Density dependence of the symmetry energy probed by β^--decay energies of odd-A nuclei

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The mass-dependent symmetry energy coefficients $a_{\mathrm{sym}}(A)$ are extracted with the β^--decay energies Q_{β^-} of heavy odd-A nuclei. The dominant position of this approach is that only the Coulomb energy survives in Q_{β^-} to determine the unknown $a_{\mathrm{sym}}(A)$. The obtained $a_{\mathrm{sym}}(A)$ is employed to analyze the density dependence of the nuclear matter symmetry energy around the saturation density. The estimated $a_{\mathrm{sym}}(A)$ of ^{208}Pb is 22.1–22.7 MeV. The slope parameter of the symmetry energy is found to be 50 ± 15 MeV, with the symmetry energy $S_0 = 32.3 \pm 1.3$ MeV at saturation density obtained from the analysis of pygmy dipole resonance (PDR) [Carbone *et al.*, Phys. Rev. C **81**, 041301(R) (2010)] as input. Furthermore, the corresponding neutron skin thickness in ^{208}Pb is estimated to be $\Delta R_{np} = 0.174 \pm 0.022$ fm.

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I. INTRODUCTION

The measured data on nuclear masses and collective excitations have allowed us to grasp some basic knowledge about nuclear matter at saturation density, such as the energy per particle, the symmetry energy, and the incompressibility. However, the variation of the equation of state (EOS) with respect to baryon density is still being intensely debated, especially the symmetry energy which characterizes its isospin dependence. In recent decades, the symmetry energy has been one of the focuses of nuclear physics because it plays a crucial role in understanding a variety of issues in nuclear physics [1–6] and astrophysics [7–12]. The behavior of density-dependent symmetry energy at low densities is routinely treated by series expansion. Around the nuclear matter saturation density $\rho_0 = 0.16$ fm^{-3}, the symmetry energy $S(\rho)$ is expanded to second order in density as

$$S(\rho) = S_0 + \frac{L}{3}\left(\frac{\rho-\rho_0}{\rho_0}\right) + \frac{K_{\mathrm{sym}}}{18}\left(\frac{\rho-\rho_0}{\rho_0}\right)^2 + \cdots, \quad (1)$$

where S_0 is the symmetry energy at density ρ_0. $L = 3\rho\partial S(\rho)/\partial\rho|_{\rho_0}$ and $K_{\mathrm{sym}} = 9\rho^2\partial^2 S/\partial\rho^2|_{\rho_0}$ are respectively the slope and curvature parameters at ρ_0 that characterize the density dependence of the symmetry energy around density ρ_0. Moreover, the slope parameter L has been found to correlate linearly with the neutron skin thickness ΔR_{np} of heavy nuclei such as ^{208}Pb [13–15]. Therefore, a measurement of ΔR_{np} with a high accuracy is an effective approach to constrain the density dependence of $S(\rho)$, but it is a difficult task because the neutron skin is not directly an observable and most of the experimental analysis that estimate its value are model dependent. Instead, one can in turn use the extracted L value to constrain the neutron skin thickness.

Many independent investigations have been performed to constrain the density dependence of the symmetry energy, such as the microscopic Brueckner-Hartree-Fock approach [16], the transport model [17,18], giant resonance [19], measured nuclear masses [20] and the neutron skin thickness of heavy nuclei [21,22]. Very recently, Sotani *et al.* constrained the slope parameter as $L \gtrsim 50$ MeV by the identification of the lowest quasiperiodic oscillation frequency observed from SGR 1806-20 [23]. On the basis of an empirical approach and the results of different energy density functionals, Agrawal *et al.* estimated a value of $L = 64 \pm 5$ MeV [24]. Möller *et al.* determined $L = 70 \pm 15$ MeV with the help of a more accurate FRDM-2011a adjustment of the model constants to new and more accurate experimental masses [25]. Steiner and Gandolfi provided a tighter constraint to $43 < L < 52$ MeV within 68% confidence via astrophysical observations of neutron star masses and radii [26]. Roca-Maza *et al.* estimated the slope parameter to be $L = 37 \pm 18$ MeV by exploiting this correlation together with the experimental values of the isoscalar and isovector giant quadrupole resonance energies [27]. A detailed summary of the recent progress can be found in Ref. [28]. Up to now, although significant progress has been achieved around subsaturation density, the problem remains

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unsolved owing to its difficulty and complexity. In the present work, we employ the β^--decay energies of heavy odd-A nuclei to derive the symmetry energy coefficient $a_{\mathrm{sym}}(A)$ of heavy nuclei and then explore the density dependence of the nuclear matter symmetry energy.

II. SYMMETRY ENERGY COEFFICIENT OF HEAVY NUCLEI AND INFINITE NUCLEAR MATTER

The $a_{\mathrm{sym}}(A)$ of finite nuclei is usually extracted by fitting the measured nuclear masses [20,29–33]. In this study, we extract the $a_{\mathrm{sym}}(A)$ with experimental β^--decay energies Q_{β^-} of heavy odd-A nuclei [34]. It has been well established that the symmetry energy and the Coulomb energy mainly determine where nuclei are stable against β decay [35,36]. The reasons why we select the β^--decay energies of odd-A nuclei are manifold. Firstly, the nuclei for which β^- decay occur are neutron rich. In other words, their isospin asymmetry $\beta = (\rho_n - \rho_p)/\rho$ is relatively large, which differs from neutron-deficient nuclei with β^+ decay, and hence it is more conducive for the present investigation. Secondly, due to such a large isospin asymmetry, the Wigner energy, which is not very well known, can be furthest depressed, which is another advantage over the β^+ emitters. Finally, the employment of odd-A nuclei makes the corresponding pairing energy negligible, which differs from the cases of even-even nuclei and odd-odd nuclei, to which the pairing energy contributes significantly, as shown later. Besides, the nuclei with magic numbers are excluded to avoid shell effects. The central purpose is to reduce the uncertainties as far as possible. The β^--decay energy is given by

$$\begin{aligned} Q_{\beta^-} &= m(Z, A) - m(Z+1, A) - m_e \\ &= (m_n - m_p - m_e) + B(Z+1, A) - B(Z, A), \end{aligned} \tag{2}$$

with $m_n - m_p - m_e = 0.782$ MeV [37]. $B(Z, A)$ is the binding energy of a nucleus that has been widely investigated [38–41], which can be described by the well known liquid drop formula

$$\begin{aligned} B(Z, A) = a_v A - a_s A^{2/3} + E_c - a_{\mathrm{sym}}(A)\beta^2 A \\ + E_p + E_W + \cdots . \end{aligned} \tag{3}$$

The Coulomb energy, including charge exchange correction, is given by

$$E_c = -a_c \frac{Z(Z-1)}{A^{1/3}(1+\Delta)}(1 - 0.76 Z^{-2/3}), \tag{4}$$

where the parameter Δ was introduced by Danielewicz [35] to describe the effect of the Coulomb interaction on the surface asymmetry and the effect of the surface diffuseness on the Coulomb energy, taking the following form:

$$\Delta = \frac{5\pi^2}{6}\frac{d^2}{r_0^2 A^{2/3}} - \frac{1}{1 + A^{1/3}/\kappa}\frac{N-Z}{6Z}. \tag{5}$$

$d \approx 0.55$ fm [35] is the diffuseness parameter in the Fermi function from the parametrization of nuclear charge distributions and r_0 is the nuclear-radius constant satisfying $3/(4\pi r_0^3) = 0.16$ fm^{-3}. The meaning of the κ is discussed later. These two effects reflected by the Δ tend to be neglected in many investigations, but cannot be discarded here due to its non-negligible contributions. The parameter $a_c = 0.71$ is known very well [29,42]; in particular it is well determined from the masses of mirror nuclei [43,44]. Thus, the contribution of the Coulomb energy to Q_{β^-} is

$$\Delta E_c = -a_c \frac{2Z}{A^{1/3}(1+\Delta)} + 1.013 a_c \frac{Z^{1/3}}{A^{1/3}(1+\Delta)}, \tag{6}$$

where the difference of the Δ between the parent nucleus and its daughter one is neglected since it is quite small.

The pairing energy is usually calculated with the following expressions used for spherical nuclei [45]:

$$-E_p = \begin{cases} 4.8/N^{1/3} + 4.8/Z^{1/3} - 6.6/A^{2/3}, & \text{odd-}Z\text{–odd-}N, \\ 4.8/Z^{1/3}, & \text{odd-}Z\text{–even-}N, \\ 4.8/N^{1/3}, & \text{even-}Z\text{–odd-}N, \\ 0, & \text{even-}Z\text{–even-}N. \end{cases} \tag{7}$$

For odd-odd and even-even nuclei, the pairing energy contributes a value of $\sim$2 MeV and $\sim$−2 MeV to Q_{β^-}, respectively, while it just contributes $\sim$0.1 MeV for odd-Z–even-N nuclei and $\sim$−0.1 MeV for even-Z–odd-N nuclei. Here, both odd-Z and even-Z nuclei belonging to odd-A nuclei are included for the calculation of the Q_{β^-}, and hence the effects of the pairing interaction are suppressed on average to a large extent. The Wigner energy leading to a stronger interaction for a neutron and proton with congruent nodal structures takes the form of $E_w = -10\exp(-4.2\beta)$ [46] in units of MeV, and hence its contribution to Q_{β^-} is estimated to be $\Delta E_w = 84\exp(-4.2\beta)/A$ with a magnitude of $\sim$0.2 MeV. The charge-asymmetry energy in the binding energy is given as $C_a(N-Z)$ where $C_a = 0.102\,89$ MeV [47], and therefore its contribution to Q_{β^-} is $-2C_a = -0.2$ MeV, which cancels out the ΔE_w of the Wigner energy. The shell correction in the binding energy of a parent nucleus and its daughter one should be close to each other because the densities of the energy levels are not expected to change distinctly when decay occurs. Accordingly, the shell energy corrections to their masses could be canceled to a large extent, leading to a negligible correction to Q_{β^-}. The small deviation of the formula for Q_{β^-} shown later supports this point. Consequently, the β^--decay energy can be rewritten as

$$Q_{\beta^-} = 0.782 + \Delta E_c + 4\left(\beta - \frac{1}{A}\right) a_{\mathrm{sym}}(A). \tag{8}$$

Different from the single-particle binding energy which is proportional to β^2, the Q_{β^-} is related to β. The contribution of the Coulomb energy is known relatively well, which is the primary advantage of this approach. The experimental Q_{β^-} values of sixty heavy nuclei around the ^{208}Pb from $A = 171$ to $A = 253$ are used for the following analysis. The parent or the daughter nuclei with magic numbers in this range are excluded to avoid strong shell effects.

TABLE I. The calculated symmetry energy coefficient $a_{\mathrm{sym}}(A)$ of ^{208}Pb, the symmetry energy S_0 of nuclear matter at saturation density ρ_0, and the reference density ρ_A for ^{208}Pb, based on the Skyrme energy density functionals. The Skyrme interactions used in [48] are employed.

	SIII	SLy4	SLy5	SKM*
a_{sym} (MeV)	22.1	23.5	23.3	20.8
S_0 (MeV)	28.2	32.0	32.0	30.0
ρ_0 (fm^{-3})	0.145	0.160	0.161	0.160
ρ_A	$0.55\rho_0$	$0.56\rho_0$	$0.56\rho_0$	$0.54\rho_0$

The mass dependence of the symmetry energy coefficient $a_{\mathrm{sym}}(A)$ of heavy nuclei in Eq. (8) is written as [49]

$$a_{\mathrm{sym}}(A) = \frac{S_0}{1+\kappa A^{-1/3}}, \quad \text{with} \quad \kappa = \frac{9}{4}\frac{S_0}{Q}, \tag{9}$$

where κ is the ratio of the surface symmetry coefficient to the volume symmetry coefficient and Q is the surface stiffness that measures the resistance of a nucleus against separation of neutrons from protons to form a neutron skin. Centelles *et al.* proposed a useful relation that the $a_{\mathrm{sym}}(A)$ of finite nuclei is approximately equal to $S(\rho_A)$ of nuclear matter at a reference density ρ_A [21]. For example, the reference density for ^{208}Pb is $\rho_A = 0.1$ fm^{-3} [21]. This relation bridges the symmetry energy of the nuclear matter and the one of finite nuclei, and thus allows us to explore the density dependence of the symmetry energy $S(\rho)$. The symmetry energy coefficient $a_{\mathrm{sym}}(A)$ was directly extracted in the framework of the Skyrme energy density functional in our previous work [48]. We would like to determine the reference density ρ_A as a continuation of the work [48], which will be used in the following part. The $a_{\mathrm{sym}}(A)$ and $S(\rho_A)$ are treated with the same interactions without introducing additional assumptions. The calculated ρ_A listed in Table I show that the $\rho_A \sim 0.55\rho_0$ for ^{208}Pb. Although the different interactions give the different symmetry energy (coefficients) of the finite nuclei as well as the nuclear matter, the obtained ρ_A are almost the same. The saturation density ρ_0 has not been completely determined yet, so the ρ_A is in units of ρ_0 in order to reduce the uncertainty to a large degree and to make Eq. (10) more convenient to use. The specific calculation process can be represented as follow: (1) With the experimental Q_{β^-} values of heavy odd-A nuclei, one needs to fit the S_0 value and the parameter κ (S_0 is not fitted but as an input in the present study as shown latter), and then obtain the $a_{\mathrm{sym}}(A)$ of ^{208}Pb. (2) With the help of the obtained $a_{\mathrm{sym}}(^{208}\mathrm{Pb})$ the density dependence of the nuclear matter symmetry energy at subnormal densities is explored. We use the formulism from the DDM3Y interaction in Ref. [30,50] to describe the density dependence of the symmetry energy $S(\rho)$,

$$S(\rho) = 13.0\left(\frac{\rho}{\rho_0}\right)^{2/3} + C_1\left(\frac{\rho}{\rho_0}\right) + C_2\left(\frac{\rho}{\rho_0}\right)^{5/3}, \tag{10}$$

which is not an empirical formula and is found to be better than the widely used expressions $S(\rho) = S_0(\rho/\rho_0)^{\gamma}$ and $S(\rho) = 12.5\,(\rho/\rho_0)^{2/3} + C_p\,(\rho/\rho_0)^{\gamma}$. Here C_1 and C_2 are linked by $S_0 = 13.0 + C_1 + C_2$ and $S(\rho_A) = 13.0\,(\rho_A/\rho_0)^{2/3} + C_1\,(\rho_A/\rho_0) + C_2\,(\rho_A/\rho_0)^{5/3}$. Thus, the two parameters C_1 and C_2 can be determined uniquely.

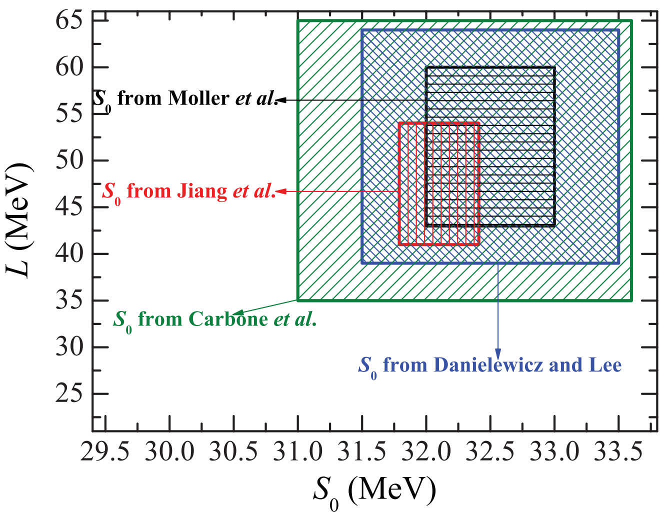

FIG. 1. (Color online) L values are obtained by using some S_0 values with narrow regions as inputs. The horizontal axis denotes the S_0 values in the range of 31.6 ± 2.2 MeV and the vertical axis denotes the correspondingly calculated L values. Some S_0 with narrow regions from Möller *et al.* [25], Jiang *et al.* [51], Danielewicz and Lee [36], and Carbone *et al.* [19] are employed to further constrain the L value.

It is difficult to fit S_0 and κ directly to obtain their optimal values since many different combinations of S_0 and κ could provide the same least deviation. Considering that S_0 has been determined as 31.6 ± 2.2 MeV at present [50], we solely determine the optimal value of κ (carrying error bars) taking the S_0 as inputs, where the uncertainty mainly results from the uncertainty of S_0 values. Recently, the S_0 value has been constrained to rather narrow regions by some authors, which will be used as inputs in the present study to fit the κ value and then to determine the slope parameter L. S_0 has been determined to be 32.5 ± 0.5 MeV by Möller *et al.* from the mass systematics [25], and the value matches very well with the one from the double differences of "experimental" symmetry energies [51]. Carbone *et al.* obtained a value of $S_0 = 32.3 \pm 1.3$ MeV using the correlation between L and S_0 [19]. Danielewicz and Lee obtained $S_0 = 31.5$–33.5 MeV from the calculations of half-infinite matter [36]. Employing these detailed S_0 values from the four independent investigations as inputs in combination with Eq. (10), the L values are calculated and the results are plotted in Fig. 1. All the calculations suggest that the most likely value of the slope parameter is $L \sim 50$ MeV. The S_0 from Carbone *et al.* covering the other three is naturally believed to be the most acceptable one. With their $S_0 = 32.3 \pm 1.3$ MeV as input, the calculated value of κ is 2.61 ± 0.46, thus the symmetry energy coefficient of ^{208}Pb is 22.1–22.7 MeV. The parameters in Eq. (10) are $C_1 = 36.4 \mp 4.3$ MeV and $C_2 = -17.1 \pm 5.6$ MeV, and correspondingly the slope and curvature parameters of nuclear symmetry energy are $L = 50 \pm 15$ MeV and $K_{\mathrm{sym}} = -197 \pm 56$ MeV, respectively. Here, we would like to stress that the determination of the L and K_{sym} values relies on the reference density ρ_A. If $\rho_A = 0.1$ fm^{-3} for ^{208}Pb and $\rho_0 = 0.16$ fm^{-3}

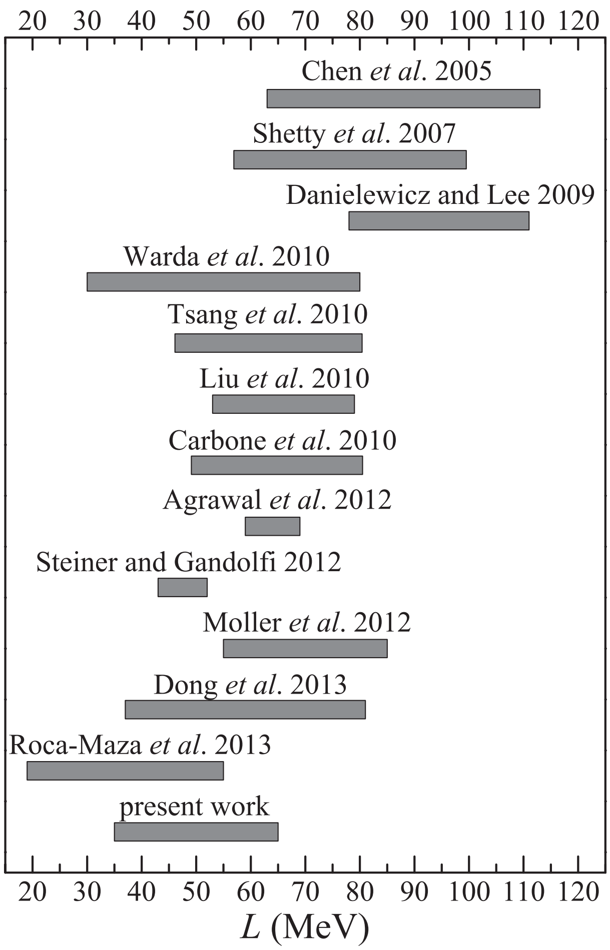

FIG. 2. Comparison between the L values obtained in the present work and those from other recent independent analyses including Chen *et al.* [17], Tsang *et al.* [18], Carbone *et al.* [19], Liu *et al.* [20], Warda *et al.* [22], Agrawal *et al.* [24], Steiner and Gandolfi [26], Roca-Maza *et al.* [27], Danielewicz and Lee [36], Möller *et al.* [25], Dong *et al.* [48], and Shetty *et al.* [52].

are employed, one obtains $L = 73 \pm 17$ MeV, which suggests that it is important to consider the ρ_A accurately. The root-mean-square deviation and mean deviation of Eq. (8) for the Q_{β^-} are 0.29 MeV and 0.22 MeV, respectively, where the values of the Q_{β^-} are ~0.1–5 MeV. This deviation, leading to a little uncertainty of $a_{\rm sym}(A)$ by just ~0.3 MeV, to some extent indicates the reliability of the present approach, and one may in turn investigate the unknown beta decay energies in heavy odd-mass nuclei.

Figure 2 shows the present estimated L values (50 ± 15 MeV) compared with those from other approaches. This figure is not exhaustive but contains several recent estimations that cover a large range of possible values of L. A more exhaustive compilation is not the aim of this work, and one can refer to Ref. [28,52,53] for a more complete review. One can see clearly that the present finding has a remarkable overlap with some recent results. The diversified approaches perhaps will allow one to determine eventually the behavior of the symmetry energy. The present approach is much more straightforward than those applying the binding energy directly. The contribution of the symmetry energy $a_{\rm sym}(A)\beta^2 A$ to the total binding energy is not very large; for example, the symmetry energy is just 14% of the total binding energy for ^{208}Pb ($\beta = 0.212$). Yet, the $4(\beta - \frac{1}{A})a_{\rm sym}(A)$ in Eq. (8) is several times larger than the Q_{β^-} value, which is much more conducive for extracting the $a_{\rm sym}$ of heavy nuclei. Furthermore, the pairing energy, Wigner energy, volume energy, and surface energy, which appear in the binding energy, are of no concern in the present method. The vanishing of these interferences reduces the uncertainty significantly.

It has been well established in Ref. [54] that the correlation between L and ΔR_{np} in ^{208}Pb is $\Delta R_{np} = 0.101 + 0.00147L$, where L and ΔR_{np} are measured in units of MeV and fm, respectively. This correction is universal in the realm of mean field theory as it is based on widely different nuclear functionals. Reference [54] did not provide the theoretical error attached to the correlation and hence we assume no theoretical error of this formula in the following calculations. With the new values of L obtained here, the neutron skin thickness in ^{208}Pb is determined to be $\Delta R_{np} = 0.174 \pm 0.022$ fm, which agrees well with 0.168 ± 0.022 fm from the electric dipole polarizability [55], 0.180 ± 0.035 from PDR [56], 0.18 ± 0.02 from exotic atom [57], and 0.17 ± 0.03 from chiral effective field theory [58]. The ΔR_{np} is related to the neutron star cooling. It was proposed in Ref. [8] that the thicker the neutron skin, the faster the electron fraction Y_e rise with density, and ΔR_{np} in ^{208}Pb of the order of 0.24 fm or larger suggests that Y_e will become large enough to allow a direct URCA process to cool down a $1.4M_\odot$ neutron star. The calculated ΔR_{np} in ^{208}Pb here is too small to allow a rapid cooling process in this canonical neutron star. Besides, it has been shown that, for neutron stars with masses above $1M_\odot$, the radius of the star varies linearly with the slope L [59]. Correspondingly, the radius is also correlated with the neutron skin thickness. These all suggest that a high precision measurement of the neutron skin thickness in ^{208}Pb can provide important information on the properties of neutron stars.

III. SUMMARY

An alternative method has been developed in the present study to determine the symmetry energy coefficient of heavy nuclei with the available experimental β^--decay energies of heavy odd-A nuclei. This approach prevents interference from other energy terms effectively, so that just a well known Coulomb energy appears in the Q_{β^-} value to determine the $a_{\rm sym}(A)$. Accordingly, the extraction of the $a_{\rm sym}(A)$ is much more straightforward. The calculated $a_{\rm sym}(A)$ of ^{208}Pb was furthermore used to probe the density dependence of the symmetry energy of nuclear matter. With the symmetry energy $S_0 = 32.3 \pm 1.3$ MeV at saturation density in Ref. [19] as an input, the estimated values of the slope and curvature parameters are $L = 50 \pm 15$ MeV and $K_{\rm sym} = -197 \pm 56$ MeV, respectively. Correspondingly, the neutron skin thickness of ^{208}Pb is determined to be $\Delta R_{np} = 0.174 \pm 0.022$ fm, which is too small to allow a rapid cooling for a $1.4M_\odot$ neutron star. If the S_0 is constrained to a relative small value, such as 29.0–32.7 MeV in Ref. [53], the L, $K_{\rm sym}$, and ΔR_{np} values are lowered correspondingly.

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