Heavy-quark dynamics for charm and bottom flavor on the Fermi surface at zero temperature

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We discuss the dynamics in a finite-density medium including a heavy impurity particle (hadron or quark) with a heavy flavor, charm and bottom, at zero temperature. As a system, we consider a $\bar{D}$ (B) meson embedded in nuclear matter as a heavy impurity boson with SU(2) isospin symmetry. As another system, we also consider a charm (bottom) quark embedded in quark matter as a heavy impurity fermion with SU(3) color symmetry. We suppose a vector current interaction with SU(n) symmetry ($n \geqslant 2$) for the fermion composing the Fermi surface and the embedded heavy impurity particle, and we calculate the scattering amplitude perturbatively for the small coupling constant up to one-loop level. We obtain that the scattering amplitude has a logarithmic enhancement in the large-mass limit of the heavy impurity particle, and we show that the perturbative calculation breaks down for any small coupling constant in this limit.

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I. INTRODUCTION

In present hadron and quark physics, heavy quarks are interesting subjects for understanding the properties of QCD. In the recent literature, there have been many discussions about charmed and bottom nuclei with bound charm and bottom hadrons. For example, charmed nuclei may have a Λ_c baryon [1–3], a Σ_c baryon [3], a D meson [4–6], or a $\bar{D}$ meson [7–21] for open charm and a J/ψ [22–26] for hidden charm, inside normal nuclei. Bottom nuclei may have a Λ_b baryon [1,2], a Σ_b baryon, a $\bar{B}$ meson, or a B meson [10,21] for open bottom and a Υ for hidden bottom. Some of these discussions were motivated by the possible attractive force between a charm (bottom) hadron and a nucleon, which was found in studies of hadronic molecules as exotic hadrons [27–33] (see also [34–37]). The results of those studies suggest that a charm (bottom) hadron can be bound in nuclear matter. Such systems are interesting, because they will help us to study, not only (i) the interaction between the heavy hadron and nucleon but also (ii) the change of the properties of the heavy hadron in nuclear medium and (iii) the change of the nuclear medium (including the partial restoration of the dynamical breaking of chiral symmetry) caused by the heavy hadron as an impurity. Exotic nuclear systems with heavy flavors can be studied experimentally at J-PARC, GSI-FAIR, and so on. The study of heavy quarks in quark matter is also interesting, when the state at high density and low temperature can be produced in heavy-ion collisions. Actually it is expected that there are rich structures such as color superconductivity in quark matter [38,39].

In systems with a single heavy meson or quark, the heavy-mass limit is very useful for analysis of their properties in vacuum [40–45]. In the present article, we study the dynamics of the heavy meson or quark with a single heavy flavor, which is embedded in a finite-density medium at zero temperature, and investigate the behavior in the large limit of their masses in the medium. As systems, we consider a $\bar{D}$ (B) meson embedded in nuclear matter and a charm (bottom) quark embedded in quark matter. Although the former and latter systems are quite different, both of them exhibit similar behavior in the large-mass limit as discussed below. We note that, in our systems, the fermions composing the Fermi surface (nucleons in nuclear matter or light quarks in quark matter) and the heavy impurity particle [$\bar{D}$ (B) meson or charm (bottom) quark] belong to the fundamental representation of SU(n) symmetry with an integer $n \geqslant 2$. In fact, a $\bar{D}$ (B) meson and nucleons are doublet states in SU(2) isospin symmetry, and a charm (bottom) quark and light quarks are triplet states in SU(3) color symmetry. In the present discussion, we suppose that SU(n) symmetry is a global symmetry, not only for isospin symmetry but also for color symmetry. For generality of the formulation, we consider that the fermions and the heavy impurity particle belong to the fundamental representation of SU(n) symmetry (isospin for $n = 2$ and color for $n = 3$). We furthermore suppose a vector current interaction between the fermion and the heavy impurity particle with a small coupling constant so that we can apply a perturbative calculation. Such a simple interaction can be used for the low-energy region near the Fermi surface. With this setup, we discuss the scattering of the fermions (nucleons or light quarks) and the heavy impurity particle [$\bar{D}$ (B) meson or charm (bottom) quark].

In condensed matter physics, such a situation has been known as the "Kondo problem." There, spin-half electrons in the conduction band are coupled to an impurity atom with a nonzero (pseudo)spin through the "(pseudo)spin-spin" interaction. In 1964, Kondo found that the interaction between the conduction electrons and the impurity atom causes a logarithmic enhancement in the temperature in the system or the energy of the scattering fermions when the one-loop scattering amplitude is considered [46]. The problem was

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analyzed in detail in the scaling method and was further developed by numerical renormalization group analysis and so on [47]. Throughout those studies, it was recognized that, even though the coupling constant is small, quantum fluctuation from creations of particles and holes near the Fermi surface enhances the scattering amplitude with higher order loops, and the perturbation breaks down in the limit of low temperature or small scattering energy.

An important comment is in order. In the original work by Kondo, it was assumed tacitly that the impurity atom has an infinitely heavy mass. This assumption will be acceptable for electron-atom systems, because of the small electron/atom mass ratio. However, it is not necessarily the case that such an assumption can also be applied to the $\bar{D}$ (B) meson in the nuclear medium and charm (bottom) quark in quark matter. Determining the role played by the large mass of the impurity particle in the medium is rather a nontrivial problem. The present study is mainly devoted to this problem. Actually, this is an important problem in hadron physics for understanding how the properties of heavy mesons, such as $\bar{D}$ and D (B and $\bar{B}$) mesons, in nuclear matter are different from those of light mesons, such as K and $\bar{K}$ mesons, in nuclear matter [48,49]. In the former the heavy-mass limit can be applied, while in the latter it cannot. It will also be important to study the difference between the charm (bottom) quark and the strange quark in quark matter. As a matter of fact, this paper covers the result given by Kondo.

The paper is organized as followings. In Sec. II, we discuss a heavy impurity boson embedded in a Fermi gas [e.g., a $\bar{D}$ (B) meson in nuclear matter], and we derive the scattering amplitude for the heavy impurity boson and the fermion composing the Fermi surface. In Sec. III, we discuss a heavy impurity fermion embedded in a Fermi gas [e.g., a charm (bottom) quark in quark matter]. In both two cases, we suppose a vector current interaction with a small coupling constant between the fermion and the heavy impurity particle, and we analyze the scattering amplitude for each heavy impurity particle up to one-loop level. Then we show that the scattering amplitudes have a logarithmic enhancement in the heavy-mass limit, and the perturbation breaks down for any small coupling constant. In Sec. IV, we discuss related topics, and in the final section we summarize the discussion and give perspectives.

II. HEAVY IMPURITY BOSON

We consider a $\bar{D}$ (B) meson embedded in nuclear matter. As a model for the interaction with SU(2) isospin symmetry between a $\bar{D}$ (B) meson and a nucleon, we consider the vector current interaction with an isospin factor of $\vec{\lambda}_{\rm f}\cdot\vec{\lambda}_{\rm B}$ (see below) and a small coupling constant, and we analyze the scattering amplitude up to one-loop level. To see how the internal degrees of freedom work in the scattering amplitude, we generally extend SU(2) symmetry to SU(n) symmetry with an arbitrary integer $n \geqslant 2$.

A. Interaction with SU(n) symmetry

We consider the interaction Lagrangian given by the vector current interaction with SU(n) symmetry for the fermion and the heavy impurity boson,

$$\mathcal{L}_{\rm B,int} = -\frac{G_{\rm B}}{2}\sum_{j=1}^{n^2-1}\left(\bar{\psi}\gamma_\mu\lambda_{\rm f}^j\psi\right)\left(-i\partial^\mu\Phi^\dagger\lambda_{\rm B}^j\Phi + \Phi^\dagger\lambda_{\rm B}^j i\partial^\mu\Phi\right), \tag{1}$$

where the fermion field $\psi = (\psi_1, \ldots, \psi_n)^{\rm t}$ and the heavy impurity boson field $\Phi = (\Phi_1, \ldots, \Phi_n)^{\rm t}$ belong to the fundamental representation of SU(n) symmetry. The $n \times n$ matrices $\lambda_{\rm f}^j/2$ and $\lambda_{\rm B}^j/2$ ($j = 1, \ldots, n^2 - 1$) are the generators of SU(n) symmetry for the fermion and the heavy impurity boson, respectively. The coupling constant $G_{\rm B}$ is assumed to be small so that the perturbation can be applied. We suppose that the initial state of the heavy impurity boson has a four-momentum P^μ and the final state of the heavy impurity boson has P'^μ, and we rewrite the interaction Lagrangian in momentum space. Considering that the mass $M_{\rm B}$ of the heavy impurity boson is very large, we decompose P^μ and P'^μ as

$$P^\mu = M_{\rm B}v^\mu + k^\mu, \tag{2}$$

$$P'^\mu = M_{\rm B}v^\mu + k'^\mu, \tag{3}$$

where v^μ is a four-velocity with an on-mass-shell condition $v^2 = 1$, and k and k' are off-mass-shell (residual) momenta. When we ignore the terms with the residual momentum, which are suppressed by the order of $1/M_{\rm B}$ from ones with v^μ, the above Lagrangian can be rewritten as

$$\mathcal{L}_{\rm B,int} = -G_{\rm B}M_{\rm B}\sum_{j=1}^{n^2-1}\left(\bar{\psi}\gamma_\mu\lambda_{\rm f}^j\psi\right) \times\left[\Phi^\dagger v^\mu\lambda_{\rm B}^j\Phi + O(k/M_{\rm B}, k'/M_{\rm B})\right]. \tag{4}$$

Dropping the terms at $O(k/M_{\rm B}, k'/M_{\rm B})$ will be justified, because the large-mass limit for the heavy impurity boson is adopted in the discussion. We assume that the heavy impurity boson is at rest in the medium and set $v^\mu = (1, \vec{0})$.

B. Scattering amplitude

Based on the above interaction Lagrangian, we consider the scattering amplitude for the fermion and the heavy impurity boson up to one-loop level as shown in Figs. 1 and 2.

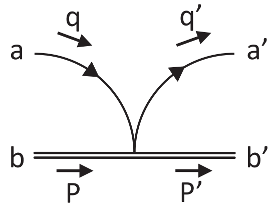

FIG. 1. Diagram of the first-order scattering amplitude (the Born term) of the fermion $\psi_{a(a')}$ ($a, a' = 1, \ldots, n$) (single line) composing the Fermi surface and the heavy impurity boson $\Phi_{b(b')}$ ($b, b' = 1, \ldots, n$) (double line) embedded in the Fermi gas. q (q') is the initial (final) four-momentum of the fermion, and P (P') is the initial (final) four-momentum of the boson.

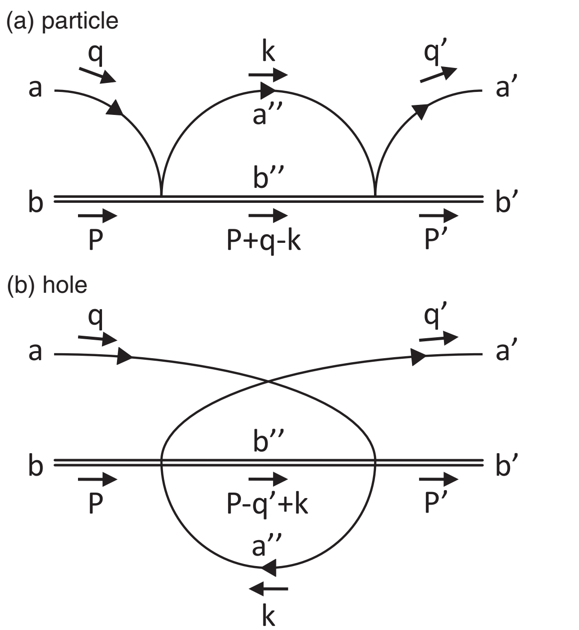

FIG. 2. Diagrams of the second-order scattering amplitude of the fermion and the heavy impurity boson. k is the internal moment in the loop. The other notations are the same as that in Fig. 1.

We introduce the initial (final) three-dimensional momentum $\vec{q}$ ($\vec{q}\,'$) of the fermion which is very close to the Fermi surface and the initial (final) momentum $\vec{P}$ ($\vec{P}\,'$) of the heavy impurity boson with $\vec{P}=0$ at rest in the medium. Then, it will be deduced that the initial and final momenta of the scattering fermion are the same ($\vec{q}\,'\simeq\vec{q}$) and the recoil of the heavy impurity boson is negligible ($\vec{P}\,'\simeq 0$). Indeed, from momentum conservation $\vec{q}+\vec{P}=\vec{q}\,'+\vec{P}\,'$ and energy conservation $\vec{q}^{\,2}/2m+\vec{P}^{\,2}/2M_{\rm B}=\vec{q}^{\,\prime 2}/2m+\vec{P}^{\,\prime 2}/2M_{\rm B}$ for the nonrelativistic fermion, we obtain the desired relation under the conditions $|\vec{q}\,|\simeq k_{\rm F}$, $\vec{P}=0$, and $|\vec{q}\,'|\gtrsim k_{\rm F}$, where the last condition is given by the Pauli blocking effect inside the Fermi sphere with Fermi momentum $k_{\rm F}$. This is also the case for the relativistic fermion. Hence, we use $\vec{q}\,'=\vec{q}$ and $\vec{P}=\vec{P}\,'=0$ throughout the present discussion. The components in the fundamental representation of SU(n) for the fermion and the heavy impurity boson are denoted by a and b (a' and b') for the initial (final) states, respectively.

To begin the first-order contribution (the Born term) in the scattering amplitude in Fig. 1 is obtained as

$$-i\mathcal{M}^{(1)}_{\rm B}=-iG_{\rm B}M_{\rm B}\,\bar{u}_{q,a'}\not{v}u_{q,a}\,(\vec{\lambda}_{\rm f})_{a'a}\cdot(\vec{\lambda}_{\rm B})_{b'b}, \tag{5}$$

with the four-velocity $v^\mu=(1,\vec{0}\,)$ for the heavy impurity boson and the four-component spinor u_q ($\bar{u}_q$) for the initial (final) fermion with momentum $q=(q_0,\vec{q}\,)$. We note again that the absolute value of the three-dimensional momentum $\vec{q}$ should be close to the Fermi momentum: $|\vec{q}\,|\simeq k_{\rm F}$. For the coupling in the interaction, we neglect the higher order terms in expansion by powers of $1/M_{\rm B}$. Apparently, the first-order scattering amplitude has no singular behavior in the large-mass limit of $M_{\rm B}$.

Next, we consider the second-order contribution, which contains the fermion propagator in the loop. In the fermion propagator in a finite-density medium at zero temperature, we use the in-medium fermion propagator,

$$(\not{p}+m)\left[\frac{i}{p^2-m^2+i\varepsilon}-2\pi\delta(p^2-m^2)\theta(p_0)\theta(k_{\rm F}-|\vec{p}\,|)\right]\times\mathbf{1}_{n\times n}, \tag{6}$$

with the fermion mass m, the four-momentum $p^\mu=(p_0,\vec{p}\,)$, and the Fermi momentum $k_{\rm F}$ [50]. Here ε is an infinitely small positive number and $\mathbf{1}_{n\times n}$ is an $n\times n$ unit matrix corresponding to SU(n) symmetry. The second term with the delta function and the step functions indicates the Pauli blocking effect for the fermions in the Fermi sphere. Because the fermions with positive energy are occupied up to the Fermi surface in momentum space, the on-mass-shell fermions in the Fermi sphere cannot propagate.

By using the in-medium fermion propagator, Eq. (6), let us consider the second-order scattering amplitude in Fig. 2. Before going to the precise calculation, it will be worthwhile to see naively how the scattering amplitude would behave for large $M_{\rm B}$. Because the strength of the interaction vertex is proportional to $G_{\rm B}M_{\rm B}$ in Eq. (4), the first-order scattering amplitude is also proportional to $G_{\rm B}M_{\rm B}$ as obtained above. For the second-order contribution, we pay attention to the heavy impurity boson propagator being proportional to $1/M_{\rm B}$ for large $M_{\rm B}$, because $(P^2-M_{\rm B}^2)^{-1}=[(M_{\rm B}v+k)^2-M_{\rm B}^2]^{-1}\simeq(2M_{\rm B}v\cdot k)^{-1}$ from Eq. (2), as long as the heavy impurity boson is close to the on-mass-shell state. Hence we might observe that the second-order scattering amplitude will be proportional to $(G_{\rm B}M_{\rm B})^2$(vertex) $\times\,M_{\rm B}^{-1}$(propagator) $=G_{\rm B}^2M_{\rm B}$, when the momentum cutoff in the loop is fixed. Then, because the power of $M_{\rm B}$ is the same in the first- and second-order scattering amplitudes, it seems for any large value of $M_{\rm B}$ that the perturbation is applicable for small $G_{\rm B}$. However, we will present that this naive analysis does not hold for the interaction of Eq. (1) in a finite-density medium at zero temperature.

The precise form of the second-order scattering amplitude is given as

$$-i\mathcal{M}^{(2)}_{\rm B}=-i\mathcal{M}^{(2)}_{\rm B}[\text{Fig. 2(a)}]-i\mathcal{M}^{(2)}_{\rm B}[\text{Fig. 2(b)}], \tag{7}$$

where each contribution is

$$-i\mathcal{M}^{(2)}_{\rm B}[\text{Fig. 2(a)}]=(-iG_{\rm B}M_{\rm B})^2\left\{4\left(1-\frac{1}{n^2}\right)\delta_{a'a}\delta_{b'b}+\left(-\frac{4}{n}\right)(\vec{\lambda}_{\rm f})_{a'a}\cdot(\vec{\lambda}_{\rm B})_{b'b}\right\}\bar{u}_{q',a'}\not{v}\int\frac{d^4k}{(2\pi)^4}(\not{k}+m)\times\left[\frac{i}{k^2-m^2+i\varepsilon}-2\pi\delta(k^2-m^2)\theta(k_0)\theta(k_{\rm F}-|\vec{k}\,|)\right]\frac{i}{(P+q-k)^2-M_{\rm B}^2+i\varepsilon}\not{v}u_{q,a}, \tag{8}$$

in Fig. 2(a), and

$$-i\mathcal{M}_{\rm B}^{(2)}[\text{Fig. 2(b)}] = (-iG_{\rm B}M_{\rm B})^2 \left\{4\left(1-\frac{1}{n^2}\right)\delta_{a'a}\delta_{b'b} + 2\left(n-\frac{2}{n}\right)(\vec{\lambda}_{\rm f})_{a'a}\cdot(\vec{\lambda}_{\rm B})_{b'b}\right\}\bar{u}_{q,a'}\not{p}\int\frac{d^4k}{(2\pi)^4}(\not{k}+m)$$
$$\times\left[\frac{i}{k^2-m^2+i\varepsilon} - 2\pi\delta(k^2-m^2)\theta(k_0)\theta(k_{\rm F}-|\vec{k}\,|)\right]\frac{i}{(P-q+k)^2-M_{\rm B}^2+i\varepsilon}\not{p}u_{q,a}, \quad (9)$$

in Fig. 2(b). k is the internal momentum in the loop. To obtain the above equations, we use the identities

$$\sum_{a'',b''=1}^{n}\sum_{i,j=1}^{n^2-1}(\lambda_{\rm f}^i)_{a'a''}(\lambda_{\rm B}^i)_{b'b''}(\lambda_{\rm f}^j)_{a''a}(\lambda_{\rm B}^j)_{b''b} = 4\left(1-\frac{1}{n^2}\right)\delta_{a'a}\delta_{b'b} + \left(-\frac{4}{n}\right)\sum_{i=1}^{n^2-1}(\lambda_{\rm f}^i)_{a'a}(\lambda_{\rm B}^i)_{b'b} \quad (10)$$

and

$$\sum_{a'',b''=1}^{n}\sum_{i,j=1}^{n^2-1}(\lambda_{\rm f}^i)_{a'a''}(\lambda_{\rm B}^j)_{b'b''}(\lambda_{\rm f}^j)_{a''a}(\lambda_{\rm B}^i)_{b''b} = 4\left(1-\frac{1}{n^2}\right)\delta_{a'a}\delta_{b'b} + 2\left(n-\frac{2}{n}\right)\sum_{i=1}^{n^2-1}(\lambda_{\rm f}^i)_{a'a}(\lambda_{\rm B}^i)_{b'b}, \quad (11)$$

in Eqs. (8) and (9), respectively. It is important in the following discussion that the sign of the coefficient $-4/n$ in the factor $(\vec{\lambda}_{\rm f})_{a'a}\cdot(\vec{\lambda}_{\rm B})_{b'b}$ in Eq. (10) is opposite to that of $2(n-2/n)$ in the corresponding factor in Eq. (11). After integrating over k_0 in the integrals, then, we obtain the result for the finite-density part:

$$-i\mathcal{M}_{\rm B}^{(2)} = (-iG_{\rm B}M_{\rm B})^2\,4\left(1-\frac{1}{n^2}\right)\delta_{a'a}\delta_{b'b}\bar{u}_{q,a'}\not{p}\left[i\int_{|\vec{k}\,|\geqslant k_{\rm F}}\frac{d^3\vec{k}}{(2\pi)^3}\frac{1}{2\epsilon_k}\frac{\epsilon_k\gamma_0-\vec{k}\cdot\vec{\gamma}+m}{(P_0+q_0-\epsilon_k)^2-E_{P+q-k}^2+i\varepsilon}\right.$$
$$\left.+(-i)\int_{|\vec{k}\,|\leqslant k_{\rm F}}\frac{d^3\vec{k}}{(2\pi)^3}\frac{1}{2\epsilon_k}\frac{\epsilon_k\gamma_0-\vec{k}\cdot\vec{\gamma}+m}{(P_0-q_0+\epsilon_k)^2-E_{P-q+k}^2+i\varepsilon}\right]\not{p}u_{q,a}$$
$$+(-iG_{\rm B}M_{\rm B})^2(\vec{\lambda}_{\rm f})_{a'a}\cdot(\vec{\lambda}_{\rm B})_{b'b}\bar{u}_{q,a'}\not{p}\left[\left(-\frac{4}{n}\right)i\int_{|\vec{k}\,|\geqslant k_{\rm F}}\frac{d^3\vec{k}}{(2\pi)^3}\frac{1}{2\epsilon_k}\frac{\epsilon_k\gamma_0-\vec{k}\cdot\vec{\gamma}+m}{(P_0+q_0-\epsilon_k)^2-E_{P+q-k}^2+i\varepsilon}\right.$$
$$\left.+2\left(2-\frac{2}{n}\right)(-i)\int_{|\vec{k}\,|\leqslant k_{\rm F}}\frac{d^3\vec{k}}{(2\pi)^3}\frac{1}{2\epsilon_k}\frac{\epsilon_k\gamma_0-\vec{k}\cdot\vec{\gamma}+m}{(P_0-q_0+\epsilon_k)^2-E_{P-q+k}^2+i\varepsilon}\right]\not{p}u_{q,a}, \quad (12)$$

where we define $P_0 = E_P$ and $q_0 = \epsilon_q$ with $E_P = \sqrt{\vec{P}^{\,2}+M_{\rm B}^2}$ for the energy of the heavy impurity boson and $\epsilon_q = \sqrt{\vec{q}^{\,2}+m^2}$ for the energy of the scattering fermion. We leave only the contribution from finite density, because we are interested in the low-energy region around the Fermi surface. In Eq. (12), the former integral with $|\vec{k}| > k_{\rm F}$ (the latter one with $|\vec{k}| < k_{\rm F}$) in each set of square brackets is the contribution from the particles (holes) in the loop shown in Fig. 2(a) [Fig. 2(b)]. For convergence of the momentum integrals, we introduce the momentum cutoffs $\Lambda_{\rm high}$ for $|\vec{k}| > k_{\rm F}$ and $\Lambda_{\rm low}$ for $|\vec{k}| < k_{\rm F}$. Finally, the scattering amplitude up to one-loop level is given by

$$-i\mathcal{M}_{\rm B} = -i\mathcal{M}_{\rm B}^{(1)} - i\mathcal{M}_{\rm B}^{(2)}. \quad (13)$$

From now on, our effort is devoted to analyzing the behavior of the second-order amplitude in the large-mass limit of the heavy impurity boson.

C. Large-mass limit of heavy impurity boson

We consider the large-mass limit for the heavy impurity boson ($M_{\rm B}\to\infty$). We expand E_P by $1/M_{\rm B}$ as $E_P \simeq M_{\rm B} + \vec{P}^{\,2}/2M_{\rm B}$ and consider only the leading term and neglect the higher order contributions. In the following, for the sake of a simple presentation, we introduce new dimensionless variables η and Λ, which are defined by $|\vec{q}\,| = k_{\rm F}(1+\eta)$ for the three-dimensional momentum $\vec{q}$ of the scattering fermion and $\Lambda_{\rm high} = k_{\rm F}(1+\Lambda)$ and $\Lambda_{\rm low} = k_{\rm F}(1-\Lambda)$ for the cutoff parameters $\Lambda_{\rm high}$ and $\Lambda_{\rm low}$. We note that $\eta \geqslant 0$ is a nonnegative small number because the initial state of the scattering fermion is supposed to lie on or outside of the Fermi sphere. We also note that the cutoff parameter Λ is fixed as a finite number ($0 < \Lambda < 1$). Because the result of the integrals in Eq. (12) is lengthy, we simplify the analysis by considering two limiting cases for scattering fermions: nonrelativistic fermions and relativistic fermions. Although nonrelativistic fermions are much more realistic in nuclear matter, relativistic fermions will be found to be useful for our analysis.

1. Nonrelativistic fermions

We consider nonrelativistic fermions. We assume that the mass of the scattering fermion is larger than a typical scale of the momentum ($\simeq k_{\rm F}$) and expand ϵ_k as $\epsilon_k \simeq m + \vec{k}^{\,2}/2m$. For the heavy impurity boson, we keep the mass of the heavy impurity boson still much heavier than the mass of the scattering fermion, and we consider $m/M_{\rm B}$ as small: $m/M_{\rm B} \ll 1$.

To begin the discussion, we assume that the initial state of the scattering fermion is the ground state and its momentum lies on the Fermi surface: $|\vec{q}\,| = k_{\rm F}$ ($\eta = 0$). Then, we take the limit $M_{\rm B} \to \infty$ ($m/M_{\rm B} \to 0$, keeping m to be a constant) and obtain the leading contribution (see also Appendix A):

$$\begin{aligned}
&\lim_{M_{\rm B}\to\infty}\lim_{\eta\to 0}\left(-i\mathcal{M}_{\rm B}^{(2)}\right)_{\rm nonrel}\\
&= (-iG_{\rm B}M_{\rm B})^2\,\delta_{a'a}\delta_{b'b}\,i\frac{1}{2M_{\rm B}}\frac{2mk_{\rm F}}{4\pi^2}4\left(1-\frac{1}{n^2}\right)\left[-4\Lambda+\log\left(1+\frac{\Lambda}{2}\right)-\log\left(1-\frac{\Lambda}{2}\right)\right]\bar{u}_{q,a'}\frac{1+\not{v}}{2}u_{q,a}\\
&\quad+(-iG_{\rm B}M_{\rm B})^2\,(\vec{\lambda}_{\rm f})_{a'a}\cdot(\vec{\lambda}_{\rm B})_{b'b}\,i\frac{1}{2M_{\rm B}}\frac{2mk_{\rm F}}{4\pi^2}\left[-4\left(n-\frac{4}{n}\right)\Lambda+2n\log\Lambda+\left(-\frac{4}{n}\right)\log\left(1+\frac{\Lambda}{2}\right)\right.\\
&\quad\left.-2\left(n-\frac{2}{n}\right)\log\left(1-\frac{\Lambda}{2}\right)-2n\log\frac{m}{M_{\rm B}}+2n(1-\log 2)\right]\bar{u}_{q,a'}\frac{1+\not{v}}{2}u_{q,a}.
\end{aligned}\tag{14}$$

The spinor for a nonrelativistic fermion is defined as $u_q \simeq \sqrt{2m}(\chi, 0)^{\rm t}$ with two-component spinor χ.

In the above result, we find the logarithmic behavior of $\log M_{\rm B}$. To see the importance of this term, let us compare the first-order scattering amplitude, Eq. (5), with the second-order scattering amplitude, Eq. (14). We notice that the first-order scattering amplitude contains the factor $G_{\rm B}M_{\rm B}$, while the second-order scattering amplitude contains not only $G_{\rm B}^2M_{\rm B}$ but also $G_{\rm B}^2M_{\rm B}\log M_{\rm B}$. When we consider the heavy-mass limit ($M_{\rm B}\to\infty$), due to the presence of $\log M_{\rm B}$, the second-order scattering amplitude overcomes the first-order one. Therefore, a perturbative calculation is not applicable for any small coupling between the fermion and heavy impurity boson in the heavy-mass limit, and the system becomes a strongly interacting one. The critical mass of $M_{\rm B}$ which gives a second-order scattering amplitude comparable with the first-order one is obtained as

$$M_{\rm B}^{\rm cr,nonrel} = m\exp\left(\frac{1}{n|G_{\rm B}|}\frac{4\pi^2}{2mk_{\rm F}}\right),\tag{15}$$

when Eq. (5) and the term proportional to $\log m/M_{\rm B}$ in Eq. (14) are compared. For $M_{\rm B} \gg M_{\rm B}^{\rm cr,nonrel}$, the second-order scattering amplitude is larger than the first-order one, and the perturbation breaks down. We note that the value of $M_{\rm B}^{\rm cr,nonrel}$ should not be considered seriously because it just gives the order of the critical mass. Nevertheless, it will be interesting to estimate the value of the critical mass at a qualitative level. Suppose nuclear matter with nucleon mass $m = m_{\rm N} =$ 940 MeV and Fermi momentum $k_{\rm F} = 270$ MeV corresponds to normal nuclear matter density. We do not know precisely the value of the coupling constant $G_{\rm B}$, which has the dimensions of the inverse of the square of energy. We may naturally assume that $G_{\rm B}$ is related to the order of $1/f_\pi^2$ with the pion decay constant $f_\pi = 132$ MeV because it represents the typical energy scale characterizing the nonperturbative QCD vacuum. So we suppose $G_{\rm B} = \alpha/f_\pi^2$ with a constant α of $O(1)$. Then, we obtain $M_{\rm B}^{\rm cr,nonrel} = 3.8m_{\rm N}$–$1.4m_{\rm N}$ in the range $\alpha = 0.5$–2. We may observe that the mass of the B meson, 5280 MeV, would be larger than the critical mass and hence that a B meson embedded in nuclear matter may exhibit characteristics of a strongly interacting system. We may expect that a $\bar{D}$ meson in nuclear matter can exhibit some characteristics of a strongly interacting system, because its mass (1870 MeV) can very close to the critical mass. We should keep in mind that our discussion is only at a qualitative level and a more quantitative estimate is awaited from future studies.

In our discussion, the factor $\vec{\lambda}_{\rm f}\cdot\vec{\lambda}_{\rm B}$ in the interaction of Eq. (1) plays a significant role. Indeed, the opposite sign in the coefficients, $-4/n$ and $2(2-2/n)$, in the terms with $(\vec{\lambda}_{\rm f})_{a'a}\cdot(\vec{\lambda}_{\rm B})_{b'b}$ in Eq. (12) induces the factor $\log M_{\rm B}$. On the other hand, in the terms with $\delta_{a'a}\delta_{b'b}$ in Eq. (12), we confirm that no logarithmic term exists. We comment that the Fermi surface is also important, because we find no logarithmic term in the vacuum part in Eqs. (8) and (9).

We should emphasize that the logarithmic behavior of $\log M_{\rm B}$ in the scattering amplitude appears only in the limit of large $M_{\rm B}$. Otherwise, there is no such singularity. To see this, let us consider the case in which the mass of the heavy impurity boson is not so large. Supposing that the heavy impurity boson mass equals the fermion mass, $M_{\rm B} = m$, we analyze the loop integrals in Eq. (12) and obtain

$$\begin{aligned}
&\lim_{M_{\rm B}\to m}\lim_{\eta\to 0}\left(-i\mathcal{M}_{\rm B}^{(2)}\right)_{\rm nonrel}\\
&= (-iG_{\rm B}M_{\rm B})^2\,\delta_{a'a}\delta_{b'b}\,i\frac{1}{2M_{\rm B}}\frac{k_{\rm F}\Lambda}{4\pi^2}4\left(1-\frac{1}{n^2}\right)\left[-\left(1+\frac{\Lambda}{2}\right)\frac{1}{2}\log\frac{2+\Lambda}{\Lambda}+\left(1-\frac{\Lambda}{2}\right)\frac{1}{2}\log\frac{2-\Lambda}{\Lambda}\right]\bar{u}_{q,a'}\frac{1+\not{v}}{2}u_{q,a}\\
&\quad+(-iG_{\rm B}M_{\rm B})^2\,(\vec{\lambda}_{\rm f})_{a'a}\cdot(\vec{\lambda}_{\rm B})_{b'b}\,i\frac{1}{2M_{\rm B}}\frac{k_{\rm F}\Lambda}{4\pi^2}\left[n-\left(-\frac{4}{n}\right)\left(1+\frac{\Lambda}{2}\right)\frac{1}{2}\log\frac{2+\Lambda}{\Lambda}+2\left(n-\frac{2}{n}\right)2\left(1-\frac{\Lambda}{2}\right)\frac{1}{2}\log\frac{2-\Lambda}{\Lambda}\right]\\
&\quad\times\bar{u}_{q,a'}\frac{1+\not{v}}{2}u_{q,a}
\end{aligned}\tag{16}$$

for a fermion on the Fermi surface. We confirm that there is no logarithmic term for $M_{\rm B} = m$. Thus, for the term $\log M_{\rm B}$ to be present, it is important that $M_{\rm B}$ be much larger than other scales in the system.

As a brief summary of the analysis for nonrelativistic fermions, starting from the interaction in Eq. (1), we have shown that the perturbation breaks down for any small coupling constant in the limit of large $M_{\rm B}$. It should be noticed that, when the interaction does not contain derivatives for the heavy impurity boson field, such as a scalar interaction, the above conclusion does not hold. This is because in the scalar interaction the first-order scattering amplitude is proportional to $G_{\rm B}$, and the second-order one contains $G_{\rm B}^2/M_{\rm B}$ or $(G_{\rm B}^2/M_{\rm B})\log M_{\rm B}$ (with the same convention of the coupling constant $G_{\rm B}$ being used in the scalar interaction). Hence, the second-order contribution can become smaller than the first-order one in the limit of large $M_{\rm B}$, and the perturbation can be applied for small coupling constant $G_{\rm B}$.

2. Relativistic fermions

Let us move to the discussion of relativistic fermions. For simplicity, we consider massless fermions. We will see that the a similar conclusion holds in the large-mass limit of the heavy impurity boson. We note that, in this case, the expansion parameter cannot be given by the ratio of the fermion mass to the heavy impurity boson mass, $m/M_{\rm B}$, because the fermions now are massless. Instead, we use the ratio of the Fermi momentum and the heavy impurity boson mass, $k_{\rm F}/M_{\rm B}$, for an expansion parameter for large $M_{\rm B}$ with finite $k_{\rm F}$.

The first-order contribution of the scattering amplitude is given by Eq. (5), provided that the nonrelativistic spinor is replaced by the relativistic spinor $u_q = \sqrt{E_q}(\chi, \vec{\sigma}\cdot\hat{q}\,\chi)^{\rm t}$ with two-component spinor χ, energy $E_q = |\vec{q}\,|$, and unit vector $\hat{q} = \vec{q}/|\vec{q}\,|$ in the $\vec{q}$ direction.

For the second-order contribution, for the scattering fermion on the Fermi surface $[|\vec{q}\,| = k_{\rm F}\ (\eta = 0)]$, we take the limit $M_{\rm B} \to \infty$ $(k_{\rm F}/M_{\rm B} \to 0)$ in the second-order scattering amplitude and obtain the result

$$\lim_{M_{\rm B}\to\infty}\lim_{\eta\to 0}\left(-i\mathcal{M}_{\rm B}^{(2)}\right)_{\rm rel}$$
$$= (-iG_{\rm B}M_{\rm B})^2\,\delta_{a'a}\delta_{b'b}\,i\,\frac{1}{2M_{\rm B}}\frac{k_{\rm F}^2}{4\pi^2}4\left(1-\frac{1}{n^2}\right)\bar{u}_{q,a'}\not{v}(-4\Lambda)\gamma^0\not{v}u_{q,a} + (-iG_{\rm B}M_{\rm B})^2\,(\vec{\lambda}_{\rm f})_{a'a}\cdot(\vec{\lambda}_{\rm B})_{b'b}$$
$$\times\, i\,\frac{1}{2M_{\rm B}}\frac{k_{\rm F}^2}{4\pi^2}\bar{u}_{q,a'}\not{v}\left[\left\{-4\left(n-\frac{4}{n}\right)+n\Lambda^2-2n\log\frac{k_{\rm F}}{M_{\rm B}}+2n\log\Lambda+2n(1-\log 2)\right\}\gamma^0 - n\hat{q}\cdot\vec{\gamma}\right]\not{v}u_{q,a}, \quad (17)$$

with the relativistic spinor u_q. As in the case of nonrelativistic fermions, we find again that the factor $\log M_{\rm B}$ exists. The second-order scattering amplitude contains $G_{\rm B}^2 M_{\rm B}\log M_{\rm B}$ for large $M_{\rm B}$, while the first-order one contains $G_{\rm B}M_{\rm B}$. Therefore, the second-order scattering amplitude can be larger than the first-order one in the limit of large $M_{\rm B}$ and the perturbation is not applicable for any small coupling constant $G_{\rm B}$ in this limit. The critical mass of $M_{\rm B}$ for which the perturbation cannot be applied is given by

$$M_{\rm B}^{\rm cr,rel} = k_{\rm F}\exp\left(\frac{1}{n|G_{\rm B}|}\frac{4\pi^2}{k_{\rm F}^2}\right). \quad (18)$$

We keep in mind that this number should be not be considered seriously, as mentioned below Eq. (15).

III. HEAVY IMPURITY FERMION

We consider a charm (bottom) quark in quark matter where up, down, and strange quarks compose the Fermi surface. Although quark matter is much different from nuclear matter, we will find again that the scattering amplitude in the large limit of heavy quark mass has a behavior similar to that obtained in the previous section. As an interaction between the charm (bottom) quark and the light quark, we suppose a vector current interaction with a color factor of $\vec{\lambda}_{\rm f}\cdot\vec{\lambda}_{\rm F}$ (see below) with SU(3) color symmetry. Assuming a small coupling constant, we analyze the scattering amplitude up to one-loop level. As in the previous section, we extend SU(3) symmetry to SU(n) symmetry with an arbitrary integer $n \geqslant 2$. We use the common notations as in the previous section, except for the coupling constant and the mass of the heavy impurity fermion.

A. Interaction with SU(n) symmetry

We consider the interaction Lagrangian for the fermion composing the Fermi surface and the heavy impurity fermion,

$$\mathcal{L}_{\rm F,int} = -\frac{G_{\rm F}}{2}\sum_{j=1}^{n^2-1}\left(\bar{\psi}\gamma_\mu\lambda_{\rm f}^j\psi\right)\left(\bar{\Psi}\gamma^\mu\lambda_{\rm F}^j\Psi\right), \quad (19)$$

where the fermion field $\psi = (\psi_1,\ldots,\psi_n)^{\rm t}$ and the heavy impurity fermion field $\Psi = (\Psi_1,\ldots,\Psi_n)^{\rm t}$ belong to the fundamental representation of SU(n) symmetry. The $n\times n$ matrices $\lambda_{\rm f}^j$ and $\lambda_{\rm F}^j$ $(j = 1,\ldots,n^2-1)$ are the generators of the SU(n) group. In this section, we use $G_{\rm F}/2$ as the coupling constant. For the heavy impurity fermion, it is convenient to separate its momentum as $P = M_{\rm F}v + k$ with heavy impurity fermion mass $M_{\rm F}$, four-velocity v with the condition $v^2 = 1$, and residual momentum k. This convention will be used later.

B. Scattering amplitude

Let us consider the scattering amplitude of the heavy impurity fermion and the fermion composing the Fermi surface. The diagrams are the same as in Figs. 1 and 2, provided that the heavy impurity boson is read as the heavy impurity fermion.

The first-order scattering amplitude (the Born term) is given as

$$-i\mathcal{M}_{\mathrm{F}}^{(1)} = -i\frac{G_{\mathrm{F}}}{2}\bar{u}_{q,a'}\gamma_\mu(\vec{\lambda}_{\mathrm{f}})_{a'a}u_{q,a}\bar{u}_{P,b'}\gamma^\mu(\vec{\lambda}_{\mathrm{F}})_{b'b}u_{P,b}, \quad (20)$$

where u_q ($\bar{u}_q$) is the spinor wave function for the initial (final) fermion with four-momentum q, and u_P ($\bar{u}_P$) is the spinor wave function for the initial (final) heavy impurity fermion with four-momentum P. As discussed in the previous section, it is reasonable to suppose that the initial and final states have the same momentum.

The second-order scattering amplitude at the one-loop level is given as

$$-i\mathcal{M}_{\mathrm{F}}^{(2)} = -i\mathcal{M}_{\mathrm{F}}^{(2)}[\text{Fig. 2(a)}] - i\mathcal{M}_{\mathrm{F}}^{(2)}[\text{Fig. 2(b)}], \quad (21)$$

with

$$\begin{aligned}-i\mathcal{M}_{\mathrm{F}}^{(2)}[\text{Fig. 2(a)}] = {} & \left(-i\frac{G_F}{2}\right)^2\left\{4\left(1-\frac{1}{n^2}\right)\delta_{a'a}\delta_{b'b} + \left(-\frac{4}{n}\right)(\vec{\lambda}_{\mathrm{f}})_{a'a}\cdot(\vec{\lambda}_{\mathrm{F}})_{b'b}\right\}\\ & \times \bar{u}_{q,a'}\gamma_\mu\int\frac{d^4k}{(2\pi)^4}(\not{k}+m)\left[\frac{i}{k^2-m^2+i\epsilon} - 2\pi\delta(k^2-m^2)\theta(k_0)\theta(k_{\mathrm{F}}-|\vec{k}\,|)\right]\gamma_\nu u_{q,a}\\ & \times \bar{u}_{P,b'}\gamma^\mu\frac{i}{(P+q-k)^2-M_{\mathrm{F}}^2+i\epsilon}(\not{P}+\not{q}-\not{k}+M_{\mathrm{F}})\gamma^\nu u_{P,b}\end{aligned} \quad (22)$$

in Fig. 2(a) and

$$\begin{aligned}-i\mathcal{M}_{\mathrm{F}}^{(2)}[\text{Fig. 2(b)}] = {} & \left(-i\frac{G_{\mathrm{F}}}{2}\right)^2\left\{4\left(1-\frac{1}{n^2}\right)\delta_{a'a}\delta_{b'b} + 2\left(n-\frac{2}{n}\right)(\vec{\lambda}_{\mathrm{f}})_{a'a}\cdot(\vec{\lambda}_{\mathrm{F}})_{b'b}\right\}\\ & \times \bar{u}_{q,a'}\gamma_\mu\int\frac{d^4k}{(2\pi)^4}(\not{k}+m)\left[\frac{i}{k^2-m^2+i\epsilon} - 2\pi\delta(k^2-m^2)\theta(k_0)\theta(k_{\mathrm{F}}-|\vec{k}\,|)\right]\gamma_\nu u_{q,a}\\ & \times \bar{u}_{P,b'}\gamma^\nu\frac{i}{(P+q-k)^2-M_{\mathrm{F}}^2+i\epsilon}(\not{P}+\not{q}-\not{k}+M_{\mathrm{F}})\gamma^\mu u_{P,b}\end{aligned} \quad (23)$$

in Fig. 2(b). In deriving the above equations, we use the identities in Eqs. (10) and (11). Because the heavy impurity fermion is sufficiently massive, the matrices γ^ρ at vertices acting for the field Ψ are replaced by v^ρ. Performing the integrals over k_0, we obtain the result for the finite-density part:

$$\begin{aligned}-i\mathcal{M}_{\mathrm{F}}{}^{(2)} = {} & \left(-i\frac{G_{\mathrm{F}}}{2}\right)^2 4\left(1-\frac{1}{n^2}\right)\delta_{a'a}\delta_{b'b}2M_{\mathrm{F}}i\left[\int_{|\vec{k}|\geqslant k_{\mathrm{F}}}\frac{d^3\vec{k}}{(2\pi)^3}\frac{1}{2\epsilon_k}\frac{1}{(P_0+q_0-\epsilon_k)^2-E_{P+q-k}^2+i\varepsilon}\bar{u}_{q,a'}\not{v}\,(\not{k}+m)|_{k_0=\epsilon_k}\,\not{v}u_{q,a}\right.\\ & \left. - \int_{|\vec{k}|\leqslant k_{\mathrm{F}}}\frac{d^3\vec{k}}{(2\pi)^3}\frac{1}{2\epsilon_k}\frac{1}{(P_0-q_0+\epsilon_k)^2-E_{P-q+k}^2+i\varepsilon}\bar{u}_{q,a'}\not{v}\,(\not{k}+m)|_{k_0=\epsilon_k}\,\not{v}u_{q,a}\right]\bar{u}_{P,b'}\frac{1+\not{v}}{2}u_{P,b}\\ & + \left(-i\frac{G_{\mathrm{F}}}{2}\right)^2(\vec{\lambda}_{\mathrm{f}})_{a'a}\cdot(\vec{\lambda}_{\mathrm{F}})_{b'b}2M_{\mathrm{F}}i\left[\left(-\frac{4}{n}\right)\int_{|\vec{k}|\geqslant k_{\mathrm{F}}}\frac{d^3\vec{k}}{(2\pi)^3}\frac{1}{2\epsilon_k}\frac{1}{(P_0+q_0-\epsilon_k)^2-E_{P+q-k}^2+i\varepsilon}\bar{u}_{q,a'}\not{v}\,(\not{k}+m)|_{k_0=\epsilon_k}\,\not{v}u_{q,a}\right.\\ & \left. - 2\left(n-\frac{2}{n}\right)\int_{|\vec{k}|\leqslant k_{\mathrm{F}}}\frac{d^3\vec{k}}{(2\pi)^3}\frac{1}{2\epsilon_k}\frac{1}{(P_0-q_0+\epsilon_k)^2-E_{P-q+k}^2+i\varepsilon}\bar{u}_{q,a'}\not{v}\,(\not{k}+m)|_{k_0=\epsilon_k}\,\not{v}u_{q,a}\right]\bar{u}_{P,b'}\frac{1+\not{v}}{2}u_{P,b},\end{aligned} \quad (24)$$

where the terms suppressed in the limit of large M_{F} are dropped. Here we define $E_P = \sqrt{\vec{P}^2 + M_{\mathrm{F}}^2}$ as the energy of the heavy impurity fermion. We use the condition that the initial and final heavy impurity fermion are in the on-mass-shell state which is resting in the medium: $P = M_{\mathrm{F}}v$ with $v^\mu = (1,\vec{0})$. Then, the scattering amplitude up to one-loop level is given by Eq. (13) by changing the subscript from B to F.

For the heavy fermion close to the on-mass-shell state, we know that the propagator is given as $(\not{P}-M_{\mathrm{F}})^{-1} \simeq (\not{v}+1)(2v\cdot k)^{-1}$ from the decomposition of the momentum $P = M_{\mathrm{F}}v + k$. Then, we may expect that the second-order scattering amplitude could be proportional to G_F^2(vertex) × M_F^0(propagator) = G_F^2 when the momentum cutoff in the loop is fixed, while the first-order one is proportional to G_F. Hence, it seems that the perturbation might be applicable for a small coupling constant G_F. However, as in the previous section, we will find that there exists a logarithmic factor in the second-order scattering amplitude and that the perturbation for any small coupling constant G_F is not applicable in the heavy-mass limit $M_F \to \infty$.

C. Large-mass limit of a heavy impurity fermion

We consider the large-mass limit for a heavy impurity fermion ($M_F \to \infty$). To analyze the second-order scattering amplitude, we consider two cases for fermions composing the Fermi surface: nonrelativistic and relativistic fermions. Although relativistic fermions are much more likely in quark matter, nonrelativistic fermions will be found to be useful.

1. Nonrelativistic fermions

We consider that the fermion mass m is sufficiently larger than the Fermi momentum k_F, and we use $u_q \simeq \sqrt{2m}(\chi, 0)^t$ as a nonrelativistic spinor. We suppose that the fermion lies on the Fermi surface ($|\vec{q}\,| = k_F$; $\eta = 0$). We note that the condition $m/M_F \ll 1$ is still kept. By considering the large-mass limit of the heavy impurity fermion, we obtain the result

$$\begin{aligned}
&\lim_{M_B\to\infty}\lim_{\eta\to 0}\left(-i\mathcal{M}_B^{(2)}\right)_{\text{nonrel}}\\
&= \left(-i\frac{G_F}{2}\right)^2 \delta_{a'a}\delta_{b'b}\, i\frac{2mk_F}{4\pi^2}4\left(1-\frac{1}{n^2}\right)\left[-4\Lambda + \log\left(1+\frac{\Lambda}{2}\right) - \log\left(1-\frac{\Lambda}{2}\right)\right]\bar{u}_{q,a'}\frac{1+\not{v}}{2}u_{q,a}\bar{u}_{P,b'}\frac{1+\not{v}}{2}u_{P,b}\\
&\quad + \left(-i\frac{G_F}{2}\right)^2 (\vec{\lambda}_f)_{a'a}\cdot(\vec{\lambda}_F)_{b'b}\, i\frac{2mk_F}{4\pi^2}\left[-4\left(n-\frac{4}{n}\right)\Lambda + 2n\log\Lambda + \left(-\frac{4}{n}\right)\log\left(1+\frac{\Lambda}{2}\right) - 2\left(n-\frac{2}{n}\right)\log\left(1-\frac{\Lambda}{2}\right)\right.\\
&\quad \left. - 2n\log\frac{m}{M_F} + 2n(1-\log 2)\right]\bar{u}_{q,a'}\frac{1+\not{v}}{2}u_{q,a}\bar{u}_{P,b'}\frac{1+\not{v}}{2}u_{P,b}.
\end{aligned} \tag{25}$$

We find the logarithmic factor $\log M_F$ in the term proportional to $(\vec{\lambda}_f)_{a'a}\cdot(\vec{\lambda}_F)_{b'b}$. The first-order scattering amplitude is proportional to G_F, while the second-order scattering amplitude contains $G_F^2 \log M_F$ in the limit of large M_F. This means that, due to the presence of $\log M_F$, the second-order scattering amplitude becomes larger than the first-order one in the limit of large M_F. Therefore, the perturbation for any small coupling G_F breaks down in the system in this limit. The critical mass of M_F is given in Eq. (15) with a replacement of G_B by G_F and M_B by M_F.

The result for a heavy impurity fermion is analogous to that for a heavy impurity boson. As discussed in the previous section, there are two reasons why the singularity from the heavy mass arises: First, the existence of the factor $\vec{\lambda}_f\cdot\vec{\lambda}_F$ in the interaction Lagrangian Eq. (19) is important. Indeed, we see that there is no logarithmic behavior in the term with $\delta_{a'a}\delta_{b'b}$ in Eq. (25). Second, the large mass of the heavy impurity fermion is also important. Indeed, when the mass of the heavy impurity fermion equals the mass of the fermion composing the Fermi surface, $M_F = m$, we obtain from Eq. (24)

$$\begin{aligned}
&\lim_{M_F\to m}\lim_{\eta\to 0}\left(-i\mathcal{M}_F^{(2)}\right)_{\text{nonrel}}\\
&= \left(-i\frac{G_F}{2}\right)^2 \delta_{a'a}\delta_{b'b}\, i\frac{k_F\Lambda}{4\pi^2}4\left(1-\frac{1}{n^2}\right)\left[-\left(1+\frac{\Lambda}{2}\right)\frac{1}{2}\log\frac{2+\Lambda}{\Lambda} + \left(1-\frac{\Lambda}{2}\right)\frac{1}{2}\log\frac{2-\Lambda}{\Lambda}\right]\bar{u}_{q,a'}\frac{1+\not{v}}{2}u_{q,a}\bar{u}_{P,b'}\frac{1+\not{v}}{2}u_{P,b}\\
&\quad + \left(-i\frac{G_F}{2}\right)^2 (\vec{\lambda}_f)_{a'a}\cdot(\vec{\lambda}_F)_{b'b}\, i\frac{k_F\Lambda}{4\pi^2}\left[n - \left(-\frac{4}{n}\right)\left(1+\frac{\Lambda}{2}\right)\frac{1}{2}\log\frac{2+\Lambda}{\Lambda} + 2\left(n-\frac{2}{n}\right)2\left(1-\frac{\Lambda}{2}\right)\frac{1}{2}\log\frac{2-\Lambda}{\Lambda}\right]\\
&\quad \times \bar{u}_{q,a'}\frac{1+\not{v}}{2}u_{q,a}\bar{u}_{P,b'}\frac{1+\not{v}}{2}u_{P,b}.
\end{aligned} \tag{26}$$

We confirm that there is no singular term for $M_F = m$.

2. Relativistic fermions

We consider that the fermions composing the Fermi surface are relativistic. Supposing massless fermions, we replace the nonrelativistic spinor by the relativistic spinor $u_q = \sqrt{E_q}(\chi, \vec{\sigma}\cdot\hat{q}\,\chi)^t$ with $E_q = |\vec{q}\,|$ and $\hat{q} = \vec{q}/|\vec{q}\,|$. We use the expansion parameter k_F/M_F in the limit of the large mass of the heavy impurity fermion. The resulting form of the second-order scattering amplitude is

$$\lim_{M_F\to\infty}\lim_{\eta\to 0}\left(-i\mathcal{M}_F^{(2)}\right)_{\rm rel} = \left(-i\frac{G_F}{2}\right)^2 \delta_{a'a}\delta_{b'b} i\frac{k_F^2}{4\pi^2}4\left(1-\frac{1}{n^2}\right)\bar{u}_{q,a'}\not{p}(-4\Lambda)\gamma^0\not{p}u_{q,a}\bar{u}_{P,b'}\frac{1+\not{v}}{2}u_{P,b} + \left(-i\frac{G_F}{2}\right)^2(\vec{\lambda}_f)_{a'a}\cdot(\vec{\lambda}_F)_{b'b}$$
$$\times i\frac{k_F^2}{4\pi^2}\bar{u}_{q,a'}\not{p}\left[\left\{-4\left(n-\frac{4}{n}\right)+n\Lambda^2-2n\log\frac{k_F}{M_F}+2n\log\Lambda+2n(1-\log 2)\right\}\gamma^0-n\hat{q}\cdot\vec{\gamma}\right]\not{p}u_{q,a}$$
$$\times\bar{u}_{P,b'}\frac{1+\not{v}}{2}u_{P,b}. \tag{27}$$

We again obtain the factor $\log M_F$. Therefore, the perturbation breaks down in the heavy-mass limit. The critical mass of M_F is given in Eq. (18) with a replacement of G_B by G_F and M_B by M_F. We may give a rough estimate for the critical mass in quark matter. Suppose that quark matter has Fermi momentum $k_F = 1$ GeV as a typical scale. The coupling constant G_F is not known. Here we simply assume that G_F is given by α/Λ_{QCD}^2 with a constant α of $O(1)$ for $\Lambda_{QCD} \simeq 200$ MeV, because it would present the relevant energy scale around the Fermi surface in momentum space. Then we obtain $M_F^{\rm cr,rel} = 2.9k_F$–$1.3k_F$ in the range $\alpha = 0.5$–2. As a result, we may observe that the mass of the bottom quark ($\simeq$4.2 GeV) will be heavier than the critical mass and hence a bottom quark in quark matter would exhibit characteristics of a strongly interacting system. We may expect that a charm quark in quark matter can show some characteristics of a strongly interacting system, because its mass ($\simeq$1.3 GeV) can be very close to the critical mass. We emphasize that this estimate is only at a qualitative level and a more quantitative analysis will be needed for future studies.

IV. DISCUSSION

We have shown that the perturbation breaks down for any small coupling constant due to the logarithmic enhancement in the limit of large mass of the heavy impurity particles for both bosons and fermions. We have obtained those results for the case in which the scattering fermion is in the ground state, namely, $|\vec{q}\,| = k_F$ ($\eta = 0$), at zero temperature. For the case in which the scattering fermion is in the excited state with $|\vec{q}\,| > k_F$ ($\eta > 0$), the result is modified in a qualitative manner. In this case, there is no logarithmic term involving the mass of the heavy impurity particles. In the limit of small η ($\eta \to 0$), instead, a new logarithmic term, $\log\eta$, arises, as shown explicitly in Appendix B. Therefore, due to the logarithmic enhancement by the energy of the scattering fermions, the system becomes a strongly interacting one. Indeed, this is the Kondo problem as mentioned in the introduction [46,47]. In both approaches in the two different limits, the reason for the presence of logarithmic terms will be found in the dynamics of the particles and the holes in the loop contribution. However, we do not pursue this problem in the present discussion, because we must go beyond the one-loop level for a more detailed analysis. Instead, we shortly discuss the possible phenomena in nuclear and quark matter including charm and bottom flavor.

Let us consider the $\bar{D}$ and B mesons embedded in nuclear matter at zero temperature. When we apply the heavy-mass limit to this system, we are inevitably faced with the strong coupling problem as we have shown in Sec. II. We remember that the second-order scattering amplitude, Eq. (14), namely, the second-order vertex, has a logarithmic enhancement by M_B in the limit of large M_B. The logarithmic behavior appears only in the term with the isospin factor $\vec{\lambda}_f\cdot\vec{\lambda}_B$, and it does not appear in the term without the isospin factor. Therefore, we expect that the isospin-dependent interaction is much more enhanced than the isospin-independent one. The strong isospin dependence may cause some change of the structure of nuclear matter, because the properties of nuclear matter are much more sensitive to the isospin symmetry. We note that, however, the mass modifications of $\bar{D}$ and B mesons in a nuclear medium are not affected regardless of the logarithmic enhancement in the scattering amplitude. This is seen by the fact that, when the fermion outer lines are closed in Fig. 2, the contribution from the term proportional to the factor $\vec{\lambda}_f\cdot\vec{\lambda}_B$ becomes zero.

As explained in the introduction, there have been discussions about charmed (bottom) nuclei where charm (bottom) hadrons, such Λ_c and Σ_c (Λ_b and Σ_b) baryons and $\bar{D}$ and D (B and $\bar{B}$) mesons, are bound in atomic nuclei. We have to note that the current formalism is not applied, unless the impurity particle belongs to the fundamental representation of SU(n) symmetry ($n \geqslant 2$). For example, the Λ_c (Λ_b) baryon is an isospin singlet state and the Σ_c (Σ_b) baryon is an isospin triplet state, and hence the formalism cannot be applied. As candidates, we may consider the D ($\bar{B}$) meson in nuclear matter, because it is an isospin doublet state. However, it is actually unstable in nuclear matter, because it can decay through the transitions by two-body processes $DN \to \pi\Sigma_c^{(*)}$ ($\bar{B}N \to \pi\Sigma_b^{(*)}$) as well as three-body absorption processes $DNN \to \Lambda_c^{(*)}N$, $\Sigma_c^{(*)}N$ ($\bar{B}NN \to \Lambda_b^{(*)}N$, $\Sigma_b^{(*)}N$), which are opened below the thresholds. We will need to extend the formalism to include such complex processes. In contrast, $\bar{D}$ and B mesons in nuclear matter have no open channel below the thresholds, because there is no annihilation channel for light quark and antiquark pairs. As a result, we conclude that $\bar{D}$ and B mesons are unique hadrons for which the present discussion can be applied.

A similar discussion will be applied to the charm and bottom quarks in quark matter at zero temperature as presented in Sec. III. We recall that, in general, the coupling in the interaction between quarks becomes smaller in the high-density limit, due to the asymptotic freedom of QCD. According to the present discussion, however, charm and bottom quarks can interact strongly with the light quarks composing the Fermi surface, because there is a logarithmic enhancement from the charm (bottom) quark mass in the second-order scattering amplitude. Hence we will need to consider the strong coupling problem for quark matter including charm and bottom flavor as long as the heavy quark mass limit is adopted.

In both cases of $\bar{D}$ and B mesons in nuclear matter and charm and bottom quarks in quark matter, we find that the scattering amplitudes have a common property: a logarithmic enhancement by the mass of the heavy impurity particle. It will be an interesting future problem to study the strong coupling effect by logarithmic enhancement in the dynamics of nuclear matter and quark matter with charm and bottom flavor.

V. SUMMARY

We discussed the dynamics of a heavy impurity particle (hadron or quark) embedded in a finite-density medium at zero temperature. We supposed that the fermions composing the Fermi surface and the heavy impurity particle belong to the fundamental representation of SU(n) symmetry ($n \geqslant 2$) and that they interact through the vector current interaction with a small coupling constant. As systems, we considered a $\bar{D}$ (B) meson in nuclear matter with isospin symmetry ($n = 2$) and a charm (bottom) quark in quark matter with color symmetry ($n = 3$). We calculated the scattering amplitude for the fermion and the embedded heavy impurity particle. We analyzed the large-mass limit of the heavy impurity particle and found that the second-order scattering amplitude at the one-loop level contains a logarithmic term of the mass of the heavy impurity particle. Due to the logarithmic enhancement in the heavy-mass limit, the perturbation breaks down for any small coupling constant and the system becomes a strongly interacting one.

When the present result is applied to the $\bar{D}$ (B) meson in nuclear matter and charm (bottom) quark in quark matter, we expect them to be strongly interacting systems in the heavy-mass limit. To study more details, for example, we will need to consider a more realistic interaction such as the long-range pion exchange potential for the $\bar{D}$ (B) meson and gluon exchange for the charm (bottom) quark. It will be interesting to study such problems in future experiments at high-energy accelerator facilities with high-momentum hadron beams such as available at J-PARC and GSI-FAIR [51] as well as by using relativistic heavy-ion collisions at the Relativistic Heavy Ion Collider and the Large Hadron Collider [52,53].

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APPENDIX A: A CALCULATION FOR MOMENTUM INTEGRALS

We give a useful equation to derive Eq. (14) from Eq. (12). The three-dimensional $\vec{k}$ integrals in Eq. (12) are reduced to two-dimensional integrals with radial component $k \equiv |\vec{k}\,|$ and angular component $t \equiv \vec{q}\cdot\vec{k}/|\vec{q}\,||\vec{k}\,|$. When we expand up to the order of m/M for $m/M \ll 1$ (at least $m/M < 1/2$ is assumed) and neglect terms with $O((m/M)^2)$, we obtain

$$\begin{aligned}
&\int_{|\vec{k}\,|\geqslant k_{\mathrm{F}}} \frac{d^3\vec{k}}{(2\pi)^3} \frac{1}{\frac{1}{2m}\vec{q}^{\,2} - \frac{1}{2m}\vec{k}^{\,2} - \frac{1}{2M}|\vec{q}-\vec{k}|^2 + i\varepsilon} \\
&= \frac{1}{4\pi^2}\int_{-1}^{1} dt \int_{k_{\mathrm{F}}}^{\Lambda_{\mathrm{high}}} dk\, k^2 \frac{1}{\frac{1}{2m}q^2 - \frac{1}{2m}k^2 - \frac{1}{2M}(q^2+k^2-2qkt) + i\varepsilon} \\
&= \frac{1}{4\pi^2}\left(-\frac{2m}{1+m/M}\right)\int_{-1}^{1} dt\Bigg[(\Lambda_{\mathrm{high}} - k_{\mathrm{F}}) + \frac{q}{2}\left(1+\frac{m}{M}(2t-1)\right)\log\frac{\Lambda_{\mathrm{high}} - q\,[1-(1-t)m/M]}{|q\,[1-(1-t)m/M] - k_{\mathrm{F}}|} \\
&\quad - \frac{q}{2}\left(1-\frac{m}{M}(2t+1)\right)\log\frac{\Lambda_{\mathrm{high}} + q\,[1-(1+t)m/M]}{k_{\mathrm{F}} + q\,[1-(1+t)m/M]}\Bigg] \\
&\quad - i\pi\frac{1}{4\pi^2}\frac{2m}{1+m/M}\begin{cases}\int_{-1}^{1} dt\frac{q}{2}\left[1+\frac{m}{M}(2t-1)\right] & \text{for} \quad q(1-2m/M) > k_{\mathrm{F}}, \\ \int_{t_{\mathrm{F}}}^{1} dt\frac{q}{2}\left[1+\frac{m}{M}(2t-1)\right] & \text{for} \quad q(1-2m/M) < k_{\mathrm{F}}\end{cases} + O((m/M)^2), \qquad \text{(A1)}
\end{aligned}$$

for $|\vec{k}\,| > k_{\mathrm{F}}$ with higher cutoff parameter $\Lambda_{\mathrm{high}} > k_{\mathrm{F}}$, and

$$\begin{aligned}
&\int_{|\vec{k}\,|\leqslant k_{\mathrm{F}}} \frac{d^3\vec{k}}{(2\pi)^3} \frac{1}{\frac{1}{2m}\vec{q}^{\,2} - \frac{1}{2m}\vec{k}^{\,2} + \frac{1}{2M}|\vec{q}-\vec{k}|^2 - i\varepsilon} \\
&= \frac{1}{4\pi^2}\int_{-1}^{1} dt \int_{\Lambda_{\mathrm{low}}}^{k_{\mathrm{F}}} dk\, k^2 \frac{1}{\frac{1}{2m}q^2 - \frac{1}{2m}k^2 + \frac{1}{2M}(q^2+k^2-2qkt) - i\varepsilon}
\end{aligned}$$

$$= \frac{1}{4\pi^2}\left(-\frac{2m}{1+m/M}\right)\int_{-1}^{1} dt\left[(k_F-\Lambda_{\rm low})+\frac{q}{2}\left(1-\frac{m}{M}(2t-1)\right)\log\frac{q\,[1+(1-t)m/M]-k_F}{q\,[1+(1-t)m/M]-\Lambda_{\rm low}}\right.$$
$$\left. -\frac{q}{2}\left(1+\frac{m}{M}(2t+1)\right)\log\frac{k_F+q\,[1+(1+t)m/M]}{\Lambda_{\rm low}+q\,[1+(1+t)m/M]}\right]+O((m/M)^2), \tag{A2}$$

for $|\vec{k}\,|>k_F$ with lower cutoff parameter $\Lambda_{\rm low}<k_F$. Here we define $q=|\vec{q}\,|$ and

$$t_F=-\frac{1}{2}\left(\frac{M}{m}-1\right)\frac{q}{k_F}+\frac{1}{2}\left(\frac{M}{m}+1\right)\frac{k_F}{q}. \tag{A3}$$

APPENDIX B: THE KONDO PROBLEM REEXAMINED

For the heavy impurity boson, in the text, we consider that the scattering fermion is in the ground state with the condition $|\vec{q}\,|=k_F$ ($\eta=0$). Let us discuss the case in which the fermion is not the ground state but is in the excited state, whose energy lies above the Fermi surface: $|\vec{q}\,|>k_F$ ($\eta>0$). We expand the second-order scattering amplitude $-i\mathcal{M}_B^{(2)}$ in Eq. (12) by large M_B, keeping η fixed, and take the limit for small η. For nonrelativistic fermions, leaving only the leading terms, we obtain

$$\lim_{\eta\to0}\lim_{M_B\to\infty}\left(-i\mathcal{M}_B^{(2)}\right)_{\rm nonrel}$$
$$=(-iG_BM_B)^2\,\delta_{a'a}\delta_{b'b}i4\left(1-\frac{1}{n^2}\right)\frac{1}{2M_B}\frac{2mk_F}{4\pi^2}\left[-4\Lambda+\log\left(1+\frac{\Lambda}{2}\right)-\log\left(1-\frac{\Lambda}{2}\right)-i\pi\right]\bar{u}_{q,a'}\frac{1+\not{v}}{2}u_{q,a}$$
$$+(-iG_BM_B)^2\,(\vec{\lambda}_f)_{a'a}\cdot(\vec{\lambda}_B)_{b'b}i\frac{1}{2M_B}\frac{2mk_F}{4\pi^2}\left[-4\left(n-\frac{4}{n}\right)\Lambda+2n\log\Lambda+\left(-\frac{4}{n}\right)\log\left(1+\frac{\Lambda}{2}\right)-2\left(n-\frac{2}{n}\right)\log\left(1-\frac{\Lambda}{2}\right)\right.$$
$$\left.-2n\log\eta-\left(-\frac{4}{n}\right)i\pi\right]\bar{u}_{q,a'}\frac{1+\not{v}}{2}u_{q,a}. \tag{B1}$$

Instead of the factor $\log M_B$, there exists the new factor $\log\eta$ in the term proportional to $(\vec{\lambda}_f)_{a'a}\cdot(\vec{\lambda}_B)_{b'b}$. This is a singular term because it prevents a smooth connection from $\eta>0$ to $\eta=0$. Thus, we find again that the perturbation is not applicable for the small coupling constant G_B due to the presence of $\log\eta$, so that the system becomes a strongly interacting one. This is in fact the original Kondo problem that the electrons are affected by (pseudo)spin of an impurity atom with infinite mass [46,47].

For relativistic fermions, when we consider the excited fermions with $|\vec{q}\,|>k_F$ ($\eta>0$), the second-order contribution is given by

$$\lim_{\eta\to0}\lim_{M_B\to\infty}\left(-i\mathcal{M}_B^{(2)}\right)_{\rm rel}=(-iG_BM_B)^2\,\delta_{a'a}\delta_{b'b}i\frac{1}{2M_B}\frac{k_F^2}{4\pi^2}4\left(1-\frac{1}{n^2}\right)\bar{u}_{q,a'}\not{v}(-4\Lambda-i\pi)\gamma^0\not{v}u_{q,a}+(-iG_BM_B)^2\,(\vec{\lambda}_f)_{a'a}\cdot(\vec{\lambda}_B)_{b'b}$$
$$\times i\frac{1}{2M_B}\frac{k_F^2}{4\pi^2}\bar{u}_{q,a'}\not{v}\left[-4\left(n-\frac{4}{n}\right)+n\Lambda^2-2n\log\eta+2n\log\Lambda+i\pi\frac{4}{n}\right]\gamma^0\not{v}u_{q,a}, \tag{B2}$$

with the relativistic spinor u_q. We obtain the factor $\log\eta$ again. This is the relativistic version of the Kondo problem for massless fermions.

For the heavy impurity fermion, we proceed in a similar way. For the nonrelativistic fermion, we obtain the result

$$\lim_{\eta\to0}\lim_{M_F\to\infty}\left(-i\mathcal{M}_F^{(2)}\right)_{\rm nonrel}$$
$$=\left(-i\frac{G_F}{2}\right)^2\delta_{a'a}\delta_{b'b}i\frac{2mk_F}{4\pi^2}4\left(1-\frac{1}{n^2}\right)\left[-4\Lambda+\log\left(1+\frac{\Lambda}{2}\right)-\log\left(1-\frac{\Lambda}{2}\right)-i\pi\right]\bar{u}_{q,a'}\frac{1+\not{v}}{2}u_{q,a}\bar{u}_{P,b'}\frac{1+\not{v}}{2}u_{P,b}$$
$$+\left(-i\frac{G_F}{2}\right)^2(\vec{\lambda}_f)_{a'a}\cdot(\vec{\lambda}_F)_{b'b}i\frac{2mk_F}{4\pi^2}\left[-4\left(n-\frac{4}{n}\right)\Lambda+2n\log\Lambda+\left(-\frac{4}{n}\right)\log\left(1+\frac{\Lambda}{2}\right)-2\left(n-\frac{2}{n}\right)\log\left(1-\frac{\Lambda}{2}\right)\right.$$
$$\left.-2n\log\eta-\left(-\frac{4}{n}\right)i\pi\right]\bar{u}_{q,a'}\frac{1+\not{v}}{2}u_{q,a}\bar{u}_{P,b'}\frac{1+\not{v}}{2}u_{P,b}. \tag{B3}$$

There exists the factor $\log\eta$ in the terms proportional to $(\vec{\lambda}_f)_{a'a}\cdot(\vec{\lambda}_F)_{b'b}$, which gives a logarithmic divergence in the limit of $\eta\to0$. This is again the Kondo problem for the heavy impurity fermion as mentioned in the case of the heavy impurity boson.

When the scattering fermion is relativistic, we set $m = 0$ for massless fermions and obtain the result

$$\lim_{\eta\to 0}\lim_{M_{\rm F}\to\infty}\left(-i\mathcal{M}_{\rm F}^{(2)}\right)_{\rm rel} = \left(-i\frac{G_{\rm F}}{2}\right)^2 \delta_{a'a}\delta_{b'b} i\frac{k_{\rm F}^2}{4\pi^2} 4\left(1-\frac{1}{n^2}\right)\bar{u}_{q,a'}\not{v}(-4\Lambda - i\pi)\gamma^0\not{v}u_{q,a}\bar{u}_{P,b'}\frac{1+\not{v}}{2}u_{P,b}$$
$$+\left(-i\frac{G_{\rm F}}{2}\right)^2 (\vec{\lambda}_{\rm f})_{a'a}\cdot(\vec{\lambda}_{\rm F})_{b'b} i\frac{k_{\rm F}^2}{4\pi^2}\bar{u}_{q,a'}\not{v}\left[-4\left(n-\frac{4}{n}\right)+n\Lambda^2 - 2n\log\eta + 2n\log\Lambda + i\pi\frac{4}{n}\right]\gamma^0\not{v}u_{q,a}\bar{u}_{P,b'}\frac{1+\not{v}}{2}u_{P,b}, \tag{B4}$$

where we find $\log\eta$ as a logarithmically divergent term. This is again the relativistic version of the Kondo problem for massless fermion scattering.

[1] K. Tsushima and F. C. Khanna, Phys. Rev. C **67**, 015211 (2003).
[2] K. Tsushima and F. C. Khanna, Prog. Theor. Phys. Suppl. **149**, 160 (2003).
[3] K. Tsushima and F. C. Khanna, J. Phys. G **30**, 1765 (2004).
[4] T. Mizutani and A. Ramos, Phys. Rev. C **74**, 065201 (2006).
[5] M. Bayar, C. W. Xiao, T. Hyodo, A. Dote, M. Oka, and E. Oset, Phys. Rev. C **86**, 044004 (2012).
[6] E. Oset, M. Bayar, C. W. Xiao, T. Hyodo, A. Dote, and M. Oka, arXiv:1209.2955 [nucl-th].
[7] K. Tsushima, D.-H. Lu, A. W. Thomas, K. Saito, and R. H. Landau, Phys. Rev. C **59**, 2824 (1999).
[8] A. Sibirtsev, K. Tsushima, and A. W. Thomas, Eur. Phys. J. A **6**, 351 (1999).
[9] K. Tsushima and F. C. Khanna, Phys. Lett. B **552**, 138 (2003).
[10] T. Hilger, R. Thomas, and B. Kampfer, Phys. Rev. C **79**, 025202 (2009).
[11] T. Hilger, R. Schulze, and B. Kampfer, J. Phys. G **37**, 094054 (2010).
[12] Z.-G. Wang and T. Huang, Phys. Rev. C **84**, 048201 (2011).
[13] A. Mishra, E. L. Bratkovskaya, J. Schaffner-Bielich, S. Schramm, and H. Stöcker, Phys. Rev. C **69**, 015202 (2004).
[14] M. F. M. Lutz and C. L. Korpa, Phys. Lett. B **633**, 43 (2006).
[15] L. Tolos, A. Ramos, and T. Mizutani, Phys. Rev. C **77**, 015207 (2008).
[16] A. Mishra and A. Mazumdar, Phys. Rev. C **79**, 024908 (2009).
[17] A. Kumar and A. Mishra, Phys. Rev. C **81**, 065204 (2010).
[18] C. E. Jimenez-Tejero, A. Ramos, L. Tolos, and I. Vidana, Phys. Rev. C **84**, 015208 (2011).
[19] A. Kumar and A. Mishra, Eur. Phys. J. A **47**, 164 (2011).
[20] C. Garcia-Recio, J. Nieves, L. L. Salcedo, and L. Tolos, Phys. Rev. C **85**, 025203 (2012).
[21] S. Yasui and K. Sudoh, Phys. Rev. C **87**, 015202 (2013).
[22] F. Klingl, S. Kim, S. H. Lee, P. Morath, and W. Weise, Phys. Rev. Lett. **82**, 3396 (1999); **83**, 4224(E) (1999).
[23] Y.-H. Song, S. H. Lee, and K. Morita, Phys. Rev. C **79**, 014907 (2009).
[24] K. Morita and S. H. Lee, Phys. Rev. C **85**, 044917 (2012).
[25] A. Hayashigaki, Phys. Lett. B **487**, 96 (2000).
[26] B. Friman, S. H. Lee, and T. Song, Phys. Lett. B **548**, 153 (2002).
[27] S. Yasui and K. Sudoh, Phys. Rev. D **80**, 034008 (2009).
[28] Y. Yamaguchi, S. Ohkoda, S. Yasui, and A. Hosaka, Phys. Rev. D **84**, 014032 (2011).
[29] Y. Yamaguchi, S. Ohkoda, S. Yasui, and A. Hosaka, Phys. Rev. D **85**, 054003 (2012).
[30] Y.-R. Liu and M. Oka, Phys. Rev. D **85**, 014015 (2012).
[31] W. Meguro, Y.-R. Liu, and M. Oka, Phys. Lett. B **704**, 547 (2011).
[32] S. Ohkoda, Y. Yamaguchi, S. Yasui, K. Sudoh, and A. Hosaka, Phys. Rev. D **86**, 014004 (2012).
[33] S. Ohkoda, Y. Yamaguchi, S. Yasui, K. Sudoh, and A. Hosaka, Phys. Rev. D **86**, 034019 (2012).
[34] E. S. Swanson, Phys. Rep. **429**, 243 (2006).
[35] M. B. Voloshin, Prog. Part. Nucl. Phys. **61**, 455 (2008).
[36] M. Nielsen, F. S. Navarra, and S. H. Lee, Phys. Rep. **497**, 41 (2010).
[37] N. Brambilla *et al.*, Eur. Phys. J. C **71**, 1534 (2011).
[38] M. G. Alford, A. Schmitt, K. Rajagopal, and T. Schafer, Rev. Mod. Phys. **80**, 1455 (2008).
[39] K. Fukushima and T. Hatsuda, Rep. Prog. Phys. **74**, 014001 (2011).
[40] A. V. Manohar and M. B. Wise, Cambridge Monogr. Part. Phys. Nucl. Phys. Cosmol. **10**, 1 (2000).
[41] A. G. Grozin, Springer Tracts Mod. Phys. **201**, 1 (2004).
[42] G. Burdman and J. F. Donoghue, Phys. Lett. B **280**, 287 (1992).
[43] M. B. Wise, Phys. Rev. D **45**, 2188 (1992).
[44] T. M. Yan, H. Y. Cheng, C. Y. Cheung, G. L. Lin, Y. C. Lin, and H. L. Yu, Phys. Rev. D **46**, 1148 (1992); **55**, 5851(E) (1997).
[45] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio, and G. Nardulli, Phys. Rep. **281**, 145 (1997).
[46] J. Kondo, Prog. Theor. Phys. **32**, 37 (1964).
[47] A. C. Hewson, *The Kondo Problem to Heavy Fermions* (Cambridge University Press, Cambridge, England, 1993).
[48] Y. Akaishi and T. Yamazaki, Phys. Rev. C **65**, 044005 (2002).
[49] See T. Hyodo and D. Jido, Prog. Part. Nucl. Phys. **67**, 55 (2012), and the references therein.
[50] N. Kaiser, S. Fritsch, and W. Weise, Nucl. Phys. A **697**, 255 (2002).
[51] Y. S. Golubeva, W. Cassing, and L. A. Kondratyuk, Eur. Phys. J. A **14**, 255 (2002).
[52] S. Cho *et al.* (ExHIC Collaboration), Phys. Rev. Lett. **106**, 212001 (2011).
[53] S. Cho *et al.* (ExHIC Collaboration), Phys. Rev. C **84**, 064910 (2011).