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Mixing of the low-lying three- and five-quark Ω states with negative parity

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Mixing of the low-lying three- and five-quark Ω states with spin-parity quantum numbers $\frac{1}{2}^-$ and $\frac{3}{2}^-$ is investigated, employing an instanton-induced quark-antiquark pair creation model, which precludes transitions between s^3 and $s^4\bar{s}$ configurations. Models with hyperfine interactions between quarks are of three different kinds, namely, one-gluon-exchange (OGE), Goldstone-boson-exchange (GBE), and an instanton-induced interaction (INS). Numerical results show that the instanton-induced pair creation causes strong mixing between the three- and five-quark configurations with spin 3/2, and that this mixing decreases the energy of the lowest spin 3/2 states in all three different hyperfine interaction models to $\sim 1750 \pm 50$ MeV. On the other hand, transition couplings between s^3 and $s^3q\bar{q}$ states with spin 1/2 caused by instanton-induced $q\bar{q}$ creation is very small and the resulting mixing of three- and five-quark configurations in the OGE and INS models is negligible, while the mixing of the spin 1/2 states in GBE model is not; but effects of this mixing on energies of mixed states are also very small. Accordingly, the lowest Ω states with negative parity in all three hyperfine interactions models have spin 3/2.

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I. INTRODUCTION

Recently, we have studied the spectrum of low-lying $s^3Q\bar{Q}$ (where $Q\bar{Q} = q\bar{q}$, $s\bar{s}$ for light and strange quark-antiquark pairs, respectively) configurations with negative parity within an extended constituent quark model with three different kinds of hyperfine interactions, namely, one-gluon-exchange (OGE), Goldstone-boson-exchange (GBE), and instanton-induced interactions (INS) [1]. Experimental data about Ω resonances is still very poor [2]: only four Ω states were found [3–6], one being the ground state Ω(1672), and all the other three states may also have positive parity [7]. A comparison of calculated results to experimental data is therefore not very conclusive. Compared to the predictions of the masses of negative parity states in traditional three-quark models, the lowest energy of $s^3Q\bar{Q}$ negative parity states is expected to be $\sim$180 MeV lower [1]. This indicates that if we consider Ω resonances as mixtures of three- and five-quark Fock components, then the latter must be relevant for the properties of negative parity Ω resonances.

In the present paper, we shall study the mixing of s^3 and $s^3Q\bar{Q}$ configurations, which involves the investigation of transitions between three- and five-quark Fock states. For such transitions, the key ingredient is the $Q\bar{Q}$ creation mechanism. Most widely accepted is the $^3P_0$ model [8], which has been successfully applied to the decays of mesons and baryons [9,10], and was also employed to analyze the sea flavor content of the ground states of the $SU(3)$ octet baryons [11,12], as well as of the spin and electromagnetic properties of baryons [13,14]. In the $^3P_0$ model, the $Q\bar{Q}$ pair is created with the quantum numbers of the QCD vacuum 0^{++}, which corresponds to $^3P_0$. There are also some other pair creation models, for instance, string-breaking models [15,16], in which the lines of color flux between quarks collapse into a string, the pair is created when the string breaks, and the created pair has as quantum numbers either $^3P_0$ [15] or $^3S_1$ [16]. In Ref. [17], the $Q\bar{Q}$ pair creation induced by a quark confinement interaction was employed to investigate mixing between three- and five-quark Fock components in the nucleon and the Roper resonance; in this case the created $Q\bar{Q}$ also possesses the quantum numbers $^3P_0$.

In case of the low-lying $s^3Q\bar{Q}$ configurations with negative parity, all the quarks and antiquarks are supposed to be relative s waves, and therefore the traditional $^3P_0$ pair creation mechanism can not contribute. Accordingly we here employ an instanton-induced interaction [18–21] for the pair creation mechanism, since this interaction also can lead to the creation of $Q\bar{Q}$ pairs with quantum numbers $^3S_1$ and $^1S_0$. The instanton-induced interaction has been used to describe the decays of (pseudo)scalar mesons [22].

The present paper is organized as follows. In Sec. II, we present our theoretical framework, which includes explicit forms of the instanton-induced quark-antiquark pair creation mechanism. Numerical results for the spectrum of the states under study and the mixing of three- and five-quark configurations in our model are shown in Sec. III. Finally, Sec. IV contains a brief conclusion.

II. THEORETICAL FRAMEWORK

In the present model, to study mixing of the three- and five-quark configurations, we describe the negative parity Ω states by the Hamiltonian

$$H = \begin{pmatrix} H_3 & V_{\Omega_3\leftrightarrow\Omega_5} \\ V_{\Omega_3\leftrightarrow\Omega_5} & H_5 \end{pmatrix} \tag{1}$$

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where H_3 is the Hamiltonian for a three-quark system and H_5 for a five-quark system, and $V_{\Omega_3\leftrightarrow\Omega_5}$ denotes the transition coupling between three- and five-quark systems. Note that in principle the number of three- and five-quark configurations can exceed two. The diagonal terms of (1), the Hamiltonian H_3 for a three-quark system, which has been discussed intensively in the literature, and the Hamiltonian H_5 for a five-quark system with the quantum numbers of negative parity Ω resonances, which was recently developed in Ref. [1], will only briefly be reviewed here in Sec. II A. The nondiagonal terms $V_{\Omega_3\leftrightarrow\Omega_5}$ will be explicitly discussed in Sec II B.

A. Diagonal terms of the Hamiltonian

The Hamiltonian for a N-particle system in the nonrelativistic constituent quark model is usually written as

$$H_N = H_o + H_{\rm hyp} + \sum_{i=1}^{N} m_i, \tag{2}$$

where H_o and $H_{\rm hyp}$ represent the Hamiltonians for the quark orbital motion and for the hyperfine interactions between quarks, respectively, m_i denotes the constituent mass of the ith quark. The first term H_o can be written as a sum of the kinetic energy and the quark confinement potential as

$$H_o = \sum_{i=1}^{N} \frac{\vec{p}_i^{\,2}}{2m_i} + \sum_{i<j}^{N} V_{\rm conf}(r_{ij}). \tag{3}$$

In Ref. [1] the quark confinement potential was taken to be

$$V_{\rm conf}(r_{ij}) = -\frac{3}{8}\lambda_i^C \cdot \lambda_j^C \left[C^{(N)}(\vec{r}_i - \vec{r}_j)^2 + V_0^{(N)}\right], \tag{4}$$

where $C^{(N)}$ and $V_0^{(N)}$ are constants. In principle these two constants can differ for three- and five-quark configurations. The hyperfine interactions between quarks $H_{\rm hyp}$, as stated in Ref. [1], can be mediated by one gluon exchange, Goldstone boson exchange, or induced by the instanton interaction. The forms of these three types of hyperfine interactions in the three-quark system are explicitly given in the literature [7,23–30]. Those in the five-quark system with the quantum numbers of the Ω resonances were explicitly discussed in Ref. [1] and will not be repeated here.

In the $N \leqslant 2$ band, e.g., of the harmonic oscillator quark model, there are two Ω states with negative parity predicted by the three-quark models corresponding to the first orbital excitation with $\ell = 1$: One has spin 1/2, the other 3/2 [7,31,32]. The energies are obtained from the eigenvalues of Eq. (2) in the case of $N = 3$. The results depend on the value of the strange constituent quark mass, the quark confinement parameters $C^{(N=3)}$ and $V_0^{(N=3)}$, as well as the strength of the hyperfine interaction. To reduce free parameters, we just take the values from Refs. [31] and [7] as matrix elements of H_3 in the OGE and GBE models, respectively, i.e., in the OGE model, $\langle H_3\rangle_{\frac{1}{2}^-} = \langle H_3\rangle_{\frac{3}{2}^-} = 2020$ MeV, and in the GBE model, $\langle H_3\rangle_{\frac{1}{2}^-} = \langle H_3\rangle_{\frac{3}{2}^-} = 1991$ MeV. In the INS model, since all three quarks in Ω states are strange and thus the flavor state is symmetric, the hyperfine interaction between quarks vanishes. Accordingly the matrix elements of H_3 in this case only depend on the constituent mass of the strange quark m_s as well as $C^{(N=3)}$ and $V_0^{(N=3)}$. If we adopt the empirical values for m_s and $C^{(N=3)}$ from Ref. [1], and take $V_0^{(N=3)}$ to be the tentative value which reproduces the mass of the ground state $\Omega(1672)$, we find $\langle H_3\rangle_{\frac{1}{2}^-} = \langle H_3\rangle_{\frac{3}{2}^-} = 1887$ MeV in the INS model.

Explicit matrix elements for $\frac{1}{2}^-$ and $\frac{3}{2}^-$ of the submatrix H_5 in (1) were already listed in Ref. [1]. In both cases $\langle H_5\rangle_{\frac{1}{2}(\frac{3}{2})}$ are 4×4 matrices. Here we just employ the results obtained within the OGE, INS, and GBE models of Ref. [1].

B. Nondiagonal terms of Hamiltonian

The nondiagonal term $V_{\Omega_3\leftrightarrow\Omega_5}$ in the Hamiltonian matrix (1) describing the transition coupling between three- and five-quark configurations depends on the explicit quark-antiquark pair creation mechanism. The most commonly accepted mechanism for quark-antiquark pair creation is the $^3P_0$ model [8–10]. In this model the created quark-antiquark pair is in its first orbitally excited state, i.e., the $Q\bar{Q}$ pair has the quantum numbers $^3P_0$. But in the present case all quarks and antiquarks in the studied five-quark configurations are assumed to be in their ground s-wave states and accordingly $^3P_0$ mechanism does not contribute to the coupling between s^3 and $s^3\,Q\bar{Q}$ states considered here.

Therefore we here adopt another quark-antiquark pair creation mechanism based on a nonrelativistic reduction of the amplitudes found from the instanton-induced interaction. This interaction was first proposed by 't Hooft [18] and developed by Shifman *et al.* [19], then Petry *et al.* applied it to the nuclear structure [20]. Explicitly, the instanton-induced interaction vertex for a quark-antiquark pair creation from a quark (antiquark) as shown in Fig. 1(a(b)), can be written [21] in terms of normal ordered products of creation and annihilation operators as

$$\begin{aligned} H_q &:= -\frac{3}{16} g_i \epsilon_{ikl}\epsilon_{imn} : q_{l+}^\dagger q_{k+}^\dagger (\gamma_0 \otimes \gamma_0 + \gamma_0\gamma_5 \otimes \gamma_0\gamma_5) \\ &\quad \left(\mathcal{P}_6^C + 2\mathcal{P}_{\bar{3}}^C\right) q_{m+} q_{n-} :, \end{aligned} \tag{5}$$

$$\begin{aligned} H_{\bar{q}} &:= -\frac{3}{16} g_i \epsilon_{ikl}\epsilon_{imn} : q_{l+}^\dagger q_{k-}^\dagger (\gamma_0 \otimes \gamma_0 + \gamma_0\gamma_5 \otimes \gamma_0\gamma_5) \\ &\quad \left(\mathcal{P}_6^C + 2\mathcal{P}_{\bar{3}}^C\right) q_{m-} q_{n-} : . \end{aligned} \tag{6}$$

Here H_q represents the pair creation from a quark, and $H_{\bar{q}}$ from an antiquark, g_i denotes strength of the instanton-induced interaction, which has been discussed in Ref. [1], $\mathcal{P}_6^C$ and $\mathcal{P}_{\bar{3}}^C$ are projector operators on color **6** and $\bar{\mathbf{3}}$ states, respectively,

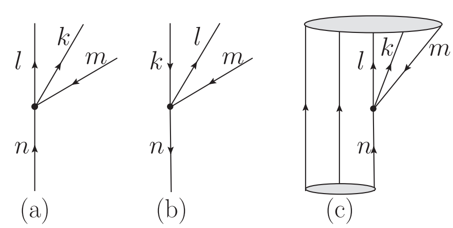

FIG. 1. (a) Quark-antiquark pair creation from a quark. (b) Quark-antiquark pair creation from an antiquark. (c) Transition coupling of three- and five-quark configurations.

which are defined as

$$\mathcal{P}^C_{\bar{3}} = \tfrac{1}{2}(\mathcal{I}d - \Pi^C_{1,2}); \quad \mathcal{P}^C_6 = \tfrac{1}{2}(\mathcal{I}d + \Pi^C_{1,2}), \tag{7}$$

where $\mathcal{I}d$ denotes the identity, and $\Pi^C_{1,2}$ is the permutation operator of two particles in color space. Finally, ϵ_{ikl} is the completely antisymmetric tensor acting on flavor space. This precludes the creation of a quark-antiquark pair whose flavor is the same as the quark n or l. Consequently, in the present case, the instanton-induced interaction does not mix s^3 and $s^4\bar{s}$ configurations. Note that the creation operator itself has only one free parameter g_i, which should be the same as that for instanton-induced hyperfine interactions between quarks [1], therefore, no additional parameter is introduced here.

It is obvious that H_q in Eq. (5) is the appropriate interaction vertex we need for the present discussion in this paper. The normal ordering in (5) leads to two contributions: If q_m has negative energy then q_n is an annihilation field operator and therefore

$$: q^\dagger_{l+} q^\dagger_{k+} q_{m-} q_{n+} := q^\dagger_{l+} q^\dagger_{k+} q_{m-} q_{n+}. \tag{8}$$

If on the other hand q_m is the annihilation field operator, then

$$: q^\dagger_{l+} q^\dagger_{k+} q_{m+} q_{n-} := -q^\dagger_{l+} q^\dagger_{k+} q_{n-} q_{m+} = q^\dagger_{k+} q^\dagger_{l+} q_{n-} q_{m+}. \tag{9}$$

Thus the sign does not change if we simultaneously interchange

$$k \longleftrightarrow l\, m \longleftrightarrow n. \tag{10}$$

Therefore one obtains

$$H_q = -\tfrac{3}{8} g_i \epsilon_{ikl}\epsilon_{imn} q^\dagger_{l+} q^\dagger_{k+} (\gamma_0 \otimes \gamma_0 + \gamma_0\gamma_5 \otimes \gamma_0\gamma_5) \left(\mathcal{P}^C_6 + 2\mathcal{P}^C_{\bar{3}}\right) q_{m+} q_{n-}. \tag{11}$$

In addition, because of the total antisymmetry of the states, one can eliminate the color projectors by replacing them by projectors in spin space. After a nonrelativistic reduction the quark-antiquark creation from a quark by the instanton-induced interaction can be compactly written as

$$H_q = -\frac{3}{64} i g_i \epsilon_{ikl}\epsilon_{imn} \sum_{t=1}^{4}\sum_{\beta=0}^{3} h^t_\beta(k,l,m,n) \times \mathcal{D}_t(\xi^\dagger_k \sigma_\beta \eta_m)(\xi^\dagger_l \sigma_\beta \xi_n), \tag{12}$$

where ξ_x and η_x represent quark and antiquark Pauli spinors. The coefficients h^t_β depend on the quark masses and are given by

$$\begin{aligned}
h^1_0(k,l,m,n) &= \frac{6}{m_k} - \frac{1}{m_l}, \quad h^1_{\beta>0}(k,l,m,n) = -\frac{1}{m_l},\\
h^2_0(k,l,m,n) &= \frac{7}{m_m}, \quad h^1_{\beta>0}(k,l,m,n) = \frac{1}{m_m},\\
h^3_0(k,l,m,n) &= \frac{6}{m_l} - \frac{1}{m_k}, \quad h^3_{\beta>0}(k,l,m,n) = -\frac{1}{m_k},\\
h^4_0(k,l,m,n) &= -\frac{7}{m_n}, \quad h^4_{\beta>0}(k,l,m,n) = -\frac{1}{m_n},
\end{aligned} \tag{13}$$

where $\mathcal{D}_t = \sigma_\alpha \frac{\partial}{\partial x_\alpha}$ acting on the quark t in Eq. (12). For instance, for $t = 3$, $\mathcal{D}_3(\xi^\dagger_k \sigma_\beta \eta_m)(\xi^\dagger_l \sigma_\beta \xi_n) = (\xi^\dagger_k \sigma_\beta \eta_m)(\frac{\partial}{\partial x_\alpha} \xi^\dagger_l \sigma_\alpha \sigma_\beta \xi_n)$. Note that here we defined σ_0 as the identity in spin space. From Eq. (12), one finds that the created $q\bar{q}$ in the instanton-induced pair creation model can have any of the quantum numbers $^3P_0$, $^1P_1$, $^3P_1$, $^1S_0$, and $^3S_1$. In the case of $s^3 \to s^3 q\bar{q}$ states with negative parity considered here only the latter two contribute.

The calculation of the transition $s^3 \to s^3 q\bar{q}$ then involves the overlap between the residual three strange quarks in the $s^3 q\bar{q}$ configuration after $q\bar{q}$ annihilation and the initial s^3 configuration, as shown in Fig. 1(c). Taking into account the overlap factors and the overall symmetry, the transition coupling $V_{\Omega_3 \leftrightarrow \Omega_5}$ in (1) reads

$$V_{\Omega_3 \leftrightarrow \Omega_5} = -\frac{9}{16} i g_i \epsilon_{ikl}\epsilon_{imn} \sum_{t=1}^{4}\sum_{\beta=0}^{3} \mathcal{C_F C_S C_C C_O} \times h^t_\beta(k,l,m,n) \mathcal{D}_t(\xi^\dagger_k \sigma_\beta \eta_m)(\xi^\dagger_l \sigma_\beta \xi_n), \tag{14}$$

where $\mathcal{C_F}$, $\mathcal{C_S}$, $\mathcal{C_C}$, and $\mathcal{C_O}$ are operators for the calculation of the corresponding flavor, spin, color, and orbital overlap factors, respectively.

III. NUMERICAL RESULTS

In the present treatment the matrix elements of transition coupling operator (14) depend on two parameters, namely the instanton-induced interaction strength g' and the oscillator parameter ω_5 for the $s^3 q\bar{q}$ configurations, if we adopt the constituent mass of the strange quark and the oscillator parameter ω_3 for the three-quark configuration as the empirical values from Ref. [1]. Notice that ω_5 is from the orbital overlap factor. In Sec. III A, we present the numerical results obtained by taking g' and ω_5 as constant as previously used in Ref. [1]. The dependence of the numerical results on the parameters is the subject of Sec. III B.

A. Numerical results with fixed parameters

In Refs. [1,33], the empirical value for the strength of the instanton-induced interactions between light and strange quarks was found to be $g' \simeq 33.3$ MeV. On the other hand, if we take the quark confinement parameters equal, i.e., $C^{(N=3)} = C^{(N=5)}$, we find a relation between the oscillator parameters of three- and five-quark configurations: $\omega_5 = \sqrt{5/6}\omega_3$ and, correspondingly, $\omega_5 \simeq 196$ MeV, as shown in Ref. [1]. We denote the three-quark configurations with quantum numbers $\frac{1}{2}^-$ and $\frac{3}{2}^-$ as $|3, \frac{1}{2}^-\rangle$ and $|3, \frac{3}{2}^-\rangle$, respectively. With the notation

$$\begin{aligned}
\left|5, \tfrac{1}{2}^-\right\rangle_1 &= |s^3 q([4]_X[211]_C[31]_{FS}[31]_F[22]_S) \otimes \bar{q}\rangle,\\
\left|5, \tfrac{1}{2}^-\right\rangle_2 &= |s^3 q([4]_X[211]_C[31]_{FS}[31]_F[31]_S) \otimes \bar{q}\rangle,\\
\left|5, \tfrac{1}{2}^-\right\rangle_3 &= |s^3 q([4]_X[211]_C[31]_{FS}[4]_F[31]_S) \otimes \bar{q}\rangle,\\
\left|5, \tfrac{1}{2}^-\right\rangle_4 &= |s^4([4]_X[211]_C[31]_{FS}[4]_F[31]_S) \otimes \bar{s}\rangle,
\end{aligned} \tag{15}$$

for the five-quark configurations with spin 1/2, and

$$\begin{aligned}
\left|5, \tfrac{3}{2}^-\right\rangle_1 &= |s^3 q([4]_X[211]_C[31]_{FS}[31]_F[31]_S) \otimes \bar{q}\rangle,\\
\left|5, \tfrac{3}{2}^-\right\rangle_2 &= |s^3 q([4]_X[211]_C[31]_{FS}[31]_F[4]_S) \otimes \bar{q}\rangle,
\end{aligned}$$

TABLE I. Energies and the corresponding probability amplitudes of three- and five-quark configurations for the obtained Ω states in three different kinds of hyperfine interaction models. The upper and lower panels are for states with quantum numbers $\frac{1}{2}^-$ and $\frac{3}{2}^-$, respectively, and for each panel, the first row shows the energies in MeV, others show the probability amplitudes.

$\frac{1}{2}^-$	OGE					INS					GBE				
	2018	2149	2453	2656	2679	1796	1888	2030	2226	2432	1835	1892	1991	2018	2163
$\lvert 3, \frac{1}{2}^- \rangle$	0.9961	−0.0774	−0.0394	0	−0.0167	0.1494	0.9854	0.0687	0.0425	−0.0096	0	0.2011	0.8964	−0.3951	0
$\lvert 5, \frac{1}{2}^- \rangle_1$	0.0454	0.8428	−0.5332	0	0.0577	0.6650	−0.1563	0.7146	0.1097	− 0.1031	1	0	0	0	0
$\lvert 5, \frac{1}{2}^- \rangle_2$	0.0727	0.5326	0.8365	0	−0.1065	0.7318	−0.0592	−0.6630	−0.1066	0.1003	0	0.9676	−0.2446	−0.0623	0
$\lvert 5, \frac{1}{2}^- \rangle_3$	0.0220	0.0068	0.1201	0	0.9925	0.0002	0.0301	0.1887	−0.9475	0.2563	0	0.1525	0.3697	0.9165	0
$\lvert 5, \frac{1}{2}^- \rangle_4$	0	0	0	1	0	−0.0036	−0.0089	0.0967	0.2775	0.9558	0	0	0	0	1
$\frac{3}{2}^-$	1727	2079	2366	2505	2519	1767	1991	2093	2193	2722	1773	1944	2010	2163	2166
$\lvert 3, \frac{3}{2}^- \rangle$	0.4989	0.8072	−0.3142	0.0299	0	0.8356	−0.0473	0.3243	−0.4353	−0.0692	−0.6389	0.2538	−0.0594	0	0.7238
$\lvert 5, \frac{3}{2}^- \rangle_1$	−0.5556	0.2510	−0.3070	−0.7308	0	−0.3013	0.7715	0.2032	−0.4772	−0.2120	0.6253	0.6984	−0.1904	0	0.2914
$\lvert 5, \frac{3}{2}^- \rangle_2$	0.6651	−0.3885	−0.0029	−0.6377	0	0.2941	0.5539	0.1523	0.5306	0.5495	−0.3637	0.6544	0.4155	0	−0.5165
$\lvert 5, \frac{3}{2}^- \rangle_3$	0.0132	−0.3668	−0.8984	0.2414	0	−0.3518	−0.3089	0.7586	−0.1450	0.4294	0.2617	−0.1395	0.8875	0	0.3528
$\lvert 5, \frac{3}{2}^- \rangle_4$	0	0	0	0	1	−0.0244	−0.0169	−0.5049	−0.5294	0.6812	0	0	0	1	0

$$\left|5, \tfrac{3}{2}^-\right\rangle_3 = |s^3 q([4]_X[211]_C[31]_{FS}[4]_F[31]_S) \otimes \bar{q}\rangle,$$
$$\left|5, \tfrac{3}{2}^-\right\rangle_4 = |s^4([4]_X[211]_C[31]_{FS}[4]_F[31]_S) \otimes \bar{s}\rangle, \qquad (16)$$

for the five-quark configurations with spin 3/2, the matrix elements of the Hamiltonian (1) in both cases are listed in Appendix A. Note that the nonzero off-diagonal matrix elements in the submatrices H_5 are caused by hyperfine interactions between quarks in the five-quark configurations, as explicitly discussed in Ref. [1]. Diagonalization of Eqs. (A1)–(A6) leads to the numerical results shown in Table I. In this table, we have ordered the states according to the energy eigenvalue: The upper panel of the table shows energies of the states with spin 1/2, and the corresponding probability amplitudes of the three- and five-quark configurations in these states, and the lower panel shows those for the states with spin 3/2.

From the upper panel of Table I we conclude that in the OGE and INS hyperfine interaction models the mixing between three- and five-quark configurations with spin 1/2 is very small, and even can be negligible. Accordingly the resulting energies are very close to those obtained in Ref. [1], in which the effects of mixing between s^3 and $s^3 q\bar{q}$ were not included. The mixing between three- and five-quark Ω configurations obtained within the GBE hyperfine interaction model is not so small that can be negligible: For instance, in the state with energy 1991 MeV, there is an 81% three-quark component and a 19% five-quark component. But also in this case the resulting energies are very close those obtained in Ref. [1].

In fact, absolute values of the transition matrix elements of V_{35} in the configurations with spin parity $\frac{1}{2}^-$ are less than 20 MeV, as shown in Eqs. (A1)–(A6), which are tiny compared to the diagonal matrix elements of the Hamiltonian matrix (1). These tiny transition coupling matrix elements lead to tiny mixing between three- and five-quark configurations with spin 1/2 in the OGE and INS models. However, in the GBE model the situation is different: The mixing depends not only on the couplings between the configurations, but also on the differences between the diagonal matrix elements. In the GBE model, as we can see in Eq. (A5), the diagonal matrix element of the third five-quark configuration is close to that of the three-quark configuration, the difference between the former and latter is only 19 MeV. Therefore the mixing of this five-quark configuration with the three-quark configuration is not as small as that in the OGE and INS models. On the other hand, because the diagonal energies of these configurations are close to each other, while matrix elements of the nondiagonal transition coupling are small, nevertheless the resulting energies are very close to those without mixing between three- and five-quark configurations.

In case of the configurations with spin parity $\frac{3}{2}^-$, as shown in the lower panel of Table I, mixing between three- and five-quark Ω configurations in all the three hyperfine interaction models are very strong. Accordingly, the resulting energies differ substantially from those without mixing between three- and five-quark configurations. The strongest mixing is obtained within the GBE model, namely the state with energy 2166 MeV: In this state, there is approximately 50% three-quark component and 50% five-quark components. A very interesting result is that the lowest states in all three models have energies lying in a narrow region around 1750 ± 25 MeV. This energy is significantly lower than the energies of the lowest states with spin parity $\frac{1}{2}^-$ in all three models.

Absolute values of the transition matrix elements of V_{35} in the configurations with spin parity $\frac{3}{2}$ are in the range of 100 ± 20 MeV, which is much larger than the 20 MeV couplings of configurations with spin parity $\frac{1}{2}^-$ and accordingly the mixing is much stronger than in the $\frac{1}{2}^-$ case. In addition, the larger nondiagonal terms lead to larger differences between the energies with and without mixing of three- and five-quark configurations. As we have discussed in Ref. [1], if we ignore the mixing between three- and five-quark configurations, the lowest state in the OGE model has spin 3/2, but in the other two hyperfine interaction models, the lowest states have spin 1/2. If we take into account the transition couplings between three- and five-quark configurations, the lowest states in all three models have spin 3/2. Here, the lowest state in the OGE model resulted partly by mixing between different five-quark configurations caused by the OGE hyperfine interactions between quarks, and partly by mixing between three- and five-quark configurations caused by the instanton-induced $q\bar{q}$ pair creation. In the other two model the lowest state is due to the action of the instanton-induced pair creation.

For both the $\frac{1}{2}^-$ and $\frac{3}{2}^-$ states, one may notice in Table I that there is at least one state that does not mix with other states in the OGE and GBE models: These are the states $|5, \frac{1}{2}^-\rangle_4$ and $|5, \frac{3}{2}^-\rangle_4$, i.e., the five-quark configurations with $s\bar{s}$. As we discussed in Sec. II B, the instanton-induced pair creation interaction does not lead to mixing between s^3 and $s^4\bar{s}$ configurations. Therefore, in the OGE and GBE models, the state with the strange quark-antiquark pair decouples from the others. However, in the INS model, because of the hyperfine interactions between quarks, there is no pure state with $s\bar{s}$ [1], i.e., the INS hyperfine interactions between quarks leads to mixing between the $s^4\bar{s}$ and the $s^3q\bar{q}$ configurations. In the GBE model, there is another configuration that does not mix with the others, namely the state $|5, \frac{1}{2}^-\rangle_2$. This is because on the one hand the GBE hyperfine interactions between quarks cannot mix this configuration with the other five-quark states, and on the other hand the matrix element of the transition coupling V_{35} between the three-quark configuration and the $|5, \frac{1}{2}^-\rangle_2$ state vanishes, as shown in Eq. (A5). In contrast, in the OGE and INS models, although the transition coupling matrix elements between three- and five-quark configurations vanish, the configuration $|5, \frac{1}{2}^-\rangle_2$ does mix with other five-quark configurations, as we can see in Eqs. (A1) and (A3), so there is no pure state $|5, \frac{1}{2}^-\rangle_2$ in the OGE and INS hyperfine interaction models.

B. Dependence of numerical results on parameters

In Sec. III A, we have shown the numerical results for a judicious choice of parameters. We still have to investigate whether the results are sensitive to the interaction parameters and on the value of the oscillator parameter ω_5, which was just taken to be a tentative value in Sec. III A. Here we discuss the dependence of the energies on these parameters. Since the mixing between three- and five-quark configurations with spin-parity quantum numbers $\frac{1}{2}^-$ was found to be very small, we refrain from a discussion on the parameter dependence in this case.

Although the value for the instanton-induced interaction strength g' is an empirical one, it is the most important parameter determining the transition between three- and five-quark configurations. Here we vary it by $\pm 20\%$ to demonstrate the dependence of the energies on its value in Fig. 2. The figures from left to right are the numerical results for states with spin parity $\frac{3}{2}^-$ obtained within the OGE, INS, and the GBE models. As is evident from Fig. 2, the energies do not change much

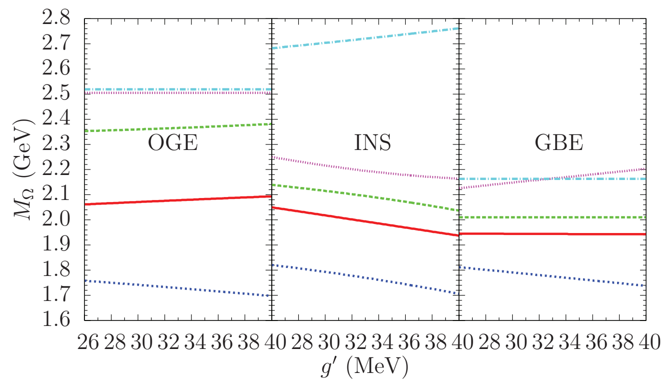

FIG. 2. (Color online) Energies of negative-parity Ω resonances with spin 3/2 as function of the instanton-induced interaction strength g'. The figures from left to right are the numerical results obtained within the OGE, INS, and GBE models, respectively.

in the OGE and GBE models within a range of ± 40 MeV. But in the INS model, the results show some more sensitivity to the coupling g', mainly because the hyperfine interactions between quarks in the INS model also depend on g'.

Another important parameter in the matrix elements of the transition coupling V_{35} is the oscillator parameter ω_5, which determines the orbital overlap of three- and five-quark configurations. As discussed in Sec. III A, once we take $C^{(N=3)} = C^{(N=5)}$ in the quark confinement potential, we get the ratio $R_{35} \equiv \omega_5/\omega_3 = \sqrt{5/6}$, but this is a tentative value only. In general, the color confinement strength $C^{(N)}$ for three- and five-quark configuration could differ, so the value of R_{35} can also differ from $\sqrt{5/6}$. Now the oscillator parameter ω_N reflects the size of the state studied. If, for instance, we take $R_{35} > 1$, i.e., $C^{(N=3)} < C^{(N=5)}$, this implies that the five-quark configurations are more compact than the three-quark configuration. In this case, an intuitive picture for our model is going to be like this: The three-quark state has a weaker potential; when quarks expand, a $q\bar{q}$ pair is pulled out via the instanton-induced pair creation mechanism and results in a $s^3q\bar{q}$ state with stronger potential; the stronger potential leads to a more compact state, which then makes the $\bar{q}$ annihilate with a quark more readily leading to the s^3 state; this leads to constant transitions between these two states and mixing. If, however, $R_{35} < 1$, i.e., $C^{(N=3)} > C^{(N=5)}$, the picture is just the opposite. In Refs. [34–37], in order to reproduce the electromagnetic and strong decays of nucleon resonances, both $R_{35} > 1$ and $R_{35} < 1$ have been suggested. In Refs. [34–37] it was shown that the most sensitive parameter is in fact the ratio $2\omega_3\omega_5/(\omega_3^2 + \omega_5^2)$. It is thus very difficult to judge whether R_{35} is less than 1 or not. Accordingly, we here vary the value of R_{35} from 0.5 to 2 keeping the instanton-induced interaction strength g' at the fixed empirical value. The dependence of the energies of states with spin parity $\frac{3}{2}^-$ on R_{35} are shown in Fig. 3. As in Fig. 2, the figures from left to right are the numerical results obtained within OGE, INS, and GBE models.

One should notice that here we only want to show the dependence of the mixing effects caused by transition couplings on the parameters, so we do not consider the variation of the

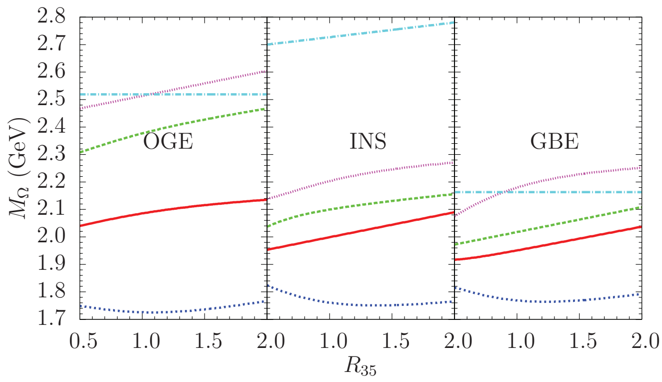

FIG. 3. (Color online) Energies of negative-parity Ω resonances with spin 3/2 as function of R_{35}. The figures from left to right are the numerical results obtained within the OGE, INS, and GBE models, respectively.

diagonal terms in Eqs. (A1)–(A6) with ω_5. As we can see in Fig. 3, the numerical results are somewhat sensitive to R_{35} in all three interaction models, the variation of the energies amount up to 160 MeV. The energies of the lowest states in the three models first decrease and then increase with increasing values for R_{35} and lie within the range $\sim 1750 \pm 50$ MeV The energies of the other states just increase with R_{35}, with the exception of the blue-dash-dotted lines in the OGE and GBE models, which represent the energy of the fourth five-quark configuration in these two models, which does not mix with any others states in the OGE and GBE models, so the energies of this state are independent of R_{35}.

IV. CONCLUSION

In this paper, we investigated the influence of the mixing of s^3 and $s^3 Q\bar{Q}$ Ω configurations on the energies of negative parity Ω states with quantum numbers $\frac{1}{2}^-$ and $\frac{3}{2}^-$. For the hyperfine interactions between quarks we investigated three alternatives: the OGE, INS, and GBE models. The $Q\bar{Q}$ pair creation is taken to be caused by the instanton-induced interaction. This mechanism has the selection rule that it precludes $s\bar{s}$ creation from a strange quark. In other words, the instanton-induced interaction does not lead to mixing between s^3 and $s^4\bar{s}$ configurations.

The matrix elements of the instanton-induced transition coupling in the spin 1/2 configurations are small, leading to negligible mixing between three- and five-quark configurations with spin 1/2 in the OGE and INS hyperfine interaction models. In the GBE model, the spin-1/2 state with strongest mixing is composed of ∼81% three-quark and ∼19% five-quark components mainly due an almost degeneracy of the corresponding unperturbed three- and five-quark configurations. Although the mixing of three- and five-quark configurations with spin 1/2 in the GBE model is not small, the resulting energies are nevertheless very close those without mixing between s^3 and $s^3 q\bar{q}$, since the transition coupling matrix elements are so small.

In the case of configurations with quantum numbers $\frac{3}{2}^-$, the matrix elements of the instanton-induced transition coupling are much larger and the resulting mixing between three- and five-quark configurations is very strong. For instance, the spin 3/2 state with the strongest mixing is composed of ∼50% three-quark and ∼50% five-quark components.

The strong mixing between three- and five-quark configurations with spin 3/2 decreases the energy of the lowest state appreciably in all the three hyperfine interaction models: The lowest states with spin 3/2 have an energy $\sim 1750 \pm 50$ MeV, which is lower than energies of all the spin-1/2 states obtained in the three different interaction models. This is different from the results of previous models [38,39] without considering the mixing between three- and five-quark configurations, which predicted the lowest Ω excitation state to be of spin 1/2. To summarize: In all interaction models the lowest states are found to be those with spin 3/2 and and lie at $\sim 1750 \pm 50$ MeV. The lowest states differ in the three models: Their major components are five-quark configurations (∼75% and ∼64%, respectively) in the OGE and GBE models, whereas in the INS model the lowest state is mainly composed of the three-quark component (∼70%).

Very recently, the BESII Collaboration at Beijing Electron Positron Collider (BEPC) reported an interesting result that $\psi(2S) \to \Omega\bar{\Omega}$ was observed with a branch fraction of $(5 \pm 2) \times 10^{-5}$ [40]. Now with the upgraded BEPC, i.e., BEPCII, BESIII Collaboration [41] is going to take billions of $\psi(2S)$ events, which is two orders of magnitude higher than what the BESII experiment got. If the lowest Ω resonance lies at $\sim 1750 \pm 50$ MeV, then it may be observed from from $\psi(2S) \to \bar{\Omega}\Omega^*$ decays. Once an Ω^* resonance is observed, its spin-parity quantum numbers can be obtained by a partial wave analysis as demonstrated for the N^* case in Refs. [42,43]. Then the most interesting result in the present paper that the lowest Ω resonance with negative parity should have spin 3/2 can be examined. However, it seems to be difficult to distinguish the three different hyperfine interaction models, since the predicted masses of the lowest state in the three models are very close to each other and the most significant difference between the three models is that the predicted probabilities of five-quark components are obviously different, but it is not easy to be examined by the present experimental measurements.

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APPENDIX: MATRIX ELEMENTS OF THE HAMILTONIAN

The matrix elements of Hamiltonian (1) in the configurations with quantum numbers $\frac{1}{2}^-$ and $\frac{3}{2}^-$ read (numbers in units of MeV)

$$\langle H^{\rm OGE}\rangle_{1/2} = \begin{pmatrix} 2020.0 & 0 & -18.5 & -13.1 & 0 \\ 0 & 2235.0 & -139.6 & 10.8 & 0 \\ -18.5 & -149.6 & 2365.4 & -25.6 & 0 \\ -13.1 & 10.8 & -25.6 & 2373.7 & 0 \\ 0 & 0 & 0 & 0 & 2654.7 \end{pmatrix}, \tag{A1}$$

$$\langle H^{\rm OGE}\rangle_{3/2} = \begin{pmatrix} 2020.0 & 115.8 & -124.5 & 81.9 & 0 \\ 115.8 & 2223.4 & 328.9 & 6.6 & 0 \\ -124.5 & 328.9 & 2095.0 & -68.0 & 0 \\ 81.9 & 6.6 & -68.0 & 2333.7 & 0 \\ 0 & 0 & 0 & 0 & 2517.1 \end{pmatrix}, \tag{A2}$$

$$\langle H^{\rm INS}\rangle_{1/2} = \begin{pmatrix} 1887.0 & 0 & -18.5 & -13.1 & 0 \\ 0 & 1928.0 & -121.5 & -30.4 & -33.3 \\ -18.5 & -121.5 & 1908.8 & 30.4 & 33.3 \\ -13.1 & -30.4 & 30.4 & 2230.3 & 47.1 \\ 0 & -33.3 & 33.3 & 47.1 & 2411.0 \end{pmatrix}, \tag{A3}$$

$$\langle H^{\rm INS}\rangle_{3/2} = \begin{pmatrix} 1887.0 & 115.8 & -124.5 & 81.9 & 0 \\ 115.8 & 2052.0 & -113.3 & -60.7 & -66.7 \\ -124.5 & -113.3 & 2250.0 & 191.9 & 210.8 \\ 81.9 & -60.7 & 191.9 & 2159.0 & 188.5 \\ 0 & -66.7 & 210.8 & 188.5 & 2411.0 \end{pmatrix}, \tag{A4}$$

$$\langle H^{\rm GBE}\rangle_{1/2} = \begin{pmatrix} 1991.0 & 0 & -18.5 & -13.1 & 0 \\ 0 & 1833.6 & 0 & 0 & 0 \\ -18.5 & 0 & 1896.6 & -16.2 & 0 \\ -13.1 & 0 & -16.2 & 2010.0 & 0 \\ 0 & 0 & 0 & 0 & 2161.6 \end{pmatrix}, \tag{A5}$$

$$\langle H^{\rm GBE}\rangle_{3/2} = \begin{pmatrix} 1991.0 & 115.8 & -124.5 & 81.9 & 0 \\ 115.8 & 1896.6 & 0 & -16.2 & 0 \\ -124.5 & 0 & 1990.2 & 0 & 0 \\ 81.9 & -16.2 & 0 & 2010.0 & 0 \\ 0 & 0 & 0 & 0 & 2161.6 \end{pmatrix}. \tag{A6}$$

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