Mirror energy differences of $2s_{1/2}$ single-particle states: Masses of ^{10}N and ^{13}F

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(Received 16 June 2013; revised manuscript received 21 July 2013; published 7 August 2013)

I have examined mirror energy differences between $2s_{1/2}$ states in neutron-excess nuclei with $N = 7$ and 9 and their proton-excess mirrors having $Z = 7$ and 9. I find they can be fitted by a simple expression, which I then use to predict the masses of ^{10}N and ^{13}F.

DOI: 10.1103/PhysRevC.88.024309 PACS number(s): 21.10.Dr, 21.10.Sf, 27.20.+n

Several neutron-excess light nuclei possess low-lying states whose dominant structure is a $2s_{1/2}$ neutron coupled to a predominantly p-shell core. For $N = 7$, these include ^{10}Li, ^{11}Be, ^{12}B, and ^{13}C. For $N = 9$, we have ^{13}Be, ^{14}B, ^{15}C, ^{16}N, and ^{17}O. For even-even cores, there will be one such state with $J^\pi = 1/2^+$. In some cases (^{11}Be and ^{15}C), this is the ground state, in others (^{13}C and ^{17}O), it is the first excited state. For odd-A cores with angular momentum J_c, there will be two states with $J = J_c \pm 1/2$. If J_c is 1/2, the 0^- and 1^- states will be relatively pure. But, for $J_c = 3/2$, the coupling $J_c \times d_{5/2}$ can produce states of the same J^π as $J_c \times s$, and the mixing could be larger.

I focus first on cases for which the proton-rich mirrors of these states have been identified. These are all isotopes of N ($Z = 7$) or F ($Z = 9$). In many of these cases, the energies of the proton-rich nuclei have been calculated with reasonable success in a simple potential model under the assumption of mirror symmetry. Here, I seek a global representation of the mirror energy differences (MEDs) without introducing a potential or a spectroscopic factor. For the present purposes, I define the MED as

$$\text{MED} = S_n(\text{neutron-excess nucleus}) - S_p(\text{proton-excess mirror}),$$

where S_n and S_p are separation energies for neutrons and protons, respectively. These nuclei and the corresponding separation energies [1] are listed in Table I. The uncertainties are listed if they are larger than 2 keV. The cases of ^{11}N and ^{15}F require special mention. Here, the experimental energies exhibit much larger variations among various experiments than would be expected from the quoted uncertainties. Presumably, this scatter is caused by differing definitions of the location of a broad resonance. For ^{11}N, three unweighted averages have recently appeared—one from the latest $A = 11$ compilation [2], one from the recent mass evaluation [1], and one by the present author [3]. The three are not very different, so I have chosen to use mine, $S_p = -1.41(10)$ MeV [3]. For ^{15}F, I earlier explored the effects of varying the definition of the energy of a broad resonance and recommended a "best value" of $S_p = -1.356(40)$ MeV [4], which I use here.

I am interested in a simple parametrization of these MEDs. From the Coulomb potential, we expect a factor $Z_c/A^{1/3}$, where Z_c is the atomic number of the core in the proton-excess

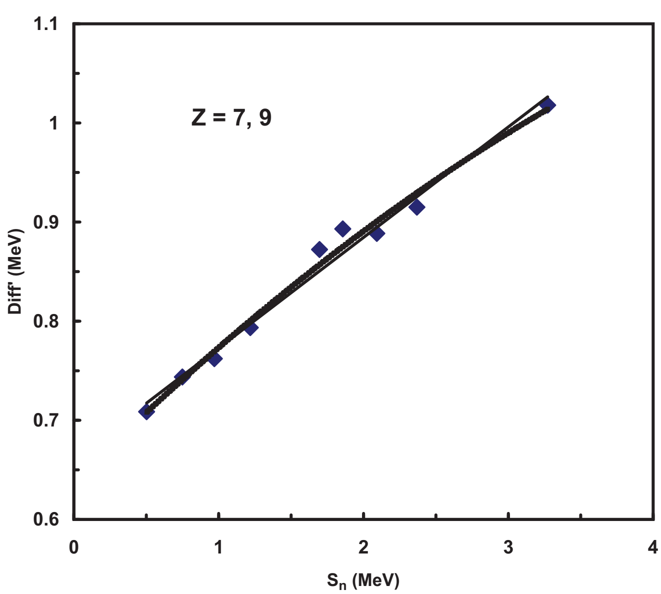

FIG. 1. (Color online) Plot of Diff′ (see text) vs neutron separation energy with linear (light) and quadratic (heavy) fits.

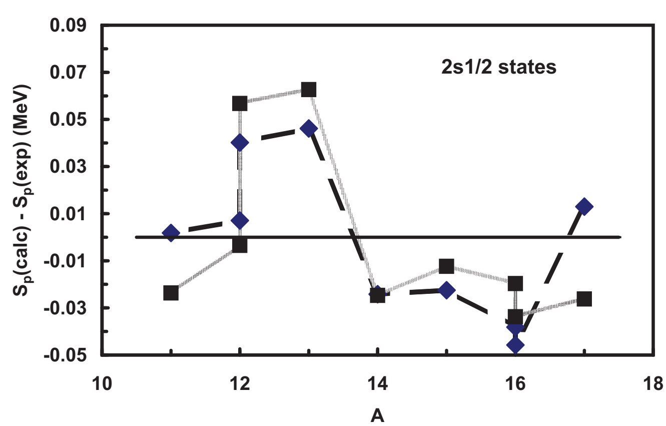

FIG. 2. (Color online) Difference between calculated and experimental proton separation energies. The solid curve is for the linear fit, and the dashed curve is for the quadratic fit of Fig. 1.

TABLE I. Energies(MeV) and J^π of the states discussed herein.

Nucleus	S_n (g.s.)[a]	J^π	E_x	S_n $(2s_{1/2})$	Mirror	S_p (g.s.)[a]	E_x	S_p $(2s_{1/2})$
^{17}O	4.143	$1/2^+$	0.871	3.272	^{17}F	0.600	0.495	0.105
^{16}N	2.4888(23)	0^-	0.120	2.3688	^{16}F	−0.536(8)	0	−0.536(8)
	2.4888(23)	1^-	0.397	2.0918		−0.536(8)	0.193(6)	−0.729(10)
^{15}C	1.218	$1/2^+$	0	1.218	^{15}F	−1.356(40)	0	−1.356(40)[b]
^{14}B	0.970(21)	2^-	0	0.970	^{14}F	−1.560(40)	0	−1.560(40)
^{13}C	4.946	$1/2^+$	3.089	1.857	^{13}N	1.943	2.365	−0.422
^{12}B	3.370	2^-	1.674	1.696	^{12}N	0.601	1.191(8)	−0.590(8)
	3.370	1^-	2.621	0.749		0.601	1.80(3)	−1.199(30)
^{11}Be	0.502	$1/2^+$	0	0.502	^{11}N	−1.41(10)	0	−1.41(10)[c]

[a]Reference [1], unless noted otherwise. Uncertainties are listed if they are larger than 2 keV.
[b]Reference [4].
[c]Reference [3].

mirror. But, we also know the MEDs will depend on S_n. Thus, I expect to have MED $= f(S_n)Z_c/A^{1/3}$. I define Diff′ = MED $A^{1/3}/Z_c$, where Z_c is 6 or 8 for N and F nuclei, respectively. These values of Diff′ vs S_n are plotted in Fig. 1. The trend is obvious. I have fitted these points with linear and quadratic curves. The two provide comparable fits with root-mean-square deviations in the resulting S_p's of about 30 keV. I, thus, assign an uncertainty of 30 keV to the calculated values in Table II. Measured proton separation energies and those computed from the fitted parameters are listed in Table II. The quality of agreement is apparent. There is some interesting structure in the differences between experimental and fitted S_p's as can be seen in Fig. 2. Presumably, this structure has to do with slight variations in parentage, which I have neglected.

I now use the established relationship to predict the energies of two unknown mirrors—^{10}N and ^{13}F, listed in Table III. In both cases, the neutron separation energy is outside the range of the other values in Fig. 1, and hence, the uncertainties here could be significantly larger than the 30 keV mentioned above. Because the separation energies of these two nuclei are outside the fitted range, the quadratic fit might be more risky, but it turns out that quadratic and linear fits produce roughly the same predictions. For ^{10}N(g.s.) (where g.s. represents ground state), these are $S_p = -1.86$ (linear) and −1.79 (quadratic) MeV. Earlier, we used a potential model to compute the proton separation energy of the lowest state in ^{10}N to be in the range of −1.81 to −1.94 MeV [5]. The compilers suggested $S_p \sim -1.8$ MeV [6]. The excited s-wave state of ^{10}Li is even further outside the fitted region. Therefore, the prediction of the energy of its mirror in ^{10}N is probably less reliable than for the g.s. Nevertheless, I list it in Table III.

Previously, we had computed the expected energy for the lowest state of ^{13}F as the mirror of a ^{13}Be s-wave structure just at threshold ($E_n \sim 0$). That procedure gave $S_p = -2.40$ MeV [7]. The current fit results in −2.25 and −2.17 MeV—not very different. If the $1/2^+$ state of ^{13}Be is somewhere else, say 0.5 or 0.7 MeV, these values would become more negative (Table III).

I encourage a search for the ground states of ^{10}N and ^{13}F. It remains to be seen if the present parametrization is a valid procedure for estimating energies of other proton-rich mirrors whose structures are predominantly s wave.

TABLE II. Separation energies (MeV) in the indicated nuclei.

Nucleus	S_n[a]	Mirror	Z_c	Diff′[b]	S_p (quad)[c]	S_p (lin)[c]	S_p (exp)[a]
^{17}O	3.272	^{17}F	8	1.018	0.118	0.079	0.105
^{16}N	2.369(2)	^{16}F	8	0.915	−0.582	−0.570	−0.536(8)
^{16}N	2.092(2)	^{16}F	8	0.889	−0.767	−0.749	−0.729(10)
^{15}C	1.218	^{15}F	8	0.794	−1.379	−1.368	−1.356(40)
^{14}B	0.970(21)	^{14}F	8	0.762	−1.584	−1.585	−1.560(40)
^{13}C	1.857	^{13}N	6	0.893	−0.376	−0.359	−0.422
^{12}B	1.696	^{12}N	6	0.872	−0.550	−0.533	−0.590(8)
^{12}B	0.749	^{12}N	6	0.744	−1.193	−1.203	−1.199(30)
^{11}Be	0.502	^{11}N	6	0.709	−1.408	−1.434	−1.41(10)

[a]From Table I.
[b]Diff′ = MED $A^{1/3}/Z_c$, where MED $= S_n - S_p$.
[c]Computed from the quadratic and linear fits. (See text and Fig. 1). Uncertainty is estimated to be ∼30 keV.

TABLE III. Predictions of proton separation energies (MeV) for ^{10}Li and ^{13}F.

Nucleus	S_n (MeV)	Mirror	S_p (quad)	S_p (lin)	S_p (previous)
^{13}Be	∼0	^{13}F	−2.17	−2.25	−2.40[a]
^{13}Be	(−0.51)	^{13}F	(−2.42)	(−2.57)	
^{13}Be	(−0.7)	^{13}F	(−2.51)	(−2.69)	
^{10}Li(g.s.)	−0.026	^{10}N(g.s.)	−1.79	−1.86	−1.8[b], −1.81 to −1.94[c]
^{10}Li(exc)	−0.726	^{10}N(exc)	−2.20	−2.34	

[a]Reference [7].
[b]Reference [6].
[c]Reference [5].

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