Nuclear equation of state in a relativistic independent quark model with chiral symmetry and dependence on quark masses

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We have calculated the properties of nuclear matter in a self-consistent manner with a quark-meson coupling mechanism incorporating the structure of nucleons in vacuum through a relativistic potential model; where the dominant confining interaction for the free independent quarks inside a nucleon is represented by a phenomenologically average potential in equally mixed scalar-vector harmonic form. Corrections due to spurious center of mass motion as well as those due to other residual interactions, such as the one gluon exchange at short distances and quark-pion coupling arising out of chiral symmetry restoration, have been considered in a perturbative manner to obtain the nucleon mass in vacuum. The nucleon-nucleon interaction in nuclear matter is then realized by introducing additional quark couplings to σ and ω mesons through mean field approximations. The relevant parameters of the interaction are obtained self-consistently while realizing the saturation properties such as the binding energy, pressure, and compressibility of the nuclear matter. We also discuss some implications of chiral symmetry in nuclear matter along with the nucleon and nuclear σ term and the sensitivity of nuclear matter binding energy with variations in the light quark mass.

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I. INTRODUCTION

The properties of nuclear matter has been an area of considerable interest for the past few decades. Such studies are quite important in nuclear physics, [e.g., in the context of nucleon-nucleon (NN) interaction, structure and properties of finite nuclei, and dynamics of heavy ion collisions], astrophysics (nucleosynthesis, structure and evolution of neutron stars [1], big bang cosmology), and also particle physics (production or interaction of hadrons). One of the fundamental concerns in the study of nuclear matter is the nature of the NN interaction. This problem is solved usually in a self-consistent manner in various different approaches which can be broadly classified into three general types, namely the *ab initio* methods, the effective field theory method, and methods based on phenomenological density functionals. The *ab initio* methods include the Brueckner-Hartree-Fock (BHF) approach [2–4], the relativistic Dirac-Brueckner-Hartree-Fock (DBHF) approach [5–8], the Green function Monte Carlo (GFMC) method [9–11] using the basic NN interactions given by boson exchange potentials. The other approach of this type pioneered by the Argonne Group [12,13] is also known as the variational approach. The effective field theory (EFT) methods [14] are based on chiral perturbation theory [15,16]. The third type of approaches are based on the phenomenological models with effective density dependent interactions such as Gogny or Skyrme forces [17] (see also [18] for a systematic analysis of Skyrme models) and the relativistic mean field (RMF) models [19]. The parameters of these models are evaluated by appealing to the bulk properties of nuclear matter and properties of closed shell nuclei.

The RMF models represent the NN interactions through the coupling of nucleons with isoscalar scalar mesons, isoscalar vector mesons, isovector vector mesons, and the photon quanta besides the self- and cross-interactions among these mesons [20,21]. Although implemented at the Hartree level only, these models have been very successful in simulating the observed bulk properties of nuclear matter including the nuclear equation of state (EOS), mass and radii of neutron star as well as in explaining properties of finite nuclei. Recently, the RMF model has also been extended to include the Hartree-Fock theory and the short range repulsion using unitary operator method [22] to study the symmetric nuclear matter.

In all these approaches mentioned above nucleons are treated as structureless point objects. However, incorporation of structure of nucleon with meson couplings at the basic quark level in the study of saturation properties of nuclear matter can provide new insight. With such a hope, there has been several attempts based on a simple bag model or some phenomenological potential models to address the nucleon structure. Using such quark-meson coupling (QMC) models nuclear equations of state (EOS) have also been constructed [23,24] and properties of nuclear matter have been studied in great detail in a series of works by Saito, Thomas, and collaborators [25] and by others [26–29].

Quantum chromodynamics (QCD) is the underlying theory of the strong forces that hopefully would also explain nuclear stability. Therefore the study of changes in nuclear properties due to the fundamental parameters of QCD in particular with the variation of light quark masses is legitimate. The sensitivity of the relative binding energy to the relative change in light quark masses was investigated for nuclei $A = 3$–8 and computed in [30] for different Argonne potential models, considering cases including the Urbana model IX three-body force, and for thorium in [31]. More recently, Ref. [32] computed the variations in the nuclear binding of light elements like deuteron, tritium, ^{7}Li, ^{12}C, and ^{16}O from the changes in quark masses in a one boson-exchange model. Although, there are sizable differences between [30] and [32], they agree within a factor less than 2, which can be due to different nuclear models, and the detailed form of computing the meson mass and coupling constant variations. In another study, the anthropomorphic principle was used to constrain variations in the quark masses and fine structure constant from the abundances of carbon and oxygen in the universe essential for life [33]. The nuclear equations of state and the nuclear matter binding energy can also be affected by quark mass variations with interesting implications for the stability of heavy nuclei and stars. This motivates a study of such effect within relativistic models of nuclear matter such as QMC models.

In the present study, we have developed a modified quark-meson coupling model (MQMC) [24,34,35], which is based on a suitable relativistic independent quark potential model rather than a bag to address the nucleon structure in vacuum. In such a picture the light quarks inside a bare nucleon are considered to be independently confined by a phenomenologically average potential with an equally mixed scalar-vector harmonic form. Such a potential has characteristically simplifying features in converting the independent quark Dirac equation into an effective Schrödinger like equation for the upper component of the Dirac spinor which can be easily solved. The implication of such potential forms in the Dirac framework has been studied earlier by several authors [36]. It has been shown that the spin-orbit interaction is absent in such models due to an exact cancellation of terms coming from the vector and the scalar part of the potential taken in equal proportion. This is a welcome feature for a baryon sector, where the contribution from spin-orbit interaction to baryon mass splittings is already known to be negligible [37].

Eichen and Feinberg [38] in a gauge invariant formalism, assuming the confinement mechanism to be purely color-electric in character, obtained a similar Lorentz structure of the potential. This typical Lorentz structure of the confining potential renders the Dirac equation solvable for all possible quark eigenmodes. Due to the harmonic nature of the potential; the quark orbitals corresponding to the lowest eigenmode is realized here in the familiar Gaussian form that makes the perturbative treatment of the residual interactions such as the short range one-gluon exchange and quark pion coupling arising out of chiral symmetry restoration in the PCAC limit as well as that due to the spurious center of mass motion in the ground state, simple and straightforward in comparison with other models. Therefore, it has provided a very suitable alternative to the otherwise successful cloudy bag models and has been extensively applied with remarkable consistency in baryonic as well as mesonic sectors [39,40].

Taking gluonic and pionic corrections together with that due to center of mass motion, baryon mass spectra in vacuum has been successfully reproduced in this potential model [24]. This model has also been quite successful in studying nucleon structure functions in deep inelastic scattering [41]. In view of this we would like to adopt this model here to address the nucleon structure properties of nucleons and nuclear matter.

Corrections due to the spurious center of mass motion as well as those due to short range one gluon exchange and quark-pion coupling would be accounted for in a perturbative manner to obtain the nucleon mass in vacuum. Then the NN interaction in nuclear matter is realized by introducing an additional quark coupling to sigma (σ) and omega (ω) mesons through mean field approximations. The relevant parameters of the interaction are obtained self-consistently while realizing the saturation properties such as the binding energy, pressure, and compressibility of the nuclear matter. We examine the effective nucleon mass, nuclear σ term as well as the effective quark condensate at saturation density in comparison with the respective values at zero density. We also study their variations including the sensitivity of the nuclear matter binding energy with the variation of the light quark masses.

The paper is organized as follows. In Sec. II, we provide a brief outline of the model describing the nucleon structure in vacuum where the nucleon mass can be obtained by appropriately taking into account the center of mass correction, pionic correction, and gluonic correction. The mean-field properties of symmetric nuclear matter in this model is discussed in Sec. III. The results and discussions are presented in Sec. IV. Finally, in the last section, the conclusions are drawn.

II. POTENTIAL MODEL

We choose from a phenomenological point of view a flavor independent potential $U(r)$ confining the constituent quarks inside the nucleon in accordance with [34], where $U(r)$ is

$$U(r) = \tfrac{1}{2}(1+\gamma^0)V(r)$$

with

$$V(r) = (ar^2 + V_0), \quad a > 0. \tag{1}$$

Here (a, V_0) are the potential parameters. This confining interaction is believed to provide phenomenologically the zeroth-order quark dynamics of the hadron, and corrections, like gluon exchange, can be added perturbatively. The quark Lagrangian density corresponding to the confining model,

$$\mathcal{L}_q^0(x) = \bar{\psi}_q(x)\left[\frac{i}{2}\gamma^\mu \overleftrightarrow{\partial}_\mu - m_q - U(r)\right]\psi_q(x), \tag{2}$$

leads to the Dirac equation for an individual quark as

$$[\gamma^0\epsilon_q - \vec{\gamma}\,\vec{p} - m_q - U(r)]\psi_q(\vec{r}) = 0. \tag{3}$$

The normalized quark wave function $\psi_q(\vec{r})$ can be written in the two-component form for the ground state as

$$\psi_q(\vec{r}) = \frac{1}{\sqrt{4\pi}}\begin{pmatrix} ig(\vec{r})/r \\ \vec{\sigma}\cdot\hat{r}\,f(\vec{r})/r \end{pmatrix}\chi_s. \tag{4}$$

Defining

$$(\epsilon_q' + m_q') = (\epsilon_q + m_q) \equiv \lambda_q, \tag{5}$$

with

$$\epsilon_q' = (\epsilon_q - V_0/2), \ \ m_q' = (m_q + V_0/2), \quad \text{and} \ \ r_{0q} = (a\lambda_q)^{-1/4}, \tag{6}$$

it can be shown that the upper and lower components of $\psi_q(r)$ corresponding to the quark-flavor q for the ground state $1s_{1/2}$ are

$$\begin{aligned} g_q(r) &= \mathcal{N}_q \left(\frac{r}{r_{0q}}\right) \exp\left(-r^2/2r_{0q}^2\right), \\ f_q(r) &= -\frac{\mathcal{N}_q}{\lambda_q r_{0q}} \left(\frac{r}{r_{0q}}\right)^2 \exp\left(-r^2/2r_{0q}^2\right), \end{aligned} \tag{7}$$

where the normalization $\mathcal{N}_q$, is given by

$$\mathcal{N}_q^2 = \frac{8\lambda_q}{\sqrt{\pi} r_{0q}} \frac{1}{(3\epsilon_q' + m_q')}. \tag{8}$$

In the above, ϵ_q is the ground state $1s_{1/2}$ individual quark energy obtained from the eigenvalue condition

$$(\epsilon_q' - m_q')\sqrt{\frac{\lambda_q}{a}} = 3. \tag{9}$$

The solution of Eq. (9) for the quark energy ϵ_q immediately leads to the zeroth-order energy of the nucleon

$$E_N^0 = \sum_q \epsilon_q. \tag{10}$$

We can now construct the nucleon state $|N\rangle$ as the symmetrized product of the spin-flavor wave function of the three independent quarks each in its ground state as in Eq. (4).

Considering the quark confinement inside the nucleon through the phenomenological interaction potential $U(r)$, the model expression for the zeroth-order energy E_N^0 of the nucleon core is obtained as in Eq. (10). However, there may be appropriate correction to E_N^0 due to possible residual interactions such as the quark-gluon interaction at short distances originating from a one gluon exchange and quark pion interaction arising out of the requirement for restoration of chiral symmetry at the $SU(2) \times SU(2)$ level as well as that coming from the spurious center of mass motion of the ground state nucleon. We next consider these corrections for the zeroth-order energy E_N^0 of the nucleon core as follows.

A. Center of mass correction

In this model, the quark constituents are independently bound by a potential with fixed center to obtain the quark orbitals in the nucleon, which are used to construct a composite nucleon wave function. If the composite nucleon is to be considered as a translationally invariant state, its wave function must be corrected for the effects of spurious center of mass (c.m.) motion. For center of mass correction, earlier workers [34] had followed the prescription given by Peierls-Yoccoz [42]. However, here we will extract the center of mass energy to first order in the difference between the fixed center and relative quark co-ordinates, using the method described by Guichon *et al.* [23].

We assume that the Hamiltonian, H_N, for the composite nucleon can be written as

$$H_N = H_{\text{in}} + H_{\text{c.m.}}, \tag{11}$$

where H_{in} is the Hamiltonian corresponding to the internal degrees of freedom and $H_{\text{c.m.}}$ is the center of mass Hamiltonian. We can write the total Hamiltonian from Eq. (2) as

$$H_N = \int \mathcal{H}_N d^3x. \tag{12}$$

Thus, the Hamiltonian density can be written as

$$\hat{\mathcal{H}}_N = \sum_{i=1}^{3} \gamma_0(i)\Big[\vec{\gamma}(i)\cdot\vec{p}_i + m_q + \frac{1}{2}(1+\gamma_0(i))U(r_i)\Big]. \tag{13}$$

The internal Hamiltonian density can be written in a similar way in terms of the relative rather than the fixed coordinates as

$$\hat{\mathcal{H}}_{\text{in}} = \sum_{i=1}^{3} \gamma_0(i)\left[\vec{\gamma}(i)\cdot\vec{\pi}_i + m_q + \frac{1}{2}(1+\gamma_0(i))U(\rho_i)\right], \tag{14}$$

where

$$\vec{\pi}_i = \vec{p}_i - \frac{1}{3}\sum_{j=1}^{3}\vec{p}_j \quad \text{and} \quad \vec{\rho}_i = \vec{r}_i - \frac{1}{3}\sum_{j=1}^{3}\vec{r}_j = \vec{r}_i - \vec{R}_{\text{c.m.}}. \tag{15}$$

The center of mass contribution to the Hamiltonian density is then the difference between the two:

$$\begin{aligned} \hat{\mathcal{H}}_{\text{c.m.}} = \hat{\mathcal{H}}_N - \hat{\mathcal{H}}_{\text{in}} = \sum_{i=1}^{3}\gamma_0(i)\Bigg[\frac{1}{3}\vec{\gamma}(i)\cdot\sum_{j=1}^{3}\vec{p}_j \\ + \frac{1}{2}(1+\gamma_0(i))\,[U(r_i) - U(\rho_i)]\Bigg]. \end{aligned} \tag{16}$$

We now estimate the center of mass contribution to the nucleon energy by calculating the expectation value of the center of mass Hamiltonian. Here, we take the solutions for the quark orbitals as given in Eq. (7), and the composite nucleon spin flavor configuration $|N\rangle$ as per $SU(6)$ prescription. Now, we have

$$\epsilon_{\text{c.m.}} = \langle N|\hat{\mathcal{H}}_{\text{c.m.}}|N\rangle = \langle N|\hat{\mathcal{H}}^{(1)}_{\text{c.m.}}|N\rangle + \langle N|\hat{\mathcal{H}}^{(2)}_{\text{c.m.}}|N\rangle, \tag{17}$$

where

$$\begin{aligned} \langle N|\hat{\mathcal{H}}^{(1)}_{\text{c.m.}}|N\rangle &= \frac{1}{3}\langle N|\sum_{i=1}^{3}\gamma_0(i)\vec{\gamma}(i)\cdot\sum_{j=1}^{3}\vec{p}_j|N\rangle \\ &= \frac{1}{3}\langle N|\sum_{i=1}^{3}\gamma_0(i)\vec{\gamma}(i)\cdot\vec{p}_i|N\rangle \\ &= \frac{6}{(3\epsilon_u' + m_u')r_{0u}^2}. \end{aligned} \tag{18}$$

In the above expression the terms $j \neq i$ in fact vanish effectively:

$$\langle N|\hat{\mathcal{H}}^{(2)}_{\text{c.m.}}|N\rangle = \frac{1}{2}\langle N|\sum_{i=1}^{3}(1+\gamma_0(i))[U(r_i)-U(\rho_i)]|N\rangle = \frac{1}{2}\langle N|\sum_{i=1}^{3}(1+\gamma_0(i))(2\vec{r}_i\cdot\vec{R}_{\text{c.m.}}-R^2_{\text{c.m.}})|N\rangle = \frac{23\epsilon'_u+13m'_u}{3(3\epsilon'_u+m'_u)^2 r^2_{0u}}. \quad (19)$$

Thus, the total center of mass correction comes out as

$$\epsilon_{\text{c.m.}} = \frac{(77\epsilon'_u+31m'_u)}{3(3\epsilon'_u+m'_u)^2 r^2_{0u}}. \quad (20)$$

B. Chiral symmetry and pionic corrections

Under the global, infinitesimal chiral transformation, we have

$$\psi_q(x) \longrightarrow \psi_q(x) - i\frac{\tau\cdot\alpha}{2}\gamma^5\psi_q(x). \quad (21)$$

Substituting the above expression in the zeroth-order Lagrangian and with little algebra, we get

$$\mathcal{L}^{0'}_q(x) \longrightarrow \mathcal{L}^0_q + i(m_q+V(r))\bar{\psi}_q(x)\gamma^5(\tau\cdot\alpha)\psi_q(x). \quad (22)$$

The axial vector current of the quarks is not conserved as the scalar term proportional to $G(r) = (m_q + V(r)/2)$ in the Lagrangian density $\mathcal{L}^0_q$ is chirally odd. The vector part of the potential poses no problem in this respect. Therefore, in the present model, chiral symmetry in $SU(2)$ sector is restored by introducing in the usual manner, an elementary pion field ϕ of small but finite mass $m_\pi \simeq 140$ MeV through the additional terms in the original Lagrangian density $\mathcal{L}_q(x)$, so as to write

$$\mathcal{L}_q(x) = \mathcal{L}^0_q(x) + \mathcal{L}^0_\pi(x) + \mathcal{L}^\pi_I(x), \quad (23)$$

where

$$\mathcal{L}^0_\pi(x) = \tfrac{1}{2}(\partial_\mu\phi)^2 - \tfrac{1}{2}m^2_\pi\phi^2. \quad (24)$$

The Lagrangian density $\mathcal{L}^\pi_I(x)$ corresponding to the quark pion interaction is taken to be linear in isovector pion field ϕ such that

$$\mathcal{L}^\pi_I(x) = -\frac{i}{f_\pi}G(r)\bar{\psi}_q(x)\gamma^5(\tau\cdot\phi)\psi_q(x) \equiv -iG_{qq\pi}\bar{\psi}_q(x)\gamma^5(\tau\cdot\phi)\psi_q(x), \quad (25)$$

where $f_\pi \simeq 93$ MeV is the phenomenological pion decay constant and $G_{qq\pi}$ is the effective quark-pion coupling strength. Then, the four-divergence of the total axial vector current becomes $\partial_\mu A^\mu(x) = -f_\pi m^2_\pi\phi(x)$ and gives the partial conserved axial current (PCAC) relation. Now the Hamiltonian from Eq. (24) in second quantized form is given by

$$H_\pi = \sum_j \int d^3k\, w_k \hat{a}_j(\vec{k})^\dagger a_j(\vec{k}), \quad (26)$$

where $\hat{a}^\dagger_j(\vec{k})$ and $\hat{a}_j(\vec{k})$ are the pion creation and annihilation operators and $w_k = (k^2+m^2_\pi)^{1/2}$ is the pion energy. Finally the interaction Hamiltonian corresponding to $\mathcal{L}^\pi_I(x)$ becomes

$$H^\pi_I = -\frac{1}{(2\pi)^{3/2}}\sum_{B,B',j}\int d^3k\big[V^{BB'}_j(k)\hat{b}^\dagger_{B'}\hat{b}_B\hat{a}_j(\vec{k}) + \text{H.c.}\big], \quad (27)$$

where j corresponds to the pion-isospin index and H.c. denotes the Hermitian conjugate. In the above equation, $\hat{b}^\dagger_B$ and $\hat{b}_B$ are the creation and annihilation operators of the baryon state with quantum numbers of $N, \Delta, \ldots$, etc. $V^{BB'}_j(k)$ represents the baryon pion absorption vertex function in the point-pion approximation and is obtained as [34]

$$V^{BB'}_j(k) = -\frac{i}{f_\pi}\frac{1}{(2w_k)^{1/2}}\int d^3r\, G(r)e^{ik\cdot r} \times \langle B'|\bar{\psi}_q(r)\gamma^5\psi_q(r)\tau_j|B\rangle. \quad (28)$$

Assuming all the quarks in the initial and final baryon are in a $1s_{1/2}$ state, then Eq. (28) becomes

$$V^{BB'}_j(k) = \frac{i}{f_\pi}\frac{1}{(2w_k)^{1/2}}\sqrt{\frac{\pi}{2}}\frac{\mathcal{N}^2_q}{k^{3/2}\lambda_q r^4_{0q}} \times \langle B'|\sum_q(\sigma_q\cdot\vec{k})\tau_j|B\rangle I(k), \quad (29)$$

where

$$I(k) = 2\int_0^\infty dr\, r^{5/2}G(r)J_{3/2}(kr)e^{-r^2/r^2_{0q}}. \quad (30)$$

The coupling of the nonstrange quarks to the pion causes a shift in the energy of the baryon core. From the second-order perturbation theory, the pionic self-energy is given by

$$\Sigma_B(E_B) = \sum_k\sum_{B'}\frac{V^{\dagger BB'}V^{BB'}}{E_B - w_k - M^0_{B'}}, \quad (31)$$

where $\sum_k = \frac{1}{(2\pi)^3}\int d^3k$ and B' is the intermediate baryon state. For degenerate intermediate states on the mass shell with $M^0_B = M^0_{B'}$, the self-energy correction becomes [34]

$$\delta M^\pi_B = \sum_B\big(E_B = M^0_B = M^0_{B'}\big) = -\sum_{k,B'}\frac{V^{\dagger BB'}V^{BB'}}{w_k}. \quad (32)$$

Using the explicit expression (29) for $V^{BB'}(k)$, one gets

$$\delta M^\pi_B = -\frac{1}{3}I_\pi\sum_{B'}C_{BB'}f^2_{BB'\pi}, \quad (33)$$

where

$$C_{BB'} = (\sigma^{BB'}\cdot\sigma^{BB'})(\tau^{BB'}\cdot\tau^{BB'}).$$

For intermediate baryon states B' we consider only the octet and decouplet ground states. Now putting the values of $f_{BB'\pi}$ and $C_{BB'}$, we get the pionic self-energy for the nucleon [34]

$$\delta M^\pi_N = -\frac{171}{25}I_\pi f^2_{NN\pi}, \quad (34)$$

where

$$I_\pi = \frac{1}{\pi {m_\pi}^2}\int_0^\infty dk\frac{k^4u^2(k)}{w^2_k} \quad (35)$$

with the axial vector nucleon form factor given as

$$u(k) = \left[1 - \frac{3}{2}\frac{k^2}{\lambda_q(5\epsilon_q' + 7m_q')}\right]e^{-k^2r_0^2/4}. \tag{36}$$

The pseudovector nucleon pion coupling constant $f_{NN\pi}$ can be obtained from the familiar Goldberg-Triemann relation using the axial vector coupling constant value g_A in the model.

C. Gluonic corrections

The one-gluon exchange interaction is provided by the interaction Lagrangian density

$$\mathcal{L}_I^g = \sum J_i^{\mu a}(x)A_\mu^a(x), \tag{37}$$

where $A_\mu^a(x)$ are the octet of gluon vector fields and $J_i^{\mu a}(x)$ is the ith quark color current. The gluonic correction can be separated in two pieces, namely, one from the color electric field (E_i^a) and another one from the magnetic one (B_i^a) generated by the ith quark color current density

$$J_i^{\mu a}(x) = g_c\bar{\psi}_q(x)\gamma^\mu\lambda_i^a\psi_q(x), \tag{38}$$

with λ_i^a being the usual Gell-Mann $SU(3)$ matrices and $\alpha_c = g_c^2/4\pi$. The contribution to the mass due to the relevant diagrams can be written as a sum of a color electric and magnetic part as

$$(\Delta E_N)_g = (\Delta E_B)_g^E + (\Delta E_B)_g^M, \tag{39}$$

where

$$(\Delta E_N)_g^E = \frac{1}{8\pi}\sum_{i,j}\sum_{a=1}^{8}\int\frac{d^3r_i d^3r_j}{|r_i - r_j|}\langle B|J_i^{0a}(r_i)J_j^{0a}(r_j)|B\rangle, \tag{40}$$

and

$$(\Delta E_N)_g^M = -\frac{1}{8\pi}\sum_{i,j}\sum_{a=1}^{8}\int\frac{d^3r_i d^3r_j}{|r_i - r_j|}\langle B|\vec{J}_i^a(r_i)\vec{J}_j^a(r_j)|B\rangle. \tag{41}$$

Finally, taking into account the specific quark flavor and spin configurations in the ground state baryons and using the relations $\langle\sum_a(\lambda_i^a)^2\rangle = 16/3$ and $\langle\sum_a(\lambda_i^a\lambda_j^a)\rangle_{i\neq j} = -8/3$ for baryons, one can write the energy correction due to color electric contribution as

$$(\Delta E_N)_g^E = \alpha_c\big(b_{uu}I_{uu}^E + b_{us}I_{us}^E + b_{ss}I_{ss}^E\big), \tag{42}$$

and due to color magnetic contributions as

$$(\Delta E_N)_g^M = \alpha_c\big(a_{uu}I_{uu}^M + a_{us}I_{us}^M + a_{ss}I_{ss}^M\big), \tag{43}$$

where a_{ij} and b_{ij} are the numerical coefficients depending on each baryon. In Fig. 1, we have shown the one gluon exchange among the quarks. The color electric contributions for the baryon masses vanishes when all the constituent quark masses in a baryon are equal, whereas it is nonzero otherwise. Therefore, we have $a_{uu} = -3$ and $a_{us} = a_{ss} = b_{uu} = b_{us} =$

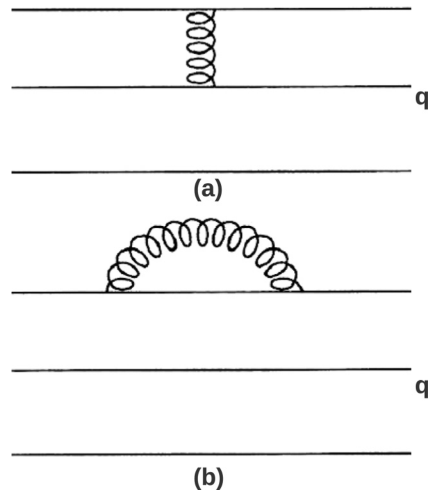

FIG. 1. One gluon exchange contributions to the baryon energy.

$b_{ss} = 0$ for the nucleon case. The quantities $I_{ij}^{E,M}$ are given in the following equation:

$$\begin{aligned} I_{ij}^E &= \frac{16}{3\sqrt{\pi}}\frac{1}{R_{ij}}\left[1 - \frac{\alpha_i + \alpha_j}{R_{ij}^2} + \frac{3\alpha_i\alpha_j}{R_{ij}^4}\right], \\ I_{ij}^M &= \frac{256}{9\sqrt{\pi}}\frac{1}{R_{ij}^3}\frac{1}{(3\epsilon_i' + m_i')}\frac{1}{(3\epsilon_j' + m_j')}, \end{aligned} \tag{44}$$

where

$$\begin{aligned} R_{ij}^2 &= 3\left[\frac{1}{\left(\epsilon_i'^2 - m_i'^2\right)} + \frac{1}{\left(\epsilon_j'^2 - m_j'^2\right)}\right], \\ \alpha_i &= \frac{1}{(\epsilon_i' + m_i')(3\epsilon_i' + m_i')}. \end{aligned} \tag{45}$$

In the calculation we have taken $\alpha_c = 0.58$ as the strong coupling constant in QCD at the nucleon scale [34]. The color electric contribution is zero here, and the gluonic corrections to the mass of the nucleon are due to color magnetic contributions only.

Finally treating all these corrections independently, one can obtain the physical mass of the nucleon as

$$M_N \equiv E_N = E_N^0 - \epsilon_{\text{c.m.}} + \delta M_N^\pi + (\Delta E_N)_g^E + (\Delta E_N)_g^M, \tag{46}$$

where $\epsilon_{\text{c.m.}}$ is the energy associated with the spurious center of mass correction, $(\Delta E_N)_g^E + (\Delta E_N)_g^M$ is the color electric and magnetic interaction energies arising out of the one-gluon exchange process, and δM_N^π is the pionic self-energy of the nucleon due to pion coupling to the nonstrange quarks. In the above M_N is the mass of the nucleon at zero density. In the next section, we will calculate the effective mass M_N^* in the medium using Eq. (46) where additional quark couplings to the mesons would be introduced in a mean field approximation.

TABLE I. Bare set parameters and energy corrections.

m_q (MeV)	V_0 (MeV)	a (fm^{-3})	ϵ_q (MeV)	$\epsilon_{c.m}$ (MeV)	δM_N^π (MeV)	$(\delta E_B)_g$ (MeV)	g_A	$g_{NN\pi}$
40	100.187229	0.892380	483.516	373.636	− 63.018	− 74.894	1.1179	10.19
50	96.287247	0.870341	482.483	369.668	− 65.749	− 73.302	1.1334	10.34
300	− 62.257187	0.534296	458.455	283.578	− 109.689	− 43.099	1.3844	12.68

III. EQUATION OF STATE FOR NUCLEAR MATTER

The Dirac equation (3) for individual quarks in the medium is now given by

$$\left[\gamma^0\left(\epsilon_q - g_\omega^q\omega_0\right) - \vec{\gamma}\,\vec{p} - \left(m_q - g_\sigma^q\sigma\right) - U(r)\right]\psi_q(\vec{r}) = 0, \tag{47}$$

where g_σ^q and g_ω^q are the quark couplings to the σ and ω mesons. In the mean field approximation, the meson fields are treated by their expectation values,

$$\sigma \to \langle\sigma\rangle \equiv \sigma_0 \quad \text{and} \quad \omega_\mu \to \langle\omega_\mu\rangle \equiv \delta_{\mu 0}\omega_0. \tag{48}$$

We can now redefine Eq. (6) in medium as

$$\epsilon_q' = (\epsilon_q^* - V_0/2) \quad \text{and} \quad m_q' = (m_q^* + V_0/2), \tag{49}$$

where the effective quark energy, $\epsilon_q^* = \epsilon_q - g_\omega^q\omega_0$, and effective quark mass, $m_q^* = m_q - g_\sigma^q\sigma_0$. Substituting this in Eq. (49), the effective mass of the nucleon at finite densities can be calculated from Eq. (46):

$$M_N^* = E_N^*. \tag{50}$$

The baryon density ρ_B, the total energy density, and pressure at a particular baryon density for the symmetric nuclear matter are given in the usual form as

$$\rho_B = \frac{\gamma}{(2\pi)^3}\sum_{N=p,n}\int_0^{k_f^N} d^3k = \frac{2k_f^3}{3\pi^2} \equiv \rho_p + \rho_n, \tag{51}$$

$$\mathcal{E} = \frac{1}{2}m_\sigma^2\sigma_0^2 - \frac{1}{2}m_\omega^2\omega_0^2 + g_\omega\omega_0\rho_B + \frac{\gamma}{(2\pi)^3}\sum_{N=p,n}\int^{k_f^N} d^3k\sqrt{k^2 + M_N^{*\,2}}, \tag{52}$$

$$P = -\frac{1}{2}m_\sigma^2\sigma_0^2 + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{\gamma}{3(2\pi)^3}\sum_{N=p,n}\int^{k_f^N}\frac{k^2d^3k}{\sqrt{k^2 + M_N^{*\,2}}}, \tag{53}$$

where $\gamma = 2$ is the spin degeneracy factor for nuclear matter and $g_\omega = 3g_\omega^q$ is the ω-nucleon coupling.

The vector mean-field ω_0 is determined through

$$\omega_0 = \frac{g_\omega\rho_B}{m_\omega^2}. \tag{54}$$

Finally, the scalar mean-field σ_0 is fixed by

$$\frac{\partial\mathcal{E}}{\partial\sigma_0} = 0. \tag{55}$$

The scalar and vector couplings g_σ^q and g_ω are fitted to the saturation density and binding energy for nuclear matter. For a given baryon density, ω_0 and σ_0 are calculated from Eqs. (54) and (55), respectively.

The compressibility modulus K is given by the standard relation:

$$K = 9\rho_B^2\frac{\partial^2(\mathcal{E}/\rho_B)}{\partial\rho_B^2}, \tag{56}$$

which measures the stiffness of nuclear matter at the saturation point.

IV. RESULTS AND DISCUSSION

Starting with this simple composite model of the nucleon in free space, we wish to study several mean field properties of the nuclear matter, where the basic NN interaction is realized through quark couplings to σ and ω mesons. We would also like to investigate the variations of these nuclear matter properties with the quark masses and their nuclear density dependance.

A. Free nucleon properties

Apart from the bulk properties like binding energy and the compressibility, we would like to address a few other properties of the nucleon in nuclear matter such as nucleon mass M_N, charge radius $\langle r^2\rangle_N^{1/2}$, axial vector coupling constant g_A, pion-nucleon coupling constant $g_{NN\pi}$, and nucleon σ term Σ_N.

The expressions for $\langle r^2\rangle_N^{1/2}$, g_A, $g_{NN\pi}$ in free space follows in the present model according to Ref. [34] as

$$\langle r_N^2\rangle_{\text{withoutc.m.}} = \frac{3}{2}\frac{11\epsilon_q' + m_q'}{(3\epsilon_q' + m_q')(\epsilon_q'^2 - m_q'^2)}, \tag{57}$$

and with c.m. correction

$$\langle r_N^2\rangle = \langle B|\frac{1}{3}\sum_{r=1}^{3}(\vec{r}_i - \vec{R}_{\text{c.m.}})^2|B\rangle = \frac{11\epsilon_q' + m_q'}{(3\epsilon_q' + m_q')(\epsilon_q'^2 - m_q'^2)}. \tag{58}$$

TABLE II. Parameters for nuclear matter. They are determined from the binding energy per nucleon, B.E. $\equiv \mathcal{E}/\rho_B - M_N = -15.7$ MeV and pressure, $P = 0$ at saturation density $\rho_B = \rho_0 = 0.15$ fm^{-3}.

m_q (MeV)	g_σ^q	g_ω
40	5.46761	3.96975
50	5.28816	4.30828
300	4.07565	9.09078

TABLE III. Properties for nuclear matter at saturation density.

m_q (MeV)	σ_0 (MeV)	ω_0 (MeV)	M_N^*/M_N	K (MeV)	$\langle r_N \rangle$ (fm)	ϵ_q (MeV)	$\epsilon_{\rm c.m.}$ (MeV)	δM_N^π (MeV)	$(\delta E_B)_g$ (MeV)	g_A	$g_{NN\pi}$
40	15.07	7.46	0.90	222.48	0.94784	447.884	376.091	– 28.909	– 82.861	0.9448	7.64
50	15.74	8.09	0.90	223.81	0.94794	445.232	373.260	– 30.545	– 81.603	0.9624	7.74
300	26.93	17.08	0.77	258.913	0.94902	382.049	303.502	– 65.282	– 53.855	1.2629	8.73

The axial-vector coupling constant g_A can also be obtained as [34]

$$g_A(n \to p) = \frac{5}{9}\frac{(5\epsilon_u' + 7m_u')}{(3\epsilon_u' + m_u')}, \tag{59}$$

without considering center of mass corrections. Another quantity of interest is the quark-pion coupling constant $G_{qq\pi}$. Using the familiar Goldberger-Treiman relation, we have

$$\frac{G_{qq\pi}}{2M_q} = \frac{1}{2f_\pi} \times \frac{3}{5} g_A. \tag{60}$$

Here M_q is the constituent quark mass $M_N/3$. The pseudoscalar pion-nucleon coupling constant $g_{NN\pi}$ which is obtained from [34] at $q^2 = m_\pi^2$:

$$G_{NN\pi}(q^2) = \frac{M_N}{f_\pi} g_A u(q), \tag{61}$$

where the axial-vector nucleon form factor is

$$u(q) = \left[1 - \frac{3}{2}\frac{q^2}{\lambda_q(5\epsilon_q' + 7m_q')}\right] e^{-q^2 r_0^2/4}. \tag{62}$$

The medium dependence of g_A, $G_{qq\pi}$, and $g_{NN\pi}$ will be discussed later.

We wish to study several mean field properties of our composite model of the nucleon by fixing first the free-space nucleon properties. Here, the quark mass m_q is kept as a free parameter. There are two unknown potential parameters (a, V_0). These are obtained by fitting the nucleon mass $M_N =$ 939 MeV and charge radius of the proton $\langle r_N \rangle = 0.87$ fm in free space. We point out here that in the present model chiral symmetry is explicitly broken since the Lagrangian $\mathcal{L}_q^0(x)$ is chirally odd with the explicit term $G(r)\bar{\psi}_q(x)\psi_q(x)$, where $G(r) = m_q + V(r)/2$. In view of PCAC, m_q in the Lagrangian density is usually expected to be the current mass. In the bag model picture, the quark masses are also taken in the current mass level. Therefore, we investigate the variations of free space nucleon properties vis-a-vis the saturation properties of nuclear matter with the light quark mass m_q taken within moderately low values $m_q = 40$ and 50 MeV. However, if we consider m_q to be an otherwise free parameter, we can also consider here $m_q = 300$ MeV at the constituent mass range. With such choices of m_q values we fit our basic inputs corresponding to the free-space nucleon properties together

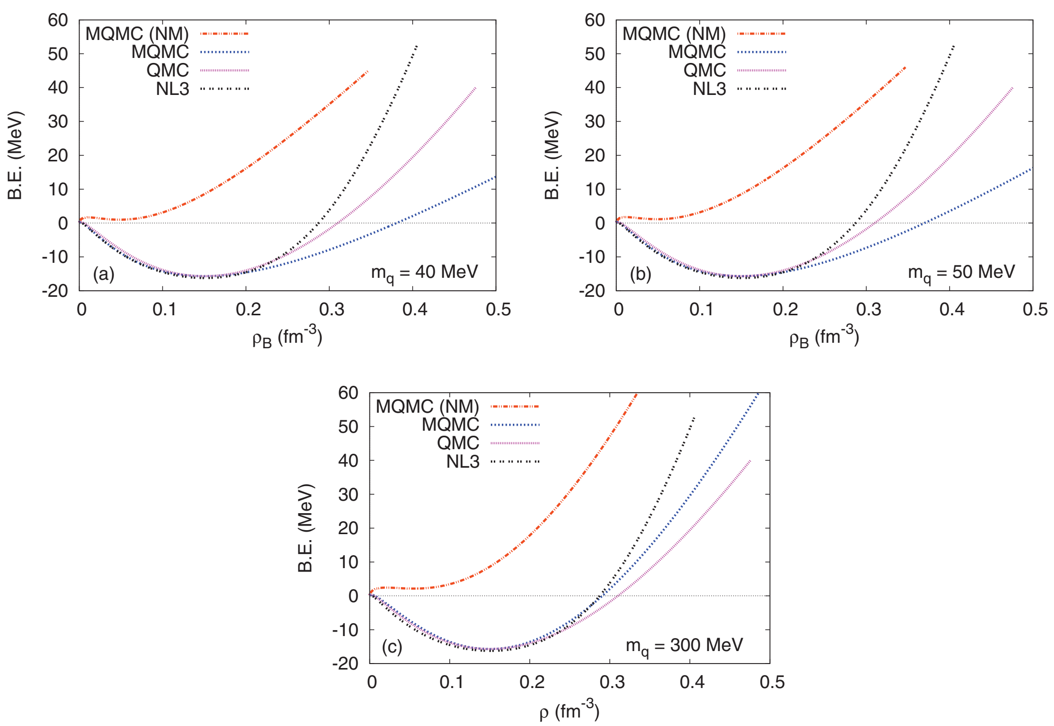

FIG. 2. (Color online) Nuclear matter binding energy as a function of density for different quark mass.

with the saturation properties of symmetric nuclear matter to determine the model parameters (a, V_0).

The parameters at zero density and the energy contributions for various corrections to the nucleon mass including the axial vector coupling constant g_A are given in Table I. Taking gluonic and pionic corrections together with those due to center of mass correction, baryon mass spectra in vacuum had been already reproduced in a similar potential model [40]. The contributions due to pionic and gluonic corrections for octet baryons and Δ have been described in detail [40].

In the present work, since our focus is on the study of the nuclear matter, we did not make fine tuning to reproduce the mass of the baryon spectra. The gluon contribution to the nucleon in the present calculation comes out to be about -75 MeV. The pionic correction to Δ is $-\frac{99}{171}\delta M_N^{\pi}$ and the gluonic correction to Δ as realized by putting $a_{uu} = 3$ and $a_{us} = a_{ss} = b_{uu} = b_{us} = b_{ss} = 0$ in Eqs. (42) and (43), we get $M_\Delta = 1115.32$ MeV.

B. Nucleons in medium and equation of state

We next fix the couplings, g_σ^q and g_ω, by fitting saturation properties of the nuclear matter. We take the standard values for the meson masses, namely $m_\sigma = 550$ MeV and $m_\omega = 783$ MeV. The quark-meson coupling constants g_σ^q, $g_\omega = 3g_\omega^q$ are fitted self-consistently to obtain the correct saturation properties of nuclear matter binding energy, B.E. $\equiv \mathcal{E}/\rho_B - M_N = -15.7$ MeV and pressure, $P = 0$ at $\rho_B = \rho_0 = 0.15$ fm^{-3}. These parameters are given in Table II. Due to the additional quark-meson coupling in the nuclear matter representing the NN interaction, the effective quark mass becomes $m_q^* = (m_q - g_\sigma^q \sigma_0)$. The compressibility comes around 223 MeV for $m_q = 40$ MeV, 224 MeV for $m_q = 50$ MeV, and 259 MeV for $m_q = 300$ MeV at nuclear matter density which is usually taken to be about 200–300 MeV. The mean field values of σ_0 and ω_0, compressibility, effective mass, the energy contributions from different corrections, g_A and $g_{NN\pi}$ at the saturation point are provided in Table III.

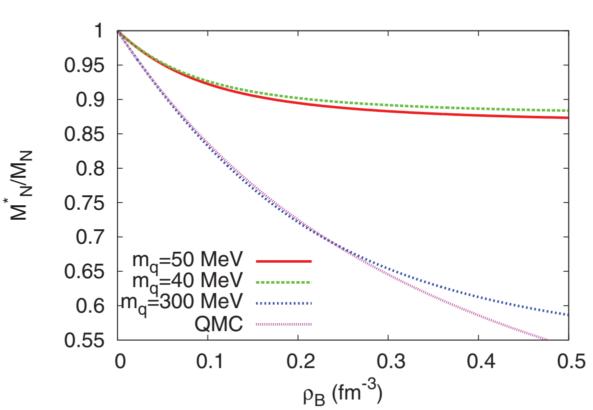

FIG. 3. (Color online) Effective mass versus density with quark mass $m_q = 40$ MeV, $m_q = 50$ MeV, and $m_q = 300$ MeV.

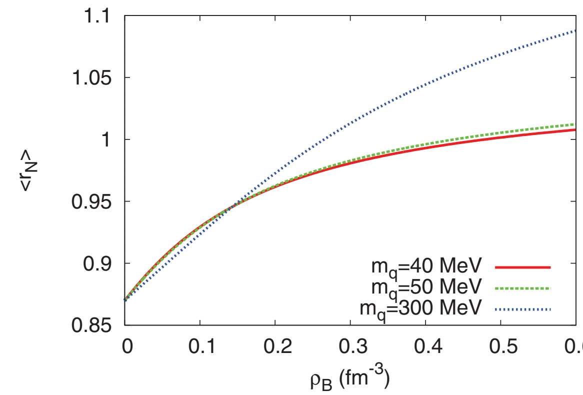

FIG. 4. (Color online) Charge radius versus density with quark mass $m_q = 40$ MeV, $m_q = 50$ MeV, and $m_q = 300$ MeV.

The binding energy per nucleon for nuclear matter as a function of nucleon density ρ_N corresponding to each of the choices of m_q values have been calculated. Therefore in Fig. 2, we plot this result for $m_q = 40$ MeV, 50 MeV, and 300 MeV to compare our result with those of NL3 [20] and QMC [29]. In the same figure, the equation of state for neutron matter in the present model with $m_q = 40$ MeV, 50 MeV, and 300 MeV has also been depicted.

Figure 3 shows the effective nucleon mass, M^*/M, as a function of baryon density for quark mass $m_q = 40$ MeV, 50 MeV, and 300 MeV. This result is compared with that obtained in QMC [29]. In all cases, the effective mass decreases as the baryon density increases and then saturates at high baryon densities. It may be noted here that the effective nucleon mass of the Walecka model [19] at saturation density is about 540 MeV, a value considered to be extremely small. A value of about 700 to 750 MeV is usually obtained in nonrelativistic calculations which is considered to be more consistent with the observed value of the density of states near the Fermi surface. In QMC, M_N^* was found to be of the order of 723 MeV. However, in the present analysis the effective mass M_N^* comes out to be 855 MeV with $m_q = 40$ MeV, 850 MeV with $m_q = 50$ MeV, and 723 MeV with $m_q = 300$ MeV. The Saito and Thomas [25] model obtained $M_N^* = 839$–856 MeV with the bag radius varying in the range of 0.6 fm to 1 fm.

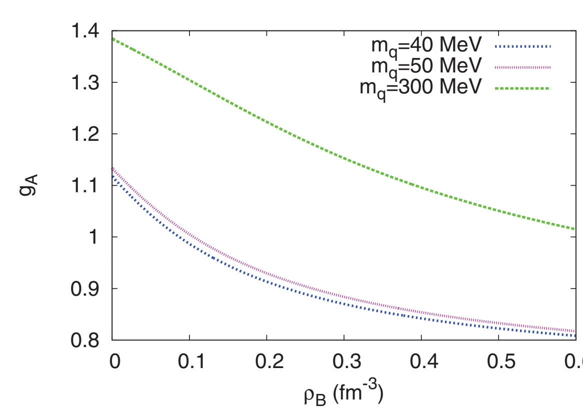

FIG. 5. (Color online) g_A versus density with quark mass $m_q = 50$ MeV, $m_q = 50$ MeV, and $m_q = 300$ MeV.

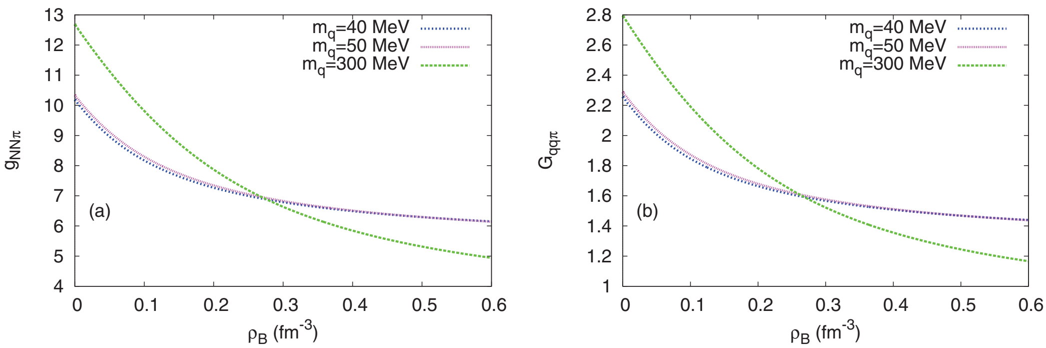

FIG. 6. (Color online) (a) $g_{NN\pi}$ versus baryon density with quark mass $m_q = 40$ MeV, $m_q = 50$ MeV, and $m_q = 300$ MeV. (b) $g_{qq\pi}$ versus baryon density with quark mass $m_q = 40$ MeV, $m_q = 50$ MeV, and $m_q = 300$ MeV.

We next calculate the spin-orbit potential strength, V_{so}, following Ref. [35] and found that $V_{so} = 0.75$ MeV for $m_q = 40$ MeV, $V_{so} = 0.83$ MeV for $m_q = 50$ MeV, and $V_{so} = 3.41$ for $m_q = 300$ MeV. It rises smoothly with increasing quark mass. Phenomenological values of the spin-orbit strength are in the range from 5 to 7 MeV.

The variations of the root mean square nucleon radius, r_N, are shown in Fig. 4 with baryon density for quark mass $m_q = 40$ MeV, $m_q = 50$ MeV, and $m_q = 300$ MeV. The nucleon radius increases with the baryon density and is approximately 0.95 fm at the saturation density. The rate of increase tends to be larger for larger values of the quark mass.

In Fig. 5, the variation of axial vector coupling constant g_A as a function of baryon density for quark mass $m_q = 40$ MeV, $m_q = 50$ MeV, and $m_q = 300$ MeV are shown. At bare level, the $g_A = 1.118$ for $m_q = 40$ MeV, $g_A = 1.133$ for $m_q = 50$ MeV, and $g_A = 1.384$ for $m_q = 300$ MeV which qualitatively agrees with the experimental value $g_A/g_V = 1.27590^{+0.00409}_{-0.00445}$ [43]. g_A is observed to decrease with increase in the density. At saturation density, $g_A = 0.945$ for $m_q = 40$ MeV, $g_A = 0.962$ for $m_q = 50$ MeV, and $g_A = 1.263$ for $m_q = 300$ MeV. Since our MQMC model is a relativistic model, the attractive scalar potential decreases the quark mass. Thus the lower component of the wave function is enhanced and hence it makes g_A decrease with density. This is similar to the observations made in [44]. The nucleon-pion coupling constant $g_{NN\pi}$ and quark pion coupling constant $g_{qq\pi}$ with $m_q = 40$, 50 MeV and 300 MeV as a function of density are plotted in Figs. 6(a) and 6(b), respectively. It is observed that both $g_{NN\pi}$ and $g_{qq\pi}$ decrease by increasing the density. This is due to the similar trend found in g_A since they are related through Goldberger-Treiman relation.

We have calculated the scalar mean field σ_0 at various densities which is plotted in Fig. 7 for $m_q = 40$ MeV, $m_q = 50$ MeV, $m_q = 300$ MeV. At saturation density, we find $\sigma_0 = 15.44$ MeV for $m_q = 40$ MeV, $\sigma_0 = 16.11$ MeV for $m_q = 50$ MeV, and $\sigma_0 = 26.93$ MeV for $m_q = 300$ MeV. It is quite interesting to note here that the effective mass of the quark that has entered in our calculation as $m'_q = m_q - g^q_\sigma \sigma_0 + V_0/2$ comes out to be 7.697 MeV at $m_q = 40$ MeV and 14.9 MeV for $m_q = 50$ MeV at saturation density. Such a low effective quark mass, which is of the order of up-down current quark masses is in quite commensurate with PCAC requirement.

In Fig. 8, the pionic corrections δM^π_N to the mass of the nucleon for quark masses 40 MeV, 50 MeV, and 300 MeV are shown for different baryon densities. It is found that δM^π_N increases with density and at saturation density the values are -29 MeV, -30 MeV, and -65 MeV for the quark masses 40, 50, and 300 MeV, respectively. Since with increase in density the quark-pion coupling strength and the pseudoscalar

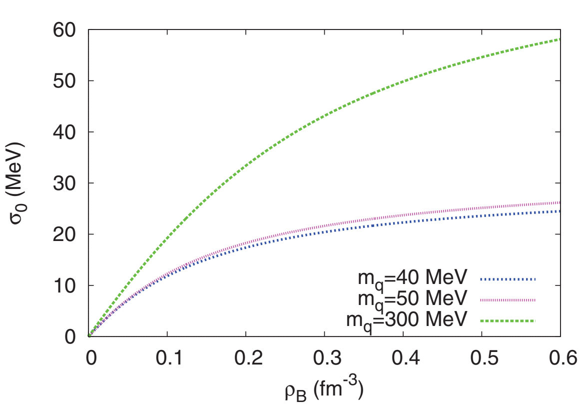

FIG. 7. (Color online) σ_0 versus baryon density with quark mass $m_q = 40$ MeV, $m_q = 50$ MeV, and $m_q = 50$ MeV.

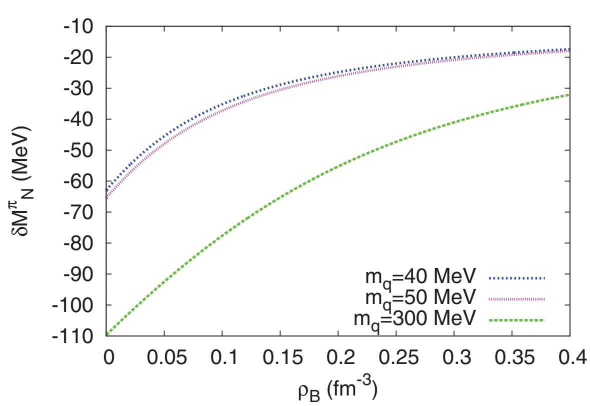

FIG. 8. (Color online) δM^π_B versus baryon density with quark mass $m_q = 40$, $m_q = 50$ MeV, and $m_q = 300$ MeV.

nucleon-pion coupling $g_{NN\pi}$ decreases, the pionic correction to the mass increases.

In Fig. 9, it is observed that the gluonic corrections to the mass of the nucleon decreases by increasing the baryon density, which is expected, because the average quark distances increase as the nucleon swells. In the same figure, the gluonic correction for the quark masses 40, 50, and 300 MeV are compared. The rate of fall appears to be the same for different quark masses.

C. Nucleon and nuclear matter σ terms

We next proceed to calculate the nucleon σ term, Σ_N which is an important property for chiral symmetry. The individual nucleon σ term in the nuclear medium can be defined as (see [45,46])

$$\Sigma_N = m_q \frac{\partial M_N}{\partial m_q}, \tag{63}$$

from the Feynman-Hellman theorem. Note that, M_N is identified with M_N^* at finite density. Alternatively, the nucleon σ term Σ_N can be related to the quark condensates at low densities as [45]

$$2m_q[\langle \bar{q}q \rangle_{\rho_B} - \langle \bar{q}q \rangle_{vac}] = \Sigma_N \rho_B + \cdots = m_q \frac{\partial \mathcal{E}}{\partial m_q}, \tag{64}$$

where $\mathcal{E}$ is the energy density of nuclear matter, which is given as

$$\mathcal{E} = M_N^* \rho_B + \delta\mathcal{E}. \tag{65}$$

Here $\delta\mathcal{E}$ is the calculation to the energy density from the nucleon kinetic energy plus the nucleon-nucleon interaction. $\delta\mathcal{E}$ is said to be small at low densities. Then, from Eq. (64) one can obtain the nuclear matter σ term per nucleon as

$$\Sigma_{NM} = m_q \frac{\partial(\mathcal{E}/\rho_B)}{\partial m_q}, \tag{66}$$

which is distinct from the individual nucleon σ term in nuclear matter, due to the nucleon kinetic energy and the interaction among the nucleons in the nuclear medium. Now, we calculate

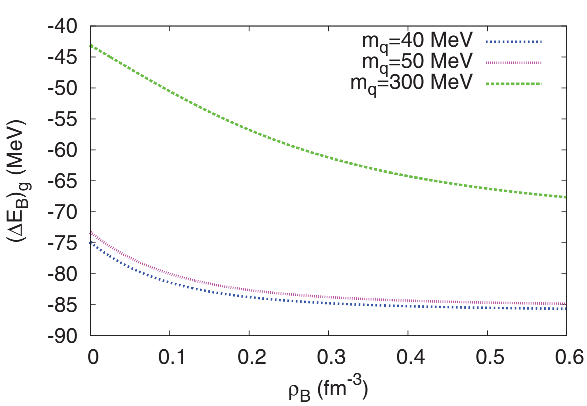

FIG. 9. (Color online) $(\Delta E_B)_g$ versus baryon density with quark mass $m_q = 40$ MeV, $m_q = 50$ MeV, and $m_q = 50$ MeV.

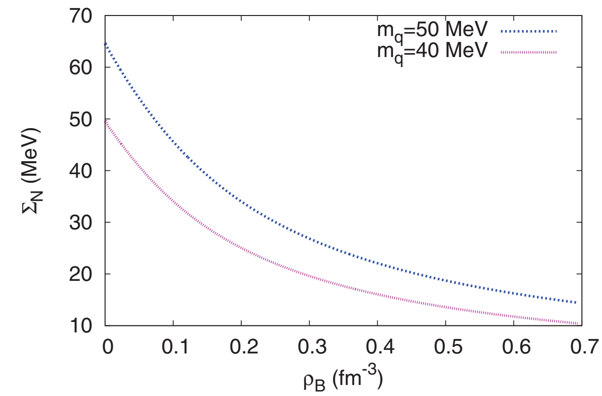

FIG. 10. (Color online) Nucleon σ term versus baryon density with quark mass $m_q = 40$ MeV and $m_q = 50$ MeV.

Σ_N using Eq. (63) and Σ_{NM} using Eq. (66) at various densities with the results shown in Figs. 10 and 11, respectively, for $m_q = 40$ and 50 MeV.

The difference in the two results from Eqs. (63) and (66) at various densities are plotted in Fig. 11. At zero density we found $\Sigma_N^0 = 49.59$ MeV for $m_q = 40$ MeV and $\Sigma_N^0 = 64.823$ MeV for $m_q = 50$ MeV. The experimental value extracted from pion-nucleon scattering is $\Sigma_N \sim 45$ MeV [47]. At saturation density, we find Σ_N to be $\Sigma_N^0 = 28.8$ MeV for $m_q = 40$ MeV $\Sigma_N^0 = 38.9$ MeV for $m_q = 50$ MeV.

The sensible quantity for the stability of nuclei is the relative variation of the binding energies with the quark mass, written below for nuclear matter:

$$\frac{\delta \mathrm{B.E.}}{\mathrm{B.E.}} = \frac{\delta\,[\mathcal{E}/\rho_B - M_N]}{\mathcal{E}/\rho_B - M_N} = K_{NM}(\rho_B)\frac{\delta m_q}{m_q}, \tag{67}$$

which gives $K_{NM} = -1.02$ at nuclear saturation density. The general trend of $K_{NM}(\rho_B)$ with the density, follows from the results presented in Fig. 10, it is negative and the magnitude increases for larger densities, as we see in Fig. 12, which suggests that compact objects could be more sensitive to variations in quark masses. We compare our result of $K_{NM} = -1.02$

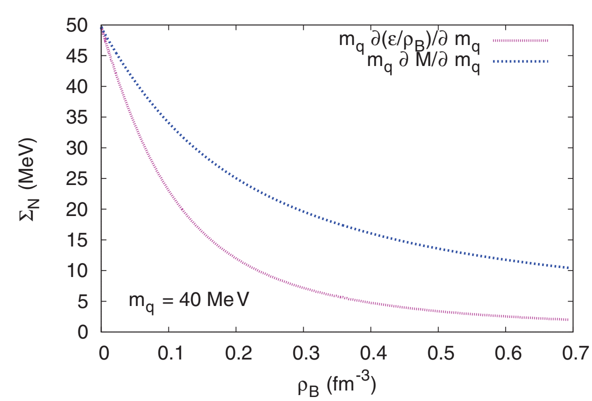

FIG. 11. (Color online) Nuclear and nucleon σ terms versus baryon density with quark mass $m_q = 40$ MeV.

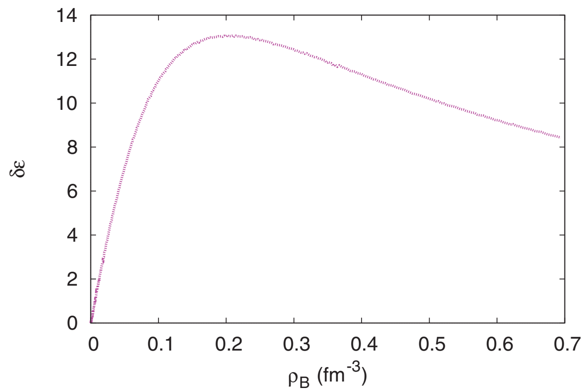

FIG. 12. (Color online) Variation of nuclear and nucleon σ term versus baryon density with quark mass $m_q = 40$ MeV.

at the saturation density with the values of the sensitivity K_A for nuclei $A = 3$–8 found in the range of 1 to 1.5, computed in [30] for different Argonne potential models, considering cases including the Urbana model IX three-body force, and for thorium $K_{^{229}\mathrm{Th}} = -1.45$ [31]. More recently, Ref. [32] computed for oxygen $K_{^{16}\mathrm{O}} = -1.082$, and the sensitivity for other light nuclei with a one boson exchange model.

It is to be noted here that in the present model the ratio of the quark condensate in the leading order

$$\frac{\langle \bar{q}q\rangle_{\rho_B}}{\langle \bar{q}q\rangle_{vac}} = 1 - \frac{\Sigma_N \rho_B}{m_\pi^2 f_\pi^2} \simeq \begin{cases} 0.80 & \text{for } m_q = 40 \text{ MeV} \\ 0.74 & \text{for } m_q = 50 \text{ MeV} \end{cases} \quad (68)$$

are somewhat smaller than the results found in Ref. [46] with a Skyrme model of the nucleon and of the nuclear force.

Comparing our result with the ratio $M_N^*/M_N = 0.9$ we find that the condensate ratio at saturation density (low density) essentially comes out as $\simeq (M_N^*/M_N)^2$ as found by Saito and Thomas [25]. This result is intermediate between the cubic dependence found by Brown and Rho [48] and the linear dependence proposed by Cohen *et al.* [45]. However, if we take the quark mass m_q as 300 MeV, we also get the saturation property with reasonable agreement with the standard values except for the nucleon σ term. At $m_q = 300$ MeV, the value of Σ_N^0 is much higher as compared to the experimental value.

Note that the quark mass at zero baryonic density is a few times larger than the current up-down quark masses, which was necessary in order to approach the nucleon σ term in the vacuum. Presumably, such value of the quark mass is parametrizing the complexity of the nucleon wave function beyond the valence state, which should contribute to the nucleon and nuclear matter matrix element of the $\bar{q}q$ operator. However, we expect that the typical changes in the σ term due to the nuclear environment will be kept in more refined description of the nucleon wave function. It is noteworthy to mention that, although we have employed such simplified nucleon, we have obtained results for the sensitivity of the nuclear binding energy comparable with the ones found in previous studies.

V. CONCLUSION

In the present paper we have studied the EOS for nuclear matter using a modified quark meson coupling model (MQMC). The properties of nuclear matter were calculated relying in a self-consistent method starting with a relativistic quark model with chiral symmetry for independent nucleons. The nucleon in the nuclear medium is composed of the three independent relativistic quarks confined by an equal admixture of a scalar-vector harmonic potential in a background of scalar and vector mean fields. We computed the corrections from the center of mass motion, pionic, and gluonic exchanges within the nucleon to obtain its effective mass. The nucleon-nucleon interaction in nuclear matter is then realized by the quark coupling to the scalar (σ) and vector (ω) mesons through a mean field approximation. Several basic characteristics of nuclear matter, such as the compressibility, the nucleon effective mass and nuclear σ term show better agreement with the experimental data than those obtained in a model with point-like nucleons. We have compared our results obtained with the quark-meson-coupling model which is based on the bag model and with the ones obtained within the nonlinear Walecka model.

The sensitivity of the nuclear binding energy was computed giving $K_{NM} \simeq -1$ ($\delta B_E/B_E = K_{NM}\delta m_q/m_q$), and we found that the sensitivity rises with density, as the nuclear σ term tends to vanish for large densities. The calculation of K_{NM} receives sizable effects from the nuclear interaction and kinetic energy, which decreases the sensitivity by almost a factor of 2 from the value computed only considering the individual nucleon σ term at the nuclear matter density. Finally, we have to mention that the model quark mass at zero baryonic density is a few times larger than the current up-down quark masses, in order to approach the nucleon σ term in the vacuum. This quark mass is effectively taken care of the complexity of the nucleon wave function beyond the valence state, assumed here, which of course should contribute to the mean value of the $\bar{q}q$ operator in the nucleon and nuclear matter states. We expect that the typical changes in the σ term due to the nuclear environment will be kept in more realistic descriptions of the nucleon, beyond the valence state, as our comparison with other models show that the values of the sensitivity are quite compatible. Further implications of this model for nuclear matter and compact stars will be taken up in our future work.

ACKNOWLEDGMENTS

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