Core-excitation three-cluster model description of ^{8}He and ^{10}He

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We introduce a new model applying to the core-nucleus and two-neutron system. The Faddeev equations of ^{6}He-n-n and ^{8}He-n-n systems for ^{8}He and ^{10}He are solved, respectively. The potential of the subsystem in the model has been determined to make a coupling both of the ground state and the excited one inside the core nucleus. By a similar mechanism the three-nucleon system is solved with the three-body force originating from an isobar excitation of the nucleon. Inputting only the information of subsystem energy levels and widths we get the coupling constants of rank 1 Yamaguchi potential between the core nucleus and neutron. We calculate the Faddeev three-cluster equations to obtain the low-lying energy levels of ^{8}He and ^{10}He. The 1^- state of ^{10}He, which has not been detected yet in experiments, is located in the energy level between the 0^+ and 2^+ states.

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I. INTRODUCTION

Due to developments of experimental technique, our knowledge of unstable nuclei has been increasing rapidly. Experimental researchers have recently reported a lot of events. Here neutron-rich nuclei are good targets for studying interesting phenomena, e.g., clustering, halos, deformation, dineutron correlation, etc. In order to look for these properties which differ from ordinary shell model study, one may need to employ cluster model calculations. However, the interactions between clusters are usually very complex, except for the α cluster model treated as the resonating group method. According to *ab initio* calculations, there are at most four-body calculations [1]. Four-nucleon scattering has been solved by the Faddeev-Yakubovsky formalism using the realistic nucleon-nucleon force including the three-body force [2]. Beyond the four-nucleon system there are computational difficulties because of limited memory size and CPU time. Nevertheless, the Green's function Monte Carlo simulation is very promising. Recent calculations show many energy spectra up to $A = 9$ [3].

There are some microscopic or effective theoretical approaches. For instance, the cluster orbital shell model (COSM), complex scaling method (CSM) [4], and the method of analytic continuation in the coupling constant (ACCC) [5] describe ^{9}He and ^{10}He nuclei by their core-nucleus + valence-neutrons model [6,7]. Systematic studies from ^{5}He to ^{8}He are reported on the basis of the tensor-optimized shell model (TOSM) [8] using a bare nucleon-nucleon interaction, of which the short-range correlation is treated by the unitary correlation operator method (UCOM) [9].

On the other hand, the three-cluster model of the Faddeev theory has been applied to the low-lying energy states of the ^{6}Li nucleus as an $\alpha + n + p$ three-body system using nonlocal separable interactions [10]. In the case of $T = 1$ the isotope ^{6}He the binding energy and widths of the resonance for the ground state $J^\pi = 0^+$ and the resonance state $J^\pi = 2^+$ agree with experiment. By the same scheme we have also been investigating other exotic nucleus $^{9}_{\Lambda}$Be of $\alpha + \alpha + \Lambda$ three-body system [11,12].

In the next section we will introduce a new model calculation based on the Faddeev theory. The three-body system is treated as a cluster model consisting of core nucleus $+n + n$ to investigate ^{8}He and ^{10}He nuclei. ^{6,8}He are so-called Borromean nuclei and ^{10}He is also regarded as the Borromean nucleus because the energy level of the ground state is much closer to the three-body breakup threshold. It is often considered that the core nucleus of the three-body model deals with only the ground state core nucleus. However, in our model not only the ground state core particle but also an excited state core nucleus are adopted. The idea [13] is also found in the case of the three-nucleon system, in which some of the nucleons become a delta isobar in ^{3}He [14].

Preliminary calculations have been carried out [15,16]. Because the excited state $J^\pi = \frac{1}{2}^-$ of ^{7}He was not found in the experiment, in the former work the ^{8}He ground state could not be described accurately. Using the presence of the excited state in the experiment [17], we recalculate with the new data of ^{7}He. Our theoretical prediction will be demonstrated in case of ^{8}He and ^{10}He nuclei in Sec. III. The conclusion is given in Sec. IV.

II. A NEW THREE-CLUSTER MODEL

In the framework of the Faddeev theory the three-body equations were represented as the Alt-Grassberger-Sandhas (AGS) equations using a separable potential of NN interaction

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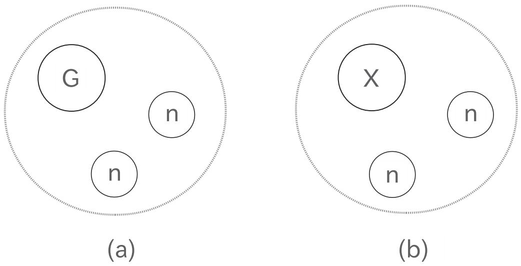

FIG. 1. Illustration of core-excitation cluster. The core cluster of the ground state and excited state are labeled G and X, respectively. Neutrons are labeled n.

[18]. The AGS equations are used in many three-body systems. They have succeeded in the calculation of a three-body breakup process for the α-n-p system first [19]. Recently the study of the system has progressed well [20]. The system was often investigated and the calculation of the resonance states $T = 1$ without Coulomb force are discussed only corresponding to the case of the ^{6}He nucleus. We verified the former work [10], and the energy level of the ground state $J^\pi = 0^+$ from the threshold of $\alpha + 2n$ is obtained as -0.56 MeV, vs data of -0.973 MeV. The energy level of the first excited state $J^\pi = 2^+$ is also obtained as 0.95 MeV ($\Gamma = 0.3$ MeV), vs data of 0.824 MeV ($\Gamma = 0.113$ MeV) [21]. The separable potential is very primitive; nevertheless, these calculations encourage us to start the neutron-rich study.

On the other hand, the research in three-nucleon scattering has made great progress according to the three-body force [22,23]. It is considered that the fundamental origin of the three-body forces comes from the delta excitation, or inner excitation, of the nucleon [24]. Study of the three-nucleon force has progressed recently, centering on the chiral symmetry that the QCD Lagrangian possesses [25,26].

If the idea of inner excitation is applied to the case of neutron-rich nuclei, more precise theoretical results would be possible, taking into consideration the inner excitation of the core cluster that constitutes the nucleus [13]. This idea has a similarity to the delta isobar excitation in the three-nucleon system [14]. Illustrations of the model that we imagine are shown in Fig. 1. Labels G of Fig. 1(a) and X of Fig. 1(b) represent the ground and excited state core nuclei, respectively.

The Hilbert space $\mathcal{H}$ of the model consists of two Hilbert ones

$$\mathcal{H} = \mathcal{H}(G) \oplus \mathcal{H}(X). \tag{1}$$

Using the wave function, we have

$$|\Psi\rangle = |G\rangle|\Psi_G\rangle + |X\rangle|\Psi_X\rangle, \tag{2}$$

where $|G\rangle$ and $|X\rangle$ are orthonormal bases to distinguish their spaces,

$$\langle G|G\rangle = \langle X|X\rangle = 1, \quad \langle G|X\rangle = \langle X|G\rangle = 0. \tag{3}$$

The free Hamiltonian $\hat{H}_0^{\text{2clust.}}$ of the subsystem consisting of the core nucleus and neutron is represented as

$$\hat{H}_0^{\text{2clust.}}|G\rangle \equiv \frac{p^2}{2\nu}|G\rangle, \quad \hat{H}_0^{\text{2clust.}}|X\rangle \equiv \left(\delta m + \frac{p^2}{2\nu}\right)|X\rangle, \tag{4}$$

where p and ν are the relative momentum and the reduced mass between the core nucleus and neutron, respectively. The mass difference δm is the energy level shift of the ground state core nucleus and the excited one.

A. Two-body interaction

In our model the potential of a two-cluster system has a rank 1 separable Yamaguchi form using a simple form factor $g(p)$. For instance, the neutron-neutron potential of the $^1S_0$ partial wave is given as

$$V_{nn}(p,p') = -\gamma_{nn}^2 g_{nn}(p) g_{nn}(p') \tag{5}$$

with

$$g_{nn}(p) = \frac{1}{p^2 + \beta_{nn}^2}, \tag{6}$$

where we choose parameters $\beta_{nn} = 1.1648$ fm^{-1} and $\gamma_{nn}^2 = 0.3943$ fm^{-3} from [10].

Let us introduce a new form factor h, which is combined with the partial waves $|l_I S_I j_I\rangle$ and the particle basis $|I\rangle$:

$$\langle p|h\rangle = \sum_{I=G,X} \sum_{l_I,S_I,j_I} \gamma_{In;l_I,S_I,j_I} g_{In;l_I,S_I,j_I}(p) |l_I S_I j_I\rangle |I\rangle \tag{7}$$

with

$$g_{In;l_I,S_I,j_I}(p) = \frac{p^{l_I}}{\left(p^2 + \beta_{In;l_I,S_I,j_I}^2\right)^{l_I+1}}, \tag{8}$$

where l_I, S_I, and j_I are angular momentum, total spin, and total angular momentum of the two-body subsystem ($j_I = l_I + S_I$), respectively. The core-nuclei neutron potential V is given by the form factor h,

$$\hat{V} = -|h\rangle\langle h|. \tag{9}$$

However, the neutron-neutron (nn) potential $\hat{V}_{nn}$ differs from this form; one writes it as

$$\hat{V}_{nn} = -|g_{nn}\rangle \gamma_{nn}^2 \langle g_{nn}| \{|G\rangle\langle G| + |X\rangle\langle X|\}. \tag{10}$$

Apparently the potential $\hat{V}_{nn}$ is not coupled between $|G\rangle$ and $|X\rangle$.

When the core-nucleus spin has the ground state 0^+ and the excited state 2^+, there are $S_G = \frac{1}{2}$ and $S_X = \frac{3}{2}$ and $\frac{5}{2}$, respectively. If one takes the same number for the parameter β, the potentials of $S_X = \frac{3}{2}$ and $S_X = \frac{5}{2}$ differ only in the coupling constants. The degenerated coupling constant $\gamma_{In;l_I,j_I}^2$ could be introduced:

$$\gamma_{Gn;l_G,j_G}^2 \equiv \gamma_{Gn;l_G,\frac{1}{2},j_G}^2, \quad \gamma_{Xn;l_X,j_X}^2 \equiv \gamma_{Xn;l_X,\frac{3}{2},j_X}^2 + \gamma_{Xn;l_X,\frac{5}{2},j_X}^2. \tag{11}$$

According to the separable scheme, the t matrix $t(p,p';E_2)$,

$$t_{In;l_I,S_I,j_I,I'n;l_I',S_I',j_I'}(p,p';E_2) \equiv \langle I|\langle l_I S_I j_I|\langle p|h\rangle \tau(E_2) \langle h|p'\rangle |l_I' S_I' j_I'\rangle |I'\rangle, \tag{12}$$

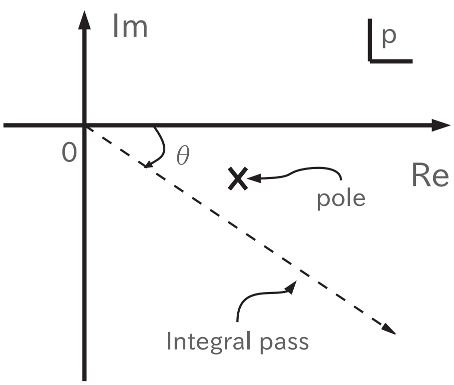

FIG. 2. Integral pass of Eq. (14). In the Riemann complex sheet of the variable p the integral pass is taken as the dashed line below the resonance pole.

fulfills the Lippmann-Schwinger equation with the result

$$\tau(E_2) = -1 - \tau(E_2)\langle h|\hat{G}_0^{\text{2clust.}}(E_2)|h\rangle. \quad (13)$$

In order to determine these coupling constants γ in Eq. (7), we introduce the following natural assumption. If the subsystem has no bound state (Borromean nuclei are in this case) but has some resonance states, the propagator $\tau(E_2)$ must diverge at the resonance energy $E_2 = E_2^{\text{res}}$, which has a real part $E_2^{(r)}$ and width Γ. Under the condition $\tau(E_2) = \infty$, Eq. (13) becomes

$$1 + \gamma^2_{Gn;l_G,j_G}\langle g_{Gn;l_G,j_G}|\frac{1}{E_2^{\text{res}} - \hat{p}^2/2\nu + i\epsilon}|g_{Gn;l_G,j_G}\rangle + \gamma^2_{Xn;l_X,j_X}\langle g_{Xn;l_X,j_X}|\frac{1}{E_2^{\text{res}} - \delta m - \hat{p}^2/2\nu + i\epsilon}|g_{Xn;l_X,j_X}\rangle = 0. \quad (14)$$

Approximately, the resonance state occurs only in two channels and there is assumed to be no absorption channel; we expect these coupling constants to be real numbers. Consequently, the condition leads to two conditions (real part and imaginary one) to subtract two unknown parameters γ_G and γ_X.

As shown in Fig. 2, one needs to take the integral pass of Eq. (14), because the resonance pole is located on the physical Riemann sheet at $p = p_{\text{pole}}$ with $p^2_{\text{pole}} = 2\nu E_2^{\text{res}}$.

In order to apply these potentials to the three-body system, we must resolve the degeneracy of S_X. Following a natural way of thinking, the weight of the couplings will be taken from the degree of multiplicity under the condition of (11),[1]

$$\gamma_{Xn;l_X,S_X,j_X} = \sqrt{\frac{2S_X+1}{10}}\gamma_{Xn;l_X,j_X}. \quad (15)$$

We will show these coupling constants of ^{6}He-n and ^{8}He-n in Sec. III.

[1]We caution that the case $(l_X, \frac{5}{2}, j_X) = (1, \frac{5}{2}, \frac{1}{2})$ does not occur; therefore, one does not need the renormalization for $(1, \frac{3}{2}, \frac{1}{2})$.

TABLE I. Parameters for ^{6}He(0^+)-n+^{6}He(2^+)-n potential. The resonance energies are measured from the ^{6}He + n threshold. The strengths γ^2 are in units of fm^{-5} for P wave, and fm^{-7} for F wave. The parameters β_G and β_X are commonly taken to be 1.5166 fm^{-1}. (β_{nn} = 1.1648 fm^{-1}.)

E_2^{res} (MeV)	Partial wave	l_G	γ_G^2	l_X	γ_X^2
$0.445 - i0.075$ [21]	$^2P_{3/2} + {}^{4,6}P_{3/2}$	1	4.1655	1	6.1580
$1.345 - i0.5$ [17,28]	$^2P_{1/2} + {}^4P_{1/2}$	1	5.3966	1	4.0418
$3.37 - i0.995$ [21]	$^2F_{5/2} + {}^{4,2}P_{5/2}$	3	116.80	1	7.6144
nn channel	$^1S_0$	0	0.3943	0	0.3943

B. Three-body integral equation

The AGS equations are well established [27], therefore, we will not repeat the same part of Ref. [10]. The following explanation is an additional part because of the extension of the core-excitation channel (G or X) and the definition of the wave function.

The total wave function $|\Psi^{J^\pi T}\rangle$ with total angular momentum J, parity π, and total isospin T consists of the Faddeev components $\psi^{J^\pi T}$ labeled by particle channels α, β, and γ:

$$|\Psi^{J^\pi T}\rangle = |\psi_\alpha^{J^\pi T}\rangle + |\psi_\beta^{J^\pi T}\rangle + |\psi_\gamma^{J^\pi T}\rangle. \quad (16)$$

The AGS equations for the Faddeev component is given by

$$|\psi_\alpha^{J^\pi T}\rangle = G_0 t_\alpha \sum_{\beta\neq\alpha} |\psi_\beta^{J^\pi T}\rangle \quad (17)$$

$$= G_0|h_\alpha\rangle\tau_\alpha\langle h_\alpha| \sum_{\beta\neq\alpha} |\psi_\beta^{J^\pi T}\rangle. \quad (18)$$

The reduced wave function $f^{J^\pi T}_{I;\tilde{K}_\alpha}(q_\alpha)$ is defined by

$$\sum_{I=G,X} \langle I|\langle\tilde{K}_\alpha|\langle q_\alpha|f_\alpha^{J^\pi T}\rangle = \langle q_\alpha|f_{\tilde{K}_\alpha}^{J^\pi T}\rangle = f_{\tilde{K}_\alpha}^{J^\pi T}(q_\alpha) \equiv \sum_{I=G,X} \gamma_{In;l_\alpha,j_\alpha}\langle g_{In;l_\alpha j_\alpha}| \sum_{\beta\neq\alpha} |\psi_{I;\beta}^{J^\pi T}\rangle, \quad (19)$$

where q_α is the Jacobi momentum designating the momentum of the particle labeled by α relative to the $(\beta\gamma)$ pair. The index

TABLE II. Sets of the quantum numbers for the $J^\pi = 0^+$ state of the ^{8}He nucleus. The quantum numbers for the particle channel $\alpha = 3$ are obtained from $\alpha = 1$ by just cyclically label replacing $s_\alpha \to s_\beta \to s_\gamma \to s_\alpha$.

K_α	$\tilde{K}_\alpha$	α	I	$\mathcal{L}_\alpha$	$\mathcal{S}_\alpha$	j_α	l_α	S_α	s_α	s_β	s_γ
1	1	1	G	1	1	3/2	1	1/2	1/2	1/2	0
2	1	1	X	1	1	3/2	1	3/2	1/2	1/2	2
3	1	1	X	0	0	3/2	1	5/2	1/2	1/2	2
4	2	1	G	0	0	1/2	1	1/2	1/2	1/2	0
5	2	1	X	0	0	1/2	1	3/2	1/2	1/2	2
6	3	1	G	0	0	5/2	3	1/2	1/2	1/2	0
7	3	1	X	0	0	5/2	1	3/2	1/2	1/2	2
8	3	1	X	0	0	5/2	1	5/2	1/2	1/2	2
9	4	2	G	0	0	0	0	0	0	1/2	1/2
10	5	2	X	2	2	0	0	0	2	1/2	1/2

TABLE III. The predicted energy levels of the ^{8}He nucleus from the ^{6}He + $n+n$ threshold. The resonance energy E equals $E^{(r)} - i\Gamma/2$. Units are MeV.

J^π	Present work		Expt.	
	$E^{(r)}$	Γ	$E^{(r)}$	Γ
0^+	−1.35		−2.14	
2^+	2.01	2.12	1.06 ± 0.5	0.6 ± 0.2

$K_\alpha = \{t_\alpha, j_\alpha, S_\alpha, l_\alpha, \mathcal{S}_\alpha, \mathcal{L}_\alpha, I\}$ is defined as the quantum numbers that label the different three-body channels $J^\pi T$. The index $\tilde{K}_\alpha = \{t_\alpha, j_\alpha, l_\alpha, \mathcal{S}_\alpha, \mathcal{L}_\alpha\}$ is also defined because of the degeneration of S_α and I.

Here, for the sake of unifying the notation the related coupling constant γ_{nn} is also written as $\gamma_{In;l_\alpha,j_\alpha}$ when the spectator of the particle channel α is the core nucleus. The following angular momentum and isospin coupling scheme is given as

$$\begin{aligned} S_\alpha &= s_\beta + s_\gamma, \quad j_\alpha = l_\alpha + S_\alpha, \quad t_\alpha = \tau_\beta + \tau_\gamma, \\ \mathcal{S}_\alpha &= j_\alpha + s_\alpha, \quad J = \mathcal{L}_\alpha + \mathcal{S}_\alpha, \quad T = t_\alpha + \tau_\alpha. \end{aligned} \tag{20}$$

Here, s_β and τ_β refer to the spin and isospin of the particle labeled by β, l_α refers to the relative orbital angular momentum of the $(\beta\gamma)$ pair, $\mathcal{S}_\alpha$ is the channel spin, and $\mathcal{L}_\alpha$ is the orbital angular momentum of the spectator particle α relative to the $(\beta\gamma)$ pair.

The AGS equations (18) are modified into equations for the reduced wave functions:

$$\begin{aligned} f^{J^\pi T}_{\tilde{K}_\alpha}(q_\alpha) &= \sum_{I,I'} \sum_{\tilde{K}_\gamma} \int_0^\infty dq_\gamma q_\gamma^2 Z^{J^\pi T}_{I;\tilde{K}_\alpha,I';\tilde{K}_\gamma}(q_\alpha, q_\gamma; E) \\ &\quad \times \tau_{l_\gamma j_\gamma}(E - \epsilon_\gamma(q_\gamma)) f^{J^\pi T}_{\tilde{K}_\gamma}(q_\gamma), \end{aligned} \tag{21}$$

where the integral kernel $Z^{J^\pi T}_{I;K_\alpha,I';K_\beta}$ is defined by

$$\begin{aligned} &Z^{J^\pi T}_{I;\tilde{K}_\alpha,I';\tilde{K}_\beta}(q_\alpha, q_\beta; E) \\ &\quad \equiv \bar{\delta}_{\alpha\beta}\delta_{II'}\gamma_{In;l_\alpha,j_\alpha}\gamma_{I'n;l_\beta,j_\beta} \\ &\quad \times \langle g_{I;l_\alpha j_\alpha}; q_\beta K_\beta JT | G_0^{(I)} | g_{I';l_\beta j_\beta}; q_\beta K_\beta JT \rangle \end{aligned} \tag{22}$$

and $\epsilon_\gamma(q_\gamma)$ is $\frac{q_\gamma^2}{2\mu_\gamma}$, and E is a total energy of the three-body c.m. system. Eqquation (22) is only changed with the parts of $\delta_{II'}$ and γ from Eq. (13) of [10]. In addition, the free three-body Green's function G_0 can be written as

$$\begin{aligned} G_0^{(G)} &\equiv \langle G|\hat{G}_0|G\rangle = \frac{1}{E - p_\alpha^2/(2\nu_\alpha) - q_\alpha^2/(2\mu_\alpha) + i\epsilon}, \\ G_0^{(X)} &\equiv \langle X|\hat{G}_0|X\rangle \\ &= \frac{1}{E + \delta m - p_\alpha^2/(2\nu_\alpha) - q_\alpha^2/(2\mu_\alpha) + i\epsilon}, \end{aligned} \tag{23}$$

where the reduced masses ν_α and μ_α are $m_\beta m_\gamma/(m_\beta + m_\gamma)$ and $m_\alpha(m_\beta + m_\gamma)/(m_\alpha + m_\beta + m_\gamma)$, respectively.

In order to find out the three-body bound state or resonance state we regard the AGS equations of Eq. (21) as the eigenvalue equation

$$\eta\vec{\psi} = \mathcal{K}(E)\vec{\psi}, \tag{24}$$

where η and $\mathcal{K}(E)$ are the eigenvalue and the integral kernel $Z(E)\tau$ in Eq. (21), respectively. We need to search for E under a constraint $\eta = 1$. Our basic technique is based on the Gauss-Seidel method to solve the eigenvalue equation. The typical procedure requires a few hundred iterations to reach the stable solutions. Performance of the integral for the complex momentum q_γ takes the integral pass as well as two-body momentum p shown in Fig. 2. The contour deformation angle θ is defined as

$$p_{\rm complex} \equiv p\exp(-i\theta), \quad q_{\rm complex} \equiv q\exp(-i\theta). \tag{25}$$

The accuracy of the calculation is sufficiently maintained within $\theta \leqslant \frac{\pi}{3}$.

III. NUMERICAL RESULTS

We applied the above-mentioned scheme to the core-nucleus + $2n$ systems of ^{8}He and ^{10}He. The results of these systems are separately demonstrated in the next subsections.

A. ^{8}He nucleus

We treat here ^{8}He as the ^{6}He-n-n three-body system. The energy shift δm between the ground state G and the first excited state X of the core nucleus ^{6}He is 1.8 MeV. There are low-lying three-resonance states in ^{7}He, which are submitted as $J^\pi = (\frac{3}{2})^-$ (g.s.: $\Gamma_{c.m.} = 0.150 \pm 0.020$ MeV [21]), $J^\pi = (\frac{1}{2})^-$ ($E_x = 0.9 \pm 0.5$ MeV, $\Gamma_{c.m.} = 1.0 \pm 0.9$ MeV [17]) and $J^\pi = (\frac{5}{2})^-$ ($E_x = 2.92 \pm 0.09$ MeV, $\Gamma_{c.m.} = 1.990 \pm 0.170$ MeV) [21]. The energy level of the ground state is 0.445 MeV [21] from the threshold of ^{6}He and neutron, we have each $E_2^{\rm res.}$ in

TABLE IV. Parameters for the ^{8}He(0$^+$)-n+^{8}He(2$^+$)-n potential. The resonance energies are measured from the ^{8}He + n threshold. The strengths γ^2 are in units of fm^{-5} for P wave, and fm^{-7} for F wave. The parameters β_G and β_X are commonly taken to be 1.5166 fm^{-1}.

$E_2^{\rm res}$ (MeV)	Partial wave	l_G	γ_G^2	l_X	γ_X^2
$1.27 - i0.05$ [29]	$^2P_{1/2} + {}^4P_{1/2}$	1	0.44601	1	10.181
$2.42 - i0.35$ [29]	$^2S_{1/2} + {}^{4,6}D_{1/2}$	0	0.016538	2	118.42

TABLE V. Sets of the quantum numbers for the $J^\pi = 0^+$ state of the ^{10}He nucleus.

K_α	$\tilde{K}_\alpha$	α	I	$\mathcal{L}_\alpha$	$\mathcal{S}_\alpha$	j_α	l_α	S_α	s_α	s_β	s_γ
1	1	1	G	1	1	1/2	1	1/2	1/2	1/2	0
2	1	1	X	1	1	1/2	1	3/2	1/2	1/2	2
3	2	1	G	0	0	1/2	0	1/2	1/2	1/2	0
4	2	1	X	0	0	1/2	2	3/2	1/2	1/2	2
5	2	1	X	0	0	1/2	2	5/2	1/2	1/2	2
6	3	2	G	0	0	0	0	0	0	1/2	1/2
7	4	2	X	2	2	0	0	0	2	1/2	1/2

TABLE VI. The predicted energy levels of the ^{10}He nucleus from the ^{8}He + $n+n$ threshold. The resonance energy E equals $E^{(r)} - i\Gamma/2$. Units are MeV.

J^π	Present work		Expt.	
	$E^{(r)}$	Γ	$E^{(r)}$	Γ
0^+	0.803	0.665	1.069	0.3 ± 0.2
1^-	1.25	0.21		
2^+	3.97	4.71	4.31 ± 0.20	0.6 ± 0.3

Table I. Using these experimental data we list the coupling constants γ_l^2 obtained by solving our model equations (14). For the sake of simplicity the reduced mass ν is $\frac{6}{7}m_N$ with nucleon mass $m_N = 939$ MeV.

The possible quantum numbers of the three-body partial wave of $J^\pi = 0^+$ are listed in Table II. There are 10 channels for the $J^\pi = 0^+$ ground state of ^{8}He, and 32 channels for $J^\pi = 2^+$. In Table III our theoretical predictions are demonstrated with the recent experimental data. Energy levels are reasonably well obtained to describe the data; however, there is a tendency of large width.

B. ^{10}He nucleus

The ^{10}He nucleus is here treated as the ^{8}He-n-n three-body system. The energy shift δm between the ground state G and the first excited state X of the core nucleus ^{8}He is 3.1 MeV. There are low-lying two-resonance states in ^{9}He, which are submitted as $J^\pi = (\frac{1}{2})^-$ (g.s.: $\Gamma_{c.m.} = 0.10 \pm 0.06$ MeV) [29] and $J^\pi = (\frac{1}{2})^+$ ($E_x = 1.15 \pm 0.10$ MeV, $\Gamma_{c.m.} = 0.7 \pm 0.2$ MeV) [29]. The energy level of the ground state is 1.27 MeV [29] from the threshold of ^{8}He and neutron, we have each $E_2^{\mathrm{res.}}$ in Table IV. Using these experimental data we obtained the coupling constants γ_l^2 by our model equations (14) as well as the case of ^{8}He. For simplicity the reduced mass ν is $\frac{8}{9}m_N$.

The possible quantum numbers of the three-body partial wave of $J^\pi = 0^+$ are listed in Table V. There are seven channels for $J^\pi = 0^+$ ground state of ^{10}He, and seven channels for $J^\pi = 2^+$. In Table VI our theoretical predictions are shown with the recent experimental data. The state (1^-) not found in the experiment is obtained. Although we would like to recommend measuring it, the clustering of the state may be poorly developed.

IV. CONCLUSION

We have been conducting research on ^{6,8,10}He isotopes based on the three-cluster model. Incorporating the core-nucleus excitation we deal with double Hilbert spaces. In the sense of *ab initio* calculation only from the fundamental NN potential double Hilbert spaces are not necessary. The three-cluster model requires effective cluster potential between the core nucleus and neutron. Even though the potential is determined by sufficient data in each space, it is not always necessarily useful in the three-cluster model. We have adopted a separable potential of rank 1, which bounds both of the Hilbert spaces. Coupling constants in the two spaces can be determined by their widths and the energy level of the resonance state in the subsystem.

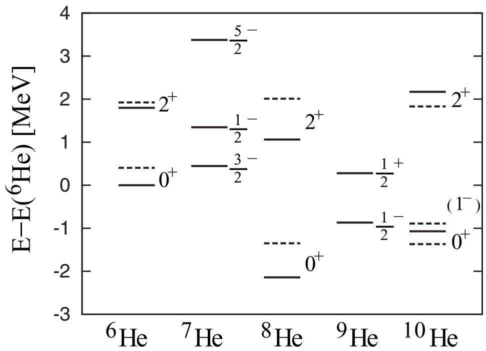

FIG. 3. Energy levels of He isotopes normalized to the ^{6}He ground state energy. The dashed lines correspond to our theoretical predictions. The solid lines are taken from experimental data [17,21,29].

There are the ground 0^+ and the excited 2^+ states in both ^{8}He and ^{10}He. In Fig. 3 their energy levels are shown. The solid (dashed) level lines correspond to experimental data (theoretical predictions). The energy levels of ^{6}He are obtained from [10], and are recalculated to check our program code. Our numbers for ^{6}He agree with [10]. The states of ^{8}He and ^{10}He mostly appear as in our theoretical prediction. Comparing with the case of ^{10}He, we obtain rather a large difference (≈1 MeV) between data and prediction in ^{8}He. The level 1^- is found, which is close to the 0^+ state. However, this might be a simple spurious state because the real state of 1^- may not be a cluster state. The expected theoretical decay width does not reproduce the experiment so much as a whole.

Although it is difficult to evaluate the accuracy of our model only by having investigated a few nuclei, we would like to mention that our results were reasonably satisfactory. For the sake of proving the effectiveness of our model we can only continue to predict unknown states which are not measured yet.

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