First observation of excited states in ^{87}Se: Collectivity and $j-1$ anomaly at $N = 53$

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The ^{87}Se nucleus has been studied via prompt γ-ray spectroscopy using the Eurogam2 Ge array to measure γ rays following fission of ^{248}Cm. Excited levels in ^{87}Se have been observed for the first time. The yrast excitation scheme in this nucleus is similar to the excitations schemes of its $N = 53$ neighbors and fits the energy systematics, indicating $j-1$ anomaly below $Z = 38$. Large-scale shell-model calculations reproduce in detail yrast excitations in ^{87}Se and other $N = 53$ isotones. The $j-1$ anomaly is explained as due to the enhancement of collectivity towards the proton midshell. The coexistence of collective and single-particle excitations is predicted in ^{87}Se.

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I. INTRODUCTION

The persistence of magic numbers far from stability has been an important subject of interest in the past two decades. Contrary to the atomic systems, magic numbers in nuclei may evolve with proton or neutron numbers. Their detailed behavior is driven by the forces at play in the nuclear many-body systems. A stringent modeling of the shell structure has a direct impact on different nucleosynthesis scenarios for the r processes occurring in type-II supernovae or neutron stars.

There are continuing theoretical and experimental efforts to describe nuclei in the vicinity of the waiting-point nucleus ^{78}Ni. Detailed structure information about its shell structure is not yet available but experimental information gathered in surrounding nuclei helps to build a consistent picture. The nucleus ^{78}Ni is expected to be doubly magic with proton, $Z = 28$ and neutron, $N = 50$ closed shells. However, the information on its stability inferred from neighboring nuclei may be blurred by the deformation arising from peculiar valence shell orbitals being degenerated both on proton and neutron sides. Specifically, the available $1f_{\frac{5}{2}}, 2p_{\frac{3}{2}}, 2p_{\frac{1}{2}}$ proton orbitals and $2d_{\frac{5}{2}}, 3s_{\frac{1}{2}}$ neutron orbitals allow recovery of pseudo-SU3 [1] and quasi-SU3 [2] schemes driving deformation as soon as a few protons and neutrons are added to the ^{78}Ni core.

In this work, continuing our systematic studies in the ^{78}Ni region [3–10], we searched for medium-spin, yrast excitations in the $^{87}_{34}$Se$_{53}$ nucleus, populated in spontaneous fission of ^{248}Cm. High-fold γ coincidence data were collected using the Eurogam2 array of anti-Compton spectrometers [11]. The details of the experiment and analysis techniques were described previously [12–14].

II. EXPERIMENTAL PROCEDURES AND RESULTS

In fission of ^{248}Cm, the most abundant complementary fragment to ^{87}Se is ^{158}Sm, accompanied by the emission of three neutrons ($3n$ channel). We gated on strong transitions of the yrast cascade in ^{158}Sm to search for the ground-state transition of about 100 keV in ^{87}Se, as suggested by the systematics of excitation energies in the $N = 53$ isotones. Figure 1 shows a spectrum doubly gated on the 167.4- and 258.2-keV lines of ^{158}Sm. In the spectrum one observes the 704.2-keV line of ^{86}Se [3,15] ($4n$ channel), a line at 886.2 keV, attributed to ^{88}Se in a study of ^{252}Cf fission [15], other lines from ^{158}Sm and ^{86}Se and contaminating lines from ^{146}Ce and ^{148}Ce isotopes, which both contain lines overlapping with gating transitions [16]. Moreover, there is also another line at 91.9 keV.

A spectrum doubly gated on the 91.9-keV line and the 167.4-keV line of ^{158}Sm is shown in Fig. 2. Present in the spectrum are the 258.2-, 344.0-, and 424.4-keV lines of the ground-state band in ^{158}Sm and, remarkably, the 886.2-keV line assigned previously to ^{88}Se [15]. A spectrum doubly gated on the 91.9- and 886.2-keV lines, displayed in Fig. 3, shows lines of ^{158}Sm at 167.4 and 258.2 keV but also lines of ^{156}Sm at 174.4 keV and ^{160}Sm at 162.3 keV. This indicates that the 91.9- to 886.2-keV cascade belongs to a Se isotope. However, the assignment of the 886.2-keV line to ^{88}Se proposed in Ref. [15] can be questioned, considering its coincidence with the 91.9-keV line. In an even-even spherical or weakly deformed nucleus, like ^{88}Se, one does not expect such a low-energy line in coincidence with the $2_1^+ \rightarrow 0_1^+$ transition. We note that the assignment of the 886.2-keV line to ^{88}Se in Ref. [15] was based on the assumption that the complementary Gd fragments are seen in the $2n$ and $4n$ channels, whereas similar intensities are also expected in the $3n$ and $5n$ channels. In the latter case the assignment of the 886.2-keV line should be to ^{87}Se.

To clear up these uncertainties we performed a mass correlation based on the variation of the intensity ratio of γ lines in two different fission fragments, complementary to Se isotopes. In Fig. 4 we show the ratio of γ intensities of the 162.3-keV line in ^{160}Sm to the intensity of the 167.4-keV line in ^{158}Sm, as seen in spectra double gated on lines from various Se isotopes and on the 91.9- to 886.2-keV cascade.

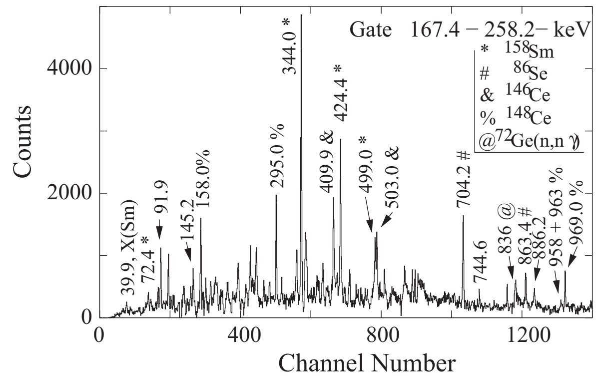

FIG. 1. γ spectrum doubly gated on the 167.4- and 258.2-keV lines of ^{158}Sm in the data from the ^{248}Cm spontaneous fission.

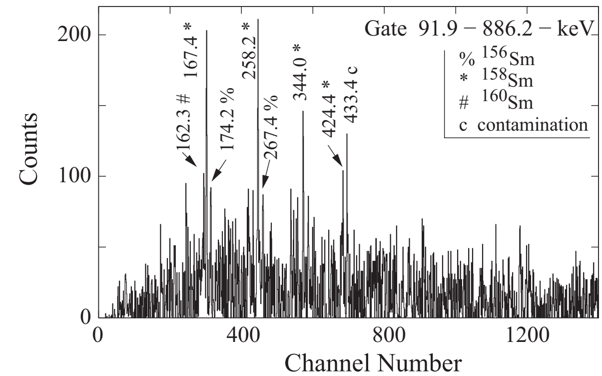

FIG. 3. γ spectrum doubly gated on the 91.9-keV and the 886.2-keV line assigned to ^{88}Se in Ref. [15]. The gate is set in data from the ^{248}Cm spontaneous fission.

Such diagrams have been used for isotope identification in other works [9,17,18]. It has been found that data points show monotonous trends, allowing us to extrapolate the correlation to, at least, the next mass number. The ratio $R = -0.30(2)$, for the 91.9- to 886.2-keV cascade, is consistent in Fig. 4 with the assignment of this cascade to ^{87}Se. Furthermore, as shown in Ref. [19] and our previous works [4,20,21], in spontaneous fission of ^{248}Cm three neutrons are emitted on average. Taking as weights the intensities of the $4^+ \rightarrow 2^+$ transitions in ^{156}Sm, ^{158}Sm, and ^{160}Sm from Fig. 3, we have calculated the average mass of 158.0(2) for the Sm isotope, complementary to the 91.9- to 886.2-keV cascade. Summarizing, we assign the 91.9- to 886.2-keV cascade to the ^{87}Se nucleus.

We have also checked that in Fig. 1 there is no any obvious candidate for the ground-state transition in ^{88}Se. Gating on lines of ^{156}Sm also did not reveal any such candidate. The search for such transition may require the data with the statistics higher than the present one or more advanced analytical techniques.

In Figs. 1 and 2 we observe another line at 744.6 keV. In the γ spectrum double gated on the 91.9- and 744.6-keV lines, which is shown in Fig. 5, one observes the 167.4- and 258.2-keV lines of ^{158}Sm, which support the assignment of the 91.9- to 744.6-keV cascade to ^{87}Se.

The observed coincidence relations and γ intensities allowed us to build the excitation scheme of ^{87}Se as shown in Fig. 6. Intensities of γ lines in ^{87}Se are drawn in square brackets, next to γ energies. The 91.9-keV transition, as the most intensive, is placed at the bottom of the scheme. The 744.6- and 886.2-keV transitions feed the 91.9-keV level. The levels of ^{87}Se fit well the energy systematics of $N = 53$ isotones, shown in Fig. 7. Based on this systematics we proposed spins and parities. Angular correlations for the 91.9- to 886.2-keV cascade, yielding $A_2/A_0 = 0.10(5)$, $A_4/A_0 = -0.11(9)$, and a large mixing ratio $\delta = 0.53(+0.31, -0.12)$ or $\delta = 5.0(16.2, -2.7)$ are consistent with these assignments.

In Figs. 1 and 2 we observe the lines at 76.9 and 145.2 keV, which are in coincidence with lines of ^{158}Sm and the 91.9-keV line. However, the spectrum double gated on the these lines is strongly contaminated, preventing their unique placement in the scheme.

We have also searched for the eventual, cross-over decay from the 836.5-keV level to the ground state. In Fig. 1 there is a strong contamination line seen at 836 keV, which is due to the ^{72}Ge$(n, n'\gamma)$ reaction, caused by fast neutrons from fission. Because of this contamination at present it is not possible to conclude the presence of the searched cross-over decay.

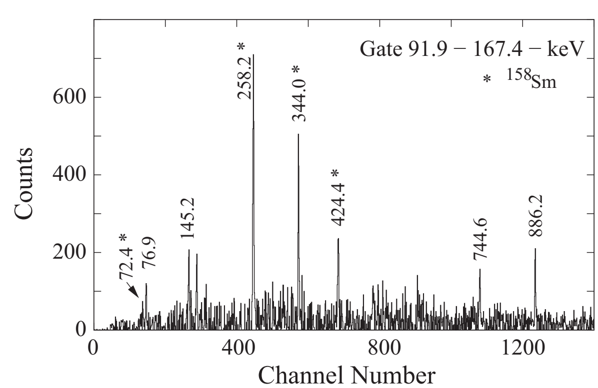

FIG. 2. γ spectrum doubly gated on the 91.9-keV line and the 167.4-keV line of ^{158}Sm, in the data from the ^{248}Cm spontaneous fission.

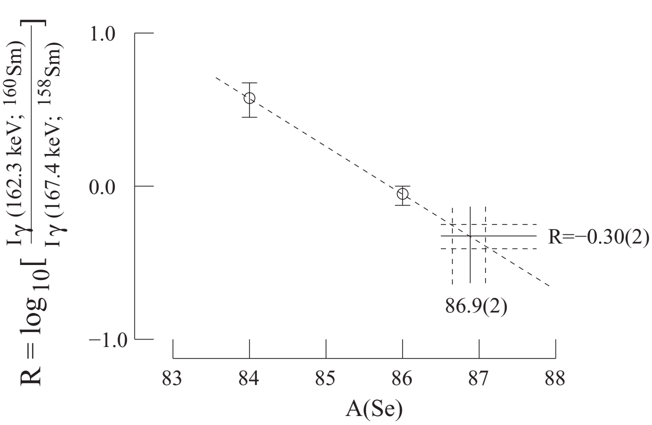

FIG. 4. Mass correlation diagram for the Se isotopes. See the text for more explanation.

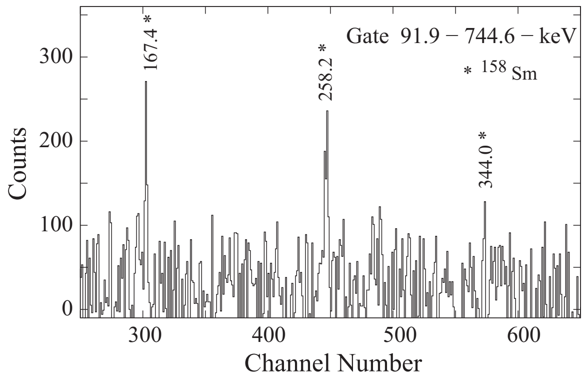

FIG. 5. γ spectrum doubly gated on the 91.9-keV and the 744.6-keV lines. The gate is set in data from the ^{248}Cm spontaneous fission.

III. DISCUSSION

It has been reported in the past that simple shell-model calculations of certain nuclei with three valence nucleons (holes) on the same orbital with spin j could not reproduce transition rates between close lying, $I = j$ and $I = j - 1$ members of the j^3 multiplet [22]. Such rates were later successfully described within the Alaga model [23–26], in which the j^3 multiplet is coupled to the vibrational phonon of the core nucleus [25,26]. One notes that the lowering of the $I = j - 1$ level results from the coupling of the three valence nucleons on the same orbital, j. However, such coupling alone will never lower the $I = j - 1$ level below the $I = j$ [27]. The extra coupling to the phonon in the Alaga model allows to reproduce the so-called $I = j - 1$ anomaly [22], i.e., when the $I = j - 1$ member of the multiplet drops below the $I = j$ state.

The $N = 53$ isotones are good candidates for such an interpretation. Indeed, the ^{95}Mo nucleus with $N = 53$ neutrons has been successfully reproduced within the Alaga model [22]. Furthermore, as can be seen in Fig. 7, whereas the $3/2^+$ level in ^{97}Ru, ^{95}Mo, and ^{93}Zr is located at nearly the same excitation energy above the $5/2^+$ level, for $Z < 40$ the $3/2^+$ level lowers quickly its energy and is located below the $5/2^+$ level in ^{89}Kr and ^{87}Se. Thus, there is clear $I = j - 1$ anomaly at $N = 53$ when moving towards $Z = 28$. As such an anomaly can be accounted for by an extra coupling of the core phonon to the $(j)^3$ system, the amplitude of the lowering of the $I = j - 1$ member can be related to the collectivity in the core nucleus [22,24]. Angular correlations for the 91.9- to 886.2-keV cascade are consistent with the 91.9-keV transition having large quadrupole component.

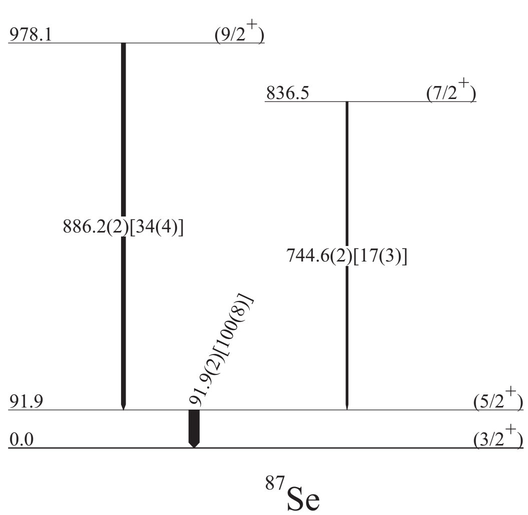

FIG. 6. Level scheme of ^{87}Se, as observed in this work.

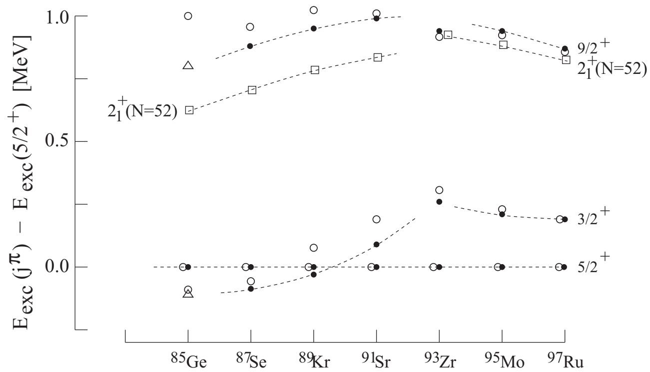

FIG. 7. Experimental yrast excitations in the $N = 53$ isotones (filled circles) as compared to the shell-model calculations performed in the present work (open circles). Open-dashed circles represent predictions for ^{85}Ge based on experimental systematics. Open squares show excitation energies of 2_1^+ levels in the respective $N = 52$ cores. The experimental data are taken from the present work and from Refs. [28–31]. Calculations for the ^{93}Zr are taken from Ref. [5]. Experimental and calculated data points are normalized at the $5/2^+$ level. Dashed lines are to guide the eye.

It is also interesting to ask about excitations in $N = 53$ isotones at $Z \leqslant 34$. In our recent work [3], we argued that the excitation energy of the 2_1^+ level in the $N = 52$ isotones has a minimum in ^{84}Ge. This is consistent with Fig. 7, according to which the $3/2^+$ excitation energy may have the minimum in ^{85}Ge and then rise again in ^{83}Zn. It is an open question if the $9/2^+$ level has lower energy in ^{85}Ge, as compared to ^{87}Se.

To interpret the structure of $N = 53$ isotones we have performed large-scale shell-model calculations in a valence space comprising proton $(1f_{5/2}, 2p_{3/2}, 2p_{1/2}, 1g_{9/2})$ and neutron $(2d_{5/2}, 3s_{1/2}, 2d_{3/2}, 1g_{7/2}, 1h_{11/2})$ orbitals outside the ^{78}Ni core. The effective Hamiltonian used in this work is the same as in our previous studies of nuclei above ^{78}Ni [3,6,8,9,32] and is described in more detail in Ref. [5]. The diagonalization of the shell-model matrices have been preformed using Strasbourg codes ANTOINE and NATHAN [2,33]. In the calculations of $E2$ transition rates the polarization charge $0.7e$ have been used, as suggested in Ref. [5]. The usual quenching factor 0.75 has been applied to the spin operator in the calculations of $M1$ transitions.

The calculated lowest energy levels of ^{87}Se and the corresponding electromagnetic transition rates are given in Table I. There is good agreement between theory and experiment for the $3/2^+$, $5/2^+$, $7/2^+$, and $9/2^+$ levels and no candidates are found in the calculations for other levels below 0.9 MeV. We note a very collective character of the $E2$ transitions between

TABLE I. Excitation energies of the lowest levels in ^{87}Se (in MeV), $B(E2)$ transition rates (in e^2 fm^4) and $B(M1)$ transition rates (in μ_N^2) obtained in shell-model calculations.

J^π	E^*	$B(E2; BJ \to J-1)$	$B(E2; J \to J-2)$	$B(M1; J \to J-1)$
$1/2^+$	0.97			
$3/2^+$	0.0	1.5		0.005
$5/2^+$	0.05	739	52	0.113
$7/2^+$	1.03	428	370	0.051
$9/2^+$	1.00	251	559	0.239

the $3/2^+$, $5/2^+$, $7/2^+$, and $9/2^+$ levels, with the largest transition rate between the $5/2^+$ and $3/2^+$ states reaching 32 W.u. In the shell-model calculations of ^{86}Se and ^{88}Se using the same model space and the same interaction, large $B(E2)$ values have been found between the yrast states [32]. Using the collective model [34] an intrinsic deformed state with $\beta \sim 0.2$ has been associated with their ground-state bands. Therefore, one may ask if ^{87}Se could be described by the particle plus rotor model in the strong coupling limit. In such a case, the spectra should follow the $J(J+1)$ law and the calculated quadrupole moments should be the same as those of the underlying rotor. It is conspicuous that the former is not fulfilled, which is often the case in lighter nuclei. On the other hand, the intrinsic quadrupole moments calculated from spectroscopic moments and derived from $B(E2)$ values assuming $K = 3/2$ are similar to each other and rather constant, especially for the higher part of the band (see Table II). The appearance of a deformed $K = 3/2$ band would naturally explain the ordering of levels in ^{87}Se with the $3/2^+$ spin below the $5/2^+$ one.

To understand better this effect we have performed calculations in two extreme limits. First, we have reduced the collectivity by imposing four particles to occupy the proton $p_{3/2}$ orbital. In this case the neutron wave functions tend to become pure $d^3_{5/2}$ configurations. As expected, the $5/2^+$ level of seniority $\nu = 1$ is placed 360 keV below the $3/2^+$ level of $\nu = 3$. The corresponding $B(E2)$ transition value of 105 e^2 fm^4 is much lower than in realistic calculations. Next, we have made degenerate the proton $p_{3/2}$, $f_{5/2}$, and $p_{1/2}$ single-particle energies in order to approach the pseudo-SU(3) limit [1]. In this case, the level sequence follows the $J(J+1)$ rule, with the $3/2^+$ level 230 keV below the $5/2^+$ level and a large $B(E2)$ value of 850 e^2 fm^4. In Table II we show the quadrupole intrinsic moments obtained for the second case. One can see that the quadrupole moments obtained from realistic SM calculations reach ~90% of the pseudo-SU(3) values. We thus conclude that ^{87}Se approaches a rotational limit and that the observed level ordering is due to a deformation.

TABLE II. Intrinsic and spectroscopic quadrupole moments (in e fm^2) obtained in this work for ^{87}Se from the shell-model calculations. $Q^{(s)}_{\rm int}$ means intrinsic moment computed from the spectroscopic one, $Q^{(t)}_{\rm int}$ computed from the $B(E2)$ transition value. $Q^{\rm SU3}_{\rm int}$ is the intrinsic quadrupole moment obtained in the pseudo-SU(3) limit; see the text for details.

J^π	$Q_{\rm spec}$	$Q^{(s)}_{\rm int}$	$Q^{(t)}_{\rm int}$		$Q^{\rm SU3}_{\rm int}$
			$\Delta J = 1$	$\Delta J = 2$	
$3/2^+$	29.6	148			161.5
$5/2^+$	−15.0	210	147		199.5
$7/2^+$	−33.5	167	142	161	183.5
$9/2^+$	−43.7	160	134	162	182.6

We have also analyzed occupation numbers of shell-model wave functions of the calculated states. In the $3/2^+, 5/2^+, 7/2^+$, and $9/2^+$ states, the dominating configurations have three neutrons in the $d_{5/2}$ orbital with the total occupancy $\geqslant$2.5 nucleons. On the proton side, the particles are predominantly confined in the $f_{5/2}$ and $p_{3/2}$ orbitals, which have the total occupancy of ≈5.5 nucleons for all the considered states. The $1/2^+$ level predicted at 0.97 MeV requires special attention in this context. Unlike the other states, it is a one-particle excitation to the neutron $s_{1/2}$ orbital. Its different nature explains the reduced $E2/M1$ transition rates to this level. A future observation of this $1/2^+$ state, which is not seen in fission due to its nonyrast character, would provide valuable information on the location of $s_{1/2}$ in the vicinity of ^{78}Ni and would allow us to verify the proposed coexistence of collective and single-particle modes at low energy in ^{87}Se. Such coexistence has been previously observed, e.g., in neutron-rich copper isotopes [7,35].

In Fig. 7 we compare experimental and calculated energies of the $3/2^+$, $5/2^+$, and $9/2^+$ levels in $N = 53$ isotones. The shell model reproduces in detail experimental trends and excitation energies of all levels with an accuracy better than 100 keV. In particular, it reproduces the change of the ordering of $3/2^+$ and $5/2^+$ states, which takes place with decreasing proton number. In ^{89}Kr the experimental ground state is already $3/2^+$, whereas the calculated one still remains $5/2^+$. However, the splitting of the $3/2^+$ and $5/2^+$ states is only 28.6 keV in the experiment and 80 keV in the calculations. The shell-model prediction for the $3/2^+$-$5/2^+$ splitting in ^{85}Ge fits well the one suggested by the experimental trend.

Table III shows the composition of the wave functions corresponding to the $3/2^+$, $5/2^+$, and $9/2^+$ levels in $N = 53$ isotones in terms of couplings of proton and neutron subspaces, which illustrates the evolution of structure of these levels with proton number. As can be seen, the highest purity of the wave functions is observed for ^{93}Zr, where the discussed states are matching well the neutron $d^3_{5/2}$ multiplet. In the remaining nuclei, the wave functions of these states are mixed, having two or three dominating components. Apparently, the coupling to the proton 2^+ excitation is an important ingredient of these wave functions. Its percentage is related to the evolution of the 2^+ energy in the $N = 50$ core nuclei. It is large for lower Z, nearly absent in ^{93}Zr due to the magicity of ^{90}Zr and rises again for $Z > 40$. The increasing percentage of the $2^+_p \otimes j_n$ component at lower Z explains the enhancement of the collectivity and large transition rates obtained in the calculations for ^{87}Se. Given the deformation effects in ^{87}Se, discussed above, one may explain the trend of the $3/2^+$ and $5/2^+$ energies in $N = 53$ nuclei as due to the transition from the seniority limit, well established in ^{93}Zr, to the deformation regime in ^{87}Se.

TABLE III. Structures of $3/2^+, 5/2^+$, and $9/2^+$ states in $N = 53$ isotones obtained from shell-model calculations in this work. Percentages of only dominating components are listed.

J^π	Z	$0^+_p \otimes \frac{3}{2}^+_n$	$2^+_p \otimes \frac{3}{2}^+_n$	$0^+_p \otimes \frac{5}{2}^+_n$	$2^+_p \otimes \frac{5}{2}^+_n$	$0^+_p \otimes \frac{9}{2}^+_n$
$3/2^+$	32	51	10		27	
	34	46	11		33	
	36	60			27	
	38	71			20	
	40	85			7	
	42	70			19	
	44	61			25	
$5/2^+$	32		21	58		
	34		23	54		
	36		14	68		
	38		9	79		
	40			89		
	42		8	79		
	44		10	74		
$9/2^+$	32				38	40
	34				46	32
	36				36	50
	38				23	67
	40					90
	42				23	64
	44				33	48

IV. CONCLUSIONS

In summary, we have observed for the first time excited states in ^{87}Se. The levels follow smoothly the systematics of excitation energies in the $N = 53$ isotones, which displays the $j - 1$ anomaly. In the past such effects, which could not be described properly by the shell model, were reproduced by the Alaga model. Large-scale shell-model calculations performed in the present work reproduce in detail excitation energies of $N = 53$ isotones, and in particular the $j - 1$ anomaly. The descending trend of the $3/2^+$ level in $N = 53$ isotones is due to the increasing coupling of the neutron $d^3_{5/2}$ configurations to the proton 2^+ excitation, the latter gaining collectivity towards the midshell. In ^{87}Se the shell model predicts highly collective $E2$ transitions between the yrast states. In addition to the observed collective excitations, a low-lying $1/2^+$ of a single-particle origin is predicted in the calculations. It is a challenge for the future experimental studies to verify the degree of collectivity in nuclei around ^{78}Ni and a possible coexistence of collective and single-particle structures at low excitation energies in this region.

In the course of this study, another work [36] reported a 92-keV line associated with β decay of ^{87}As. This supports our assignment of the 91.9- to 886.2-keV cascade to ^{87}Se. No comments on excited states in ^{87}Se were made in Ref. [36].

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