Role of neutrons and protons in entropy, spin cut off parameters, and moments of inertia

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The nuclear level densities, spin cut off parameters, and entropies have been extracted in $^{116-119}$Sn and ^{162,163}Dy nuclei using superconducting theory, which includes nuclear pairing interaction. The results agree well with the recent data obtained from experimental level densities by the Oslo group for these nuclei. Also, the entropy excess ratio proposed by Razavi *et al*. [R. Razavi, A.N. Behkami, S. Mohammadi, and M. Gholami, Phys. Rev. C **86**, 047303 (2012)] for a proton and neutron as a function of nuclear temperature have been evaluated and are compared with the spin cut off excess ratio. The role of the neutron (proton) system is well determined by the entropy excess ratio as well as the spin cut off excess ratio. The moment of inertia for even-odd and even-even nuclei are also compared. The moment of inertia carried by a single hole is smaller than the single particle moment of inertia.

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I. INTRODUCTION

The nuclear level density is a topic of current research interest and plays a special role in many branches of nuclear physics. Pairing correlations have a strong influence on nuclear level densities and special importance for many fermion systems [1,2]. Pairing correlations have been successfully described by the Bardeen-Cooper-Schrieffer (BCS) theory [3] of superconductivity.

The Oslo cyclotron group has developed a new method to extract level densities from measured gamma ray spectra using a 45 MeV ^{3}He beam [4], which has provided new experimental data in $^{116-119}$Sn [5,6] and ^{162,163}Dy nuclei [7,8].

In this work, the nuclear level densities and entropies have been computed for $^{116-119}$Sn and ^{162,163}Dy nuclei using the BCS Hamiltonian with the inclusion of pairing effects. The results are compared to their corresponding experimental values by the Oslo group. In our recent paper [9], we have studied the neutron and proton entropy excess ratio in ^{121}Sn compared to ^{122}Sn. We have shown that the proton system at low temperatures plays a minor role in entropy excess. In the present study, we have extended our investigation of the neutron and proton systems' role. A novel formula for the spin cut off excess ratio has been introduced. Then the ratio of proton as well as neutron spin cut off excess in ^{121}Sn compared to ^{122}Sn has been determined and the results are compared to the proton as well as neutron entropy excess ratios, respectively. Then calculations of the entropy excess ratio and spin cut off excess ratio are executed for ^{117}Sn compared to ^{118}Sn, ^{117}Sn compared to ^{116}Sn, ^{119}Sn compared to ^{118}Sn, and ^{163}Dy compared to ^{162}Dy.

II. SUMMARY OF THE THEORY

A brief review of the microscopic model to calculate the level density, spin cut off, and thermodynamic quantities of the system is presented in the following. The logarithm of the grand partition function of a paired system for one type of fermion is [10]

$$\Omega(\alpha,\beta) = -\beta\sum_k(\epsilon_k-\lambda-E_k) + 2\sum_k Ln[1+\exp(-\beta E_k)] - \beta\frac{\Delta^2}{G}. \quad (1)$$

In this expression $E_k = [(\varepsilon_k-\lambda)^2+\Delta^2]^{1/2}$ is the quasiparticle energy where ϵ_k is the fermion energy of single particle k. The quantity β is the inverse of the statistical nuclear temperature T, $\lambda = \alpha/\beta$ is related to the chemical potential, and Δ is the gap parameter that is a measure of nuclear pairing, while G is the pairing strength. The gap parameter Δ is obtained by solving the gap equation [11]

$$\frac{2}{G} = \sum_k \frac{1}{E_k}\tanh\frac{\beta E_k}{2}. \quad (2)$$

The state density can be calculated by an inverse Laplace transform of the grand partition function [12]

$$\omega(N,E) = \frac{\exp(S)}{2\pi|D|^{1/2}}, \quad (3)$$

where D is a determinant of the second derivations of the grand partition function taken at the saddle point and the entropy S can be written as [9,13]

$$S = 2\sum_k Ln[1+\exp(-\beta E_k)] + 2\beta\sum_k\frac{E_k}{1+\exp(\beta E_k)}. \quad (4)$$

The occupational probability of level k is given by the authors of Ref. [13]

$$n_k = 1 - \frac{\varepsilon_k-\lambda}{E_k}\tanh\frac{\beta E_k}{2} \quad (5)$$

and the saddle point conditions that must be satisfied are

$$N = \sum_k n_k, \quad (6)$$

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$$E = \sum_k n_k E_k - \frac{\Delta^2}{G}. \tag{7}$$

For a system of N neutrons and Z protons, the thermodynamical quantities Ω, the total energy E, and the total entropy S are additive. Therefore the solution is determined by

$$\Omega = \Omega_n + \Omega_p, E = E_n + E_p, \tag{8}$$

$$S = S_n + S_p. \tag{9}$$

The total level density is given by

$$\rho(N, Z, U) = \frac{\omega(N, Z, U)}{(2\pi\sigma^2)^{1/2}}, \tag{10}$$

where σ^2 is the spin cut off parameter defined as

$$\sigma^2 = \sigma_n^2 + \sigma_p^2 \tag{11}$$

with [14]

$$\sigma_p^2 = \frac{1}{2}\sum_k m_k^2 \mathrm{sech}^2\left(\frac{\beta E_k}{2}\right), \tag{12}$$

and a similar equation for σ_n^2. m_k is the magnetic momentum spin quantum number of the state k.

III. CALCULATIONAL PROCEDURE

We assume Z protons and N neutrons as two noninteracting systems. In performing calculations of level densities and entropies the single particle spins and energies were first extracted for a specified deformation. The calculations have been based on the modified harmonic oscillator potential according to the Nilsson potential [15]. The oscillator quantum number $\hbar\omega$ has been assigned the value of $41A^{-1/3}$ MeV. The quantities κ and μ for the proton system and the neutron system, which enter in the Nilsson potential, are taken from Ref. [16]. We calculate the neutron and proton pairing gap at zero temperature using the three point method [17] and known nuclear masses [18]. Then the values of parameters $\lambda(T)$ and $\Delta(T)$ and a set of occupational probabilities are estimated, and the saddle point conditions are checked for a given nucleon number. The procedure is repeated until the saddle point conditions are satisfied. The quantities S_p, E_p, and σ_p^2 are determined by applying Eqs. (4), (7), and (12), respectively. The excitation energy U_p can be determined by subtracting the energy at $T = 0$. A similar set of calculations is used to compute S_n, E_n, σ_n^2, and U_n for neutrons. The total spin cut off parameter σ^2 and total entropy S, at excitation energy $U = U_n + U_p$, are obtained using Eqs. (11) and (9), respectively. For more information on calculational procedures see Refs. [9,10,19].

IV. SUMMARY AND RESULTS

Numerical calculations of the level densities are performed for deformed nuclei $^{116-119}$Sn and ^{162,163}Dy with Eq. (10). However, the single particle spins and energies were first obtained for a specified deformation. The results of level densities as a function of excitation energy for ^{116,117}Sn and ^{163}Dy are shown in Figs. 1 to 3. Their corresponding experimental [5–8] values are also plotted for comparison. As can be seen from Figs. 1 to 3, the overall agreement between the experimental level densities and the microscopic theoretical with pairing correlations is satisfactory. Similar results were observed for ^{118,119}Sn and ^{162}Dy. We have also obtained entropies for $^{116-119}$Sn and ^{162,163}Dy nuclei with microscopic theory using known values of the single particle energy and magnetic spin quantum numbers. The evaluated entropies for ^{118,119}Sn nuclei as a function of nuclear temperature T, up to 1.0 MeV are plotted in Fig. 4. Our results show the entropy does not increase as smoothly as was explained in the macroscopic theory [20]. The step-like structures are interpreted as a signature of neutron pair breaking and, at higher energies, the possible quenching of pair correlations [21,22]. This structure

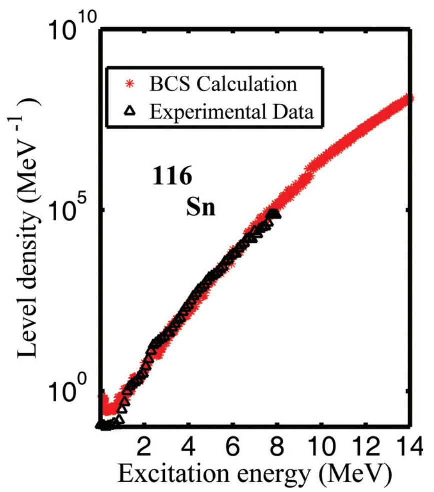

FIG. 1. (Color online) The experimental [6] and the calculated level density as a function of the excitation energy in ^{116}Sn nucleus. The open triangles are data from the (^{3}He, ^{4}He) reaction. The filled stars denote the BCS calculations.

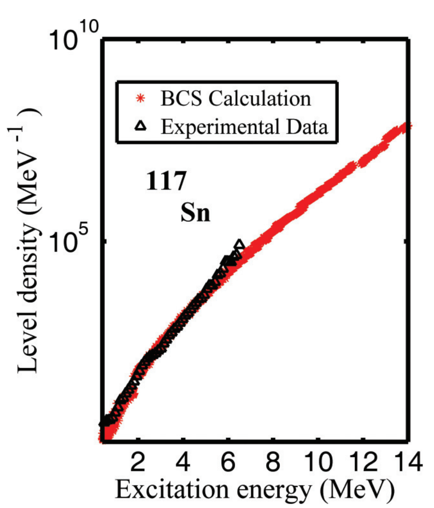

FIG. 2. (Color online) The experimental [6] and the calculated level density as a function of the excitation energy in ^{117}Sn nucleus. The open triangles are data from the (^{3}He, ^{3}He) reaction. The filled stars denote the BCS calculations.

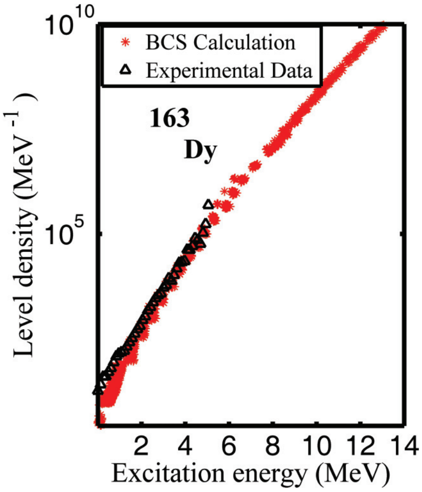

FIG. 3. (Color online) The experimental [8] and the calculated level density as a function of the excitation energy in ^{163}Dy nucleus. The open triangles are data from the (^{3}He, ^{4}He) reaction. The filled stars denote the BCS calculations.

is reflected in both the entropies and level densities. Similar behavior for ^{121,122}Sn has recently been described within the BCS theory in our previous publication [9]. The entropy of the odd A system follows closely the entropy for its even-even neighbors' nuclei, although there is an almost constant value between the entropy of the odd A nucleus and its even-even neighbors' nuclei. The entropy excess interpreted as the single hole and single particle entropy [19]

$$\Delta S(\text{hole}) = S(\text{odd}A) - S(A+1),$$
$$\Delta S(\text{particle}) = S(\text{odd}A) - S(A-1). \quad (13)$$

Figure 5 shows the calculated entropy excess between ^{163}Dy and ^{162}Dy. The entropy of the ^{163}Dy is about two units higher than for ^{162}Dy at temperatures below the critical temperature T_c. $\Delta S \sim 2$ is observed in several experiments of the rare earth nuclei [23–25]. The critical temperature in the combined

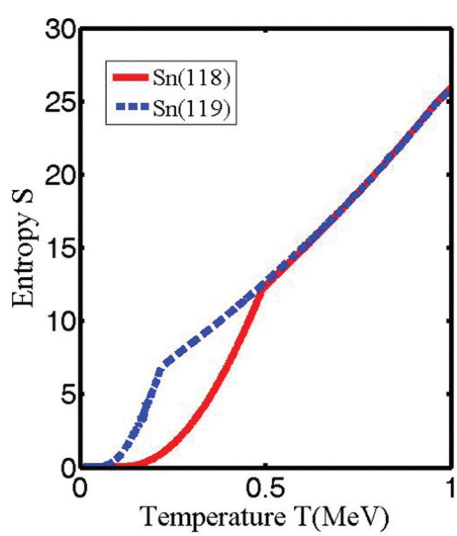

FIG. 4. (Color online) Relation between the entropy and nuclear temperature for ^{118,119}Sn.

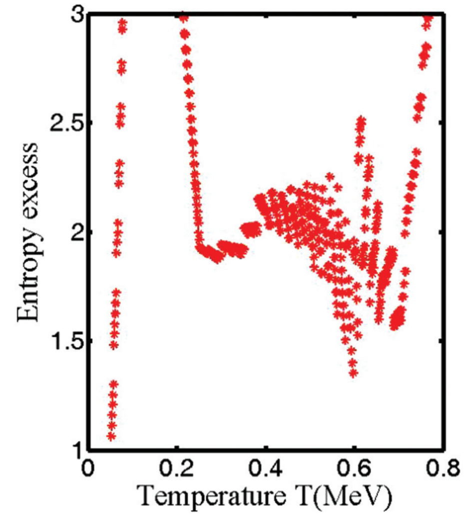

FIG. 5. (Color online) Calculated entropy excess between ^{163}Dy and ^{162}Dy.

system of both protons and neutrons is [26]

$$T_c = \frac{T_c^n + T_c^p}{2}. \quad (14)$$

The role of neutron and proton systems in entropy excess were investigated by using the neutron and proton entropy excess ratio $R_i (i = p, n)$ [9]

$$R_i = \frac{\Delta S_i}{\Delta S} \quad (15)$$

with

$$\Delta S_i = S_i(\text{odd}A) - S_i(A \pm 1). \quad (16)$$

We have examined this finding in more detail by a novel formula for the neutron and proton spin cut off excess ratio Rs_i

$$Rs_i = \frac{\Delta\sigma_i^2}{\Delta\sigma^2} \quad (17)$$

with

$$\Delta\sigma_i^2 = \sigma_i^2(\text{odd}A) - \sigma_i^2(A \pm 1) \quad (18)$$

and $\Delta\sigma^2$ is the total spin cut off excess. In Fig. 6 we have shown the evaluated spin cut off excess ratio for proton and neutron systems in ^{121}Sn compared to ^{122}Sn. It is seen that the spin cut off excess ratio Rs_i corresponds well with the calculated entropy excess ratio R_i [9]. The role of the neutron (proton) system is well described by the spin cut off excess ratio, as well as by the entropy excess ratio. The obtained results are also very similar for ^{119}Sn compared to ^{118}Sn, ^{117}Sn compared to ^{118}Sn, and ^{117}Sn compared to ^{116}Sn. The spin cut off excess ratio for ^{163}Dy compared to ^{162}Dy is shown in Fig. 7. The result shows that the protons make rather small contributions to the entropy excess and spin cut off excess, as expected. An examination of Figs. 6 and 7 reveals that the role of the proton system is slightly different in Dy isotopes compared to the magic tin isotopes and negative ratio in Fig. 7 is apparent. We have investigated in more detail as follows: The spin cut off

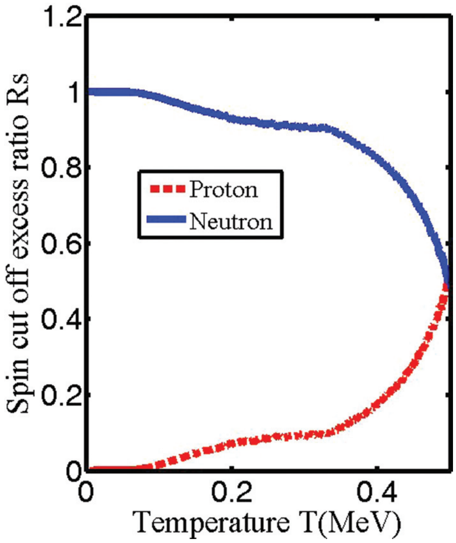

FIG. 6. (Color online) Spin cut off excess ratio for proton and neutron systems as a function of nuclear temperature in ^{121}Sn compared to ^{122}Sn.

σ^2 is related to an effective moment of inertia [27,28]

$$\sigma^2 = \frac{I_{\text{eff}} T}{\hbar^2}. \tag{19}$$

We introduce the neutron and proton moment of inertia ratio Rm_i as

$$Rm_i = \frac{I_i}{I}. \tag{20}$$

Figure 8 shows the extracted moment of inertia ratio for ^{121,122}Sn nuclei. The values of Rm_n for ^{121}Sn are higher than those for ^{122}Sn at intermediate energies and odd neutrons play a greater role in the moment of inertia. The obtained results for $^{116-119}$Sn are also very similar to the observed results in Fig. 8. If we denote the neutron and proton moment of inertia

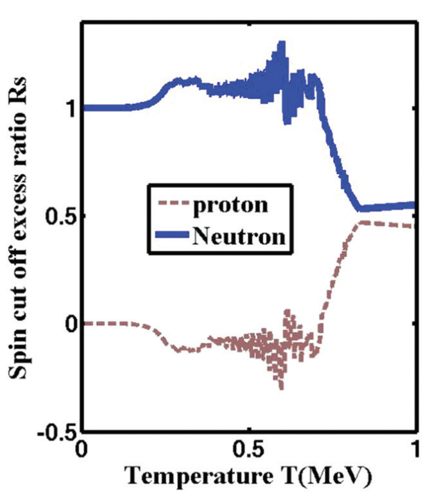

FIG. 7. (Color online) Spin cut off excess ratio for proton and neutron systems as a function of nuclear temperature in ^{163}Dy compared to ^{162}Dy.

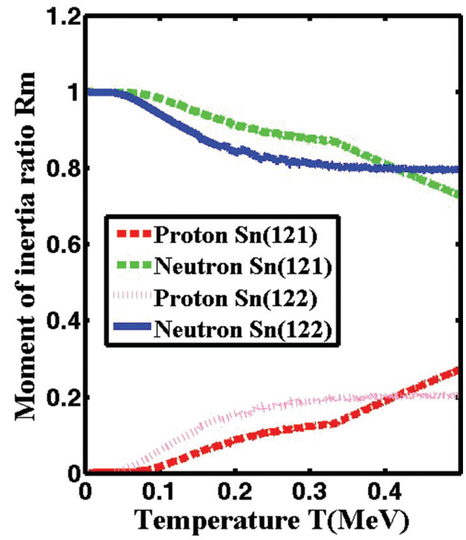

FIG. 8. (Color online) Proton and neutron moment of inertia ratio for ^{121,122}Sn nuclei.

excess

$$\Delta I_i = I_i(\text{odd}A) - I_i(A \pm 1) \tag{21}$$

and the moment of inertia carried by single-particle or hole moment of inertia ΔI as

$$\Delta I = I(\text{odd}A) - I(A \pm 1). \tag{22}$$

The moment of inertia excess ratio Rdm_i is given by

$$Rdm_i = \frac{\Delta I_i}{\Delta I} \tag{23}$$

and equal to the values of spin cut off excess ratio Rs_i (see Figs. 6 and 7).

An examination of these figures reveals that the major contribution to the total moment of inertia at low temperatures come mainly from the neutrons. The real nucleon-nucleon

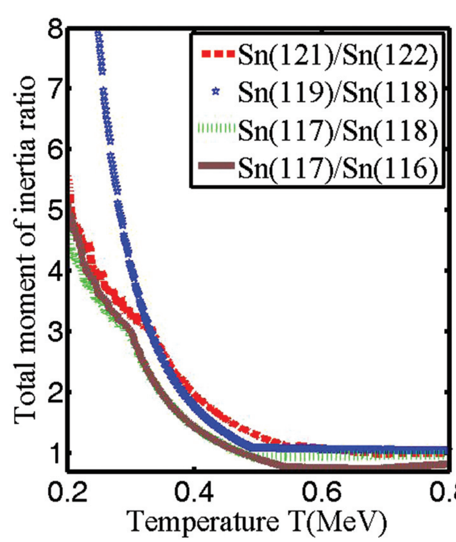

FIG. 9. (Color online) The total moment of inertia in odd mass nuclei compared to even mass nuclei for tin isotopes.

interaction is much more complicated than a short-range attractive effective interaction between identical valence nucleons that are defined by a δ-function potential. The negative values of the proton moment of inertia excess ratio Rdm_p (or Rs_p) are apparent from Fig. 7. We interpret the negative values at intermediate energies as a proton system deformation against the increase of the total moment of inertia. The total moment of inertia in odd mass nuclei compared to even mass nuclei for tin isotopes is shown in Fig. 9. Similar results are obtained for Dy isotopes. Our results show that the moment of inertia of the odd A nucleus is much larger than the even A nucleus at low energy.

In summary, we have shown that the superconducting theory, which includes the nuclear pairing interaction based on the modified harmonic oscillator according to the Nilsson potential, describes well the observed level densities and entropies for $^{116-119}$Sn and ^{162,163}Dy nuclei. The role of neutrons and protons in entropy, spin cut off, and moment of inertia have been investigated. The entropy excess ratio as well as the spin cut off excess ratio is a suitable factor for calculating the role of the neutron or proton system in nuclei.

We have shown a clear evidence of a phase transition from the superfluid state to the normal state and breaking of Cooper pairs. On the other hand, the total moment of inertia in odd mass nuclei has been compared to even mass nuclei. It is found that the single particle moment of inertia is much larger than the single hole moment of inertia.

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