M}_-^2\Lambda_{\rm mes}^*+\frac{\Lambda_{\rm env}^{*2}}{2\Lambda^*}+\frac{\hat{\boldsymbol T}^2}{2\Lambda^*}-\left(a^*+\frac{\Lambda_{\rm env}^*}{\Lambda^*}\right)\hat T_3. \quad (26)$$

By sandwiching the Hamiltonian between the appropriate baryon states one determines the energy of a nucleon in nuclear matter and the corresponding neutron-proton mass difference due to the strong interactions:[4]

$$\Delta m_{\rm np}^*=a^*+\frac{\Lambda_{\rm env}^*}{\Lambda^*}. \quad (27)$$

Finally, one gets the in-medium masses of the nucleons,

$$m_{\rm n,p}^*=m_{\rm N}^{\rm S*}-\Delta m_{\rm np}^*T_3, \quad (28)$$

where

$$m_{\rm N}^{\rm S*}=M_{\rm NP}^*+\frac{3}{8\Lambda^*}+\frac{\Lambda^*}{2}\left(a^{*2}+\frac{\Lambda_{\rm env}^{*2}}{\Lambda^{*2}}\right) \quad (29)$$

is the isoscalar part of the nucleon's mass and T_3 is the third component of isospin.

[4]During our practical calculations, we will ignore the electromagnetic part of the neutron-proton mass difference, while its density dependence is found to be very weak (e.g., see Ref. [19]).

IV. NUCLEAR MATTER

In order to discuss the properties of nuclear matter, let us introduce two free parameters: the density parameter $\lambda=(\rho_{\rm n}+\rho_{\rm p})/\rho_0$ in terms of the normal nuclear matter density ρ_0 and the isospin asymmetry parameter $\delta=(\rho_{\rm n}-\rho_{\rm p})/\rho$.

It is known that the ground-state properties of finite nuclei are described very well by the semiempirical mass formula. Consequently, one can write the binding energy per nucleon in the form

$$\varepsilon(A,Z)=-a_V+a_S\frac{(N-Z)^2}{A^2}+\cdots, \quad (30)$$

where the first term is the volume part and the second term is the asymmetric part. The surface, Coulomb, and pairing terms are indicated by dots. They can be ignored at zero temperature and in the thermodynamic limit with an infinite number of baryons distributed in an infinite volume. Therefore, the isospin asymmetry parameter δ can be expressed as $\delta=(N-Z)/A$. In terms of the asymmetry parameter, the binding energy formula takes the form

$$\varepsilon(\lambda,\delta)=-a_V(\lambda)+\varepsilon_S(\lambda)\delta^2+O(\delta^4)$$
$$\equiv\varepsilon_V(\lambda)+\varepsilon_A(\lambda,\delta), \quad (31)$$

where $\varepsilon_S(\lambda)$ is known as the symmetry energy.

A. Symmetric matter

In the case of symmetric matter ($N=Z$), the volume term in Eq. (30) is a functional of the nuclear density and its minimization gives the desired equation of state (EoS). Here, in order to perform the similar task, we define the volume term as

$$\varepsilon_V(\lambda)=\tfrac{1}{2}[m_{\rm p}^*(\lambda,0)+m_{\rm n}^*(\lambda,0)]-m_{\rm N}^{\rm S}$$
$$=m_{\rm N}^{\rm S*}(\lambda,0)-m_{\rm N}^{\rm S}, \quad (32)$$

where $m_{\rm n,p}^*(\lambda,0)$ are the masses of nucleons in symmetric nuclear matter and $m_{\rm N}^{\rm S}=(m_{\rm p}+m_{\rm n})/2$ is the isospin-averaged mass of the nucleons in free space.

If the volume term is fixed one can discuss the thermodynamic properties of symmetric nuclear matter. First of all it will be interesting to analyze the pressure. The density dependence of the pressure is given by the formula

$$p=\rho\,\frac{\partial\tilde{\mathcal{E}}_V(\rho)}{\partial\rho}-\tilde{\mathcal{E}}_V(\rho)=\rho_0\lambda^2\frac{\partial\varepsilon_V(\lambda)}{\partial\lambda}, \quad (33)$$

where

$$\tilde{\mathcal{E}}_V\equiv\frac{\mathcal{E}_V(\rho)}{V}=\varepsilon_V(\rho)\frac{A}{V}=\lambda\varepsilon_V(\lambda)\rho_0 \quad (34)$$

is the binding energy per unit volume in symmetric matter.

Another important quantity is the compressibility of nuclear matter, K_0, defined as

$$\frac{1}{9}\,\rho K_0=\frac{1}{K^{\rm th}}. \quad (35)$$

Here $K^{\rm th}$ is the thermodynamic isothermal compressibility and is defined as

$$\frac{1}{K^{\rm th}}=\rho\,\frac{\partial p}{\partial\rho}=\rho^2\left(2\,\frac{\partial\varepsilon_V}{\partial\rho}+\rho\,\frac{\partial^2\varepsilon_V}{\partial\rho^2}\right). \quad (36)$$

Consequently, at the equilibrium point ($\partial\varepsilon_V/\partial\rho = 0$), one can get the widely used expression of the compressibility

$$K_0 = 9\rho^2 \frac{\partial^2 \varepsilon_V}{\partial \rho^2}\bigg|_{\rho=\rho_0} = 9\lambda^2 \frac{\partial^2 \varepsilon_V}{\partial \lambda^2}\bigg|_{\lambda=1}. \quad (37)$$

B. Asymmetric matter

In order to reproduce the symmetry energy and discuss the properties of related quantities, we rewrite the binding energy per nucleon given in Eq. (31) as

$$\varepsilon(\lambda,\delta) = \frac{Zm_{\rm p}^*(\lambda,\delta) + Nm_{\rm n}^*(\lambda,\delta)}{A} - \frac{Zm_{\rm p} + Nm_{\rm n}}{A}. \quad (38)$$

Then, using the definition of the volume term in Eq. (32), one can define the asymmetric part of the binding energy as

$$\begin{aligned}\varepsilon_A(\lambda,\delta) &= \varepsilon(\lambda,\delta) - \varepsilon_V(\lambda)\\ &= m_{\rm N}^{\rm S*}(\lambda,\delta) - m_{\rm N}^{\rm S*}(\lambda,0) + [\Delta m_{\rm np}^*(\lambda,\delta) - \Delta m_{\rm np}]\frac{\delta}{2}.\end{aligned} \quad (39)$$

Consequently, the symmetry energy is defined by the formula

$$\begin{aligned}\varepsilon_S(\lambda) &= \frac{1}{2}\frac{\partial^2\varepsilon_A(\lambda,\delta)}{\partial\delta^2}\bigg|_{\delta=0}\\ &= \frac{1}{2}\frac{\partial^2}{\partial\delta^2}\left(m_{\rm N}^{\rm S*}(\lambda,\delta) + \Delta m_{\rm np}^*(\lambda,\delta)\frac{\delta}{2}\right)_{\delta=0}.\end{aligned} \quad (40)$$

The peculiarities of the symmetry energy can be studied by considering the Taylor series near the normal nuclear matter density $\lambda = 1$:

$$\varepsilon_S(\lambda) = \varepsilon_S(1) + \frac{L_S}{3}(\lambda-1) + \frac{K_S}{18}(\lambda-1)^2 + \cdots. \quad (41)$$

Here L_S and K_S are the quantities related to the slope and the curvature of the symmetry energy, respectively.

V. RESULTS AND DISCUSSION

It is necessary to note that our goal is not to fine-tune and reproduce the exact quantitative values of experimental observables; we will rather concentrate on the analysis at a qualitative level. Nevertheless, when possible, we try to be more accurate at a quantitative level too.

A. Symmetric matter

In the case of symmetric nuclear matter ($\Delta\chi = 0$) the solutions depend only on one parameter β, $F = F(x;\beta,0)$, and one can easily calculate the β dependence of m^*, Λ_2, and Λ_4. One can also ignore the term proportional to $\mathcal{M}_-^2$ in Eq. (29) while this term is several orders smaller relative to the first and second terms. This situation corresponds to the isospin-symmetric mesonic sector, i.e., $m_{\pi^\pm} = m_{\pi^0}$, and, consequently, $\mathcal{M}_- = 0$. Further, in order to simplify calculations we assume that $\alpha_s^{02} = \zeta_\tau^{-1}\alpha_p^0\zeta_s$. Then the symmetric nuclear matter can be parametrized in terms of three variational parameters C_1, C_2, and C_3 as shown below. If we define the

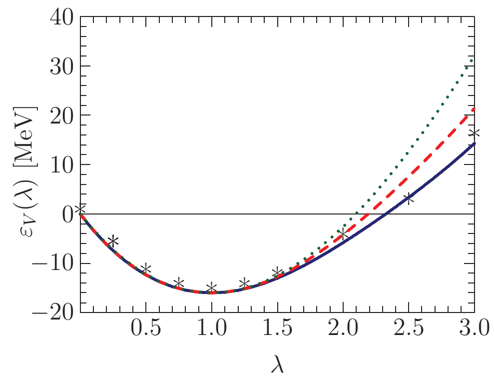

FIG. 1. (Color online) The volume energy ε_V as a function of normalized density $\lambda = \rho/\rho_0$. The parameters of medium functionals are given in Table I: the solid curve corresponds to Set I, the dashed one to Set II, and the dotted curve to Set III. Akmal-Pandharipande-Ravenhall predictions [68] are marked by stars.

three medium functions

$$1 + C_1\lambda = f_1(\lambda) \equiv \sqrt{\frac{\alpha_p^0}{\zeta_s}}, \quad (42)$$

$$1 + C_2\lambda = f_2(\lambda) \equiv \frac{\alpha_s^{00}}{(\alpha_p^0)^2\zeta_s}, \quad (43)$$

$$1 + C_3\lambda = f_3(\lambda) \equiv \frac{(\alpha_p^0\zeta_s)^{3/2}}{\alpha_s^{02}}, \quad (44)$$

the binding energy per nucleon in symmetric matter Eq. (32) becomes

$$\varepsilon_V = f_1 m^*(f_2) + \frac{3f_3}{8\{\Lambda_2(f_2) + \Lambda_4(f_2)\}} - m_{\rm N}, \quad (45)$$

while $\beta = \beta(f_2)$ [see Eq. (22)]. Now one can easily reproduce the symmetric-matter properties by relating the variational parameters C_1, C_2, and C_3 to the value of the volume term $\varepsilon_V(\rho_0)$, to the stability of nuclear matter $p(\rho_0) = 0$, and to the compressibility of nuclear matter, $K_0 = K(\rho_0)$, at the saturation density ρ_0.

The results of the minimization procedure are presented in Fig. 1, where the density dependence of the volume term $\varepsilon_V(\lambda)$ in terms of the normalized nuclear matter density λ is shown. For comparison, we present also the Akmal-Pandharipande-Ravenhall (APR) predictions (see the model A18 + δv + UIX* in Ref. [68]). The values of the corresponding variational parameters and the coefficients of symmetric matter at saturation density are listed in Table I. One can see that the

TABLE I. The variational parameters and the coefficients of the volume term at the saturation density ρ_0. Here the isospin-breaking effect in the mesonic sector is ignored, i.e., $\mathcal{M}_- = 0$.

Set	C_1	C_2	C_3	$\varepsilon_V(\rho_0)$ (MeV)	K_0 (MeV)	Q (MeV)
I	−0.279	0.737	1.782	−16	240	−410
II	−0.273	0.643	1.858	−16	250	−279
III	−0.277	0.486	2.124	−16	260	−178

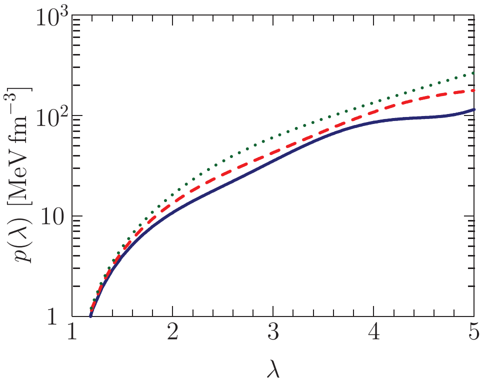

FIG. 2. (Color online) The pressure p in symmetric matter as a function of normalized density $\lambda = \rho/\rho_0$. The parameters of medium functionals are given in Table I: the solid curve corresponds to Set I, the dashed one to Set II, and the dotted curve to Set III.

parameters defined in Set III reproduce results similar to the APR predictions.

Here it is necessary to note that the compressibility of symmetric and infinite nuclear matter, within the various approaches [69–74], is found to be $K_0 \sim 290 \pm 70$ MeV. Therefore, by fitting the symmetric-matter properties we choose the compressibility value in that given range.

The quantity in the last column of Table I is proportional to the third derivative of the volume term at saturation density ρ_0 and is defined as

$$Q = 27\lambda^3 \frac{\partial^3 \varepsilon_V(\lambda)}{\partial \lambda^3}\bigg|_{\lambda=1}. \tag{46}$$

Our predictions for Q are qualitatively similar to the results from other approaches. For example, the Hartree-Fock approach, based on Skyrme interactions [75] and the isospin- and momentum-dependent interaction (MDI) model [76] gives qualitatively similar results. Another example is the ratio Q/K_0; in the present model one has $Q/K_0 \approx -1.71$ for $K_0 = 240$ MeV while the phenomenological momentum-independent model (MID) [77] gives the result $Q/K_0 \approx -1.6$.

It is also interesting to analyze the density dependence of pressure. The pressure p in symmetric matter as a function of normalized density λ is shown in Fig. 2. Our results are consistent with the results deduced from experimental flow data and simulations studies by Danielewicz *et al.* [78].

B. Asymmetric nuclear matter

In isospin-asymmetric matter, there will be additional term proportional to δ^2, i.e., the term containing γ^2 in Eq. (23). Consequently, the solutions and the corresponding integrals, m^*, Λ_2, and Λ_4, become γ dependent. In the present section we mainly concentrate on the symmetry energy and discuss the solutions corresponding to $\delta \simeq 0$. Therefore, in the first approximation one may assume that the solutions are still δ independent, i.e., $F \simeq F(x;\beta,0)$. We note that if $\mathcal{M}_- = 0$ then the solutions become exactly γ independent [see the Lagrangian in Eq. (14)]. In this case, the symmetry energy has the form

$$\begin{aligned}\varepsilon_S(\lambda) &= \frac{1}{4}\left\{\frac{\partial^2}{\partial\delta^2}\left[\frac{\Lambda^*_{\mathrm{env}}}{\Lambda^*}(\Lambda^*_{\mathrm{env}}+\delta)\right]\right\}_{\delta=0}\\ &= \frac{C_4\lambda m_\pi \Lambda_2}{8\alpha_s^{02}\Lambda}\left(\frac{C_4\lambda m_\pi\Lambda_2}{4\alpha_s^{02} f_3(\lambda)}+1\right),\end{aligned} \tag{47}$$

where we defined

$$\Delta\chi = m_\pi^{-1} C_4\,\delta\rho = 0.5 m_\pi^2 C_4 \lambda\delta \tag{48}$$

and introduced the new variational parameter C_4. To parametrize asymmetric matter we need one more parameter C_5. Here we consider two possible variants. In the first case, one can assume that the medium function α_p^0 is given by pion-nucleus scattering data and has the form [24]

$$\alpha_p^0 = 1 - \frac{4\pi c_0 \rho}{\eta + 4\pi g' c_0 \rho}, \tag{49}$$

where $c_0 = C_5 m_\pi^{-3}$ is the p-wave scattering volume, $g' = 0.7$ is the correlation parameter, and $\eta = 1 + m_\pi/m_{\mathrm{N}}^{\mathrm{S}}$ is the kinematic factor. This situation corresponds to the in-medium spatial part of the pion decay constant: $F^*_{\pi,s} = F_\pi\sqrt{\alpha_p^0}$. In the second case, one can assume that α_s^{02} is given by an approximate expression [19]

$$\alpha_s^{02} = 1 + C_5 m_\pi^{-1}\rho, \tag{50}$$

which corresponds to the in-medium temporal part of the pion decay constant: $F^*_{\pi,t} = F_\pi\sqrt{\alpha_s^{02}}$. As soon as α_p^0 or α_s^{02} is defined, all medium functionals are completely defined according to Eqs. (42)–(44).

Now let us discuss the asymmetric-matter properties. The symmetry energy ε_S as a function of the normalized nuclear matter density λ is given in Fig. 3 and the additional parameters are listed in Table II. The quantities in the last two columns,

$$K_\tau = K_S - 6L_S, \quad K_{0,2} = K_\tau - \frac{Q}{K_0}L_S, \tag{51}$$

represent the correlations between symmetric- and asymmetric-matter properties.

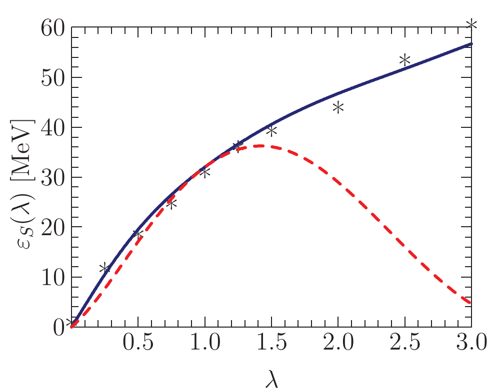

FIG. 3. (Color online) The symmetry energy as a function of normalized density $\lambda = \rho/\rho_0$. The solid curve corresponds to Eq. (49) and to the parameters defined by Set II, while the dashed one corresponds to Eq. (50) and to the parameters defined by Set II (see Tables I and II). APR predictions [68] are marked by stars.

TABLE II. Three sets of parameters which reproduce the asymmetric-matter properties. The variational parameters C_4 and C_5 are chosen in such a way that, at saturation point $p(\rho_0) = 0$, the values of the symmetry energy ε_S and its first derivative $L_S/3$ are reproduced in the commonly adopted range. Other parameters are defined in Table I and are used in the approximation $\mathcal{M}_- = 0$.

Set	C_4	C_5	$\varepsilon_S(\rho_0)$ (MeV)	L_S (MeV)	K_S (MeV)	K_τ (MeV)	$K_{0,2}$ (MeV)
			C_5 is defined by Eq. (49)				
I	2.367	0.037	32	60	−411	−591	−488
II	2.356	0.036	32	60	−405	−585	−518
II	2.228	0.035	32	60	−418	−598	−557
			C_5 is defined by Eq. (50)				
I	4.249	−0.1370	32	60	−127	−307	−204
II	4.258	−0.1372	32	60	−126	−306	−239
III	4.290	−0.1373	32	60	−124	−304	−263

There are different predictions [69–72,79–84] of nuclear symmetry energy at densities higher than the normal nuclear matter density and, in general, they are classified into two types: stiff or soft. Stiff symmetry energy is characterized by a monotonically increasing behavior when the density of nuclear matter increases, while soft symmetry energy, after increasing at subnormal nuclear matter densities, starts to decrease at supranormal densities. Our results belong to the soft symmetry energy class if one starts from Eq. (49) and assumes that the spatial part of the pion decay constant decreases in the nuclear medium. In contrast, if one starts from Eq. (50) and assumes that the temporal part of the pion decay constant decreases in the nuclear medium the results belong to the stiff symmetry energy class. In both situations, either Eq. (49) or (50) is used as an input form, the symmetry energy in the present model is quiet insensitive to the different model parameters presented in Table I. It is necessary to note that all three sets of parameters in the lower part of Table II well fit the APR predictions [68] (see Fig. 3). The results are consistent also with the result from the MDI [76] and MID models [77].

The parameter L_S is related to the neutron-skin thickness of the nuclei. The recent analysis of the data from heavy-ion collisions [79] and neutron-skin experiments [80] give the prediction $L_S^{\rm exp} = 70 \pm 20$ MeV. In general, our results are consistent with the results from other approaches. For example, in relativistic mean-field approaches there are mainly two classes: (i) small $\varepsilon_S(\rho_0) \sim 30$ MeV and small $L_S \sim 50$ MeV (BSP, IUFSU*, and IUFSU) and (ii) large $\varepsilon_S(\rho_0) \sim 37$ MeV and large $L_S \sim 110$ MeV (G1, G2, TM1*, and NL3) [85]. Our results correspond to the region somewhere in between those two classes.

Let us discuss the third coefficient, the compressibility of asymmetric matter. Usually, the value of the combination $K_\tau = K_S - 6L_S$ is discussed instead of K_S (e.g., see Ref. [81]). There are also considerations including up to third-order derivatives of binding energy on the nuclear density (for example, see Ref. [82]). We follow the definition of K_τ used in Ref. [81] in discussing the third parameter in the symmetry-energy term. But here one essential point must be underlined: Experimental knowledge about the third parameter K_τ is very poor. Some predictions for estimated errors may exceed several times the absolute value of K_τ. This is because the compressibility of the asymmetry term K_τ, at normal nuclear matter density is strongly correlated with the behavior of asymmetric matter at high densities. As we already mentioned, the properties of asymmetric matter at supranormal nuclear matter densities are not well established. Analysis of the data related to the phenomenology of giant monopole resonances give quite different predictions for K_τ. For example, $K_\tau \sim -320 \pm 180$ MeV is predicted in Ref. [69], and values range from -566 ± 1350 MeV to 139 ± 1617 MeV in Ref. [70] to -550 ± 100 MeV in Ref. [83]. Recent analysis of the isotopic transport ratios in medium-energy heavy-ion reactions predict the value -500 ± 50 MeV [81] and we also have more or less similar results in the case of the soft symmetry energy class (see the upper part of Table II).

As we already mentioned, the quantity $K_{0,2}$ represents the correlations between symmetric- and asymmetric-matter properties [see Eq. (51)]. For example, the phenomenological momentum-independent model predicts $-477 \leqslant K_{0,2} \leqslant -241$ MeV [77]. Our predictions from Sets II and III corresponding to the stiff symmetry energy class also belong to that range (see lower part of Table II).

It is interesting also to analyze the low-density behavior of the symmetry energy. An analysis of the giant dipole resonance (GDR) of ^{208}Pb with Skyrme forces predicts the following values of symmetry energy at subnormal nuclear matter density: $23.3 < \varepsilon_S(\rho = 0.1\ \mathrm{fm}^{-3}) < 24.9$ MeV [86]. For comparison, our results are 23.218 MeV (Set I), 23.223 MeV (Set II), and 23.238 MeV (Set III), respectively.

The pressure p in asymmetric matter as a function of normalized density λ is shown in Fig. 4. Again our results are consistent with the results deduced from experimental flow data and simulations studies by Danielewicz *et al.* [78].

For completeness, the binding energy per nucleon $\varepsilon(\lambda, \delta)$ in symmetric matter (solid curve) and in neutron matter (dashed curve) are shown in Fig. 5. One can see that our results are consistent with the APR predictions [68] in both cases.

C. $\mathcal{M}_- \neq 0$ corrections

If one tries to be more correct, without ignoring the term proportional to $\mathcal{M}_-^2$ in Eq. (29), the binding energy per nucleon in symmetric matter, Eq. (32), takes the form

$$\varepsilon_V = f_1 m^*(f_2) + \frac{3 f_3}{8\{\Lambda_2(f_2) + \Lambda_4(f_2)\}} + \left(\alpha_p^0 \zeta_s\right)^{-3/2} \mathcal{M}_-^2\, \Lambda_2 - m_{\rm N}. \tag{52}$$

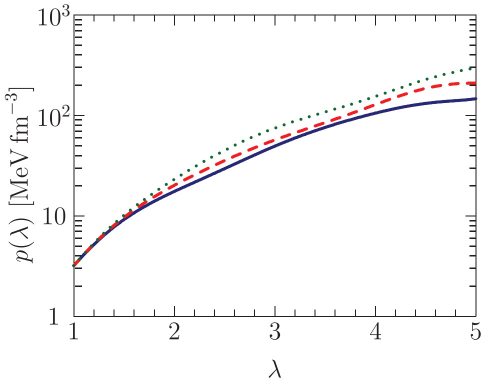

FIG. 4. (Color online) The pressure p in asymmetric matter as a function of normalized density $\lambda = \rho/\rho_0$. The parameters of medium functionals are given in Table II [C_5 is defined by Eq. (50)]: the solid curve corresponds to Set I, the dashed one to Set II, and the dotted curve to Set III.

The results of a minimization procedure in this case are given in Table III. One can see that ignoring isospin-breaking effects in the mesonic sector practically does not change the results.

The symmetry energy is defined by asymmetric matter near $\delta = 0$. Therefore, the values of its coefficients presented in Table II also practically do not change after $\mathcal{M}_- \neq 0$ corrections have been taken into account. However, in the pure neutron matter case one should estimate the contributions to the asymmetry energy $\varepsilon_A(\lambda, \delta)$ from terms higher than $O(\delta^2)$. This is now an easy task while all of the medium functions are already defined. Let us estimate the contribution from the term containing γ^2 in Eq. (23). First of all we note that α_s^{00} increases with increasing density and remains at the order of 1 in the present work. Furthermore, a^* is at the order of an MeV and in the case of neutron matter one has

$$2a^*\gamma^2 = \frac{a^* C_4 \lambda}{12 m_\pi \alpha_s^{00}(\lambda)} \sim 10^{-3}\lambda \ll 1$$

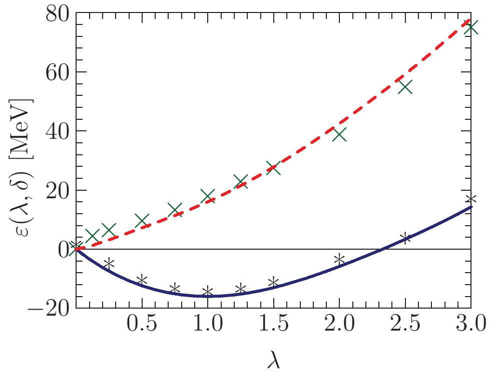

FIG. 5. (Color online) The energy per nucleon $\varepsilon(\lambda, \delta)$ as a function of normalized nuclear matter density $\lambda = \rho/\rho_0$. The parameters are taken from Model II (see the lower part of Table II). The solid curve represents symmetric nuclear matter ($\delta = 0$) while the dashed curve represents neutron matter ($\delta = 1$). For comparison, APR predictions [68] are marked by crosses and stars.

TABLE III. The variational parameters and the coefficients of the volume term at the saturation density ρ_0. Here the isospin-breaking effect in the mesonic sector is taken into account, i.e., $\mathcal{M}_- \neq 0$. C_5 is defined by Eq. (50) and its values are given in Table II.

Set	C_1	C_2	C_3	$\varepsilon_V(\rho_0)$ (MeV)	K_0 (MeV)	Q (MeV)
I	−0.279	0.737	1.782	−16	240	−411
II	−0.274	0.644	1.858	−16	250	−280
III	−0.276	0.488	2.120	−16	260	−179

for the given values of C_4 in Table II. Consequently, one can ignore the γ dependence in solutions $F = F(x;\beta)$ and ignore terms higher than $O(\delta^2)$ in the EoS for pure neutron matter.

VI. SUMMARY AND OUTLOOK

We have considered an in-medium modified chiral solitonic model which takes into account the isospin-breaking effects in the mesonic sector and reproduces the corresponding isospin-breaking effects in the baryonic sector. We have discussed the interplay between the isospin-breaking effects in the nuclear medium and the modifications of asymmetric nuclear matter properties. The results are in qualitative and quantitative agreement with the phenomenological indications and the results from other approaches.

The approach can be extended to the case of finite nuclei using the local density approximation as has been done in Refs. [19,20]. At this stage one should adjust the densities of the finite nuclei and, then, one can evaluate the contributions from the surface and Coulomb terms. Moreover, the calculations of the polarization-transfer double ratio $\mathcal{R}$ during the proton knock-out, ${}^4\text{He}(\vec{e}, e', \vec{p})^3\text{H}$ and ${}^{16}\text{O}(\vec{e}, e', \vec{p})^{15}\text{N}$, reactions can also be done easily.

In relation to the properties of finite nuclei one can make here a quick comment. Initial studies of mirror nuclei properties in the framework of an in-medium modified Skyrme model could qualitatively describe the Nolen-Schiffer anomaly, but at the quantitative level there was a big discrepancy between the theoretical calculations and the experimental indications [20]. That big discrepancy was a result of the large renormalization of the nucleon's in-medium effective mass. Artificial corrections of the in-medium nucleon mass lead to the correct description of NSA at the quantitative level too (see Table II in Ref. [20] and the corresponding discussions). It is expected that the present version of the model naturally reproduces the correct order of the NSA while the effective mass of the in-medium nucleon is not much changed.

It will be also interesting to apply the ideas of the present work to solitonic approaches where the Skyrme term is replaced by the explicit mesonic degrees of freedom, as is done in Ref. [87] in the case of symmetric nuclear matter. Then the model can be used for discussions of vector meson properties in asymmetric nuclear matter.

Summarizing the peculiarities of our model one may ask, "How far can one go in studying low-energy nuclear many-body problems at least at the qualitative level?" The answer may indicate whether the approach can be used for providing descriptions at the quantitative level. Our main purpose in future work will be to address these questions. The ideas and outcome results from the present approach would be useful in a more consistent theory describing nuclear many-body systems, their constituents, and interactions among the constituents on the same footing, starting from the same Lagrangian. The in-medium modified Skyrme model may serve as a simple case with the simplest self-consistent Lagrangian.

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