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Hauser-Feshbach calculations in deformed nuclei

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Hauser-Feshbach calculations are frequently done in deformed nuclei. Although modifications are made in the level density to reflect the rotational bands for deformed nuclei, these calculations are in error if a conventional Hauser-Feshbach code is used. A modification to the Hauser-Feshbach formalism is proposed. The new version of the formalism was tried, both with K conserved and K mixed. It is found that even in the limit of K mixing the results do not agree completely with the calculations using a spherical (conventional) Hauser-Feshbach code with the same input. A formula frequently used to calculate the J dependence of level densities in deformed nuclei is found to be in error.

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I. INTRODUCTION

Calculation of spectra and cross sections for reactions proceeding under conditions such that the Bohr independence hypothesis is valid were first carried out by Weisskopf [1,2]. This formalism assumed that the relative decay probabilities of the compound nucleus were independent of the angular momentum of the nucleus. Somewhat later, Wolfenstein [3] and Hauser and Feshbach [4] extended the formula to include the possibility that branching ratios vary with J, the spin of the compound nucleus. The resulting form is

$$\sigma_{ab} = \frac{\pi \lambdabar^2}{(2I_1+1)(2I_2+1)} \sum_{J,\pi} (2J+1) \frac{\sum_{\alpha,\beta} T_{a\alpha} T_{b\beta}}{\sum_{\gamma c} T_{c\gamma}}. \quad (1)$$

Here, σ_{ab} is the cross section for the reaction $A(a,b)B$, where a is the bombarding particle, A is the target, b is the emitted particle, and B is the final nucleus. J is the total angular momentum of the compound nucleus and π denotes the parity. I_1 is the spin of the target and I_2 is the spin of the projectile. $T_{a\alpha}$ is the transmission coefficient for the entrance channel, α the angular momentum in this channel, $T_{b\beta}$ is the transmission coefficient in the exit channel, and β denotes the angular momentum in the exit channel. $\lambdabar$ is the reduced wavelength of the incident particle. Finally, the sum in the denominator is over all possible (energetically available) exit channels, with γ the associated angular momentum. Since there is no sum over angular momentum projections of the total angular momentum of the compound nucleus, it is obvious that the formalism assumes that the branching ratio does not depend on the projection. Thus, this formula has the assumption of spherical symmetry "built in."

This assumption is clearly inappropriate for a deformed system. In a spherical odd-A nucleus, a 5/2 level is sixfold degenerate, with states of $J_z = -5/2, -3/2, -1/2, 1/2, 3/2,$ and $5/2$ all sharing the same energy. The corresponding situation in a deformed nucleus would have the 5/2 level split into a $J = 5/2$, $K = 1/2$, a $J = 5/2$, $K = 3/2$, and a $J = 5/2$, $K = 5/2$ group. The K value is the spin projection on the symmetry axis. Each of these three levels has a different energy and each is twofold degenerate. Thus, an energy level in an odd-A deformed nucleus is twofold degenerate. For even-A deformed nuclei, all levels with $K \neq 0$ are doubly degenerate, while those with $K = 0$ are singly degenerate.

In addition to removing the $(2J+1)$ degeneracy, deformation has an additional effect on the level density. On each level of spin J and spin projection K, a rotational band is built. For $J = 0$, the states in the band are $J = 2, 4, 6, 8, 10, \ldots$ with spacing proportional to $J(J+1)$. For larger J, the bands have J values $J+1, J+2, J+3, \ldots$. For each band, the excited levels in the band have the same K value as the level which is the band head. The consequence of the deformation is that the density of levels as a function of J increases more rapidly at a given excitation energy than is the case for a spherical nucleus [5]. Also, because of the bands, for a given J the density of levels as a function of K at a given energy is smallest for $K = J$ and increases for smaller K. For large J, the ratio at a given energy can approach a factor of 10.

For a Hauser-Feshbach calculation in a spherical nucleus, it is found that levels of the same J at approximately the same excitation energy will have about the same cross section, and that for levels with different J (where $J \leqslant \sigma$, the spin cutoff parameter) the population varies roughly as $(2J+1)$. The modification of the level densities now gives a dependence of the cross section on K as well as J. The much higher level density for $K \ll J$ compared to $K \sim J$ makes the competition more severe for low K, making the cross section per level larger for levels with $K \sim J$ than $K \sim 0$. The larger number of the latter type of level, however, enhances the total cross section populating levels of $K \sim 0$ relative to the total population of $K \sim J$ levels. Both of these results are in conflict with the predictions of a spherical Hauser-Feshbach code.

The modifications to Eq. (1) needed to deal with deformed nuclei are straightforward. The primary sum must be extended to K as well as J and π. Each transmission coefficient must be replaced by the corresponding transmission coefficient multiplied by the squares of the appropriate Clebsch-Gordan coefficients. The revised Hauser-Feshbach equation then becomes

$$\sigma_{ab} = \frac{\pi \lambdabar^2}{(2I_\alpha+1)} \sum_{JK\Pi} \frac{\sum_{\alpha\beta} \tau^{\Pi}_{a\alpha} \tau^{\Pi}_{b\beta}}{\sum_{\gamma c} \tau^{\Pi}_{c\gamma}}. \quad (2)$$

In Eq. (2),

$$\tau_{a\alpha}^{\Pi} = \sum_{J_\alpha K_\alpha} \langle J_\alpha K_\alpha J_1 K_1 | J K\rangle^2 T_{\alpha J_\alpha K_\alpha}^{\Pi'}. \quad (3)$$

In this sum, J_1 and K_1 are the J and K values of the target, J_α and K_α are the J and K of the projectile, and $T_{\alpha J_\alpha K_\alpha}^{\Pi'}$ is the transmission coefficient for particle a in the entrance channel with parity Π if the target has positive parity in the ground state and $-\Pi$ if the parity of the ground state is -1. $\langle J_\alpha K_\alpha J_1 K_1 | J K\rangle$ is the Clebsch-Gordan coefficient for coupling the angular momentum J_α with Z projection K_α to angular momentum J_1 with projection K_1 to give total angular momentum J with projection K. Similarly,

$$\tau_{b\beta}^{\Pi} = \sum_{J_\beta K_\beta} \langle J_\beta K_\beta J_2 K_2 | J K\rangle^2 T_{\beta J_\beta K_\beta}^{\Pi''}, \quad (4)$$

where J_β is the angular momentum of the outgoing particle and K_β is its spin projection. J_2 is the angular momentum of the final state and K_2 its projection. The transmission coefficient is in the β channel and has J_β and K_β as its spin and spin projection, respectively. Finally Π'' is Π if the parity of the state in nucleus b is positive and $-\Pi$ if it is -1. Finally,

$$\tau_{c\gamma}^{\Pi} = \sum_{c}\sum_{\gamma}\sum_{J_\gamma K_\gamma} \langle J_\gamma K_\gamma J_2 K_2 | J K\rangle^2 T_{\gamma J_\gamma K_\gamma}^{\Pi'''}. \quad (5)$$

In this expression, c denotes an index including all exit channels (particle type and energy), γ denotes the associated angular momentum parameters of the final state. Π''' will be Π if the parity of the final state is positive, otherwise it will be $-\Pi$. In both the expressions for τ_b and τ_c there is an implicit delta function for the final state [in terms of the final energy, J (J_2 or J_3), K (K_2 or K_3), and parity]. In practice, the delta function will frequently be replaced by a level density. Note that Eq. (2) is appropriate if the target is deformed. If the target is spherical (leading to a deformed compound nucleus), then the leading factor becomes $\pi\lambda\!\!\!^{-2}/(2I_\alpha+1)(2I_1+1)$ in Eq. (2) and the sum in the expansion for $\tau_{a\alpha}$ must be extended to cover a sum over the values for K_1.

It is remarkable that these changes are analogous to the changes in the Hauser-Feshbach equations introduced to deal with isospin. Miller, *et al.* [6] measured (p,p') and (p,α) spectra and compared them with (α,α') and (α,p) spectra, with the targets chosen so that the same compound nucleus was formed by the proton beams as by the alpha beam. The authors determined the ratios $[(p,p')/(p,\alpha)]/[(\alpha,p)/(\alpha,\alpha')]$. The independence hypothesis would predict that this ratio is 1. In fact, because of the change in branching ratio with J, the Hauser-Feshbach calculations predicted that it would be about 1.2. The measured value was about 1.4. The authors interpreted this enhancement as being due to isospin selection rules. A paper [7] subsequently introduced a revised Hauser-Feshbach formalism, which corresponded exactly to the changes just described but with the spin projection replaced by the isospin projection. A number of subsequent papers [8] have used this formalism to infer evidence of partial mixing of isospin in states before decay, and another paper [9] has shown that these effects can be seen in some cases for neutron- or alpha-induced reactions.

One question which was raised by the isospin treatment was the issue of isospin mixing. It is clear that at very low energies the level density is so small that mixing is minimal, i.e., the mixing matrix element is much smaller than the separation of the states which are mixing. As the energy increases,the mixing would be expected to increase and the measurements done for excitation energies about 20 MeV show that the mixing is roughly 0.5. Approximately half of the higher isospin states damp into lower isospin states before they decay. The mixing is asymmetric, since the ratio of level densities is about 1:10 or higher with the largest fraction of the states having $T = T_0 = (N-Z)/2$ and a smaller number having $T = T_0 + 1$. At very high energies, the mixing will be suppressed by the fact that the decay width becomes larger than the mixing width.

A similar situation exists with respect to the J dependence of mixing. The level density at a given energy first increases with J and then drops once J exceeds σ, the spin cutoff parameter. Thus, one would expect to find that mixing is largest for $J \sim \sigma$ and would be smaller for smaller or larger J. To this point, the tests for isospin mixing have not looked at either J or excitation energy dependence.

Since the K mixing matrix elements are expected to be about the same magnitude as the isospin mixing matrix elements, it is likely that the mixing is energy dependent. The present code allows for K mixing if desired.

In analogy to the situation for isospin, it is likely that the K mixing will vary with excitation energy and J. Because of level density arguments, it is plausible that mixing for a given energy and J may depend on K as well.

A similar code has been described by Charity [10]. He applied the code to the analysis of evaporation spectra at about 100 MeV. Clearly, the issues involved at this energy are not the same as those addressed here. At low energies, cross sections can be measured for the population of individual states of known J and K; this was not possible at 100 MeV. It is also less likely that the compound states will live long enough for K mixing to occur at the higher energies. Charity concluded that for the systems he examined the only significant effect was a change in the shape of the alpha emission spectrum at low outgoing energy. This was caused by the change in transmission with K for a given J.

II. CALCULATIONS

A new Hauser-Feshbach code has been written [11]. This code differs from a conventional Hauser-Feshbach code in the following ways:

(1) The level densities are input as functions of J, K, and π for a given nucleus at a given energy rather than just J and π.

(2) The transmission coefficients are input for a given channel as functions of energy, J', K, and π rather than just J' and π and energy. J' is the total angular momentum in the appropriate channel ($= \vec{\ell} + \vec{s}$). The appropriate Clebsch-Gordan coefficients are also included.

(3) The decay width is summed over J, K, and π.
(4) It can be assumed that K is conserved or mixed; J is conserved.

This code is formulated in such a way that axial symmetry is assumed, but nuclear level densities can be input which are either spherical or deformed. This allows a calculation to be made in which the nucleus reached by neutron decay is deformed while the one reached by alpha decay is spherical. Furthermore, a calculation could be made for the case where a particular nucleus is deformed up to a particular excitation energy and becomes spherical beyond this energy. An additional possibility is to treat a nucleus which has some deformed states embedded in a group of spherical states at a given energy. The spherical Hauser-Feshbach calculations were done using the Hauser-Feshbach code described in [12].

A. Level densities

Level density refers to a density of levels as a function of excitation energy, spin ($= J$), and parity (π) for a given nucleus. In this paper, we will also specify the spin projection on the symmetry axis (K). At low energies, there is usually information on the excitation energy, J, K, and π of the levels. These levels (called the resolved levels) are used in each channel at low energies. As the energy increases, some levels are known by excitation energy but J, K, or π are not known. At even higher energies, the levels will overlap. In the present calculations, the low-lying states which have complete spectroscopic information are included in the calculation as delta functions. Once the energy is increased for a given nucleus so that spectroscopic information appears incomplete, the level density is represented by an analytic function. This region is called the "continuum," even though the lowest energies in this region have levels which strictly are not overlapping.

Level densities are calculated in three steps. First the "intrinsic" state density is calculated from the expression

$$\rho(U) = \frac{\sqrt{\pi}}{12a^{1/4}} \frac{\exp(2\sqrt{aU})}{U^{5/4}}. \tag{6}$$

In this expression, $\rho(U)$ is the total density of states at energy U and a is the level density parameter. For a spherical nucleus, it is assumed that the states have a Gaussian distribution in J_z, which leads to a factor multiplying Eq. (6) of

$$\frac{1}{\sqrt{2\pi}} \frac{1}{\sigma} \exp\left[\frac{-J_z^2}{2\sigma^2}\right], \tag{7}$$

where $\sigma^2 = \langle J_z^2 \rangle$. The level density is obtained by differentiation and becomes

$$\rho_L(U, J) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma} \frac{\left(J + \frac{1}{2}\right)}{\sigma^2} \exp\left[\frac{-\left(J + \frac{1}{2}\right)^2}{2\sigma^2}\right] \rho(U). \tag{8}$$

If we now calculate the density of states as a function of J, we obtain

$$\rho_S(U, J) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma^3} 2 \left(J + \frac{1}{2}\right)^2 \exp\left[\frac{-\left(J + \frac{1}{2}\right)^2}{2\sigma^2}\right] \rho(U). \tag{9}$$

Note that, if this is integrated from $J = 0$ to infinity, we recover $\rho(U)$. Finally, we can calculate the density of states as a function of both J and J_z at a specific U by dividing the above expansion by $(2J + 1)$:

$$\rho_{Sz}(U, J, J_z) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma^3} \left(J + \frac{1}{2}\right) \exp\left[-\frac{\left(J + \frac{1}{2}\right)^2}{2\sigma^2}\right] \rho(U). \tag{10}$$

For a deformed nucleus, the notation is changed so that J_z becomes K. Each level with specified J and K is doubly degenerate ($\pm K$), unless $K = 0$, which makes the level singly degenerate. We introduce the factors $\sigma_\parallel^2$ ($= \langle K^2 \rangle = \frac{I_\parallel T}{\hbar^2}$) and $\sigma_\perp^2$ ($= \frac{I_\perp T}{\hbar^2}$), where T is the nuclear temperature, $I_\parallel$ is the moment of inertia about an axis parallel to the axis of symmetry, and $I_\perp$ is the moment of inertia about an axis perpendicular to the axis of symmetry. Thus, the intrinsic level density is

$$\rho_{LK}(U, J, K) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma_\perp^2 \sigma_\parallel} \left(J + \frac{1}{2}\right) \exp\left[-\frac{\left(J + \frac{1}{2}\right)^2}{2\sigma_\perp^2}\right] \times \exp\left[-K^2 \left(\frac{1}{2\sigma_\parallel^2} - \frac{1}{2\sigma_\perp^2}\right)\right] \rho(U). \tag{11}$$

The final step is to construct a rotational band on each of the levels given by Eq. (11). The final level density is then

$$\rho(U, J, K) = \rho_{LK}(U, J, K) R(J, K) \tag{12}$$

$$= \rho(U) G(J) S(K) R(J, K), \tag{13}$$

where $\rho(U)$ is the intrinsic state density at energy U,

$$G(J) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma_\perp^2 \sigma_\parallel} (J + 1/2) \exp \frac{-(J + 1/2)^2}{2\sigma_\perp^2}, \tag{14}$$

and

$$S(K) = \exp\left[-K^2 \left(\frac{1}{2\sigma_\parallel^2} - \frac{1}{2\sigma_\perp^2}\right)\right], \tag{15}$$

and $R(J, K)$ is approximately

$$R(J, K) \approx \frac{[(J + 1)^2 - K^2]}{(2J + 1)}. \tag{16}$$

$R(J, K)$ is the enhancement factor for the levels added in rotational bands. It is the factor by which the intrinsic deformed level density of levels with spin J and spin projection K must be multiplied by to give the total level density of levels with spin J and projection K.

If $K = 0$, only alternate J values appear in the rotational band. Thus, if $K = 0$,

$$R(J, 0) = \frac{J^2 + 4J + 2}{2(2J + 1)}. \tag{17}$$

The approximation made in obtaining Eq. (16) is that the spin cutoff factors are the same at the energy of the band head or at the energy of the state in the band.

It can be seen that the $R(J, K)$ factor goes to 1 if $J = K$. This is expected because rotational bands are built on band heads with J and K such that the added levels have the same K but have J values larger than the band head J. Thus, no levels will be added with $K = J$. This form predicts that the density of levels for a given J as a function of K increases as K decreases. The total enhancement of the level density is approximately $\sigma_\perp/3$ due to the added levels and $(\sqrt{\pi/2})\sigma_\perp$ due to the breaking of the K degeneracy. This results in a multiplication of the state density by about $\sigma_\perp/3$ and the level density by about $\sigma_\perp^2(5/12)$.

The present analytical form is based on two steps. The intrinsic state density is calculated in a deformed basis. This makes it a function of K as well as J [the J_z is degenerate for a spherical well but leaves the total number of states the same (unitarity)]. On the other hand, the level density increases substantially because of the removal of the K degeneracy. Each level of spin J (spherical basis) becomes $J + \frac{1}{2}$ (odd A) or $J + 1$ (even A) levels in the deformed basis. As a last step, rotational bands are added. To check the R factors, a calculation was done which iteratively added the rotational bands. This showed the $R(J, K)$ factors are accurate to about 2%.

It should be noted that the behavior observed here is not consistent with a widely used formula proposed by Bohr and Mottelson [13] and Huizenga *et al.* [14]. They conclude that for deformed nuclei the level density formula is Eq. (8) multiplied by a factor of σ^2 with the σ^2 in the exponential replaced by $\sigma_\perp^2$. This would mean that the relative distribution of levels as a function of J is unchanged. It also leads to a multiplication factor enhancing the level density over the spherical case which is about a factor of 2 larger than the present result [$\sigma_\perp^2$ compared to ((5/12)$\sigma_\perp^2$].

A further problem with the previously used formula is that it predicts the K distribution for a given J at a specified energy is the same for $0 \leqslant K \leqslant J$. The present numerical results and the approximate expression both give differences of up to a factor of 5 as K varies from 0 to J.

It is interesting that the revised form for the level density in deformed nuclei helps resolve a puzzle. Komarov *et al.* [15] note that deformed nuclei are predicted to become spherical at high energies ($U >\sim 50$ MeV). If the level density for a deformed nucleus is enhanced by σ^2, the authors argue that a substantial drop in the level density should occur when the nucleus becomes spherical. No such drop or even leveling out was found in evaporation spectra examined in Ref. [15]. It was subsequently noted [5] that a Hauser-Feshbach calculation essentially populates a given channel based on the state density rather than the level density. This would reduce the factor from σ^2 to $\sqrt{2/\pi}\sigma$ for the Bohr-Mottelson expression. This makes the expected factor more like 7–10 rather than 50–100. The present results indicate the factor is about $\sigma_\perp/3$. This lowers that ratio to 2–4. If the transition occurs over 5–10 MeV, this change could more easily disappear in the shape of an evaporation spectrum and could be more easily lost than the factor of σ^2 originally expected.

The fact that the present level density form disagrees with that proposed in Refs. [13] and [14] has additional consequences. The previous form has frequently been used to infer level density parameters at the binding energy. If the interaction of the neutrons with the nucleus is limited to s waves, the levels populated will be $\frac{1}{2}^+$ for the spin and parity if the target was even-Z–even-N and $J = J_T \pm \frac{1}{2}$ if either Z or N or both are odd, where J_T is the J of the target. Assuming both parities are equally likely, the total level density for a spherical nucleus will be

$$\rho_T(U) = \frac{2\rho\left(U, \frac{1}{2}\right) \sum_{J=\frac{1}{2}}^{\infty} G_1(J)}{G_1(\frac{1}{2})} \tag{18}$$

if the target is even-even and

$$\rho_T(U) = \frac{2\left(\rho\left(U, J_T - \frac{1}{2}\right) + \rho\left(U, J_T + \frac{1}{2}\right)\right)}{G_1\left(J_T - \frac{1}{2}\right) + G_1\left(J_T + \frac{1}{2}\right)} \times \sum_{J=0}^{\infty} G_1(J) \tag{19}$$

where $G_1(J) = (J + \frac{1}{2}) \exp\left[-(J + \frac{1}{2})^2/2\sigma^2\right]$. The sum over $G_1(J)$ in Eqs. (18) and (19) yields σ^2. Here the ρ_T is the total level density. If the deformed level density is simply $\sigma_\perp^2 \rho_s(U, J)$, the $\sigma_\perp^2$ factors cancel out, leaving Eqs. (18) and (19) the same if σ^2 is replaced by $\sigma_\perp^2$.

The present results would require that Eq. (18) be modified to

$$\rho_T(U) = \frac{2\rho\left(U, \frac{1}{2}\right) \sum_J G(J) \sum_K R(J, K) S(K)}{G\left(\frac{1}{2}\right) R\left(\frac{1}{2}, \frac{1}{2}\right) S\left(\frac{1}{2}\right)} \tag{20}$$

for a deformed even-Z–even-N target and

$$\rho_T(U) = A \times \frac{B}{C}, \tag{21}$$

where

$$A = 2\left[\rho\left(U, J - \tfrac{1}{2}\right) + \rho\left(U, J + \tfrac{1}{2}\right)\right],$$
$$B = \sum_J G(J) \sum_K R(J, K) S(K),$$

and

$$C = G\left(J - \frac{1}{2}\right) \sum_K \left[R\left(J - \frac{1}{2}, K\right) S(K)\right] + G\left(J + \frac{1}{2}\right) \sum_K \left[R\left(J + \frac{1}{2}, K\right) S(K)\right]$$

if N and Z are not both even. These equations assume that K is mixed at the binding energy for Eq. (21). Comparison of Eqs. (18) and (20) with Eqs. (19) and (21) shows a systematic distortion of the correction for low-J_T relative to high-J_T results from the use of the Bohr-Mottelson equation. Level density parameters in Ref. [14] have been obtained using Eqs. (18) and (19) rather than Eqs. (20) and (21). The results of the present paper are insensitive to this problem. Calculations using the Bohr-Mottelson level density form and those using the current numerically generated form gave similar correction factors (within 10%) for the deformed versus spherical Hauser-Feshbach calculations.

Even though the difference between the spherical and deformed Hauser-Feshbach code was similar for the two level density approaches, the cross sections for resolved levels are enhanced about a factor of 2 to 3 using the present level density formulation over that of Bohr and Mottelson for the same level density parameters. This occurs with both Hauser-Feshbach codes. The enhancement comes at the expense of the continuum.

Because of the difference in level density between the Bohr-Mottelson form and the present results for the same level density parameter, it is best to correct the results of level density compilations based on level counting to the present form if they are used in a K-dependent Hauser-Feshbach code.

Level density parameters used in the calculations reported here were taken from Rohr [16]. In addition to a level density (continuum), specific levels were included. Various calculations included between 10 and 15 low-lying levels for the various individual nuclei.

B. Transmission coefficients

Transmission coefficients are obtained through calculations based on optical models. The most common procedure is to subject elastic scattering data to an analysis based on spherical optical model calculations. A comparable analysis can also be done with a coupled-channel code which specifically couples in certain strongly coupled excited states and solves the resulting coupled equations. Comparisons have been made for protons [17,18] and neutrons [19,20] of the changes in transmission coefficients caused by the use of a coupled-channel code compared to a spherical code. At high energies, inclusion of the coupled channels reduces the transmission coefficients by about 5% to 10%. This is due to the fact that flux is removed in populating the coupled excited states. At energies near the centripetal barrier (neutrons) or the centripetal plus Coulomb barrier (protons) the two calculations are equal. Below this energy the coupled channel transmission is higher than that for a spherical calculation. This is apparently due to the fact that population of an excited state when the particle has an energy at or below the barrier makes it more difficult for the particle to emerge. Thus, the channel coupling makes it more probable that a compound nucleus is formed rather than less likely. Although the best choice for a deformed nucleus would seem to be to use a transmission coefficient set generated with a couple channel code, the above analysis suggests that this will not be true if the optical parameters were generated through fits with a spherical code. Since the present optical parameters were generated with a spherical code, it is more reasonable to use a spherical code to calculate the transmission coefficients. In cases where cross sections for deformed nuclei are being fit, there is an obvious superiority to using a coupled channel code to get both transmission coefficients and the direct contribution, but this should utilize an optical potential derived from fits to the elastic cross section with a coupled-channel code.

The gamma-ray channel was also included in the calculations. Gamma-ray transmission coefficients were based on the parameters of Kopecky and Uhl [21]. This allowed for $E1$, $M1$, and $E2$ decay branches. Proton and neutron transmission coefficients were calculated with the parameters proposed by Koning and Delaroche [22], while the potential of McFadden and Satchler [23] was used for alpha particles. The deuteron, triton, and ^{3}He channels were not included in these calculations.

C. Specific calculations

Specific calculations were done for the $n + {}^{168}$Er, $\alpha + {}^{22}$Ne, $n + {}^{25}$Mg, $n + {}^{182}$W, and $n + {}^{183}$W reactions. Table I shows the results for the ^{168}Er $+ n$ reaction. The numbers quoted indicate the ratio of the Hauser-Feshbach calculation of a specified cross section with a spherical (conventional) code divided by the cross section calculated with the deformed Hauser-Feshbach code. An entry of 1.7, for example, indicates that the spherical Hauser-Feshbach code gave a value 70% larger than the deformed Hauser-Feshbach code. The values labeled (n, n') include the first fifteen levels in ^{168}Er; the values of J, K, and parity are listed for these levels, A strong tendency for the spherical code to overpredict cross sections for large J is seen, with the largest ratios listed for the fourth level ($J = 6$, $K = 0, +$) and the seventh level ($J = 8$, $K = 0, +$).

The lowest ratios, indicating underprediction of the cross section by the spherical code, come for the first and twelfth states, both of which are ($J = 0$, $K = 0, +$). Note also the difference for the cross section for levels 3, 8, and 10. These are all $J = 4$, but have K and parity of 0^+, 2^+, and 4^- respectively. A similar difference is seen between the fourth, thirteenth, and fifteenth levels, which have $J = 6$, $K = 0^+$; $J = 6$, $K = 2^+$; and $J = 6$ $K = 4^-$, respectively. Although there is a general tendency for cross sections to depend on K as well as J, it is particularly obvious that cross sections for $K = 0$ are smaller than for $K \neq 0$.

Cross sections for (n, α), (n, p) $(n, 2n)$, $(n, 3n)$, and (n, γ) are also compared in Table I. Since these involve a number of final levels, they show a tendency to be closer to unity than the ratios for cross sections to single levels. The (n, γ) reaction ratio below 1 MeV neutron bombarding energy approaches 1 as $E_n \to 0$, because it becomes the largest cross section. If the largest cross section has a change of 20% in its sum, then the cross section itself changes less, because the change in the denominator largely compensates the change in the numerator. Similarly, the $(n, 2n)$ and $(n, 3n)$ cross section ratios deviate from 1 by more near threshold than at energies high enough that they were the dominant cross sections. This is due both to the "denominator effect" described above and also the fact that at higher energies more final levels are being summed over, reducing the sensitivity to the spins of the levels in the first 1–2 MeV.

Table II presents similar information for the reactions ^{25}Mg(n, n') and ^{22}Ne(α, n). In this case, the compound nucleus is the same (^{26}Mg) and the binding energies of a neutron and an alpha particle in ^{26}Mg are almost identical. Thus, a 5 MeV (11 MeV) neutron produces essentially the same compound nuclear excitation energy as a 5 MeV (11 MeV) alpha particle. The cross-section ratios for the first ten levels in ^{25}Mg are shown, as well as their spin, K, and parity. Again, large J levels have cross sections which are predicted to be too large by the spherical Hauser-Feshbach code, while the $J = 1/2$ levels are underpredicted. It is somewhat surprising that the ratios vary considerably between the neutron-induced and alpha-induced

TABLE I. Cross-section ratio values for $n + {}^{168}$Er.

Reaction	Bombarding energy (MeV)				
	1	2	6	9	16
(n, α)			1.5	1.3	1.07
(n, p)			0.98	0.98	1.01
$(n, 2n)$				0.89	0.99
$(n, 3n)$					1.11
(n, γ)	1.08	1.05	1.02	1.01	1.01
The following are for the lowest fifteen levels in ^{168}Er:					
(n, n') $J = 0$ $K = 0^+$	0.53	0.39	0.33	0.29	0.23
(n, n') $J = 2$ $K = 0^+$	2.34	1.80	1.58	1.37	1.11
(n, n') $J = 4$ $K = 0^+$	3.33	2.68	2.59	2.33	1.91
(n, n') $J = 6$ $K = 0^+$	3.69	2.83	3.17	3.14	2.65
(n, n') $J = 2$ $K = 2^+$		0.8	0.75	0.63	0.52
(n, n') $J = 3$ $K = 2^+$		1.05	0.96	0.86	0.71
(n, n') $J = 8$ $K = 0^+$		2.22	3.7	3.83	3.08
(n, n') $J = 4$ $K = 2^+$		1.23	1.18	1.08	0.90
(n, n') $J = 5$ $K = 2^+$		1.44	1.37	1.29	1.09
(n, n') $J = 4$ $K = 4^-$		1.0	0.88	0.86	0.76
(n, n') $J = 5$ $K = 4^-$		0.87	1.06	1.06	0.90
(n, n') $J = 0$ $K = 0^+$		0.4	0.29	0.29	0.24
(n, n') $J = 6$ $K = 2^+$		1.42	1.5	1.46	1.26
(n, n') $J = 2$ $K = 0^+$		1.81	1.53	1.39	1.13
(n, n') $J = 6$ $K = 4^-$		1.06	1.21	1.22	1.07

reactions at a given energy. This is due to the fact that the alpha and ^{22}Ne nucleus in its ground state both have zero spin. Thus, only natural parity states are populated in ^{26}Mg. The neutron-induced reactions populated both natural and unnatural parity states. A close examination of the calculation shows that the differences in ratios for the same cross section between the alpha-induced and neutron-induced reactions are largely due to this parity selectivity in the alpha channel. A further difference is primarily seen at energies below 12 MeV. The distribution of strength as a function of J in the compound nucleus is different in the two reactions at low energy. The neutron brings in less angular momentum but is incident on a $5/2^+$ ground state. At low energies, the average J in the compound nucleus is slightly higher for the neutron channel, but the two become more comparable above 10 MeV, with the alpha entrance channel eventually overtaking the neutron channel.

TABLE II. Cross-section ratios for the ^{26}Mg system.

Reaction	Bombarding energy (MeV)			
	$n + {}^{25}$Mg		$\alpha + {}^{22}$Ne	
	5	11	5	11
(x,α)	0.75	0.58	0.67	0.62
(x, p)	0.6	0.68	1.02	0.89
(x, n)	1.03	1.02	1.12	1.07
$(x, 2n)$		1.28		1.21
(x,γ)	1.08	1.02	1.14	1.01
The following are the lowest ten levels in ^{25}Mg:				
(x, n) $J = 5/2$ $K = 5/2^+$	0.24	0.2	0.53	0.44
(x, n) $J = 1/2$ $K = 1/2^+$	0.12	0.12	0.35	0.25
(x, n) $J = 3/2$ $K = 1/2^+$	0.47	0.45	0.31	0.30
(x, n) $J = 7/2$ $K = 5/2^+$	0.88	0.52	0.71	0.29
(x, n) $J = 5/2$ $K = 1/2^+$	1.45	1.1	0.84	0.72
(x, n) $J = 1/2$ $K = 1/2^+$	0.11	0.12	0.48	0.32
(x, n) $J = 7/2$ $K = 1/2^+$	4.93	2.43	1.15	0.78
(x, n) $J = 3/2$ $K = 3/2^+$	0.54	0.45	0.29	0.37
(x, n) $J = 9/2$ $K = 1/2^+$	10.3	4.5	1.96	0.92
(x, n) $J = 3/2$ $K = 3/2^-$	0.2	0.2	0.75	0.56

Results for $^{182}\mathrm{W} + n$ are shown in Table III. This target is even-even so the results are similar to those for ^{168}Er. In particular, the tendency to overestimate cross sections for larger J (6 and 8) with calculations from a conventional Hauser-Feshbach code is evident. Large J levels with $K \neq 0$ are not found in the lowest 10 levels, but note the substantial difference between ratios for levels 7 and 8, which have $J = 2$, $K = 0$ and $J = 2$, $K = 2$, respectively. The spherical code predicts identical cross sections for these levels, while the cross section predictions differ by about a factor of 2 with the new code. The degeneracy ratio is, of course, a factor of 2. Cross sections for capture are slightly overestimated by the spherical code. This reflects the fact that the degeneracy of the entrance (and elastic exit) channel is 1 for both the spherical and deformed calculations. Capture occurs to 1/2, 3/2, and 5/2 levels if $E1$, $M1$, and $E2$ gamma strength functions are used. The degeneracy of the 1/2, 3/2, and 5/2 levels varies substantially in a spherical basis (2, 4, 6). The K must be 1/2 for $J = 1/2$ levels; the degeneracy for these levels is unchanged. 3/2 and 5/2 levels have degeneracy of 4 and 6, respectively, in the spherical calculation and

TABLE III. Cross-section ratio values for $n + {}^{182}$W.

Reaction	Bombarding energy (MeV)					
	0.5	1	4	7	10	14
(n, α)	1.67	1.77	1.85	1.7	1.51	1.3
(n, p)			0.98	1.0	1.0	1.0
$(n, 2n)$						0.97
$(n\alpha, \alpha n)$						1.52
(n, γ)	1.12	1.07	1.03	1.01		
The following are the ten lowest levels of ^{182}*W:*						
(n, n') $J = 0$ $K = 0^+$	0.56	0.52	0.36	0.30	0.27	0.24
(n, n') $J = 2$ $K = 0^+$	2.12	2.21	1.25	1.42	1.26	1.14
(n, n') $J = 4$ $K = 0^+$	3.25	3.44	2.84	2.3	2.15	1.96
(n, n') $J = 6$ $K = 0^+$		3.56	3.75	3.16	2.93	2.71
(n, n') $J = 0$ $K = 0^+$			0.377	0.322	0.273	0.24
(n, n') $J = 8$ $K = 0^+$			3.6	3.87	3.59	3.58
(n, n') $J = 2$ $K = 0^+$			1.68	1.45	1.28	1.14
(n, n') $J = 2$ $K = 2^+$			0.75	0.67	0.59	0.54
(n, n') $J = 1$ $K = 1^-$			0.54	0.46	0.40	0.35
(n, n') $J = 3$ $K = 2^+$			0.99	0.89	0.8	0.73

2 ($K = 1/2, 3/2, 5/2$) in the deformed calculation. The role of 5/2 levels is small, since they require $E2$ emission. The net effect is to reduce the gamma width. This overestimate is reduced as E drops below 0.5 MeV, because the gamma width eventually dominates, causing changes in the denominator which approximately cancel those in the numerator. Similar arguments explain why the ratio for $(n, 2n)$ is so close to 1 at 14 MeV.

Table IV lists the corresponding results for neutron bombardment of ^{183}W; as in the other cases, the (n, n') cross sections are underestimated by the spherical code if J is small and overestimated if J is large. In this case, the (n, γ) cross section is closer to being the same with the two codes. This is because the target has $J = 1/2$, which has no change in degeneracy. Capture proceeds through transitions to $J = 0$, 1, 2, or 3 levels. These have degeneracies of 1, 3, 5, or 7 in a spherical calculation. The degeneracies will all be 2 in the deformed cases. Thus, some have increased width and some decreased width, making the net change small. For energies below 0.5 MeV, the ratio becomes very close to 1.

D. Related questions

The calculations tabulated in Sec. II C show some consistent tendencies. In each system, the cross sections for population of specific final levels are affected most. Cross sections for states of large J are reduced and those of small J are increased. Cross sections to particular channels [e.g., (n, p)] are also changed, but by smaller amounts. An effort was made to see if some of these changes could be attributed to other effects:

(1) Incomplete gamma strength functions
The present calculations utilized $E1$, $M1$, and $E2$ gamma-ray strength functions. In a spherical Hauser-Feshbach calculation, this leaves a small number of levels without a decay channel (mostly in odd-A final nuclei). If the ground state is $7/2^-$ and the next level is $1/2^-$ then this level cannot decay (a transition of $\Delta\ell = 3$ would be needed). Approximately 4% of the final levels were affected. Using the deformed Hauser-Feshbach code, this number increased to about 6%. As an example, a 3^- state in an even-even nucleus is often the lowest negative parity state. If the next lowest state is 2^+, an $E1$ decay can occur. For the deformed calculation, if the 2^+ has $K = 0$ and the 3^- has $K = 3$, $E1$ decay is not allowed and the lowest order is $E3$. This change is too small to explain the calculated differences. The ambiguity would be eliminated if one included the specific relative branching of each of the low-lying levels.

(2) Level density form
The calculations reported here were done with level densities generated with the level density parameters a and δ in a spherical basis. The state density and level density were then converted to a deformed basis and rotational bands were then added (Method A). An alternative would be to use the Bohr and Mottelson formula relating the deformed level density $\rho_D(U, J)$ to the spherical level density ρ_{Sp} with $\rho_D(U, J) = \sigma_\perp^2 \rho_{Sp}(U, J)$. This was called Method B. Finally, an analytic approximation to Method A was developed as described in Sec. II B (Method C). Use of the same a and δ with Method B gave a consistent reduction of the resolved level cross sections by 45 to 55%. It also enhanced the neutron continuum by 2%–3% and reduced the charged particle continuum by about 1%–2%. Use of Method C produced very similar results to Method A. None of these methods for obtaining level density can make the deformed Hauser-Feshbach agree with the spherical Hauser-Feshbach. An additional difference between the Bohr-Mottelson and present level density form is that the J and K distribution of levels populated in the continuum is changed. This may be important in calculating cross sections like $(n, 2n)$ just above threshold or in calculating particle emission cross sections in competition with fission (this channel is not yet in the present code).

TABLE IV. Cross-section ratio values for $n + {}^{183}W$.

Reaction	Bombarding energy (MeV)				
	0.6	1	4	7	14
(n, α)	1.9	2.05	2.2	2.0	1.46
(n, p)				1.0	1.0
$(n, 2n)$				1.07	1.01
$(n, \alpha n + n\alpha)$				1.52	1.44
(n, γ)	1.1	1.04	1.015	1.01	
The following are the twelve lowest levels of ^{183}W:					
(n, n') $J = 1/2$ $K = 1/2^-$	0.18	0.19	0.18	0.16	0.13
(n, n') $J = 3/2$ $K = 1/2^-$	0.68	0.71	0.59	0.47	0.37
(n, n') $J = 5/2$ $K = 1/2^-$	1.14	1.18	1.0	0.81	0.64
(n, n') $J = 7/2$ $K = 1/2^-$	1.6	1.61	1.38	1.12	0.88
(n, n') $J = 3/2$ $K = 3/2^-$	0.5	0.61	0.54	0.46	0.38
(n, n') $J = 5/2$ $K = 3/2^-$	0.94	1.0	0.9	0.75	0.61
(n, n') $J = 9/2$ $K = 1/2^-$	1.77	1.78	1.83	1.48	1.13
(n, n') $J = 11/2$ $K = 11/2^+$	0.56	0.65	1.05	0.94	0.83
(n, n') $J = 7/2$ $K = 3/2^-$	1.32	1.42	1.29	1.05	0.84
(n, n') $J = 7/2$ $K = 7/2^-$	0.87	1.02	0.97	0.52	0.69
(n, n') $J = 11/2$ $K = 1/2^-$	2.15	2.1	2.06	1.70	1.37
(n, n') $J = 13/2$ $K = 11/2^+$	0.73	0.8	1.17	1.13	1.01

(3) Level density parameters
Changes in a and δ can change Hauser-Feshbach cross sections. Changes of 5% in a caused changes in the relative cross section of the resolved levels to the cross section to the continuum of about 20%. It did not change the relative cross sections to the resolved levels in a manner similar to the difference between the deformed and spherical Hauser-Feshbach.

(4) Parity ratio changes
Hauser-Feshbach calculations are usually done with level density functions for the continuum which have equal densities for positive and negative parities. A paper by Al Quraishi *et al*. [24] has proposed a function for the parity ratio as a function of energy for the level density. If this function is put into the deformed and spherical Hauser-Feshbach calculations, it is found that cross sections for resolved levels change by as much as 10%, but the ratio between the spherical and deformed Hauser-Feshbach changed by 5% or less for each level. The effects are larger for the $A = 26$ calculations than for these for heavy nuclei. The ambiguity could be eliminated if the level spin and parities are known and input for levels at higher excitation energies. The best solution would be to use level density values derived from microscopic calculations which have appropriate parity ratios.

(5) Level density: collective states
The calculations described in Sec. II C are based on a level density constructed from deformed levels on which rotational bans are built. To test if other collective levels might change these results, level densities were constructed with 5% or 10% of the deformed levels assumed to have β vibrational bands built on them instead of rotational bands. Also, a calculation was done with the deformed basis level density used but without the rotational bands. The first change made very little difference in either the resolved level or continuum cross section. The second enhanced the resolved level cross sections but left the ratios only slightly changed (∼3%–5%).

(6) Spin cutoff parameters
The calculations in Sec. II C assumed a rigid body moment of inertia. Reducing the moment of inertia by 10% enhanced the large J level cross sections and reduced the small J resolved level cross sections. This is in the opposite direction to the present changes. A corresponding increase in the moment of inertia will add to small J and reduce large J cross sections. A 10% change in the moment of inertia (5% in σ) causes a much smaller change in cross sections as a function of J than found in Sec. II C. It also does not cause cross sections in the spherical Hauser-Feshbach to vary with K as well as J.

(7) Transmission coefficients
As has already been noted, the use of the same optical parameters in a coupled channel code and a spherical optical code gives different energy dependence to the transmission coefficients. The most accurate results would be obtained if optical model parameters for deformed nuclei were derived from fits with a coupled channel code to calculate transmission coefficients. The present calculations are based on optical parameters obtained from fits with spherical optical codes so transmission coefficients were calculated for the calculations reported in Tables I–IV with a spherical optical code. Use of transmission coefficients with the energy dependence of a coupled channel calculation raises the continuum cross section for neutrons below 2 MeV, protons below 5 MeV, and alpha particles below 8 MeV. Cross sections for low-energy particles are enhanced and those for high-energy particles are slightly suppressed. No J selectivity similar to that

observed in Sec. II C was observed. Use of the coupled channel energy dependence does enhance the reaction cross section at low energies and lowers it at high energies.

Individual cross sections can be changed by 10%–15% by using transmission coefficients from the coupled channel code, but these are small changes relative to the 40% to 100% changes caused by using the deformed Hauser-Feshbach code instead of the spherical one.

(8) K mixing

The calculations reported in Sec. II C were for no mixing of K. Calculations for mixing fractions up to 90% showed small changes. Mixing was applied to all excited levels above 4 MeV, since it is expected that at low energies the level density is so low that mixing is suppressed. The mixing tended to remove the differences between levels of a given J with different K values unless the K was zero. It did not change the basic tendency for the deformed Hauser-Feshbach to lower large-J cross sections and raise low-J cross sections. This is due to the fact that the primary difference between the two calculations is not the relative branching as a function of K but rather the change in degeneracy of the levels. This latter change is not affected by K mixing.

(9) Isospin

As mentioned previously, the spherical Hauser-Feshbach code can be run with or without inclusion of isospin, while the new deformed Hauser-Feshbach code does not yet have isospin inclusion as an option. Calculations with the spherical code showed no significant isospin effects for compound nuclei with $165 \leqslant A \leqslant 184$ and also no effects for $A = 26$. In each case, this conclusion would be changed if the proton entrance channel is used. For the $A = 24$ compound system, formation of the compound system through alpha bombardment only makes $T = 0$ compound states. Thus, $T = 1$ states in ^{20}Ne are not populated through (α,α') processes nor are $T = 3/2$ states in ^{23}Na or ^{23}Mg. Formation of the compound nucleus through the $p + {}^{23}$Na channel populates both $T = 0$ and $T = 1$ states in ^{24}Mg; both $T = 0$ and $T = 1$ states in ^{20}Ne as well as both the $T = 3/2$ and $T = 1/2$ states in ^{23}Na and ^{23}Mg, can be populated, removing the above restriction. Although the present deformed code does not include isospin, it appears the effects would be similar to those for the spherical code.

It appears that the effects 1 though 9 all influence the compound nuclear cross sections but none seem likely to change the specific signature of the deformed Hauser-Feshbach calculations relative to the spherical.

III. CONCLUSIONS

A Hauser-Feshbach code has been written which is based on an extension of the traditional formalism to include the K quantum number. The modification of level degeneracy in a deformed nucleus is also taken into account. The following changes are observed:

(1) The degeneracy change causes cross sections for small J to increase and large J to decrease. In some cases, the ratio changes by more than a factor of 2.
(2) In even A residual nuclei, there is a tendency for levels of a given J to be populated more strongly if $K \neq 0$ than for $K = 0$ levels of the same J. The K dependence is smaller for odd A nuclei, but is not damped out completely by K mixing in the compound nucleus.
(3) Cross sections which are sums over many final levels (e.g., n, p) show smaller changes because they include both small and large J final levels.
(4) A number of assumptions affecting compound calculations were examined. The changes caused by these modifications do not change the basic conclusion of the paper.
(5) An error was found in a frequently used level density formula for deformed nuclei. A principal problem is in the K dependence; this would never have been seen in Hauser-Feshbach calculations which ignored K. There are also changes in the J distribution and rotational enhancement factor, however.

Further plans include the addition of isospin, the fission channel, and the calculation of angular distributions.

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