PHYSICAL REVIEW C **88**, 014901 (2013)

Longitudinal fluctuations of the center of mass of the participants in heavy-ion collisions

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(Received 19 April 2013; published 1 July 2013)

A model for computing the probability density of event-by-event participant center-of-mass rapidity $y^{\text{c.m.}}$ is presented. The evaluations of the $y^{\text{c.m.}}$ distribution are performed for different collision energies and different centralities. We show that for certain conditions the rapidity distribution is described by a Gaussian with a variance determined mostly by the collision centrality. It is found that the width of the $y^{\text{c.m.}}$ distribution increases strongly for more peripheral collisions, while it depends weakly on the collision energy. Other theoretical estimates of rapidity distribution are presented and questions of interaction and separation between spectators and participants are discussed.

DOI: 10.1103/PhysRevC.88.014901 PACS number(s): 25.75.Ag, 24.10.Lx

I. INTRODUCTION

To describe the many-particle interacting system created in heavy-ion collisions (participant system) different models—such as hydrodynamics or kinetic transport models—are used. Along with the participants there are also spectators, which are nucleons emerging from the colliding nuclei that do not take part in any reaction with other nucleons during the collision process and move with their initial momenta. The number of spectators from each of the nuclei changes event by event (EbE) and, due to this fluctuation, the center of mass of participant system does not coincide with the collider center-of-mass system (cms); i.e., the participant c.m. rapidity, $y^{\text{c.m.}}$, may be nonzero in a particular event. The EbE fluctuations of $y^{\text{c.m.}}$ can be especially significant in peripheral collisions when the impact parameter of colliding nuclei is large and the mass of the spectators is essential.

When comparing different observables which depend on rapidity, for instance collective flow (calculated, e.g., in a hydrodynamical model) with experimental measurements, it could be important to account for participant c.m. fluctuations, which may influence the results [1–3]. The possible influence of EbE longitudinal fireball density fluctuations on the measurable two-particle rapidity correlation function was recently studied in [4].

In the present work a simplified model for the calculation of the EbE $y^{\text{c.m.}}$ distribution is presented, and center-of-mass rapidity fluctuations are discussed.

II. THE MODEL

A. Participant rapidity from spectators

We consider the collision of two identical heavy nuclei with mass number A, and we analyze the probability for a nucleon to become a spectator or a participant. The many-particle system created in heavy-ion collisions can be divided into three subsystems (Fig. 1): spectators from the projectile (A), spectators from the target (B), and participant particles (P). The conservation of four-momentum provides us with the following expressions for the total energy and longitudinal momentum in the collider cms:

$$E_{\text{tot}} = E_A + E_B + E_P, \tag{1}$$

$$P^z_{\text{tot}} = P^z_A + P^z_B + P^z_P = 0. \tag{2}$$

The c.m. rapidity of the participant system can then be expressed as

$$y_P = \frac{1}{2}\ln\frac{E_P + P^z_P}{E_P - P^z_P}. \tag{3}$$

Using Eqs. (1) and (2) we can express y_P in terms of spectator energy and momentum [3]:

$$y_{\text{c.m.}} \approx y_P = \text{arctanh}\left[\frac{-\left(P^z_A + P^z_B\right)}{E_{\text{tot}} - E_A - E_B}\right]. \tag{4}$$

Next we will neglect the initial Fermi motion of nucleons in the colliding nuclei compared to their collective collision energy. In this case we can express E_{tot}, $E_{A(B)}$, and $P^z_{A(B)}$ in terms of spectator numbers N_A and N_B as

$$E_{\text{tot}} = 2A\, p^0_{\text{i}}, \tag{5}$$

$$E_A = N_A\, p^0_{\text{i}}, \tag{6}$$

$$E_B = N_B\, p^0_{\text{i}}, \tag{7}$$

$$P^z_A = N_A\, p^z_{\text{i}}, \tag{8}$$

$$P^z_B = -N_B\, p^z_{\text{i}}, \tag{9}$$

where $p^0_{\text{i}} = \sqrt{s}/2$ and $p^z_{\text{i}} = \sqrt{s/4 - m_N^2}$ are the initial nucleon energy and momentumm respectively, and hence the spectator nucleon energy and momentum. Here $m_N = 938$ MeV/c^2 is the nucleon mass. The c.m. rapidity can be expressed now in terms of the spectator numbers N_A and N_B as

$$y_P(N_A,\, N_B) = \text{arctanh}\left(\frac{N_B - N_A}{2A - N_A - N_B}\, v_{\text{i}}\right), \tag{10}$$

where $v^{\text{i}} = p^z_{\text{i}}/p^0_{\text{i}}$ is the initial velocity of nucleons. It is seen from this relation that within our model only discrete sets of values of $y^{\text{c.m.}}$ are possible. This is a consequence of neglecting the Fermi motion of nucleons, which would smear the momenta of spectators if accounted for, and also a

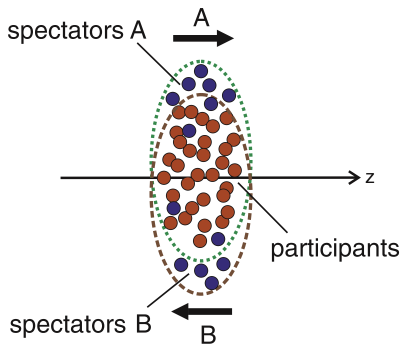

FIG. 1. (Color online) Decomposition of the system into spectators and participants. The possibility for nucleons to become spectators even if they are in the overlap region of colliding nuclei is specially illustrated.

consequence of neglecting the interaction between spectators and participants. Thus, we can determine the probabilities of different participant rapidities, $y^{\mathrm{c.m.}}$, if we can determine the probabilities of spectator numbers N_A and N_B.

B. Spectator number probability

The transverse distribution of spectators in the collision of heavy ions can be evaluated from the Glauber-Sitenko approach [5–7]:

$$\frac{d^2 N_{\mathrm{spec}}}{dx dy} = T_A(x - b/2, y)\left[1 - \frac{\sigma_{NN} T_B(x + b/2, y)}{A}\right]^A + T_B(x + b/2, y)\left[1 - \frac{\sigma_{NN} T_A(x - b/2, y)}{A}\right]^A, \tag{11}$$

where b is the impact parameter, σ_{NN} is the nucleon-nucleon reaction cross section, and

$$T_{A(B)}(x, y) = \int dz\, \rho_{A(B)}(x, y, z)$$

is the thickness function of the projectile (target) nucleus. Here

$$\rho_{A(B)}(x, y, z) \propto \left[1 + \exp\left(\frac{r - R}{\alpha}\right)\right]^{-1} \tag{12}$$

is the Woods-Saxon nuclear density distribution in nucleus. For large mass number, A, we have $(1 - \sigma_{NN} T_B/A)^A \approx \exp(-\sigma_{NN} T_B)$ and in this case Eq. (11) is often written in terms of exponents. The first term on the right-hand side of Eq. (11) is the transverse distribution of the spectators from the projectile nucleus and the second term is the transverse distribution of the spectators from the target nucleus. The probability that a nucleon from the projectile will become a spectator (which is the same as the probability for a nucleon from the target due to symmetry) can be expressed as

$$p_A = p_B = p = \frac{1}{A}\int dx\, dy\, T_A\,(x - b/2, y) \times \left(1 - \frac{\sigma_{NN} T_B(x + b/2, y)}{A}\right)^A. \tag{13}$$

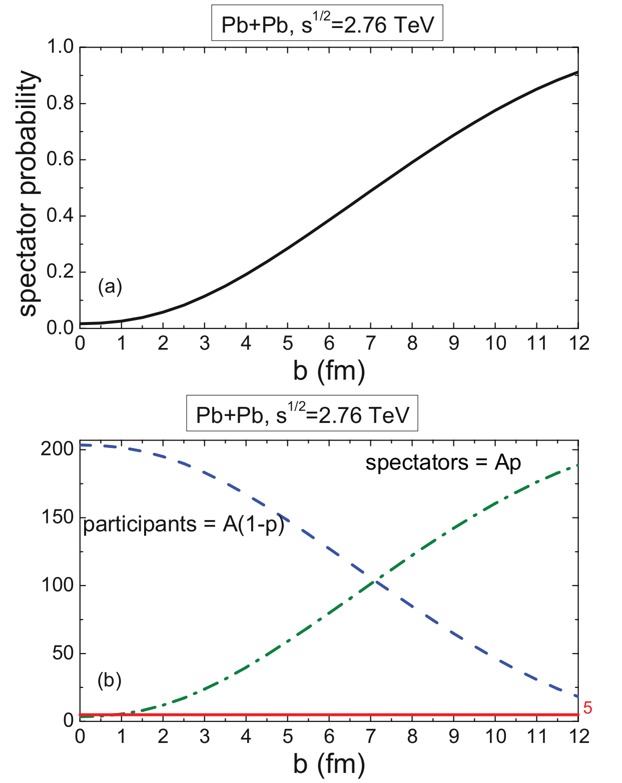

FIG. 2. (Color online) (a) The nucleon spectator probability dependence on impact parameter for Pb + Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV. (b) The same dependence of the average number of spectators (dotted green line) and participants (dashed blue line). The solid red line indicates the lower threshold value for these numbers, which determines the conditions when the Gaussian approximation of the Poisson distribution is applicable.

The dependence of this probability on the impact parameter for Pb + Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV is depicted in Fig. 2. The values of parameters used for calculations are $\sigma_{NN} = 70$ mb [8,9], $A = 208$, $R = 6.53$ fm, and $\alpha = 0.545$. Using parameter p from (13) we can determine the probability that there will be $N_{A(B)}$ spectators in the projectile (target) as a binomial distribution:

$$p(N_A) = \binom{A}{N_A} p^{N_A} (1 - p)^{A - N_A}, \tag{14}$$

$$p(N_B) = \binom{A}{N_B} p^{N_B} (1 - p)^{A - N_B}. \tag{15}$$

Here we have assumed that the initial many-nucleon distribution function can be approximately expressed as a product of one-nucleon distribution functions; i.e., the momenta and spatial positions of nucleons are uncorrelated. Next, one can take the number of spectators in the projectile as independent of the number of spectators in the target. This is not exactly true: e.g., if there are participants from one nucleus then there were reactions between the colliding nucleons, and there should also be participants from the other nucleus. This implies that, for a fixed number, N_A, of spectators in the projectile nucleus we can expect the number of spectators, N_B, in the target nucleus to fluctuate around the value $\langle N_B \rangle \approx N_A$. An analogous statement can be found in Ref. [10] where this subject was analyzed within the microscopic transport models. So, the numbers of spectators from different nuclei are not fully uncorrelated. Meanwhile, for the sake of simplicity, we assume that the number of spectators in the projectile is independent of the number of spectators in the target. But we can expect this approximation to work well if we have colliding heavy ions with large mass numbers. Using this approximation we can write

$$p(N_A,\ N_B) \approx p(N_A)\,p(N_B). \tag{16}$$

Using this probability one can then determine the distribution function of the corresponding c.m. rapidity of the participants [see Eq. (10)].

C. Gaussian approximation and rapidity distribution

As mentioned above, in our approach y^P takes discrete set of values. Because we neglect the smearing of momentum of spectators, the rapidity is defined solely by the spectator numbers, N_A and N_B, which so far take discrete sets of values. It is possible to obtain a continuous rapidity distribution if we allow the quantities N_A and N_B to take continuous values. It is well known that for some conditions the binomial distributions $p(N_A)$ and $p(N_B)$ can be accurately approximated by the Gaussian distribution with mean Ap and variance $Ap(1-p)$ as

$$p(N_{A(B)}) \Rightarrow \rho(N_{A(B)}) = \frac{\exp\left(-\frac{(N_{A(B)}-Ap)^2}{2Ap(1-p)}\right)}{\sqrt{2\pi Ap(1-p)}}. \tag{17}$$

In our case these conditions are the following: the average spectator and participant numbers, Ap and $A(1-p)$, are large enough, e.g., $Ap > 5$ and $A(1-p) > 5$. It is seen from Fig. 2 that these conditions are quite well satisfied in our model for heavy ions, especially for noncentral collisions. Using the Gaussian approximation (17) we can write the rapidity distribution function as

$$\begin{aligned} f_P(y) &= \int_0^A dN_A \int_0^A dN_B\, \rho(N_A)\,\rho(N_B) \\ &\quad\times \delta[y - y_P(N_A, N_B)] \\ &= \int_{-\infty}^{\infty} dN_A \int_{-\infty}^{\infty} dN_B\, \tilde{\rho}(N_A)\,\tilde{\rho}(N_B) \\ &\quad\times \delta[y - y_P(N_A, N_B)], \end{aligned} \tag{18}$$

where to switch to infinite integration limits we introduce

$$\tilde{\rho}(N_{A(B)}) = \rho(N_{A(B)})\,\theta(A - N_{A(B)})\,\theta(N_{A(B)}). \tag{19}$$

Let us now define the new variables

$$\begin{aligned} N &= \frac{1}{2}(N_A + N_B), \quad n = N_B - N_A, \\ f_P(y) &= \int_{-\infty}^{\infty} dN \int_{-\infty}^{\infty} dn\, \tilde{\rho}(N + n/2)\,\tilde{\rho}(N - n/2) \\ &\quad\times \delta\left[y - \operatorname{arctanh}\left(\frac{n}{2(A-N)}\,v_{\rm i}\right)\right]. \end{aligned} \tag{20}$$

Then we make a transformation to a new variable in the δ function in accordance with the rule $\delta[y - f(n;N)] = \delta[n - F(y;N)]/|f'(\overline{n})|$, where

$$\overline{n} = F(y;N) = \frac{1}{v_{\rm i}} 2(A-N)\tanh y. \tag{21}$$

After $f'(\overline{n})$ is introduced explicitly, the rapidity distribution becomes

$$f_P(y) = \int_{-\infty}^{\infty} dN\, \tilde{\rho}(N + \overline{n}/2)\,\tilde{\rho}(N - \overline{n}/2)\,\frac{2(A-N)}{v_{\rm i}\cosh^2 y}. \tag{22}$$

In order to compute the integral in (22) we will use the following approximation:

$$\tilde{\rho}(N \pm \overline{n}/2) \approx \rho(N \pm \overline{n}/2). \tag{23}$$

This approximation works well in the case when the original binomial distribution (15) is well approximated by the Gaussian (17). The presence of the Gaussian allows one to neglect the Heaviside theta functions in the integration in (22), which we perform using (17) for $\rho(N \pm \overline{n}/2)$ and obtain

$$f_P(y) = \sqrt{\frac{A(1-p)}{\pi p}}\,\frac{v_{\rm i}^2 \exp\left[-\frac{A(1-p)}{p}\,\frac{\tanh^2 y}{v_{\rm i}^2+\tanh^2 y}\right]}{\cosh^2 y\left[v_{\rm i}^2 + \tanh^2 y\right]^{\frac{3}{2}}}. \tag{24}$$

D. Ultrarelativistic limit and distribution at midrapidity

It is useful to analyze Eq. (24) in the ultrarelativistic limit, i.e., when $v_{\rm i} \to 1$. We obtain

$$f_P^{\rm UR}(y) = \sqrt{\frac{A(1-p)}{\pi p}}\,\frac{\exp\left[-\frac{A(1-p)}{p}\,\frac{\tanh^2 y}{1+\tanh^2 y}\right]}{\cosh^2 y[1+\tanh^2 y]^{\frac{3}{2}}}. \tag{25}$$

If we now consider collisions of identical nuclei at fixed impact parameter for different collision energies then we can see that the only parameter in Eq. (25) which depends on collision energy is the single-nucleon spectator probability p. It depends on collision energy only due to possible energy dependence of the nucleon-nucleon cross section σ_{NN} [see Eq. (13)]. It is known that for a wide energy range [e.g., for Super Proton Synchrotron (SPS) and Relativistic Heavy Ion Collider (RHIC) energies] σ_{NN} depends very weakly on collision energy and therefore we can claim that under such conditions the participant center-of-mass rapidity distribution is invariant of collision energy.

Another limit which can be explored is the distribution at midrapidity, i.e., around $y = 0$. We can expect that the c.m. rapidity fluctuations in heavy-ion collisions should be quite small (and subsequent calculations in next section seem to confirm this) and for small c.m. rapidity values $y^{\rm c.m.}$ we can approximate hyperbolic functions in Eq. (24) as $\cosh y \approx 1$ and $\tanh y \approx y$. Also, since we deal with relativistic collision energies $v_{\rm i}$ should be close to 1, which allows us to write $(\tanh y)^2 + v_{\rm i}^2 \approx v_{\rm i}^2$. With these approximations the rapidity distribution becomes

$$f_P(y) = \sqrt{\frac{A(1-p)}{\pi p\, v_{\rm i}^2}}\,\exp\left[-\frac{A(1-p)}{p\, v_{\rm i}^2}\,y^2\right]. \tag{26}$$

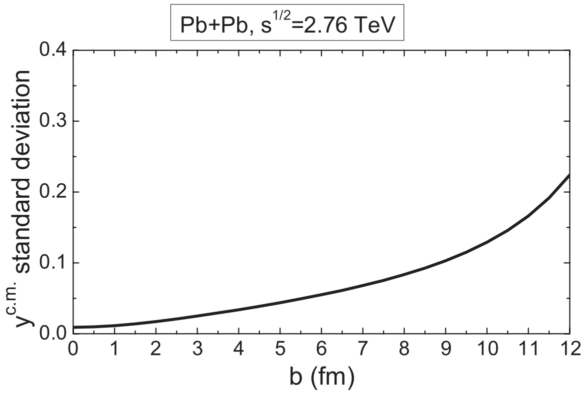

FIG. 3. The dependence of the standard deviation on the impact parameter. The calculations are made for Pb + Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV.

This is actually a Gaussian distribution around $y = 0$ with variance

$$\delta y^2 = \frac{p\, v_\text{i}^2}{2A(1-p)}. \tag{27}$$

The expression for the variance gives the following result: rapidity fluctuations are stronger for higher nucleon spectator probability p; i.e., they are increasing with the increase of impact parameter. For $p = 0$ there are no spectators in the system and therefore the collider cms and the participant cms coincide. This result is reproduced by Eq. (26) in our model.

The dependence of standard deviation, $\sqrt{\delta y^2}$, given by this Gaussian distribution on collision impact parameter for Pb + Pb collisions at $\sqrt{s} = 2.76$ TeV is depicted in Fig. 3. It is seen that the standard deviation of rapidity (which is basically a distribution width) stays significantly smaller than 1, hence justifying our approximation of rapidity distribution for small values of y. We can expect the total rapidity distribution given by the Gaussian in Eq. (26) to work well for most conditions in heavy-ion collisions.

Expression (27) for $\sqrt{\delta y^2}$ can be rewritten in terms of mass number A and average number of participants, $N_p = A(1 - p)$. It reads as

$$\delta y^2 = \frac{v_\text{i}^2}{2}\left(\frac{1}{N_p} - \frac{1}{A}\right). \tag{28}$$

It is interesting to explore the dependence of rapidity fluctuations on the average number of participants, N_p, for different pairs of colliding nuclei $A + A$. This dependence is depicted in Fig. 4 for Pb + Pb, In + In, S + S, and C + C collisions. There we take the initial nucleon velocity $v_\text{i} = 1$, since just ultrarelativistic collision energies are considered. Notice that, for fixed average number of participants, N_p, the rapidity fluctuations are stronger in collisions of heavier nuclei. For instance, if we consider central collisions of light nuclei, then the rapidity fluctuations in "equivalent" noncentral collisions of heavier nuclei will be bigger. Here "equivalent" means that in both colliding systems the average number of participants, N_p, is the same. A similar amplification of fluctuations with respect to the mass number was obtained in Ref. [10].

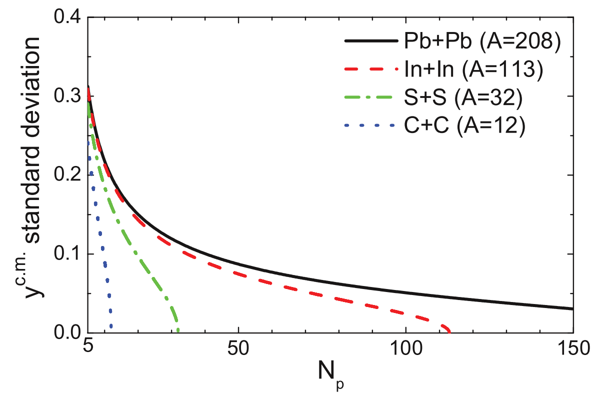

FIG. 4. (Color online) The dependence of standard deviation, $\sqrt{\delta y^2}$ [see Eq. (28)], of the rapidity distribution given by the Gaussian (26) on the average number of participants, N_p, for different colliding nuclei: Pb + Pb, In + In, S + S, and C + C.

III. CALCULATION RESULTS

Let us calculate the participant c.m. rapidity distribution for various collision conditions. The dependence of nucleon spectator probability p from (13) on the impact parameter was considered in Sec. II. This dependence for Pb + Pb collisions at $\sqrt{s} = 2.76$ TeV is depicted in Fig. 2(a). It is seen that the nucleon spectator probability strongly depends on the centrality of the collision: it is small, but nonzero, for central collisions, about 0.5 for midcentral collisions, and closer to unity for peripheral collisions.

Next, we will explore the participant c.m. rapidity distribution for different centralities but for the same collision energy using Eq. (24) for calculations. The rapidity distributions for Pb + Pb collisions at $\sqrt{s} = 2.76$ TeV are depicted in Fig. 5 for three different centralities: central ($b = 0$ fm), midcentral ($b = 6$ fm), and peripheral ($b = 9.5$ fm). It is seen that the rapidity distribution $f_P(y)$ depends strongly on the impact parameter, just as the nucleon spectator probability. At small impact parameter the $y^{\text{c.m.}}$ fluctuations are small and appear to be insignificant. However, increasing the impact parameter up to 9.5 fm (peripheral collisions) results in a significant increase of the $f_P(y)$ distribution width compared to central

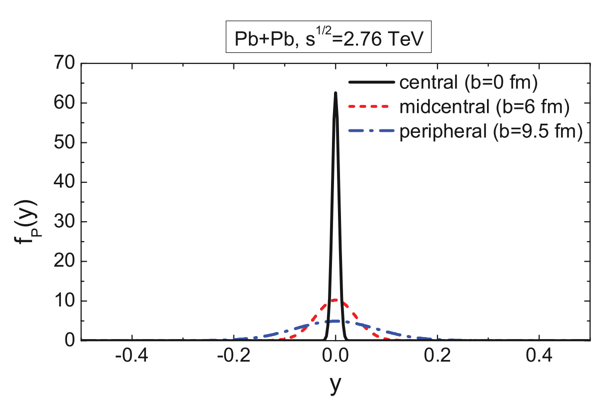

FIG. 5. (Color online) Participant center-of-mass rapidity distribution for Pb + Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV for three different centralities.

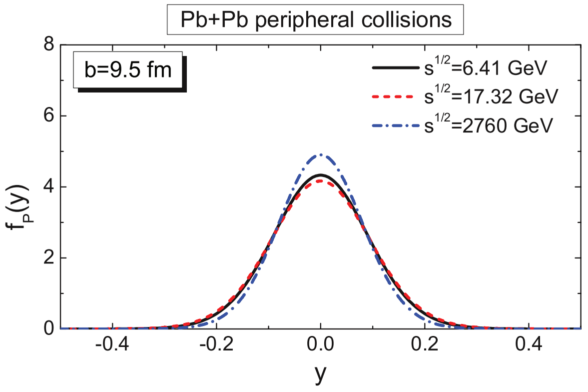

FIG. 6. (Color online) Participant center-of-mass rapidity distribution for peripheral Pb + Pb collisions at $\sqrt{s_{NN}} = 6.41$ GeV ($E_{\rm kin} = 20$ GeV), $\sqrt{s_{NN}} = 17.32$ GeV ($E_{\rm kin} = 158$ GeV), and $\sqrt{s_{NN}} = 2.76$ TeV.

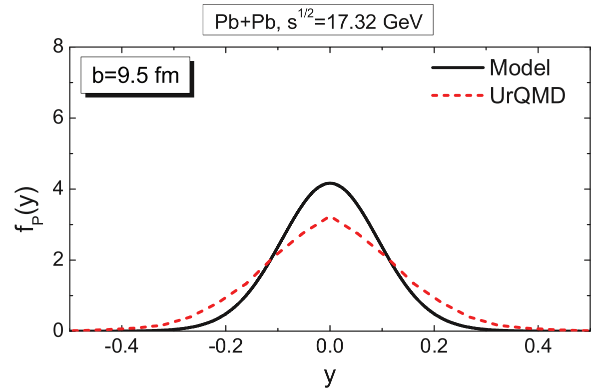

FIG. 7. (Color online) Participant center-of-mass rapidity distribution for Pb + Pb peripheral collisions at $\sqrt{s_{NN}} = 17.32$ GeV ($E_{\rm kin} = 158$ GeV) within the present model and in the UrQMD model.

collisions. So, in peripheral collisions the c.m. rapidity fluctuations may play an important role when calculating different measurable rapidity distributions [3]. As was discussed, the rapidity fluctuations are smaller in central collisions of light nuclei compared to the fluctuations in noncentral collisions of heavier nuclei when the number of participants is the same in both cases. It was also checked that in all three cases depicted in Fig. 5 the calculated rapidity distribution virtually coincides with the Gaussian distribution given by (26).

It is interesting to explore the influence of the collision energy on the participant c.m. rapidity fluctuations. To do that we consider Pb + Pb collisions at three different energies: $\sqrt{s_{NN}} = 6.41$ GeV ($E_{\rm kin} = 20$ GeV), $\sqrt{s_{NN}} = 17.32$ GeV ($E_{\rm kin} = 158$ GeV), and $\sqrt{s_{NN}} = 2.76$ TeV. The first two energies correspond to CERN SPS experiments and the third one to the CERN Large Hadron Collider (LHC) experiment. We take $\sigma_{NN} = 33$ mb for both SPS energies [8] and $\sigma_{NN} = 70$ mb for LHC energy. The calculation results for different energies for peripheral collisions ($b = 9.5$ fm) are presented in Fig. 6.

We can see that the collision energy influence on participant c.m. rapidity fluctuations is rather weak, especially compared to the centrality dependence. This can be explained by the fact that for high energies the rapidity distribution is well described in the ultrarelativistic limit (25), and the difference between LHC energy and SPS energies is due to doubling of the nucleon-nucleon cross section, which still does not lead to a significant change in rapidity fluctuations.

A. Other theoretical estimates

Longitudinal fluctuations arising from initial state fluctuations in the PACIAE parton and hadron molecular dynamics model were analyzed recently [11], and the fluctuation of the center-of-mass rapidity of the system was conservatively estimated to be $\Delta y^{\rm c.m.} = 0.1$, by neglecting all pre-equilibrium emission effects that increase the $y^{\rm c.m.}$ fluctuations.

The unique separation of participants and spectators in realistic situations is not trivial. Between the participants and spectators some level of interaction may remain at the separation, and a small number of nucleons cannot be classified definitely as either participants or spectators. This indefiniteness may result in an increase or a decrease of $y^{\rm c.m.}$ fluctuations. Longitudinal fluctuations may influence other observables also [12,13].

Next we will compare our model calculations with corresponding calculations within the ultrarelativistic quantum molecular dynamics (UrQMD) microscopic transport model [14,15]. The EbE c.m. rapidity can be computed in the UrQMD model by using Eq. (3). There we can account for the Fermi motion as well as for initial nucleon correlations; however, a large number of simulated UrQMD events are necessary to obtain a smooth distribution. The comparison of the participant c.m. rapidity distribution calculated within our model and with the UrQMD model for Pb + Pb collisions at $\sqrt{s_{NN}} = 17.32$ GeV ($E_{\rm kin} = 158$ GeV) with $\sigma_{NN} = 33$ mb for peripheral collisions ($b = 9.5$ fm) is presented in Fig. 7.

One can see that there is a difference in the distributions calculated within these two models: the distribution from the UrQMD model is wider. The difference is relatively small compared to the difference arising from changing the collision centrality (see Fig. 5). The difference, which is seen in Fig. 7, can be attributed to neglecting the initial many-nucleon correlations as well as the spectator number correlations for nucleons from the colliding nuclei, which were assumed in our model. It could also be questioned whether nucleons, which did not take part in any reaction in the UrQMD model, may be correctly identified as spectators in the Glauber-Sitenko approach.

The separation of spectators from participants is studied in Ref. [16]. Here the pre-equilibrium emission of one or two nucleons plays a non-negligible role. The (thermal) equilibration is demonstratively not present for particles, which interacted fewer than 4–6 times. These cannot be considered as parts of a participant system and usually have large longitudinal and small transverse momenta, although these do not reach the zero-degree calorimeters, so experimentally these are not identified as spectators. Similar considerations were used to describe the strangeness enhancement within the core-corona picture [17], where nucleons which have

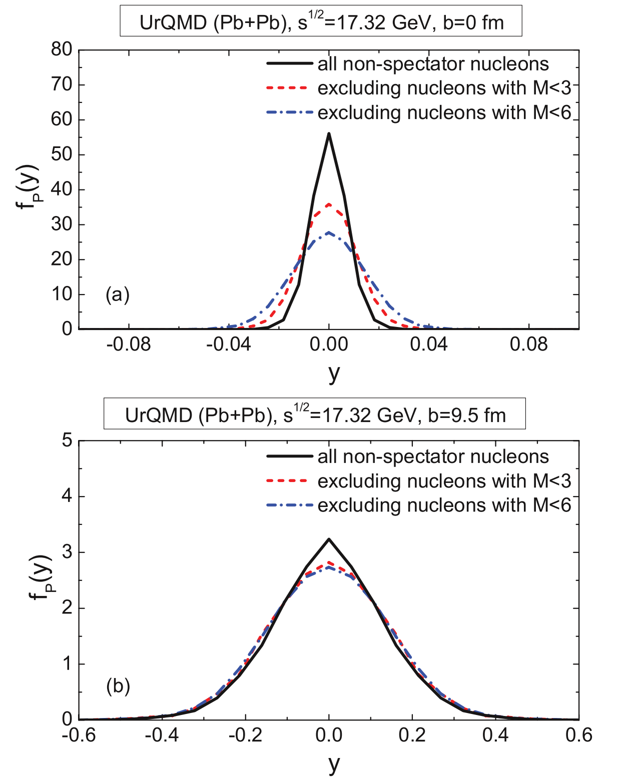

FIG. 8. (Color online) Participant center-of-mass rapidity distribution for (a) central and (b) peripheral Pb + Pb collisions calculated in the UrQMD model for different participant definitions, where nucleons which collided less often than the given limit are excluded from the participants.

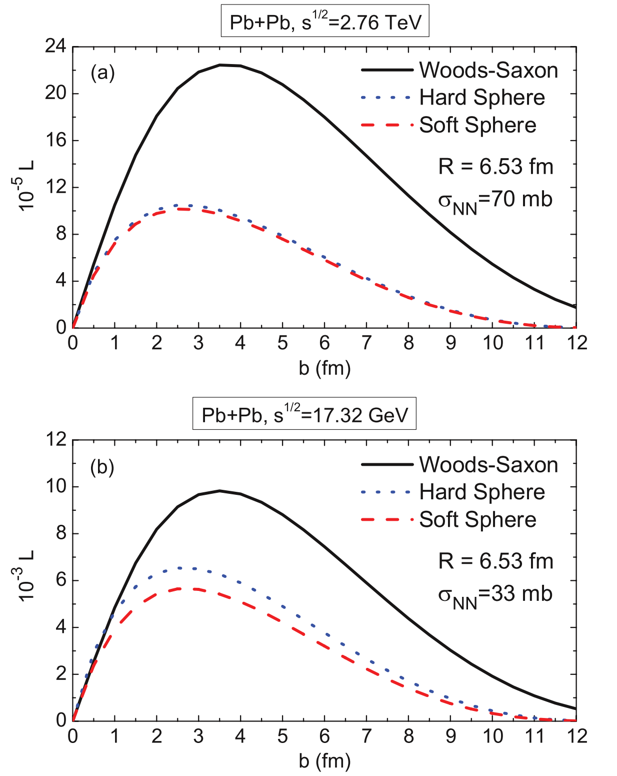

FIG. 9. (Color online) The dependence of total angular momentum of the participant system on impact parameter in Pb + Pb collisions for (a) LHC and (b) SPS conditions for different nuclear density profiles.

scattered only once were regarded as corona nucleons and were not part of a fireball. In central and semiperipheral reactions these pre-equilibrium particles may influence the $y^{\text{c.m.}}$ fluctuations considerably. For example, if we exclude nucleons from the participant system which collided fewer than six times, $M < 6$, then in central collisions the center-of-mass rapidity fluctuation doubles [see Fig. 8(a)]. However, there is little change in the rapidity distribution in case of peripheral collisions [Fig. 8(b)].

B. Participant angular momentum

The model also provides an estimate of the total angular momentum of the initial participant system. The angular momentum L_{tot}^P of the participant system can be calculated as the difference of total angular momentum L_{tot} and the angular momentum of spectators, L_{tot}^S. The quantities L_{tot} and L_{tot}^S can be calculated with the use of nuclei thickness functions $T_{A(B)}(x, y)$ and using the transverse distribution of spectators, $T_{A(B)}^S(x, y)$, as

$$L_{\text{tot}} = p_{\text{i}}^z \int dx\, dy\, x\, [T_A(x - b/2, y) - T_B(x + b/2, y)]\,,$$

$$L_{\text{tot}}^S = p_{\text{i}}^z \int dx\, dy\, x \left[T_A^S(x, y) - T_B^S(x, y)\right],$$

$$L_{\text{tot}}^P = L_{\text{tot}} - L_{\text{tot}}^S.$$

The transverse distribution of spectators can be determined from the Glauber-Sitenko model as (11). Another approach is to consider as participants all nucleons in the overlap region of colliding nuclei [1,2,18,19].

First of all the angular momentum for LHC Pb + Pb reactions at $\sqrt{s_{NN}} = 2.76$ TeV is about two orders of magnitude larger than that at SPS energy of $\sqrt{s_{NN}} = 17.32$ GeV (see Fig. 9) and one order of magnitude larger than for Au+Au reactions at RHIC energy of $\sqrt{s_{NN}} = 200$ GeV [19].

The angular momentum is the largest for nuclei with a Woods-Saxon radial density profile [see Eq. (12)], due to the presence of the diffusion zone with a tail, which effectively increases angular momentum. For nuclei with homogeneous nuclear density where the density profile has a sharp boundary we consider all nucleons from the overlap region as participants and all other nucleons as spectators (hard sphere nuclei). In this case the angular momentum is about a factor of 2 less than for a Woods-Saxon profile (see Fig. 9).

If, in addition, transparency in the overlap region is assumed due to the finite NN cross section (soft sphere nuclei) then the angular momentum is further reduced by 2% and 15% at LHC and SPS energies, respectively (see Fig. 9).

Thus, in fluid dynamical and in molecular dynamics models, the assumed initial state leaves some freedom for the angular momentum of the participant system.

IV. CONCLUSIONS

A simple model to calculate the participant c.m. rapidity distribution is developed and used to analyze the rapidity fluctuations for different conditions in heavy-ion collisions. In the model a weak initial nucleon-nucleon correlation in colliding nuclei and weak correlations between spectator numbers from different nuclei are assumed and the interaction between spectators and participants is neglected. The main input parameter in the model is the probability for a nucleon to be a spectator, which is determined from the Glauber-Sitenko approach in the present work. Different models for calculating this probability are applicable.

It is shown that for small rapidity values the rapidity distribution can be well approximated by the Gaussian distribution with variance determined by the nucleon spectator probability and by initial nucleon velocities. The calculation results confirm that this approximation works well in a wide range of collision energies and centralities.

It is shown that rapidity fluctuations strongly depend on impact parameter—they are stronger for more peripheral collisions and these fluctuations should be taken into account in calculation and interpretation of various rapidity-dependent observables [3]. It is necessary to note that, if we consider collisions of two different pairs of nuclei, for instance $A_1 + A_1$ and $A_2 + A_2$ with $A_1 > A_2$, where the number of participants is the same in both collisions, then the rapidity fluctuations are smaller in collisions $A_2 + A_2$ of lighter nuclei. Recent studies [3] indicate a possibility of experimental measurement of the $y^{\text{c.m.}}$ fluctuations.

The collision energy dependence of rapidity fluctuations appears to be weak. Comparison with similar c.m. rapidity distribution calculations within the UrQMD model shows qualitative agreement; however, some indefiniteness in identification of spectators and participants, for instance pre-equilibrium emission of nucleons, may lead to extra sources of participant c.m. fluctuations, especially at more central collisions.

ACKNOWLEDGMENTS

L.P.C. is thankful for the enlightening discussions and kind hospitality at the Frankfurt Institute for Advanced Studies, where he has done part of this work. D.A. was supported by the National Academy of Sciences of Ukraine (Project No. 0113U001092).

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