Energy dependence of pion in-medium effects on the π^-/π^+ ratio in heavy-ion collisions

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Within the framework of a thermal model with its parameters fitted to the results from an isospin-dependent Boltzmann-Uehling-Uhlenbeck transport model, we have studied the pion in-medium effect on the charged-pion ratio in heavy-ion collisions at various energies. We find that due to the cancellation between the effects from pion-nucleon s-wave and p-wave interactions in nuclear medium, the π^-/π^+ ratio generally decreases after including the pion in-medium effect. The effect is larger at lower collision energies as a result of narrower pion spectral functions at lower temperatures.

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The density dependence of nuclear symmetry energy $E_{\text{sym}}(\rho)$ is important for understanding isospin effects in heavy-ion reactions and properties of rare isotopes as well as many phenomena in nuclear astrophysics [1–6]. Extensive studies using various probes during past years have significantly constrained the values of E_{sym} at subsaturation densities (for a recent review, see, e.g., Ref. [7]). The behavior of $E_{\text{sym}}(\rho)$ at suprasaturation densities remains, however, uncertain despite many studies based on the π^-/π^+ ratio [8–14], the K^0/K^+ ratio [14], the n/p or $t/^3$He ratio [15], and the ratio of neutron and proton elliptic flows [16] in heavy-ion collisions as well as on the properties of neutron stars [17]. In particular, contradictory conclusions were obtained on $E_{\text{sym}}(\rho)$ from the π^-/π^+ ratio [8–14]. To describe the FOPI data from GSI, results by Xiao *et al.* [8] based on the isospin-dependent Boltzmann-Uehling-Uhlenbeck (IBUU) transport model together with an isospin- and momentum-dependent interaction favor a supersoft symmetry energy, while those of Feng and Jin [11] based on an improved isospin-dependent quantum molecular dynamics model requires a stiff symmetry energy. A more recent study by Xie *et al.* [12] based on an improved isospin-dependent Boltzmann-Langevin approach also favors a supersoft symmetry energy.

In all these studies of the symmetry energy effect on the π^-/π^+ ratio, medium effects on the pion production threshold in nucleon-nucleon collisions that may lead to an opposite effect from that of the symmetry energy [14] were neglected. Also, pions were treated as free particles in nuclear medium. It is known that due to its coupling to the nucleon-particle–nucleon-hole and Δ-particle–nucleon-hole (Δ-hole) excitations, the dispersion relation of a pion in nuclear medium is softened and the strength of its spectral function is enhanced at lower energies [18]. Because of the isospin dependence of pion-nucleon interactions, π^- couples more strongly with neutrons than π^+ and thus has an even softer dispersion relation in neutron-rich nuclear matter than π^+. On the other hand, the pion mass in neutron-rich matter is larger for π^- than for π^+ as a result of the pion s-wave interaction as shown in Ref. [19] based on the chiral perturbation calculation. Within a thermal model for heavy-ion collisions [20], effects from the pion-nucleon p-wave and s-wave interactions were, however, found to largely cancel out, resulting in only a slight reduction of the π^-/π^+ ratio compared to that without the pion in-medium effects.

The pion in-medium effect studied in Ref. [20] was for Au + Au collisions at only one energy of $400A$ MeV, using $2\rho_0$ ($\rho_0 = 0.16$ fm^{-3} being the normal nuclear matter density), $T = 43.6$ MeV, and $\delta_{\text{like}} = 0.135$ for the density, temperature, and isospin asymmetry of produced matter. As different densities and temperatures are reached in heavy-ion collisions at different beam energies, the π^-/π^+ ratio is expected to be modified differently by pion in-medium effects. It is therefore worth investigating the beam energy dependence of pion in-medium effects to find the most suitable collision energies where the π^-/π^+ ratio is more sensitive to the density dependence of the nuclear symmetry energy than the pion in-medium effects. In this paper, we carry out such a study within a framework similar to that in Ref. [20], but using thermal model parameters that are more accurately determined from the IBUU transport model calculations.

In high-energy heavy-ion collisions, pions and Δ resonances are mostly produced from the initial high-density ($\rho > \rho_0$) phase. Assuming that this high-density matter is in thermal equilibrium with baryon density $\rho_B = \rho_n + \rho_p + \rho_\Delta^- + \rho_\Delta^0 + \rho_\Delta^+ + \rho_\Delta^{++}$, isospin asymmetry $\delta_{\text{like}} = (\rho_n - \rho_p + \rho_{\Delta^-} - \rho_{\Delta^{++}} + \rho_{\Delta^0}/3 - \rho_{\Delta^+}/3)/\rho_B$, and temperature T. The densities of nucleons, Δ resonances, and pions with isospin states m_τ, m_T, and m_t can then be expressed,

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respectively, as

$$\rho_N^{m_\tau} = 2 \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{e^{(m_N + p^2/2m_N + U_N^{m_\tau} - \mu_B - 2m_\tau\mu_Q)/T} + 1},$$

$$\rho_\Delta^{m_T} = 4 \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{P_\Delta^{m_T}(M)dM}{z_\Delta^{-1} e^{[M + p^2/2M + U_\Delta^{m_T} - \mu_B - (m_T + \frac{1}{2})\mu_Q]/T} + 1},$$

$$\rho_\pi^{m_t} = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{S_\pi^{m_t}(\omega, p)d\omega^2}{z_\pi^{-1} e^{(\omega - m_t\mu_Q)/T} - 1}. \tag{1}$$

In the above, $U_N^{m_\tau}$ is the momentum-independent nucleon mean-field potential with an isoscalar part as well as an isovector part. The former is fitted to the binding energy $E_0 = -16$ MeV and incompressibility $K_0 = 230$ MeV of normal nuclear matter at saturation density ρ_0. The latter is taken to be from either the supersoft symmetry energy that vanishes at about $3\rho_0$ and is denoted by $x = 1$ or a stiff one that increases almost linearly with density and is denoted by $x = 0$ in Ref. [21]. These symmetry energies correspond to current uncertainties in $E_{\text{sym}}(\rho)$ at suprasaturation densities. $U_\Delta^{m_T} = \sum_{m_\tau, m_t} |\langle \frac{3}{2} m_T | 1 m_t \frac{1}{2} m_\tau \rangle|^2 U_N^{m_\tau}$ is the Δ mean-field potential with $\langle \frac{3}{2} m_T | 1 m_t \frac{1}{2} m_\tau \rangle$ being the Clebsch-Gordan coefficient from the isospin coupling of a Δ resonance with a nucleon and a pion. μ_B and μ_Q are the baryon and charge chemical potentials, respectively. $P_\Delta^{m_T}(M)$ and $S_\pi^{m_t}(\omega, p)$ are the mass distribution of Δ resonances and the pion spectral function including effects due to both pion-nucleon s-wave and p-wave interactions, and they are determined from a self-consistent calculation as described in detail in Ref. [20].

The z_π and z_Δ in Eq. (1) are the fugacity parameters of pions and Δ resonances, respectively, and their values at a given temperature depend on the pion and Δ resonance numbers relative to their equilibrium values. Because of the short chemical equilibrium times estimated from the pion and Δ resonance production rates in the absence of in-medium effects, we assume that both z_π and z_Δ are 1 in this case. To determine their values when in-medium effects are included,
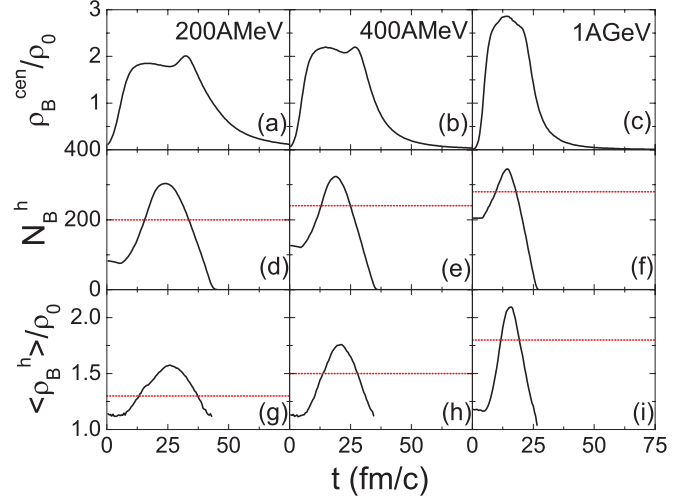

FIG. 1. (Color online) Time evolution of the central baryon density (top panels), the baryon multiplicity (middle panels), and the average baryon density (bottom panels) in the high-density ($\rho > \rho_0$) phase of central Au + Au collisions at different beam energies. Horizontal lines indicate the average values used in the thermal model.

we first determine the baryon density, isospin asymmetry, and temperature of the nuclear medium from the IBUU transport model calculations with the same nucleon and Δ resonance mean-field potentials as used in the thermal model but without the pion in-medium effects. Figure 1 displays the time evolution of the central baryon density, the baryon multiplicity, and the average baryon density in the high-density phase of central Au + Au collisions at three beam energies of $200A$ MeV, $400A$ MeV, and $1A$ GeV. Higher central densities are reached at higher collision energies but last a shorter duration than at lower collision energies. Also, the baryon number in the high-density region changes with time, and the average values are about 200 for $200A$ MeV, 240 for $400A$ MeV, and 280 for $1A$ GeV as given in Table I. They

TABLE I. Baryon number N_B^h, pion-like particle multiplicity N_π, their ratio N_π/N_B^h, average baryon density $\langle \rho_B^h \rangle$, and π^-/π^+ ratio in the high-density phase from the IBUU model with free pions and symmetry energy $x = 1$ ($x = 0$) for central Au + Au collisions at different beam energies; and temperature T, isospin asymmetry δ_{like}, baryon and charge chemical potentials μ_B and μ_Q, fugacity parameters z_π and z_Δ for pions and Δ resonances, and π^-/π^+ ratio with pion in-medium effects in the corresponding thermal model.

E_{beam}	$200A$ MeV	$400A$ MeV	$1A$ GeV
N_B^h	200	240	280
N_π	0.52 (0.47)	9.59 (9.25)	70.8 (70.2)
N_π/N_B^h ($\times 10^{-3}$)	2.60 (2.36)	40.0 (38.5)	253 (251)
$\langle \rho_B^h \rangle / \rho_0$	1.3	1.5	1.8
Free π^-/π^+	4.34 (4.05)	2.74 (2.63)	1.79 (1.76)
T (MeV)	32.6 (32.2)	53.3 (52.9)	90.8 (90.5)
δ_{like}	0.152 (0.128)	0.163 (0.136)	0.211 (0.185)
μ_B (MeV)	919.9 (920.6)	894.3 (895.8)	814.9 (817.0)
μ_Q (MeV)	-23.6 (-22.5)	-24.8 (-24.5)	-23.1 (-23.8)
z_π	0.115 (0.116)	0.190 (0.189)	0.392 (0.391)
z_Δ	0.422 (0.417)	0.667 (0.634)	0.813 (0.806)
In-med. π^-/π^+	4.15 (3.85)	2.70 (2.59)	1.80 (1.77)

are, however, similar when different $E_{\text{sym}}(\rho)$ are used in the IBUU model.

The multiplicities of pion-like particles at different collision energies are also given in Table I, and they are slightly larger for $x = 1$ than for $x = 0$. For the temperature of the high-density phase, it is determined from fitting the ratio of the pion-like particle number to the baryon number using the thermal model. Since the average baryon density of the high-density phase changes with time as shown in the bottom panels of Fig. 1, we use the mean values of $1.3\rho_0$ for $200A$ MeV, $1.5\rho_0$ for $400A$ MeV, and $1.8\rho_0$ for $1A$ GeV for the thermal model calculations. As expected, the resulting temperature and multiplicity of pion-like particles in the high-density phase increase with increasing collision energy as shown in Table I. The isospin asymmetry δ_{like} used in the thermal model is determined by fitting the final π^-/π^+ ratio from the IBUU calculations shown in Table I. It is seen that δ_{like} is larger for $x = 1$, consistent with the argument that a softer symmetry energy leads to a more neutron-rich high-density phase and thus a larger π^-/π^+ ratio as π^- (π^+) is mostly produced from neutron-neutron (proton-proton) scatterings.

Further shown in Table I are the baryon and charge chemical potentials μ_B and μ_Q determined from the chemical equilibrium conditions. Because the multiplicity of pion-like particles is much smaller than the total nucleon number in heavy-ion collisions at energies considered here, including pion in-medium effects has negligible effects on the extracted values for ρ_B, δ_{like}, and T as the dynamics of heavy-ion collisions is hardly modified. Indeed, the multiplicities of final pions at different collision energies are reasonably reproduced by transport model calculations with free pions [8,11,12], and including pion in-medium effects does not affect much the total pion yield [22]. Therefore, the same values of μ_B and μ_Q are obtained with and without the pion in-medium effects. However, due to the softening of pion spectral functions and the broadening of Δ mass distributions from pion in-medium effects, the multiplicities of pion-like particles would increase if they are assumed to be in chemical equilibrium with nucleons. As in Ref. [20], to reproduce the pion and Δ resonance multiplicities from the IBUU model with free pions, their fugacity parameters used in the thermal model become much smaller than one if pion in-medium effects are included as shown in Table I. Their values increase, however, with increasing collision energy due to a shorter chemical equilibration time.

In Fig. 2, we show the pion spectral functions at a typical pion momentum $p = m_\pi$, obtained with the supersoft symmetry energy $x = 1$, in the presence of pion-nucleon s-wave and p-wave interactions at different temperatures corresponding to different collision energies of $200A$ MeV, $400A$ MeV, and $1A$ GeV. It is seen that the π^+ spectral function peaks at lower energies compared to that of π^-, resulting in an increase of the π^+ multiplicity and a decrease of that of π^-. Also, the widths of pion spectral functions become broader with increasing temperature, leading to larger isospin-dependent pion in-medium effects at lower collision energies.

The temperature dependence of the pion spectral functions is related to that of Δ mass distributions, as displayed in Fig. 3 for the supersoft symmetry energy $x = 1$ as well. Similar to the pion spectral functions, the Δ mass distribution is broader
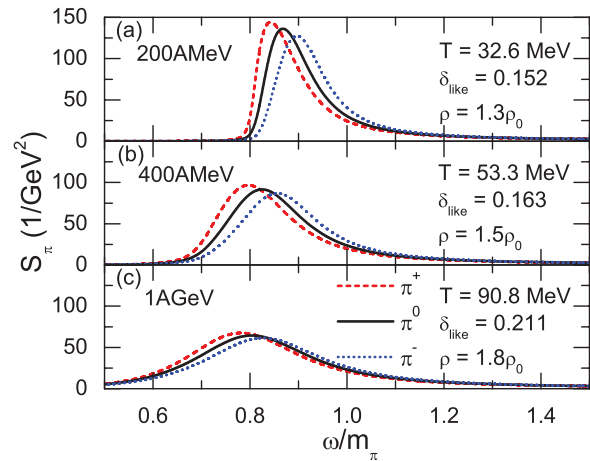

FIG. 2. (Color online) Pion spectral functions at momentum $p = m_\pi$ with the supersoft symmetry energy $x = 1$.

at higher temperatures. At lower temperatures or collision energies, there is a larger probability for Δ^{++} to have lower masses than for Δ^- as shown in the insets. As a result, the production of positive pion-like particles is more enhanced than that of negative ones. This isospin-dependent in-medium effect is, however, rather small at higher temperatures. Since pions in the IBUU transport model are produced from Δ resonance decay, the mass distribution of Δ resonances near the nucleon and pion threshold is important for determining the pion yield in heavy-ion collisions near and below the threshold energy.

The final π^-/π^+ ratios after including both pion-nucleon s-wave and p-wave interactions at different collision energies with different $E_{\text{sym}}(\rho)$ are given in Table I and compared to their values using free pions in the left panel of Fig. 4. It is seen that the pion in-medium effects generally reduce the π^-/π^+ ratio, leading to the need for an even softer symmetry energy to reproduce their ratio using free pions. In addition, although the π^-/π^+ ratio at lower collision energies is larger and more sensitive to the symmetry energy in the absence of
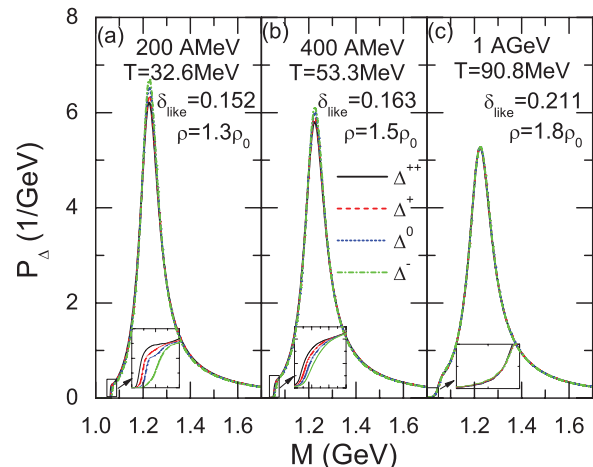

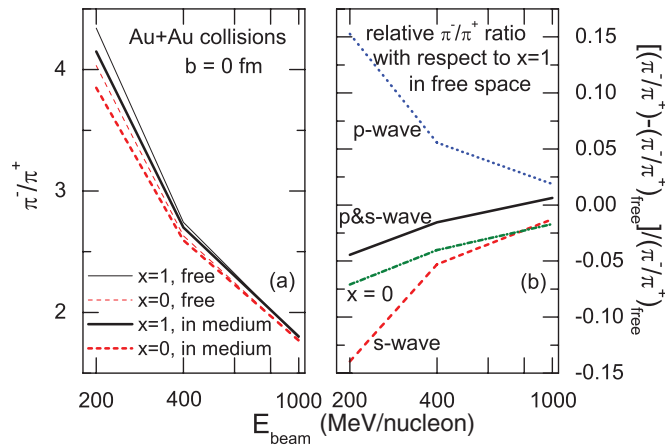FIG. 3. (Color online) Same as Fig. 2 for the Δ resonance mass distributions.

FIG. 4. (Color online) Collision energy dependence of (a) π^-/π^+ ratios with and without pion in-medium effects and (b) relative π^-/π^+ ratios from pion-nucleon s-wave interaction, p-wave interaction, both p- and s-wave interactions, and $x = 0$ with respect to that from $x = 1$ in free space from the thermal model for central Au + Au collisions at different energies.

medium modifications of the production threshold, the pion in-medium effects are also larger, especially at energies below the pion production threshold. This is consistent with the stronger isospin-dependent pion in-medium effects at lower temperatures shown in Figs. 2 and 3.

To understand the relative effect due to the pion-nucleon s-wave and p-wave interactions, we compare in the right panel of Fig. 4 the π^-/π^+ ratios due to the symmetry energy effect with those due to only the s-wave interaction, only the p-wave interaction, and both interactions. With only the s-wave interaction, the π^-/π^+ ratio is significantly reduced. At the collision energy of $200A$ MeV, this reduction is about 14% and is larger than the symmetry energy effect of about 7%. Including only the p-wave interaction, the π^-/π^+ ratio is, on the other hand, increased appreciably, reaching about 15% at

the collision energy of $200A$ MeV. After including both s-wave and p-wave interactions, the effect of the s-wave interaction turns out to dominate over that of the p-wave interaction, leading to a 4% reduction in the π^-/π^+ ratio. This value is still comparable to that due to the symmetry energy effect. At higher collision energies, the pion in-medium effect on the π^-/π^+ ratio becomes, however, very small.

To summarize, we have found via a thermal model that the pion in-medium effects reduce the π^-/π^+ ratio in high-energy heavy-ion collisions compared to that using free pions despite the cancellation between the pion-nucleon s-wave interaction, which reduces the π^-/π^+ ratio, and the p-wave interaction, which increases the π^-/π^+ ratio. Although at lower energies the charged-pion ratio is more sensitive to the symmetry energy, the pion in-medium effect is also larger, especially at collision energies below the pion production threshold. Our results thus indicate that to understand quantitatively the symmetry energy effect on pion production in heavy-ion collisions, it is important to include the isospin-dependent pion in-medium effects, although this is highly nontrivial in the transport model.

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