Density determinations in heavy ion collisions

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The experimental determination of freeze-out temperatures and densities from the yields of light elements emitted in heavy ion collisions is discussed. Results from different experimental approaches are compared with those of model calculations carried out with and without the inclusion of medium effects, which become of relevance for baryon densities above $\approx 5 \times 10^{-4}$ fm^{-3}. A quantum statistical (QS) model incorporating medium effects is in good agreement with the experimentally derived results at higher densities. A densitometer based on medium modified chemical equilibrium constants is proposed.

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I. INTRODUCTION

Heavy ion collisions (HICs) serve as tools to investigate the properties of excited nuclear matter. Measured yields of different ejectiles as well as their energy spectra and their correlations in momentum space can be used to infer the properties of the emitting source. Despite the fact that a great deal of experimental data has been accumulated from HIC during the past few decades, reconstruction of the properties of the hot expanding nuclear system remains a difficult task. Two major problems are (i) the complications inherent in incorporating nonequilibrium effects and (ii) the treatment of strong correlations that are already present in equilibrated nuclear matter.

In the present work we focus on the latter item, the treatment of strong correlations that are responsible for the formation of clusters in dense matter. For future progress, the consistent description of correlations in equilibrium will be a prerequisite to work out a nonequilibrium approach to HICs.

An often-employed simple approach to handling these effects is the freeze-out approximation. Starting from hot dense matter produced in HICs, this approach assumes the attainment of local thermodynamic equilibrium after a short relaxation time. Chemical equilibrium may also be established in the expanding fireball as long as the reaction rates in the expanding hot and dense nuclear system are above a critical value.

While more microscopic approaches employing transport models that describe the dynamical evolution of the many-particle system are being pursued, a freeze-out approach provides a very efficient means to get a general overview of the reaction. Such approaches have been applied in heavy ion reactions to analyze the equation of state of nuclear matter; see Ref. [1]. Similar concepts are also used in high-energy experiments (Relativistic Heavy Ion Collider, Large Hadron Collider) to describe the abundances of emitted elementary particles; see Refs. [2–4] and further references given therein. For a critical consideration of deriving unbiased freeze-out parameters from particle yield ratios, see Ref. [5]. In warm nuclear matter, much information on the symmetry energy, on phase instability, etc., has been obtained using the freeze-out concept; examples are given in the following section.

Within the freeze-out approximation to expanding excited nuclear matter, the abundances of emitted particles and clusters at freeze-out are determined by the temperature T, the baryon density n_B, and the isospin asymmetry $\delta = (n_n - n_p)/n_B$, which is related to the total proton fraction $Y_e = (1-\delta)/2$. In this work we discuss the extraction of densities and temperatures from the measured yields of ejectiles in HIC. We focus on the information content of neutrons (n), protons (p), deuterons (d), tritons (t), ^{3}He (h), and ^{4}He (α) particles, emitted in near-Fermi-energy reactions. To extract the relevant information we optimize the freeze-out approach by including correlations and density effects using systematic, consistent quantum statistical approaches.

We are considering only the yields Y_i of these particles; the energy spectra are established by long-range interactions and are not discussed here. It is possible to extend the approach also to other situations where not only particles with $A \leqslant 4$ are of relevance. Whereas the asymmetry is easily obtained from the proton and neutron numbers of all emitted fragments, for the determination of the temperature many efforts have been made. In particular, double ratios have been considered. We do not discuss these results here. In contrast, the determination of the density from the measured yields is a serious problem that has not been solved in a satisfactory manner until now. We give the reason and propose a solution to this problem.

II. EXPERIMENTS AND DATA ANALYSIS

A. Nuclear statistical equilibrium (NSE)

The NIMROD Collaboration has recently measured yields of light particles in three different experiments performed at energies near the Fermi energy. Collisions of ^{64}Zn projectiles with ^{92}Mo and ^{197}Au target nuclei [6] and the collisions ^{70}Zn + ^{70}Zn, ^{64}Zn + ^{64}Zn, and ^{64}Ni + ^{64}Ni were studied at $E/A = 35$ MeV/nucleon [7]. Collisions of ^{40}Ar + ^{112}Sn, ^{124}Sn and ^{64}Zn + ^{112}Sn, ^{124}Sn [8] were studied

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at 47 MeV/nucleon. These experiments have been described in several papers [6,8–11].

Our goal is to infer parameter values T, n_B, and δ, characterizing the freeze-out state, from the five experimental yields, Y_p, Y_d, Y_t, Y_h, Y_α, of the light charged $Z = 1, 2$ species. (The neutron yields are not accurately measured but, under equilibrium assumptions, can be ascertained from the proton yields combined with t/h ratios as indicated below.) This problem is easily solved in the low-density limit where the NSE can be applied, i.e., below $n_B \approx 10^{-4}$ fm^{-3}, and at moderate temperatures where medium effects can be neglected. In chemical equilibrium, simple relations for the nondegenerate ideal mixture of reacting components

$$n_i = \frac{2s_i + 1}{\Lambda_i^3} e^{(E_i + Z_i\mu_p + N_i\mu_n)/T} \tag{1}$$

hold, where $\Lambda_i = (2\pi\hbar^2/m_i T)^{1/2}$ is the thermal wave length, m_i is the mass, s_i is the spin, and E_i is the binding energy of the different components (including excited states). Then, one can construct expressions that are almost directly related to the different thermodynamic parameters.

In particular the ratio Y_h/Y_t can be used to determine the asymmetry of the nuclear system. It can also be used to give an estimate of the neutron yield

$$Y_n = Y_p \frac{Y_t}{Y_h} f_\delta(T), \tag{2}$$

where $f_\delta(T) = \exp[(E_h - E_t)/T][(m_n m_h)/(m_p m_t)]^{3/2}$ is a correction that accounts for the difference in the binding energies of ^{3}H and ^{3}He. For the sake of simplicity we use in the following the approximation $m_A = Am$ with the average baryon mass, m.

The temperature can be determined by a double ratio of yields chosen so that the chemical potentials are compensated in the NSE. For instance, according to Albergo [12] the H-He temperature can be obtained from bound hydrogen and helium states,

$$T_{\rm HHe} = 14.3\ \text{MeV}\left(\ln\left[1.59\frac{Y_\alpha Y_d}{Y_t Y_h}\right]\right)^{-1/2}. \tag{3}$$

Within the NSE framework, knowledge of the temperature allows the extraction of the baryon density. In Ref. [6], the yield ratio of ^{4}He to ^{3}H was used to determine the free proton density according to

$$n_p = 0.62 \times 10^{36}\, T^{3/2} \exp[-19.8/T]\, \frac{Y_\alpha}{Y_t}, \tag{4}$$

and similarly

$$n_n = 0.62 \times 10^{36}\, T^{3/2} \exp[-20.6/T]\, \frac{Y_\alpha}{Y_h}. \tag{5}$$

Here T is the temperature in MeV, and n_i has units of nucleons/cm^3. The total baryon density follows as $n_B = (n_p/Y_p)\sum_i A_i Y_i$.

B. Consistency test for the NSE

Note that only ratios of yields of bound states were used to infer the temperature, Eq. (3), and the chemical potentials, Eqs. (4) and (5). To infer the thermodynamic parameter, other ratios that contain the free nucleon (p,n) yields can also be considered. If we focus on five measured yields, $Y_p, Y_d, Y_t, Y_h, Y_\alpha$, we have four ratios that are of relevance to infer the three parameters T, n_B, δ that characterize the thermodynamic state of the nuclear system. There is one additional degree of freedom that can be used for a consistency check. In particular, we can consider the ratio

$$R_{\rm test} = 4^\epsilon \left(\frac{27}{16}\right)^{3\epsilon/2} \frac{3}{4}\left(\frac{8}{9}\right)^{3/2} \frac{Y_\alpha^{2\epsilon-1} Y_p^\epsilon}{Y_h^{2\epsilon-1} Y_t^{\epsilon-1} Y_d} \tag{6}$$

with $\epsilon = (E_\alpha + E_d - E_t - E_h)/(2E_\alpha - E_t - 2E_h) = 0.438\,33$, where the prefactor of the yield fraction has the value 1.627 96.

From NSE it follows that $R_{\rm test}^{\rm NSE} = 1$. This quantity is also easily determined from measured yields. In particular, the data obtained in the experiment Ref. [6] give in total (summed over the surface velocity v_s) $R_{\rm test} = 1.22$, the data of Ref. [8] lead to $R_{\rm test} = 1.36$, and the data of Ref. [11] lead to 1.147 (summed over all excitation energies). As will be shown, in comparison with the yields of bound nucleons, the yield Y_p is higher than expected within NSE.

Different reasons can be given for this deviation:

(i) The assumption of thermodynamic equilibrium is not realized. One has to investigate the dynamical nonequilibrium expansion of the fireball produced in HIC.
(ii) The source is more complex.
(iii) The assumption of an ideal mixture of reacting but otherwise noninteracting components (free nucleons and clusters) must be improved.

We do not discuss how the freeze-out concept has to be modified when nonequilibrium and finite-size effects are taken into account. Rather here we focus on the last point—improving the approximation of an ideal mixture by considering effects of correlations in the medium. This can be done within a systematic quantum statistical approach.

As shown in the following section, the account of medium effects leads to a suppression of bound states due to Pauli blocking. As a consequence, the fraction of nucleons that are found in single-particle states ("free nucleons") is larger than expected within the NSE.

III. QUANTUM STATISTICAL (QS) APPROACH

A. Pauli blocking and Mott effect

Within a quantum statistical approach to nuclear matter, correlations and bound-state formation are treated using Green's functions to derive in-medium few-body wave equations; see Refs. [13,14]. After a cluster decomposition of the single-nucleon self-energy, the spectral function contains the contribution of the A-nucleon propagators. The A-particle wave function $\psi_{A\nu P}(1\ldots A)$ (with total momentum P and internal quantum number ν) and the corresponding eigenvalues $E_{A\nu}^{\rm qu}(P)$ follow from solving the in-medium Schrödinger equation

$$\begin{aligned}&\left[E_1^{\mathrm{qu}}(1)+\cdots+E_1^{\mathrm{qu}}(A)-E_{A\nu}^{\mathrm{qu}}(P)\right]\psi_{A\nu P}(1\ldots A)\\&\quad+\sum_{1'\ldots A'}\sum_{i<j}[1-f_1(i)-f_1(j)]V(ij,i'j')\\&\quad\times\prod_{k\neq i,j}\delta_{kk'}\psi_{A\nu P}(1'\ldots A')=0.\end{aligned}\tag{7}$$

This equation contains the effects of the medium in the single-particle shift $E_1^{\mathrm{qu}}(1)$ as well as in the Pauli blocking terms due to the phase space occupation $f_1(p)$. The single-particle shift was taken according to the relativistic mean-field approximation [15]. The Fermi-like distribution function $f_1(p)$ depends on the baryon densities n_n, n_p and the temperature T. Obviously the bound-state wave functions and energy eigenvalues as well as the scattering phase shifts become dependent on temperature and density. The medium dependent bound-state energies $E_{A\nu}^{\mathrm{qu}}(P;T,n_n,n_p)$ may be considered as quasiparticle energies. In the mean-field approximation (7), two effects have been taken into account: the single-nucleon quasiparticle energy shift and the Pauli blocking. Diagrams that contribute in higher orders to the Green's functions have been neglected. For the light elements $A \leqslant 4$, the quasiparticle shifts have been evaluated (see Ref. [14]) where interpolation formulas for the dependence on P, T, n_n, n_p can be found.

After performing the cluster decomposition of the spectral function using the quasiparticle picture for the in-medium bound states, the evaluation of the equation of state is straightforward; see Ref. [13]. We obtain the result (Ω is the volume)

$$\begin{aligned}n_p(T,\mu_n,\mu_p)&=\frac{1}{\Omega}\sum_{A,\nu,P}Zf_{A,Z}\left[E_{A,\nu}^{\mathrm{qu}}(P;T,\mu_n,\mu_p)\right],\\n_n(T,\mu_n,\mu_p)&=\frac{1}{\Omega}\sum_{A,\nu,P}(A-Z)f_{A,Z}\left[E_{A,\nu}^{\mathrm{qu}}(P;T,\mu_n,\mu_p)\right],\end{aligned}\tag{8}$$

for the equation of state describing a mixture of components (cluster quasiparticles) obeying Fermi or Bose statistics,

$$f_{A,Z}(\omega)=[\exp\{\beta[\omega-Z\mu_p-(A-Z)\mu_n]\}-(-1)^A]^{-1}.\tag{9}$$

The NSE is obtained in the low-density limit if the in-medium energies $E_{A,\nu}^{\mathrm{qu}}(P;T,\mu_n,\mu_p)$ can be replaced by the binding energies of the isolated nuclei $E_{A,\nu}^{(0)}(P)$. Note that at low temperatures Bose-Einstein condensation may occur.

Within the present work, we consider in Eq. (8) only the contribution of light clusters ($A = 1,2,3,4$). The approach can be extended to include also larger clusters $A > 4$ if their contribution becomes of relevance. Furthermore, we restrict ourselves to only the contribution of bound states. The internal quantum number ν covers also excited bound states and scattering states. These contributions are neglected here but are necessary to derive the exact expression for the second virial coefficient as known from the generalized Beth-Uhlenbeck formula [16].

Comparing to the NSE that results in the zeroth-order approach with respect to medium corrections, improvements are obtained. In particular, we find the following:

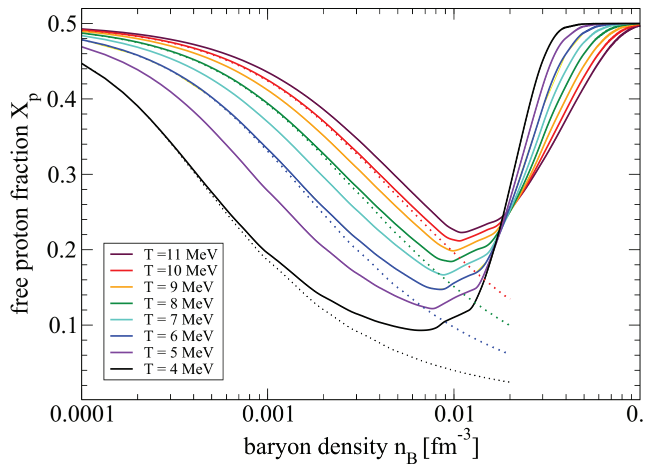

FIG. 1. (Color online) Free proton fraction as function of density and temperature in symmetric matter. Restricting components to light elements $A \leqslant 4$, the QS calculations (solid lines) are compared with the NSE results (dotted lines). No continuum contributions are included. The Mott effect and its temperature dependence is clearly seen near 0.01 fm^{-3} where the bound state fraction decreases and the free proton fraction rises.

(i) The classical Boltzmann distribution is replaced by the Fermi or Bose distributions if degenerate effects are to be accounted for. This follows immediately from a quantum statistical approach. In a similar spirit, the momentum quadrupole and normalized number fluctuations for light particle emission in HIC have been analyzed in Ref. [11]. In that work it has been proposed to use the reduction of fluctuations for Fermi systems or enhancement of fluctuations for Bose systems to estimate the thermodynamic parameters.

(ii) With increasing density, medium effects have to be included. Within a quasiparticle picture, the binding energies of the bound states are decreasing with increasing density due to Pauli blocking. Depending on temperature and center-of-mass momentum, the bound states merge in the continuum at the so-called Mott density. Since the composition is determined by the quasiparticle energies, the cluster abundances are suppressed. As a consequence, the mass fraction of free nucleons is enhanced compared to the NSE; see Fig. 1. The medium effects become of relevance when the baryon density n_B exceeds a value of about 5×10^{-4} fm^{-3}. The expressions (3), (4), and (5) used to derive the thermodynamic parameters based on the NSE have to be correspondingly corrected, as shown in the following. (See also Ref. [17].)

(iii) In a QS approach, contributions of the continuum (continuous internal quantum number ν) to the density also arise (scattering states). Within a virial expansion, for each channel where a bound state is formed, scattering states will also contribute to the equation of state. An upper limit for the contributions of the continuum can be given by subtracting for each bound state the same term with zero binding energy. These continuum contributions are small in the region considered here and are neglected in the present work. Future investigations are needed to account for the continuum correlations.

Because the ratio of free nucleons to bound clusters is strongly influenced by medium effects, the use of the NSE is limited to very small densities. Compared to the NSE, in the QS approach the concentration of bound states goes down, whereas the fraction of free nucleons increases. This modifies the yield fractions that contain the free nucleon yields.

B. Temperature determinations in low-density nuclear matter

At densities below the Mott point the effect of medium modifications on the double isotope ratios is not strong [17]. Thus, to a good approximation the determination of the temperature can be performed employing the double ratios, Eq. (3). In Refs. [6,8–10] this technique is employed to characterize the temperature evolution of the expanding nascent fireball (the intermediate velocity or nucleon-nucleon source) by associating particle velocity with emission time. (The Albergo expression, Eq. (3), is modified by a factor $(9/8)^{1/2}$ in front of the double ratio when applied to particles with the same surface velocity; see Ref. [6].) In Ref. [11], which focuses on quasiprojectile sources of different excitation energy, temperatures have been calculated employing the momentum quadrupole fluctuation method. In the comparisons which follow, the temperatures are those derived in the quoted references.

C. Density determinations

The main problem is the determination of the density because the influence of medium effects can be strong. In the following we compare results from four different approaches to determination of the density: (i) the Albergo NSE-based relations [6], (ii) the Mekjian coalescence model, which takes into account three-body terms which might mimic either a higher density (three-body collisions) or Pauli blocking [8,18], (iii) the quantum fluctuation analysis method [11], and (iv) an approach based on use of the Chemical equilibrium constant employed in Refs. [8–10].

The use of the first three of these techniques to extract temperatures and densities have been well described in the references cited. The use of the chemical equilibrium constant, introduced in Ref. [8], to characterize the relative yields

$$K_c(A,Z) = \frac{n_{A,Z}}{n_p^Z n_n^{(A-Z)}}, \tag{10}$$

has some particular advantages. In contrast to the free proton fraction, these chemical equilibrium constants, while sensitive to the effects of the medium, are not dependent on the asymmetry parameter or the choice of competing species present in a model in the low-density limit where the NSE can be applied. Specifically, to infer the values for the thermodynamic parameters of nuclear matter in HIC at freeze-out from experimental data we define the quantity $\tilde{K}_\alpha$ that is related to the chemical equilibrium constant for α

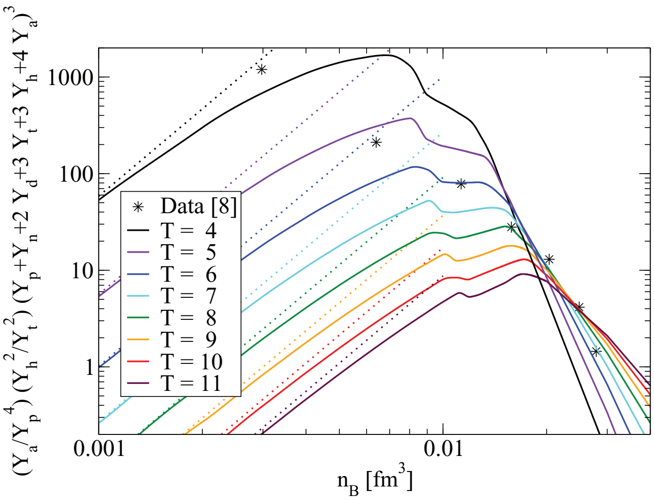

FIG. 2. (Color online) Chemical constant $\tilde{K}_\alpha$, Eq. (11), as function of density and temperature (T in MeV). Data (stars) [8] for $T = 5, 6, 7, 8, 9, 10, 11$ MeV (increasing density) in comparison with the NSE values (thin dotted lines) and QS calculations (bold straight lines).

particle formation and can be directly determined from the observed experimental yields,

$$\tilde{K}_\alpha = \frac{Y_\alpha}{Y_p^4}\frac{Y_h^2}{Y_t^2}\left(\sum_i A_i Y_i\right)^3 = \frac{n_\alpha}{n_p^4}\frac{n_h^2}{n_t^2}n_B^3. \tag{11}$$

The second relation is found by dividing the particle numbers by their common volume. This modified chemical constant $\tilde{K}_\alpha$ does not depend on the volume of the system. Note that the baryon density equals $n_B = n_p + n_n + 2n_d + 3n_t + 3n_h + 4n_\alpha$, if the ejectiles are restricted to $A \leqslant 4$. In general, clusters with higher A must be included if they are formed from the source under consideration.

Within NSE we can show that

$$\ln \tilde{K}_\alpha^{\rm NSE} = 3 \ln n_B + f_\alpha(T), \tag{12}$$

is applicable for the low-density region, $f_\alpha(T) = (E_\alpha + 2E_h - 2E_t)/T + (9/2)\ln[2\pi\hbar^2/(mT)] - \ln 2$. The quasiparticle shifts, which we have previously calculated for the single nucleons as well as for the light clusters [14], indicate that medium effects are relevant above the density of about $n_B = 5 \times 10^{-4}$ fm^{-3}. In Fig. 2 we present theoretical values of $\tilde{K}_\alpha$ which have been calculated, accounting for the QS corrections (symmetric matter). The decrease of $\tilde{K}_\alpha$ for densities above 10^{-2} fm^{-3} is due to the Mott effect in which bound states disappear because of Pauli blocking; see Ref. [9].

In our calculations we find essentially no dependence of $\tilde{K}_\alpha$ on the asymmetry parameter as should be expected for the chemical equilibrium expression. In principle this plot constitutes a densitometer which may be employed to estimate the density from experimental yields if the temperature has been determined. However, in general there are two solutions so that one has to select out the correct one. For comparison to the theoretical values presented in Fig. 2 we present also in that figure experimental values for $T = 5$ to 11 MeV, derived from the measured data discussed in Refs. [8–10]. These

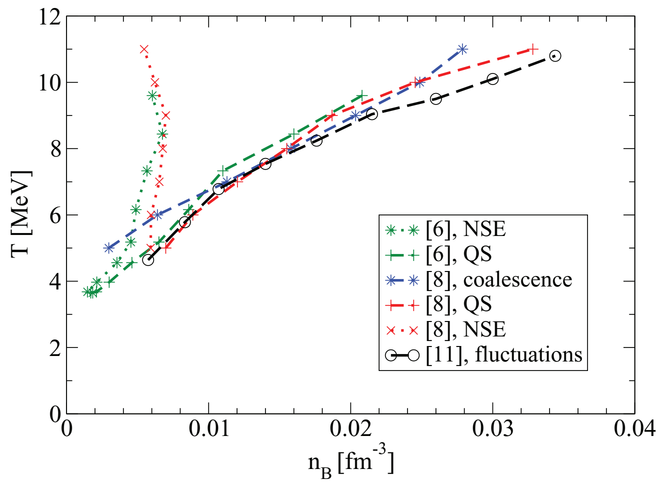

FIG. 3. (Color online) Baryon density derived from yields of light elements. Data according to Refs. [6,8,11] are compared with the results of the analysis of yields using NSE and QS calculations for $\tilde{K}_\alpha$.

data reinforce the interpretation that the natural evolution of the systems under investigation in those works encompasses densities approaching the Mott point as was previously concluded.

To compare the results of using this densitometer with results from the other three techniques in our list of possible techniques for density determinations, we now use comparisons to the theoretical curves to derive densities from the observed experimental values of reference [8]. These derived values (QS) are only slightly different than those extracted using a coalescence model. The comparison of results from different techniques of extracting T and n_B from experimental data are presented in Fig. 3. The use of the basic NSE gives unrealistically low densities reflecting the limitations of that model and its region of applicability [8,18,19]. This point was already apparent in the results for laboratory tests of the astrophysical equation of state (EoS) [8] that also demonstrate the relevance of medium effects above $n_B \approx 10^{-3}$ nuc/fm^3. Interestingly, the results of the coalescence model analysis and the quantum fluctuation analysis presented in Fig. 3 lead to very similar results even though different systems and sources have been explored. Both are quite similar to the densitometer analysis based on QS model results. We return to this point below.

IV. DISCUSSION

Substantial progress has been made in the effort to explore nuclear matter at subsaturation densities. There is now experimental evidence that proves the relevance of incorporating medium effects such as Pauli blocking and the Mott effect into theoretical treatments. As expected from a quantum statistical approach, the NSE based on noninteracting components is not sufficient to explain the data from experiments that investigate nuclear systems at densities around one tenth of saturation density and above. Considering the clusters as quasiparticles, a smooth transition from the NSE at low densities to mean-field approaches at the saturation density can be modeled [15]. The Albergo densitometer is restricted to very low densities. The densitometer proposed here, based upon chemical equilibrium constants calculated within the framework of the QS model, can be applied at significantly higher densities.

According to Fig. 2, measured yields of light elements can be used to infer the baryon density if the temperature is known. Despite the double valued solution, this diagram may serve as an important tool to derive densities from measured yields. Two other independent methods have been used to infer densities from the yields of light clusters:

(i) The Mekjian coalescence model [18] has been used. Coalescence parameters P_0 were calculated for the different clusters (see Ref. [10]) and used to determine the volumes. The corresponding volume was used to convert the measured yields into densities. The results are shown in Figs. 2 and 3.

(ii) An alternative approach to infer the parameter values for density and temperature, proposed in Ref. [11], employs quadrupole momentum fluctuations and the fluctuations of fermion and boson numbers in the nuclear system. Compared with classical systems number fluctuations are decreased for fermion systems and increased for boson systems if the temperature approaches the degeneration temperature.

The density values derived by both the coalescence and fluctuation methods are in rather good agreement with QS results that include medium effects but in disagreement with the values derived from NSE. Only below densities of about 5×10^{-4} fm^{-3} is the NSE applicable.

The discrepancies with NSE are substantially reduced if medium effects such as Pauli blocking [14] or, alternatively, excluded volume [18,20] are taken into account. The fact that the different experimental results for the temperature and density regions explored are consistent with each other, despite the fact that they are obtained from quite different emitting sources and analyses, suggests that an underlying unifying feature of the EOS is responsible. Indeed, further analysis by Mabiala *et al.* [11] indicates that the data are sampling the vapor branch of the liquid gas coexistence curve and within the framework of the Guggenheim systematics may be employed to determine the critical temperatures of mesoscopic nuclear systems, in a manner analogous to previous treatments [21,22].

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