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Nucleon-meson couplings in a one-boson-exchange potential using noncritical string theory

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A noncritical holographic QCD model constructed in the six dimensional anti–de Sitter (AdS_6) supergravity background is employed to study a baryon. It is shown that the size of the baryon is of order one with respect to the λ, however, it is smaller than the scale of the dual QCD. An effective four-dimensional action for the nucleon is obtained in terms of the meson exchange potentials. All meson-nucleon couplings in the noncritical AdS_6 background are calculated. Results obtained using our model are compared with predictions of four modern phenomenological interaction models. Also, our numerical results are compared with the results of the Sakai-Sugimoto (SS) model which indicate that the noncritical holographic QCD model can be a good toy to calculate the meson-nucleon couplings.

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I. INTRODUCTION

The construction of a nucleon-nucleon (NN) potential has a long history in nuclear physics due to its role in understanding the nuclear force. Many potential models have been constructed from the 1950s which have been composed to fit the available NN scattering data. The newer potentials have only slightly improved with respect to the previous ones in describing the recent much more accurate data. As it is shown in Ref. [1], all of these potential models do not have good quality with respect to the pp scattering data below 350 MeV and just a few of them are of satisfactory quality. These models are the Reid soft-core potential Reid68 [2], the Nijmegen soft-core potential Nijm78 [3], the new Bonn pp potential Bonn89 [4], and also the parametrized Paris potential Paris80 [5]. These familiar one-boson-exchange potentials (OBEP) contain a relatively small number of free parameters (about 10 to 15 parameters), but do not have a reasonable description of the empirical scattering data. Also, most of these potentials which have been fitted to the np scattering data, unfortunately do not automatically fit to the pp scattering data even by considering the correction term for the Coulomb interaction [1]. Of course, new versions of these potentials have been constructed such as Nijm I, Nijm II, Reid93 [6], CD-Bonn [7], and AV18 [8] which explain the empirical scattering data successfully. But they contain a large number of purely phenomenological parameters. For example, an updated (Nijm92pp [9]) version of the Nijm78 potential contains 39 free parameters.

On the other hand, there are many attempts to impose the symmetries of QCD using an effective Lagrangian of pions and nucleons [10,11]. These models only capture the qualitative features of the nuclear interactions and cannot compete with the much more successful potential models mentioned above. Despite many efforts, no potential model has yet been constructed which gives a high-quality description of the empirical data, obeys the symmetries of QCD, and contains only a few number of free phenomenological parameters.

One of the applications of anti–de Sitter space/conformal field theory (AdS/CFT) duality [12–14] is a holographic QCD introduced recently to solve the strong-coupling QCD problems such as the chiral dynamics of hadrons in particular baryons [15–38]. The Sakai-Sugimoto (SS) [39,40] and Klebanov-Strassler (KS) models [41] are the most interesting holographic models.

The predictions of the SS model are in a good agreement with the lattice simulations such as a glueball spectrum of pure QCD [42,43]. Also this model describes baryons and their interactions with mesons [22–24,39,40]. It is shown that the baryons can be taken as point-like objects at distances larger than their sizes, so their interactions can be described by the exchange of light particles such as mesons. Therefore, one can find the baryon-baryon potential from the Feynman diagrams using the interaction vertices including baryon currents and light mesons [23]. But there are some inconsistencies. For example, the size of the baryon is proportional to $\lambda^{-1/2}$. Consequently in the large 't Hooft coupling (large λ), the size of the baryon becomes zero and the stringy corrections have to be taken into account. Another problem is that the scale of the system associated with the baryonic structure is roughly half the one needed to fit to the mesonic data [44].

Also there is another problem for such holographic models arising from critical string theory. In these models, the color brane backgrounds are ten-dimensional so the dual gauge theories are supersymmetric. In order to break the supersymmetry, some parts of such backgrounds need to be compacted on some manifolds. This causes the production of some Kaluza-Klein (KK) modes with the mass scale of the same order as the masses of the hadronic modes. These unwanted modes are coupled to the hadronic modes, and there is no mechanism to disentangle them from the hadronic modes yet. In order to overcome this problem, it is possible to consider the color brane configuration in noncritical string theory. The result is a gravitational background located at the low dimensions [45–48]. In this background the string coupling constant is proportional to $\frac{1}{N_c}$, so the large N_c limit

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corresponds to the small string coupling constant. However, contrary to the critical holographic models, in the large N_c limit, the 't Hooft coupling is of order one instead of infinity and the scalar curvature of the gravitational background is also of order one. So, it seems the noncritical gauge-gravity correspondence is not very reliable. But studies show that the results of these models for some low energy QCD properties such as the meson mass spectrum, Wilson loop, and the mass spectrum of glueballs [49–51] are comparable with lattice computations. Therefore noncritical holographic models still seem useful to study QCD.

One of the noncritical holographic models is composed of a $D4$ and anti-$D4$ brane in six-dimensional noncritical string theory [47,49]. The low energy effective theory on the intersecting brane configuration is a four-dimensional QCD-like effective theory with the global chiral symmetry $\mathrm{U}(N_f)_L \times \mathrm{U}(N_f)_R$. In this brane configuration, the six-dimensional gravity background is the near horizon geometry of the color $D4$ branes. This model is based on the compactified AdS_6 space-time with constant dilaton. So the model does not suffer from the large string coupling as the SS model. The meson spectrum [49] and the structure of thermal phase [52] are studied in this model. Some properties, like the dependence of the meson masses on the stringy mass of the quarks and the excitation number are different from the critical holographic models such as the SS model.

In this paper, we are going to obtain the NN potential using the noncritical AdS_6 background. We study the gauge field and its mode expansion in this noncritical holography model and obtain the pion action. The model has a mass scale M_{KK} like the SS model in which we set its value by computing the pion decay constant. Then, we study the baryon and obtain its size. We show that the size of the baryon is of order one with respect to the 't Hooft coupling, so the problem of the zero size of the baryon in the critical holography model is solved. But the size of the baryon is still smaller than the mass scale of holographic QCD, so we treat it as a point-like object and introduce an isospin 1/2 Dirac field for the baryon. We write a 5D effective action for the baryon field and reduce it to the 4D using the mode expansion of a gauge field and baryon field and obtain the NN potential in terms of the meson exchange interactions. We calculate the meson-nucleon couplings using the suitable overlapping wave function integrals and compare them with the results of SS model. Also, our results are compared with predictions of some phenomenological models and also the SS models for the couplings. Our study shows that the noncritical results are in good agreement with the other available models.

This paper is organized as follows. In Sec. II we briefly review the noncritical model and mode expansion of the gauge field. We analyze the baryon and extract its mass and size in Sec. III. In Sec. IV, an effective action for the baryon is considered and a noncritical prescription of the nucleon-nucleon potential in terms of the meson exchange interactions is obtained. In Sec. V, the nucleon-meson couplings are calculated and compared with predictions of four modern phenomenological models [Nijmegen (93), Paris, CD-Bonn, and AV 18 models]. Section VI is devoted to a brief summary and conclusions.

II. HOLOGRAPHIC QCD FROM THE NONCRITICAL STRING THEORY

In the presented noncritical model, the gravity background is generated by near-extremal $D4$ branes wrapped over a circle with antiperiodic boundary conditions. Two stacks of flavor branes, namely $D4$ branes and anti-$D4$ branes, are added to this geometry and are called flavor probe branes. The color branes extend along the directions t, x_1, x_2, x_3, and τ while the probe flavor branes fill the whole Minkowski space and stretch along the radius U which is extended to infinity. The strings attaching a color $D4$ brane to a flavor brane transform as quarks, while strings hanging between a color $D4$ and a flavor $\overline{D4}$ transform as antiquarks. The chiral symmetry breaking is achieved by a reconnection of the brane/antibrane pairs. Under the quenched approximation ($N_c \gg N_f$), the reactions of flavor branes and the color branes can be neglected. Just like the SS model, the τ coordinate is wrapped on a circle and the antiperiodic condition is considered for the fermions on the thermal circle. The final low energy effective theory on the background is a four-dimensional QCD-like effective theory with the global chiral symmetry $\mathrm{U}(N_f)_L \times \mathrm{U}(N_f)_R$.

In this model, the near horizon gravity background at low energy is [49]

$$ds^2 = \left(\frac{U}{R}\right)^2(-dt^2 + dx_i dx_i + f(U)d\tau^2) + \left(\frac{R}{U}\right)^2\frac{dU^2}{f(U)}, \tag{1}$$

where R is the radius of the AdS space. Also $f(U)$ and RR six-form field strength, $F_{(6)}$ are defined by the following relations:

$$\begin{aligned} F_{(6)} &= Q_c\left(\frac{U}{R}\right)^4 dt \wedge dx_1 \wedge dx_2 \wedge dx_3 \wedge du \wedge d\tau, \\ f(U) &= 1 - \left(\frac{U_{KK}}{U}\right)^5. \end{aligned} \tag{2}$$

In order to obtain solutions of near extremal flavored AdS_6, the values of dilaton and R_{AdS} are considered as

$$\begin{aligned} e^{\phi} &= \frac{2}{3}\frac{Q_f}{Q_c^2}\left(\sqrt{1+\frac{6Q_c^2}{Q_f^2}} - 1\right), \\ R_{\mathrm{AdS}}^2 &= \frac{90}{12 + \frac{Q_f^2}{Q_c^2} - \frac{Q_f^2}{Q_c^2}\sqrt{1+\frac{6Q_c^2}{Q_f^2}}}. \end{aligned} \tag{3}$$

This relation indicates that the R_{AdS} and dilaton depend on the ratio of the number of colors ($\sim Q_c$) and flavors ($\sim Q_f$). Under the quenched approximation, the values of the dilaton and AdS radius can be rewritten as

$$R_{\mathrm{AdS}}^2 = \frac{15}{2}, \quad e^{\phi} = \frac{2\sqrt{2}}{\sqrt{3}Q_c}, \tag{4}$$

where Q_c is proportional to the number of color branes, N_c.

To avoid singularity, the coordinate τ satisfies the following periodic condition:

$$\tau \sim \tau + \delta\tau, \quad \delta\tau = \frac{4\pi R^2}{5U_{KK}}. \tag{5}$$

Also, the Kaluza-Klein mass scale of this compact dimension is

$$M_{KK} = \frac{2\pi}{\delta\tau} = \frac{5}{2}\frac{U_{KK}}{R^2}, \tag{6}$$

and dual gauge field theory for this background is non-supersymmetric. Also, the Yang-Mills coupling constants can be defined as a function of string theory parameters using the DBI action as follows:

$$g_{YM}^2 = \frac{g_s}{\mu_4\,(2\pi\alpha')^2\,\delta\tau}, \tag{7}$$

where $\alpha' = l_s^2$ is the Regge slope parameter and l_s is the string length. Also, the 't Hooft coupling is $\lambda = g_{YM}^2\,N_c$.

In AdS/QCD, there is gauge field living in the bulk AdS whose dynamics is dual to the meson sector of QCD such as pions and higher resonances. The gauge field on the $D4$ brane includes five components, $A_\mu(\mu = 0, 1, 2, 3)$ and A_U. The $D4$ brane action is given by

$$S_{D4} = -\mu_4 \int d^5x e^{-\phi}\sqrt{-\det(g_{MN} + 2\pi\alpha' F_{MN})} + S_{CS}, \tag{8}$$

where $F_{MN} = \partial_M A_N - \partial_N A_M - i[A_M, A_N]$, $(M, N = 0, 1, \ldots, 5)$ is the field strength tensor, and the A_M is the $U(N_f)$ gauge field on the $D4$ brane. The second term in the above action is the Chern-Simons action and $\mu_4 = 2\pi/(2\pi ls)^5$. It is useful to define the new variable z as

$$U_z = \left(U_{KK}^5 + U_{KK}^3\,z^2\right)^{1/5}. \tag{9}$$

Then by neglecting the higher order of F^2 in the expansion, the $D4$ brane action can be written as

$$S_{D4} = -\widetilde{\mu}_4(2\pi\alpha')^2 \int d^4x dz \left[\frac{R^4}{4U_z^{5/2}}\eta^{\mu\nu}\eta^{\rho\sigma}F_{\mu\rho}F_{\nu\sigma} + \frac{25}{8}\frac{U_z^{9/2}}{U_{kk}^3}\eta^{\mu\nu}F_{\mu z}F_{\nu z}\right] + \mathcal{O}(F^3), \tag{10}$$

where $\widetilde{\mu}_4$ is

$$\widetilde{\mu}_4 = \sqrt{\frac{3}{2}}\,\frac{N_c U_{KK}^{3/2}}{5R^3}\,\mu_4. \tag{11}$$

The gauge fields A_μ $(\mu = 0, 1, 2, 3)$ and A_z have a mode expansion in terms of complete sets $\{\psi_n(z)\}$ and $\{\phi_n(z)\}$ as

$$A_\mu(x^\mu, z) = \sum_n B_\mu^{(n)}(x^\mu)\psi_n(z)\,, \tag{12}$$

$$A_z(x^\mu, z) = \sum_n \varphi^{(n)}(x^\mu)\phi_n(z)\,. \tag{13}$$

After calculating the field strengths, the action (10) is rewritten as

$$S_{D4} = -\widetilde{\mu}_4(2\pi\alpha')^2 \int d^4x dz \sum_{m,n}\left[\frac{R^4}{4U_z^{5/2}}F_{\mu\nu}^{(m)}F^{\mu\nu(n)}\psi_m\psi_n + \frac{25}{8}\frac{U_z^{9/2}}{U_{kk}^3}(\partial_\mu\varphi^{(m)}\partial^\mu\varphi^{(n)}\phi_m\phi_n + B_\mu^{(m)}B^{\mu(n)}\dot{\psi}_m\dot{\psi}_n - 2\partial_\mu\varphi^{(m)}B^{\mu(n)}\phi_m\dot{\psi}_n)\right], \tag{14}$$

where the overdot denotes the derivative respect to the z coordinate.

Let us consider first the vector meson field $B_\mu^{(m)}$. So, we need to keep the following part of action:

$$S_{D4} = -\widetilde{\mu}_4(2\pi\alpha')^2 \int d^4x dz \times \sum_{m,n}\left[\frac{R^4}{4U_z^{5/2}}F_{\mu\nu}^{(m)}F^{\mu\nu(n)}\psi_m\psi_n + \frac{25}{8}\frac{U_z^{9/2}}{U_{kk}^3}B_\mu^{(m)}B^{\mu(n)}\dot{\psi}_m\dot{\psi}_n\right]. \tag{15}$$

We introduce the following dimensionless parameters:

$$\widetilde{z} \equiv \frac{z}{U_{KK}}, \quad K(\widetilde{z}) \equiv 1 + \widetilde{z}^2 = \left(\frac{U_z}{U_{KK}}\right)^5, \tag{16}$$

and using these parameters, we rewrite the action (15) as

$$S_{D4} = -\widetilde{\mu}_4(2\pi\alpha')^2\frac{R^4}{U_{KK}^{3/2}} \int d^4x d\widetilde{z} \times \sum_{n,m}\left[\frac{1}{4}K^{-1/2}F_{\mu\nu}^{(n)}F^{(m)\mu\nu}\psi_n\psi_m + \frac{1}{2}M_{KK}^2K^{9/10}B_\mu^{(n)}B^{(m)\mu}\partial_{\widetilde{z}}\psi_n\partial_{\widetilde{z}}\psi_m\right]. \tag{17}$$

Functions ψ_n $(n \geqslant 1)$ satisfy the normalization condition as

$$\widetilde{\mu}_4(2\pi\alpha')^2\frac{R^4}{U_{KK}^{3/2}} \int d\widetilde{z}\; K^{-1/2}\,\psi_n\psi_m = \delta_{nm}\,. \tag{18}$$

Also, we suppose the functions ψ_n $(n \geqslant 1)$ satisfy the following condition:

$$\widetilde{\mu}_4(2\pi\alpha')^2\frac{R^4}{U_{KK}^{3/2}} \int d\widetilde{z}\; K^{9/10}\,\partial_{\widetilde{z}}\psi_m\,\partial_{\widetilde{z}}\psi_n = \lambda_n\delta_{nm}. \tag{19}$$

Using Eqs. (18) and (19), an eigenvalue equation is obtained for the functions ψ_n $(n \geqslant 1)$ as

$$-K^{1/2}\,\partial_{\widetilde{z}}(K^{9/10}\,\partial_{\widetilde{z}}\psi_m) = \lambda_m\psi_m\,. \tag{20}$$

Considering the above conditions, the action becomes canonically normalized

$$S_{D4} = \int d^4x \sum_{n=1}^{\infty}\left[\frac{1}{4}F_{\mu\nu}^{(n)}F^{\mu\nu(n)} + \frac{1}{2}m_n^2\,B_\mu^{(n)}B^{\mu(n)}\right], \tag{21}$$

where $B_\mu^{(n)}$ is a massive vector meson of mass $m_n \equiv \lambda_n^{1/2}M_{KK}$ for all $n \geqslant 1$. Let us consider $\varphi^{(n)}$ and rewrite the pseudoscalar part of action (14) in terms of new variables, Eq. (16):

$$S_{D4} = -\widetilde{\mu}_4(2\pi\alpha')^2 \int d^4x d\widetilde{z}\,\frac{25}{4}\,U_{KK}^{3/2}\,K^{9/10} \times \sum_{m,n}\left[\frac{1}{2}\,U_{KK}\partial_\mu\varphi^{(m)}\partial^\mu\varphi^{(n)}\phi_m\phi_n - \partial_\mu\varphi^{(m)}B^{\mu(n)}\phi_m\partial_{\widetilde{z}}\psi_n\right]. \tag{22}$$

In order to normalize the kinetic part of the above action, we consider the following orthonormal condition for ϕ_n:

$$(\phi_m, \phi_n) \equiv \frac{25}{4}\,\widetilde{\mu}_4(2\pi\alpha')^2 U_{KK}^{5/2} \int d\widetilde{z}\; K^{9/10}\,\phi_m\phi_n = \delta_{mn}. \tag{23}$$

By multiplying Eq. (23) by λ_n and comparing it with Eq. (19), we find that the functions $\phi_{(n)}$ and $\dot{\psi}_n$ are related together. In fact, we can consider $\phi_n = m_n^{-1}\,\dot{\psi}_n$ $(n \geqslant 1)$. Also, there exists a function $\phi_0 = C/K^{9/10}$ which is orthogonal to $\dot{\psi}_n$ for all $n \geqslant 1$:

$$(\phi_0, \phi_n) \propto \int d\widetilde{z}\; \partial_{\widetilde{z}}\psi_n = 0, \quad (\text{for} \quad n \geqslant 1). \tag{24}$$

We use the normalization condition $1 = (\phi_0, \phi_0)$ to obtain the normalization constant C. Finally by using an appropriate gauge transformation, the action (10) becomes

$$S_{D4} = -\int d^4x \left[\frac{1}{2}\,\partial_\mu\varphi^{(0)}\partial^\mu\varphi^{(0)} + \sum_{n\geqslant 1}\left(\frac{1}{4}F_{\mu\nu}^{(n)}F^{\mu\nu(n)} + \frac{1}{2}m_n^2\,B_\mu^{(n)}B^{\mu(n)}\right)\right], \tag{25}$$

where $\varphi^{(0)}$ is the pion field, which is the Nambu-Goldstone boson associated with the chiral symmetry breaking. An interpretation of this field is the same as the critical SS model [39]. Therefore it is not necessary to repeat it here.

To ensure that the field strengths vanish at $z \to \pm\infty$, it is useful to make another gauge choice, namely the $A_z = 0$ gauge. Actually, we can transform to the new gauge through the following gauge transformation:

$$A_M \to A_M - \partial_M \Lambda\,, \tag{26}$$

and obtain the following new gauge fields:

$$\begin{aligned} &A_z(x^\mu, z) = 0,\\ &A_\mu(x^\mu, z) = -\partial_\mu\varphi^{(0)}(x^\mu)\psi_0(z) + \sum_{n\geqslant 1} B_\mu^{(n)}(x^\mu)\psi_n(z)\,. \end{aligned} \tag{27}$$

Function $\psi_0(z)$ is calculated through

$$\psi_0(z) = \int_0^z dz'\,\phi_0(z') = C\,U_{KK}\;\widetilde{z}\;F_1(0.5, 0.9, 1.5, -\widetilde{z}^2), \tag{28}$$

where F_1 is well-known hypergeometric function. It should be noted that the massless pseudoscalar meson appears in the asymptotic behavior of A_μ, since we have

$$A_\mu(x^\mu, z) \to \pm 1.8 C U_{KK}\,\partial_\mu\varphi^{(0)}(x^\mu) \quad (\text{as } z \to \pm\infty). \tag{29}$$

In order to calculate the meson spectrum, it is necessary to solve Eq. (20) numerically by considering the normalization condition (18).

Since Eq. (20) is invariant under $\widetilde{z} \to -\widetilde{z}$, we can assume ψ_n to be an even or odd function. In fact, the $B_\mu^{(n)}$ is a four-dimensional vector and axial vector if ψ_n is an even or odd function, respectively. Equation (20) is solved numerically using the shooting method to obtain the mass of lightest mesons. Our results are compared with the results of the SS model and experimental data in Table I. As is clear, our result are in good agreement with experimental data. Also, the same values have been obtained in Ref. [49] using the AdS_6 background which is exactly coincident with our results.

TABLE I. The ratio of the obtained eigenvalues of Eq. (20) compared with the results of the SS model [39] and the ratio of meson masses.

	Our model	SS model	Experiment
$\frac{\lambda_2}{\lambda_1}$	2.78	2.4	$\frac{m^2_{a_1(1260)}}{m^2_\rho} \simeq \frac{(1230\ \mathrm{MeV})^2}{(776\ \mathrm{MeV})^2} \simeq 2.51$
$\frac{\lambda_3}{\lambda_1}$	5.5	4.3	$\frac{m^2_{\rho(1450)}}{m^2_\rho} \simeq \frac{(1465\ \mathrm{MeV})^2}{(776\ \mathrm{MeV})^2} \simeq 3.56$
$\frac{\lambda_3}{\lambda_2}$	1.98	1.8	$\frac{m^2_{\rho(1450)}}{m^2_{a_1(1260)}} \simeq \frac{(1465\ \mathrm{MeV})^2}{(1230\ \mathrm{MeV})^2} \simeq 1.41$

It is straightforward to generalize the above analyses to the case of $N_f > 1$ flavor QCD by introducing N_f probe $D4$ branes. In order to obtain a finite four-dimensional action for the modes localized around $z = 0$, the field strength F_{MN} should vanish at $z = \pm\infty$. This implies that the gauge field A_M must asymptotically take a pure gauge configuration

$$A_M(x^\mu, z) \to U_\pm^{-1}(x^\mu, z)\partial_M U_\pm(x^\mu, z), \quad (\text{as} \quad z \to \pm\infty). \tag{30}$$

In analogy to the SS model [39], we can write

$$A_\mu(x^\mu, z) = U^{-1}(x^\mu)\partial_\mu U(x^\mu)\psi_+(z) + \sum_{n\geqslant 1} B_\mu^{(n)}(x^\mu)\psi_n(z), \tag{31}$$

where

$$\psi_\pm(z) = \tfrac{1}{2} \pm \widehat{\psi}_0(z), \tag{32}$$

$$\widehat{\psi}_0(\widetilde{z}) = \tfrac{1}{3.6}\;\widetilde{z}\;F_1(0.5, 0.9, 1.5, -\widetilde{z}^2). \tag{33}$$

Now, by neglecting the vector meson fields, $B_\mu^{(n)}$ $(n \geqslant 1)$, the field strengths can be written as

$$\begin{aligned} F_{\mu\nu} &= [U^{-1}\partial_\mu U, U^{-1}\partial_\nu U]\,\psi_+(\psi_+ - 1)\,,\\ F_{z\mu} &= U^{-1}\partial_\mu U\,\partial_{\widetilde{z}}\widehat{\psi}_0(\widetilde{z}). \end{aligned} \tag{34}$$

Substituting these quantities into the non-Abelian generalization of Eq. (10), we obtain

$$S_{D4} = -\widetilde{\mu}_4(2\pi\alpha')^2 \int d^4x\ \mathrm{tr}(A(U^{-1}\partial_\mu U)^2 + B\,[U^{-1}\partial_\mu U, U^{-1}\partial_\nu U\,]^2)\,, \tag{35}$$

where the coefficients A and B are defined by the following relations:

$$\begin{aligned} A &\equiv 2\,\frac{25}{8}\frac{1}{U_{KK}^3}\int d\widetilde{z}\;U_z^{9/2}(\partial_{\widetilde{z}}\widehat{\psi}_0(\widetilde{z}))^2 = \frac{25}{4}\frac{U_{KK}^{1/2}}{3.6}\,,\\ B &\equiv 2\,\frac{R^4}{4}\int dz\,\frac{1}{U_z^{5/2}}\psi_+^2(\psi_+ - 1)^2 = \frac{0.16R^4}{2U_{KK}^{3/2}}\,. \end{aligned} \tag{36}$$

If we compare Eq. (35) with the familiar action of the Skyrme model [53]

$$S = \int d^4x\left(\frac{f_\pi^2}{4}\,\mathrm{tr}(U^{-1}\partial_\mu U)^2 + \frac{1}{32e^2}\,\mathrm{tr}[U^{-1}\partial_\mu U, U^{-1}\partial_\nu U]^2\right), \quad (37)$$

it is possible to calculate the pion decay constant f_π and dimensionless parameter e in terms of the noncritical model parameters

$$f_\pi^2 = 4\,\tilde{\mu}_4(2\pi\alpha')^2\,A = \sqrt{\frac{3}{2}}\,\frac{45\,\mu_4(2\,\pi\,\alpha')^2}{3.6\,R^3}N_c M_{KK}^2, \quad (38)$$

and

$$\frac{1}{e^2} = 32\,\tilde{\mu}_4(2\pi\alpha')^2\,B = \sqrt{\frac{3}{8}}\mu_4(2\,\pi\,\alpha')^2\,R\,N_c. \quad (39)$$

It is clear from the above equations that the parameters f_π and e depend on N_c as $f_\pi \sim \mathcal{O}(\sqrt{N_c})$ and $e \sim \mathcal{O}(1/\sqrt{N_c})$, respectively. It is coincident with the result obtained from the SS model and also QCD in large N_c. We fix the M_{KK} such that the $f_\pi \sim 93$ MeV for $N_c = 3$. So, we obtain $M_{KK} = 395$ MeV for our holographic model. It should be noted that M_{KK} is the only mass scale of the noncritical model below which the theory is effectively pure Yang-Mills in four dimensions.

III. BARYON IN AdS$_6$

In this section we aim to introduce baryon configuration in the noncritical holographic model. As is known, in the SS model the baryon vertex is a $D4$ brane wrapped on a S^4 cycle. Here in six-dimensional configuration, there is no compact S^4 sphere. So, we introduce an unwrapped $D0$ brane as a baryon vertex instead [26]. In analogy with the SS model, there is a Chern-Simons term on the vertex world volume as

$$S_{CS} \propto \int dt\, A_0(t), \quad (40)$$

which induces N_c units of electric charge on the unwrapped $D0$ brane. In accordance with the Gauss constraint, the net charge should be zero. So, one needs to attach N_c fundamental strings to the $D0$ brane. In turn, the other side of the strings should end up on the probe $D4$ branes. The baryon vertex looks like an object with N_c electric charge with respect to the gauge field on the $D4$ brane whose charge is the baryon number. This $D0$ brane dissolves into the $D4$ brane and becomes an instanton soliton [26]. It is important to know the size of the instanton in our model. In the SS model, it is shown that the size of an instantonic baryon goes to zero at a large 't Hooft coupling limit which is one of the problems of the SS model in describing the baryons [23].

Let us consider the DBI action in the Yang-Mills approximation for the $D4$ brane

$$S_{YM} = -\frac{1}{4}\,\mu_4(2\pi\alpha')^2\int e^{-\phi}\sqrt{-g_{4+1}}\,\mathrm{tr}\,F_{mn}F^{mn}. \quad (41)$$

The induced metric on the $D4$ brane is

$$g_{4+1} = \left(\frac{U}{R}\right)^2\left(\eta_{\mu\nu}dx^\mu dx^\nu + \left(\frac{R}{U}\right)^4\frac{dU^2}{f(U)}\right). \quad (42)$$

It is useful to define the new coordinate w

$$dw = \frac{R^2\,U^{1/2}\,dU}{\sqrt{U^5 - U_{KK}^5}}. \quad (43)$$

Using this coordinate, the metric (42) transforms to a conformally flat metric

$$g_{4+1} = H(w)(dw^2 + \eta_{\mu\nu}dx^\mu dx^\nu), \quad (44)$$

where

$$H(w) = \left(\frac{U}{R}\right)^2. \quad (45)$$

Also, the w coordinate can be rewritten in terms of the z coordinate introduced in Eq. (16) as

$$dw = \frac{2}{5}\frac{R^2\,U_{KK}^3\,dz}{\left(U_{KK}^5 - U_{KK}^3\,z^2\right)^{7/10}}. \quad (46)$$

Note that in the new conformally flat metric, the fifth direction is a finite interval $[-w_{max}, w_{max}]$ because

$$w_{max} = \int_0^\infty \frac{R^2\,U^{1/2}\,dU}{\sqrt{U^5 - U_{KK}^5}} = \frac{R^2}{U_{KK}}\int_1^\infty \frac{d\tilde{U}}{\sqrt{\tilde{U}^5 - 1}} \simeq \frac{R^2}{U_{KK}}1.25 < \infty. \quad (47)$$

We can approximate w near the origin $w \simeq 0$, as

$$w \simeq \frac{2}{5}\left(\frac{R}{U_{KK}}\right)^2 z, \quad (48)$$

and using relation (6), we obtain

$$w \simeq \frac{z}{M_{KK}\,U_{KK}} \quad \text{or} \quad M_{KK}\,w \simeq \frac{z}{U_{KK}}, \quad (49)$$

or equivalently,

$$U^5 \simeq U_{KK}^5\left(1 + M_{KK}^2\,w^2\right). \quad (50)$$

In analogy with the SS model, this relation implies that M_{KK} is the only mass scale that dictated the deviation of the metric from the flat configuration and it is the only mass scale of the theory in the low energy limit. (It should be noted that the $D4$ branes come with two asymptotic regions at $w \to \pm w_{max}$ corresponding to the ultraviolet and infrared region near the $w \simeq 0$.)

Equation (41) is rewritten in the conformally flat metric (44) as

$$S_{YM}^{D4} = -\frac{1}{4}\mu_4(2\pi\alpha')^2\int d^4x dw e^{-\phi}\left(\frac{U(w)}{R}\right)\mathrm{tr}F_{mn}F^{mn} = -\int dx^4 dw\,\frac{1}{4e^2(w)}\,\mathrm{tr}F_{mn}F^{mn}. \quad (51)$$

Thus, the position dependent electric coupling $e(w)$ of this five-dimensional Yang-Mills is equal to

$$\frac{1}{e^2(w)} \equiv \frac{\sqrt{3/2}\,\mu_4\,(2\pi\,\alpha')^2\,R\,N_c}{5}\,M_{KK}\left(\frac{U}{U_{KK}}\right). \quad (52)$$

Also, for a unit instanton we have

$$\frac{1}{8\pi^2}\int \mathrm{tr}F \wedge F = \frac{1}{16\pi^2}\int \mathrm{tr}F_{mn}F^{mn} = 1. \quad (53)$$

Inserting the above relations into Eq. (51), we obtain the energy of a point-like instanton localized at $w = 0$ as

$$m_B^{(0)} = \frac{\sqrt{3/2}\,4\pi^2\mu_4\,(2\pi\,\alpha')^2 R}{5}\,N_c\,M_{KK}. \quad (54)$$

By increasing the size of the instanton, more energy is needed because $1/e^2(w)$ is an increasing function of $|w|$. So the instanton tends to collapse to a point-like object. On the other hand, N_c fundamental strings attached to the $D4$ branes behave as N_c units of electric charge on the brane. The Coulomb repulsions among them prefer a finite size for the instanton. Therefore, there is a competition between the mass of the instanton and Coulomb energy of fundamental strings. For a small instanton of size ρ with the density $D(x^i, w) \sim \rho^4/(r^2 + w^2 + \rho^2)^4$, the Yang-Mills energy is approximated as

$$\sim \tfrac{1}{6}\,m_B^{(0)} M_{KK}^2 \rho^2, \quad (55)$$

and the five-dimensional Coulomb energy is

$$\sim \frac{1}{2} \times \frac{e(0)^2 N_c^2}{10\pi^2\rho^2}. \quad (56)$$

The size of a stable instanton is obtained by minimizing the total energy

$$\rho_{baryon}^2 \simeq \frac{1}{\sqrt{3/2}\,2\pi^2\mu_4\,(2\pi\,\alpha')^2}\,\frac{1}{M_{KK}^2}. \quad (57)$$

As it is stated in the previous section, in the SS model (the critical version of dual QCD) the size of the instanton goes to zero because of the large ’t Hooft coupling limit. However in the noncritical string theory, the ’t Hooft coupling is of order one. So, the size of the instanton is also of order 1 but it is still smaller than the effective length of the fifth direction $\sim 1/M_{KK}$ of the dual QCD.

IV. NUCLEON-NUCLEON POTENTIAL

In the previous section, we demonstrated that the size of the baryon in the noncritical holographic model is smaller than the scale of the dual QCD and we can assume that the baryon is a point-like object in five dimensions. Thus as a leading approximation, we can treat it as a point-like quantum field in five dimensions. In the rest of this paper, we will restrict ourselves to fermionic baryons because we intend to study the nucleons. So, we consider the odd N_c to study a fermionic spin-1/2 baryon. We choose $N_c = 3$ in our numerical calculations for realistic QCD. Also, we will assume $N_F = 2$ and consider the lowest baryons which form the proton-neutron doublet under SU($N_F = 2$). All of these assumptions lead us to introduce an isospin-1/2 Dirac field, $\mathcal{N}$ for the five-dimensional baryon.

The leading 5D kinetic term for $\mathcal{N}$ is the standard Dirac action in the curved background along with a position dependent mass term for the baryon. Moreover, there is a coupling between the baryon field and the gauge filed living on the flavor branes that should be considered. Therefore, a complete action for the baryon reads as

$$\int d^4x dw\Big[-i\bar{\mathcal{N}}\gamma^m D_m\mathcal{N} - i m_b(w)\bar{\mathcal{N}}\mathcal{N} + g_5(w)\frac{\rho_{baryon}^2}{e^2(w)}\bar{\mathcal{N}}\gamma^{mn}F_{mn}\mathcal{N}\Big] - \int d^4x dw\frac{1}{4e^2(w)}\,\mathrm{tr}\,F_{mn}F^{mn}, \quad (58)$$

where D_m is a covariant derivative, ρ_{baryon} is the size of the stable instanton, and $g_5(w)$ is an unknown function with a value at $w = 0$ of $2\pi^2/3$ [23]. γ^m are the standard γ matrices in the flat space and $\gamma^{mn} = 1/2[\gamma^m, \gamma^n]$.

The factor $\frac{\rho_{baryon}^2}{e^2(w)}$ is used for convenience. Usually, the first two terms in the action are called the minimal coupling and the last term in the first integral refers to the magnetic coupling.

A four-dimensional nucleon is the localized mode at $w \simeq 0$ which is the lowest eigenmode of a five-dimensional baryon along the w direction. So, the five-dimensional action for the baryon must be reduced down to the four dimensions. In order to do this, one should perform the KK mode expansion for the baryon field $\mathcal{N}(x_\mu, w)$ and the gauge field $A(x_\mu, w)$. The gauge field has a KK mode expansion which studied in Sec. III in detail. The baryon field also can be expanded as

$$\mathcal{N}_{L,R}(x^\mu, w) = N_{L,R}(x^\mu)\,f_{L,R}(w), \quad (59)$$

where $N_{L,R}(x^\mu)$ is the chiral component of the four-dimensional nucleon field. Also the profile functions, $f_{L,R}(w)$ satisfy the following conditions:

$$\begin{aligned} \partial_w f_L(w) + m_b(w) f_L(w) &= m_B f_R(w), \\ -\partial_w f_R(w) + m_b(w) f_R(w) &= m_B f_L(w), \end{aligned} \quad (60)$$

in the range $w \in [-w_{max}, w_{max}]$, and the eigenvalue m_B is the mass of the nucleon mode, $N(x)$. Moreover, the eigenfunctions $f_{L,R}(w)$ obey the following normalization condition:

$$\int_{-w_{max}}^{w_{max}} dw\,|f_L(w)|^2 = \int_{-w_{max}}^{w_{max}} dw\,|f_R(w)|^2 = 1. \quad (61)$$

It is more useful to consider the following second-order differential equations for $f_{L,R}(w)$:

$$\begin{aligned} \big[-\partial_w^2 - \partial_w m_b(w) + (m_b(w))^2\big] f_L(w) &= m_B^2 f_L(w), \\ \big[-\partial_w^2 + \partial_w m_b(w) + (m_b(w))^2\big] f_R(w) &= m_B^2 f_R(w). \end{aligned} \quad (62)$$

As we approach $w \to \pm w_{max}$, $m_b(w)$ diverges as $m_b(w) \sim \frac{1}{(w \mp w_{max})^2}$ and the above equations have normalizable eigenfunctions with a discrete spectrum of m_B. Note that the term $-\partial_w m_b(w)$ is asymmetric under $w \to -w$. It causes that $f_L(w)$ tends to shift to the positive side of w and the opposite behavior happens for $f_R(w)$. It is important in the axial coupling of the nucleon to the pions.

It is mentioned in Sec. II that the gauge field has a mode expansion (31) at $A_z = 0$ gauge which can be rewritten as

$$A_\mu(x, w) = i\alpha_\mu(x)\psi_0(w) + i\beta_\mu(x) + \sum_n B_\mu^{(n)}(x)\psi_{(n)}(w), \tag{63}$$

where α_μ and β_μ are related to the pion field $U(x) = e^{2i\pi(x)/f_\pi}$ by the following relations:

$$\alpha_\mu(x) \equiv \{U^{-1/2}, \partial_\mu U^{1/2}\}, \quad \beta_\mu(x) \equiv \tfrac{1}{2}[U^{-1/2}, \partial_\mu U^{1/2}]. \tag{64}$$

Here, we use the above expansion along with the properties of $f_L(w) = \pm f_R(-w)$, ψ_0, and $\psi_{(n)}$ under the $w \to -w$ transformation to calculate the four-dimensional action. It is worthwhile to note that again $\psi_{(2k+1)}(w)$ is even, while $\psi_{(2k)}(w)$ is odd under $w \to -w$, corresponding to vector $B_\mu^{(2k+1)}(x^\mu)$ and axial-vector mesons $B_\mu^{(2k)}(x^\mu)$, respectively. For simplicity, we neglect the Chern-Simons term in the baryon equation (58). By inserting the mode expansion of the baryon field in the action, we obtain the minimal coupling as

$$\begin{aligned} S^{min} &= \int d^4x dw[-i\bar{N}\bar{f}\gamma^m(\partial_m - iA_m)Nf \\ &\quad - im_b(w)\bar{N}\bar{f}Nf] \\ &= \int d^4x[-i\bar{N}\gamma^\mu\partial_\mu N - im_B\bar{N}N] \\ &\quad - \int d^4x dw[\bar{N}\bar{f}\gamma^\mu A_\mu N f]. \end{aligned} \tag{65}$$

Now, we expand the gauge field presented in the last integral using Eq. (63). Since the parity of $\psi_{(n)}(w)$ depends on n, it is possible to separate the odd and even n. After taking the integrals over w, we obtain the four-dimensional minimal action for the nucleon as

$$S^{min} = \int d^4x\left[-i\bar{N}\gamma^\mu\partial_\mu N - im_B\bar{N}N + \mathcal{L}^{min}_{\rm vector} + \mathcal{L}^{min}_{\rm axial}\right], \tag{66}$$

where the minimal vector and axial interactions are

$$\begin{aligned} \mathcal{L}^{min}_{\rm vector} &= -i\bar{N}\gamma^\mu\beta_\mu N - \sum_{k\geqslant 0} g^{(k)}_{V,min}\bar{N}\gamma^\mu B_\mu^{(2k+1)}N, \\ \mathcal{L}^{min}_{\rm axial} &= -\frac{ig_{A,min}}{2}\bar{N}\gamma^\mu\gamma^5\alpha_\mu N \\ &\quad - \sum_{k\geqslant 1} g^{(k)}_{A,min}\bar{N}\gamma^\mu\gamma^5 B_\mu^{(2k)}N. \end{aligned} \tag{67}$$

The various minimal couplings constants $g^{(k)}_{V,min}$, $g^{(k)}_{A,min}$ as well as the pion-nucleon axial coupling $g_{A,min}$ are calculated by the following suitable overlap integrals of wave functions:

$$\begin{aligned} g^{(k)}_{V,min} &= \int_{-w_{max}}^{w_{max}} dw\,|f_L(w)|^2\psi_{(2k+1)}(w), \\ g^{(k)}_{A,min} &= \int_{-w_{max}}^{w_{max}} dw\,|f_L(w)|^2\psi_{(2k)}(w), \\ g_{A,min} &= 2\int_{-w_{max}}^{w_{max}} dw\,|f_L(w)|^2\psi_0(w). \end{aligned} \tag{68}$$

Also, the magnetic interaction term in Eq. (58) becomes

$$S^{magnetic} = -\int d^4x dw\left(g_5(w)\frac{\rho^2_{baryon}}{e^2(w)}\bar{N}\bar{f}\gamma^{w\mu}F_{w\mu}Nf\right). \tag{69}$$

Inserting the gauge field expansion into Eq. (69), the magnetic interaction reads as

$$S^{magnetic} = \int d^4x\left(\mathcal{L}^{magnetic}_{\rm vector} + \mathcal{L}^{magnetic}_{\rm axial}\right), \tag{70}$$

where

$$\begin{aligned} \mathcal{L}^{magnetic}_{\rm vector} &= -\sum_{k\geqslant 0} g^{(k)}_{V,mag}\bar{N}\gamma^\mu\gamma^5 B_\mu^{(2k+1)}N, \\ \mathcal{L}^{magnetic}_{\rm axial} &= -\frac{ig_{A,mag}}{2}\bar{N}\gamma^\mu\gamma^5\alpha_\mu N \\ &\quad - \sum_{k\geqslant 1} g^{(k)}_{A,mag}\bar{N}\gamma^\mu\gamma^5 B_\mu^{(2k)}N, \end{aligned} \tag{71}$$

and the magnetic couplings are defined as

$$\begin{aligned} g_{A,mag} &= 4\int_{-w_{max}}^{w_{max}} dw\left(g_5(w)\frac{\rho^2_{baryon}}{e^2(w)}\right)|f_L(w)|^2\partial_w\psi_0(w), \\ g^{(k)}_{A,mag} &= 2\int_{-w_{max}}^{w_{max}} dw\left(g_5(w)\frac{\rho^2_{baryon}}{e^2(w)}\right)|f_L(w)|^2\partial_w\psi_{(2k)}(w), \\ g^{(k)}_{V,mag} &= 2\int_{-w_{max}}^{w_{max}} dw\left(g_5(w)\frac{\rho^2_{baryon}}{e^2(w)}\right)|f_L(w)|^2\partial_w\psi_{(2k+1)}(w). \end{aligned} \tag{72}$$

Using Eq. (52), we can rewrite the magnetic couplings as

$$\begin{aligned} g^{(k)}_{V,mag} &= 2\,C_{mag}\int_{-w_{max}}^{w_{max}} dw\left(\frac{g_5(w)}{g_5(0)}\right)\left(\frac{U(w)}{U_{KK}}\right) \\ &\quad \times |f_L(w)|^2\partial_w\psi_{(2k+1)}(w), \\ g^{(k)}_{A,mag} &= 2\,C_{mag}\int_{-w_{max}}^{w_{max}} dw\left(\frac{g_5(w)}{g_5(0)}\right)\left(\frac{U(w)}{U_{KK}}\right) \\ &\quad \times |f_L(w)|^2\partial_w\psi_{(2k)}(w), \\ g_{A,mag} &= 4\,C_{mag}\int_{-w_{max}}^{w_{max}} dw\left(\frac{g_5(w)}{g_5(0)}\right)\left(\frac{U(w)}{U_{KK}}\right) \\ &\quad \times |f_L(w)|^2\partial_w\psi_0(w), \end{aligned} \tag{73}$$

where we define C_{mag} as

$$C_{mag} = \frac{\sqrt{3/2}\mu_4\,(2\pi\,\alpha')^2}{5}\,R\,N_c\,g_5(0)\,M_{KK}\,\rho^2_{baryon}. \tag{74}$$

Also, there is a next-to-leading order term in the magnetic coupling equation which is responsible for the derivative couplings. Finally by considering the derivative terms, the Lagrangian of the nucleon is obtained as

$$\mathcal{L}_{\rm nucleon} = -i\bar{N}\gamma^\mu\partial_\mu N - im_B\bar{N}N + \mathcal{L}_{\rm vector} + \mathcal{L}_{\rm axial}, \tag{75}$$

where

$$\mathcal{L}_{\text{vector}} = -i\bar{N}\gamma^{\mu}\beta_{\mu}N - \sum_{k\geqslant 0} g_V^{(k)}\bar{N}\gamma^{\mu}B_{\mu}^{(2k+1)}N + \sum_{k\geqslant 0} g_{dV}^{(k)}\bar{N}\gamma^{\mu\nu}\partial_{\mu}B_{\nu}^{(2k+1)}N,$$
$$\mathcal{L}_{\text{axial}} = -\frac{ig_A}{2}\bar{N}\gamma^{\mu}\gamma^5\alpha_{\mu}N - \sum_{k\geqslant 1} g_A^{(k)}\bar{N}\gamma^{\mu}\gamma^5B_{\mu}^{(2k)}N + \sum_{k\geqslant 0} g_{dA}^{(k)}\bar{N}\gamma^{\mu\nu}\gamma^5\partial_{\mu}B_{\nu}^{(2k)}N. \quad (76)$$

Also, $g = g_{min} + g_{mag}$ stands for all the couplings. We neglect the derivative couplings in the following calculations as a leading approximation.

Since the instanton carries only the non-Abelian field strength, the isoscalar mesons couple to the nucleon in a different formalism than the isovector mesons. Therefore for the isoscalar mesons, such as the $\omega^{(k)}$ meson, only the minimal couplings contribute:

$$g_A^{isoscalar} = g_{A,min},\quad g_A^{(k),isoscalar} = g_{A,min}^{(k)},\quad g_V^{(k),isoscalar} = g_{V,min}^{(k)}. \quad (77)$$

However, the isovector mesons couple to the nucleon from both the minimal and magnetic channels. Thus, isovector meson couplings are

$$g_A^{isovector} = g_{A,min} + g_{A,mag},\quad g_A^{(k),isovector} = g_{A,min}^{(k)} + g_{A,mag}^{(k)},\quad g_V^{(k),isovector} = g_{V,min}^{(k)} + g_{V,mag}^{(k)}. \quad (78)$$

The isoscalar and isovector mesons have the same origin in the five-dimensional dynamics of the gauge field. In fact, if we write the gauge field in the fundamental representation, we could decompose the massive vector mesons as

$$B_{\mu}^{(2k+1)} = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}\omega_{\mu}^{(k)} + \rho_{\mu}^{(k)}, \quad (79)$$

where $\omega_{\mu}^{(k)}$ and $\rho_{\mu}^{(k)}$ are the isoscalar and the isovector parts of a vector meson, respectively. Since the baryon is made out of N_c product quark doublets, the above composition for nucleon should be written as

$$B_{\mu}^{(2k+1)} = \begin{pmatrix} N_c/2 & 0 \\ 0 & N_c/2 \end{pmatrix}\omega_{\mu}^{(k)} + \rho_{\mu}^{(k)}. \quad (80)$$

Therefore, there is an overall factor N_c between the isoscalar $\omega_{\mu}^{(k)}$ and isovector $\rho_{\mu}^{(k)}$ mesons. Indeed, there is a universal relation between the Yukawa couplings involving the isoscalar and isovector mesons

$$|g_{\omega^{(k)}NN}| \simeq N_c \times |g_{\rho^{(k)}NN}|. \quad (81)$$

We will be back to this relation later. Here we need to solve the eigenvalue equation (62) numerically to obtain the wave function $f_{L,R}$ and the mass m_B of the nucleon. It is useful to define the dimensionless variables $\tilde{w} = M_{KK}w$, $\tilde{U} = U/U_{KK}$, and $\tilde{z} = z/U_{KK}$ which are related together by

$$\tilde{w} = \int_0^{\tilde{z}} \frac{d\tilde{z}}{[1+\tilde{z}^2]^{\frac{7}{10}}} = \frac{5}{2}\int_1^{\tilde{U}} d\tilde{U}\sqrt{\frac{\tilde{U}}{\tilde{U}^5 - 1}}, \quad (82)$$

and rewrite the $m_b(w)$ in terms of these dimensionless variables as

$$m_b(w) \simeq m_b^{(0)} \cdot \tilde{U} = M_{KK}\,\tilde{m}_b(\tilde{w}), \quad (83)$$

where

$$\tilde{m}_b(\tilde{w}) = \frac{\sqrt{3/2}\,4\pi^2\mu_4\,(2\pi\alpha')^2\,R}{5}\,N_c\,\tilde{U}(\tilde{w}). \quad (84)$$

After rewriting Eq. (62) in terms of $\tilde{w}$, we obtain

$$\left[-\partial_{\tilde{w}}^2 - \partial_{\tilde{w}}\tilde{m}_b(\tilde{w}) + (\tilde{m}_b(\tilde{w}))^2\right]f_L(\tilde{w}) = \left(\frac{m_B}{M_{KK}}\right)^2 f_L(\tilde{w}). \quad (85)$$

The key idea for using dimensionless variables is that the functions $f_L(\tilde{w})$ do not depend on the scales further. Now, we use the shooting method again to solve the above equation numerically and find $f_L(\tilde{w})$ and its eigenvalue, m_B/M_{KK}. In order to do the numerical calculation, we assume $N_c = 3$ for realistic QCD. Also was mentioned in the previous section, we choose the value of $M_{KK} = 0.395$ GeV to have the pion decay constant $f_{\pi} = 0.093$ GeV. We obtain the various couplings by evaluating integrals (68) and (73) and compare some of our results with the results of the SS model [23] in Table II.

TABLE II. Numerical results for axial and vector meson couplings in the noncritical holographic model of QCD. The values of vector couplings are compared with the SS model results [23].

k	$g_{A,min}^{(k)}$	$g_{A,mag}^{(k)}$	$g_{V,min}^{(k),a}$	$g_{V,min}^{(k),b}$	$g_{V,mag}^{(k),a}$	$g_{V,mag}^{(k),b}$
0	1.16	1.86	8.30	5.933	−1.988	−0.816
1	1.07	1.44	1.6488	3.224	−6.83	−1.988
2	0.96	0.862	1.9	1.261	−7.44	−1.932
3	0.67	0.14	0.688	0.311	−4.60	−0.969

[a]Presented model results.
[b]SS model results.

Also, using this noncritical model, the axial couplings are obtained as

$$g_{A,mag} = 1.582, \quad g_{A,min} \simeq 0, \quad (86)$$

while in the previous analyses [23] using the SS model, these couplings are reported as

$$g_{A,mag} = 0.7\,\frac{N_c}{3}, \quad g_{A,min} \simeq 0.13. \quad (87)$$

If we choose $N_c = 3$, then the SS model predicts $g_{A,mag} = 0.7$ and $g_A = 0.83$. It should be noted that the higher order of $1/N_c$ corrections can be used to improve this result but the lattice calculations indicate that higher order of $1/N_c$ corrections are suppressed. Our results are a good approximation of the experimental data at leading order $g_A^{\exp} = 1.2670 \pm 0.0035$.

TABLE III. The values of different effective meson-nucleon couplings in the phenomenological interaction models [54], SS model [23], and our model.

g	V18	CD-Bonn	Nijm(93)	Paris	SS model	Our model
g_{a^0}	9.0	9.0	9.0	10.4	–	–
g_σ	9.0	11.2	9.8	7.6	–	–
g_π	13.4	13.0	12.7	13.2	16.48	15.7
g_η	8.7	0.0	1.8	11.7	16.13	0.0
g_ω	12.2	13.5	11.7	12.7	12.6	11.57
g_ρ	–	3.19	2.97	–	3.6	3.15
g_{a^1}	–	–	–	–	3.94	1.51
g_{f^1}	–	–	–	–		1.74

V. NUCLEON-MESON COUPLINGS

In the previous section, we explained that the NN interaction can be interpreted as meson exchange potentials. We showed that the nucleons couple to the meson through the minimal and magnetic couplings. Our holographic NN potential contains just the vector, axial-vector, and pseudoscalar meson exchange potentials which have the isospin dependent and isospin independent components.

All of the leading order meson-nucleon couplings are calculated numerically and compared with the predictions of the four modern phenomenological NN interaction models such as the AV 18 [8], CD-Bonn [7], Nijmegen(93) [6], and Paris [5] potentials in Table III. Also, results of the SS model are presented in the table. It is necessary to mention here that the components of the phenomenological models are very different in strength, and if parametrized in terms of single meson exchange, give rise to effective meson-nucleon coupling strengths, which also are similar. We explain different components of the NN potential below in detail.

A. Scalar potential

In the phenomenological interaction models, the exchange of a single scalar meson produces the isospin independent scalar potential. The mass of the lowest scalar meson is not established well [55], but in the phenomenological NN interaction models it is typically taken to be of order 500–700 MeV. By considering m_S to be roughly 600 MeV, the effective scalar meson coupling constant for these interaction models differs from 7.6 (Paris) to 11.2 (CD-Bonn). It should be noted again that these values are the effective couplings. In fact two of these models do not contain any scalar meson in their parametrized forms. In our holographic model based on the noncritical string theory, there is no scalar interaction term either.

The isospin dependent component of the scalar potential can be interpreted as a scalar spin-1 meson exchange. In N_c counting, this component is of order $1/N_c$, so it is weaker than the isospin independent scalar meson by an order of its magnitude. The effective values for the lowest scalar meson [$a_0(980)$], range from 9.0 to 10.4 in the various phenomenological NN interaction models.

B. Vector potential

The vector component of the phenomenological NN interaction models is equal to the scalar component with the minus sign. It means that the strength of the $a_0(980)$ exchange interaction is equal to the exchange of $\rho(770)$ in the large N_c limit. Indeed, the vector potential arises from a stronger ω-meson exchange (isospin independent component) and a weaker ρ-meson exchange (isospin dependent component) interaction. In our model the vector meson couplings are related to the minimal and magnetic couplings as follows:

$$g_{\omega^{(k)}NN} \equiv \frac{N_c\, g_V^{(k),isoscalar}}{2} = \frac{N_c\, g_{V,min}^{(k)}}{2},$$
$$g_{\rho^{(k)}NN} \equiv \frac{g_V^{(k),isovector}}{2} = \frac{g_{V,min}^{(k)} + g_{V,mag}^{(k)}}{2}. \quad (88)$$

The value of effective ω-nucleon coupling ranges from 11.7 [Nijmegen(93)] to 13.5 (CD-Bonn), while in the original version of these models this value varies from 10.35 (Nijmegen) to 15.85 (Bonn) [56]. In the SS model, g_ω is equal to 12.6 by considering $M_{KK} = 940$ MeV, $N_c = 3$, and $\lambda = 17$. We also obtain the value of $g_\omega = 11.57$ which is in the range of the values anticipated from the phenomenological potential models. We have used $N_c = 3$ and $M_{KK} = 395$ MeV in our calculations. The obtained value for the nucleon mass in our model is roughly 920 MeV which is very realistic and close to the familiar nucleon mass.

The isospin dependent component of the vector potential which arises from a ρ-meson exchange is roughly three times weaker than the isospin independent component. In a chiral quark model, it is expected to have $g_\omega = 3\, g_\rho$, but the value of the $\mathcal{R} = g_\omega/3\, g_\rho$ differs from the one in the above phenomenological interaction models. It is 1.66 for the CD-Bonn, 1.5 for the Nijmegen, and 0.77 in the Paris model. This ratio is about 1.2 in the SS model and equals to $\mathcal{R} = 1.33$ in our model. Actually, the NN phase shifts uniformly require a larger $\mathcal{R}$ than the chiral quark model prediction which is a mystery. However in the resultant potential of the holographic QCD model, it can be explained by the contribution of the magnetic coupling in the vector channel.

C. Axial-vector potential

The Nijmegen(93) and CD-Bonn models do not contain any single axial vector meson exchange, so there is no axial vector interaction in their structures. The phenomenological AV 18 and Paris potentials predict a small value for the axial vector interactions too.

In our holographic potential model, the axial vector mesons $a^{(k)}$ and $f^{(k)}$ couple to the nucleon with the following couplings:

$$g_{f^{(k)}NN} \equiv \frac{N_c\, g_A^{(k),isoscalar}}{2} = \frac{N_c\, g_{A,min}^{(k)}}{2},$$
$$g_{a^{(k)}NN} \equiv \frac{g_A^{(k),isovector}}{2} = \frac{g_{A,min}^{(k)} + g_{A,mag}^{(k)}}{2}. \quad (89)$$

The values of the $a_{(1)}$-nucleon and $f_{(1)}$-nucleon couplings are obtained at about 1.51 and 1.74, respectively, in our holographic potential.

D. Pseudoscalar potential

The isospin independent pseudoscalar interaction comes from an η'-meson exchange. This component is not well constrained by NN scattering data and the phenomenological interaction models give extremely different values for this component ranging from 1.8 [Nijmegen(93)] to 11.7 (Paris). While analyses of other observables such as η'-meson photoproduction suggest that the coupling constant value should not go beyond 2.2 [57]. In our holographic potential, the pseudoscalar mesons such as the pion π and η' couple to the nucleon as

$$\frac{g_{\pi^{(k)}NN}}{2m_N}M_{KK} \equiv \frac{g_A^{isovector}}{2f_\pi}M_{KK} = \frac{g_{A,min}+g_{A,mag}}{2f_\pi}M_{KK},$$
$$\frac{g_{\eta'^{(k)}NN}}{2m_N}M_{KK} \equiv \frac{N_c\, g_A^{isoscalar}}{2\, f_\pi}M_{KK} = \frac{N_c g_{A,min}}{2\, f_\pi}M_{KK}. \tag{90}$$

We obtain $g_\eta = 0$ at the leading order. Despite the isospin independent component, the isospin dependent component is strong and spread on a long range. All of the models considered here have a main component for this interaction which is a pion exchange potential. The values of the pion-nucleon coupling constant, g_π, vary from 12.7 to 13.4 effectively. In our calculation, g_π is evaluated as 15.7 while in the calculations based on the SS model it is obtained at 16.48 [23].

VI. CONCLUSION

In this study, we used the noncritical holographic duality of gauge theories in the background of the near-extremal AdS_6. We obtained the mass scale of the model by a comparison of the pion field action with the usual Skyrme model action. We showed that the size of the baryon is of order one in respect to the t' Hooft coupling but it is still smaller than the scale of the dual QCD. So we introduced an isospin-1/2 Dirac field for the five-dimensional baryon and wrote an effective action for it. Then, we reduced the five-dimensional action down to the four dimensions using the mode expansion and obtained the NN interaction in terms of the meson exchange potentials.

In our analyses such as the critical ones, the NN potential involves only the pseudoscalar, vector, and axial vector meson interactions. In fact these meson exchange potentials play a special role in producing the NN potential. The long-range part of the potential ($r > 3$ fm) is mostly due to the one-pion-exchange mechanism which is the strongest component among the isospin dependent components. In the phenomenological potential models, the isospin dependent pseudoscalar meson exchange potential is of order N_c^3 while the isospin independent component is of order N_c. So it is expected that the η-meson exchange potential would be much weaker than the π-meson exchange potential. We obtained $g_\pi = 15.7$ and $g_\eta = 0.0$ at the leading order that is consistent with the phenomenological interaction models.

Isospin independent scalar mesons are responsible for the attractive interaction in the intermediate range ($0.7 < r < 2$ fm) of the potential. These components are the main reason for the nuclear binding. Also in the phenomenological interaction models, the strength of this interaction is equal to the vector meson exchange with a minus sign. In fact, the radial shapes differ considerably at short distances, ranging from the attractive to repulsive area. Some of these phenomenological potential models involve only the scalar meson exchange and the others contain the vector meson exchange interaction term. We considered the vector meson exchange potential in our analyses which produces the strong short-range repulsion. By exchanging the vector meson, ρ can explain the small attractive behavior of the odd-triplet state.

We compared our results with the available values of coupling constants predicted in the four modern phenomenological interaction models [Nijmegen(93), Paris, CD-Bonn, and AV 18 models] in Table III. The remarkable point is that all of the meson-nucleon couplings are calculated directly in our model whereas in the phenomenological interaction models these values were obtained by fitting the NN scattering data. It is also obvious from Table III that the values of the coupling constants are very different in the various interaction models.

We believe that our noncritical holography model is more reliable than the critical SS model to study the NN interactions for these reasons:

(i) Just like the SS model, there exist some KK modes which come from the antiperiodic boundary conditions over the circle S^1. These modes have the masses of the same order of magnitude as the lightest glueballs of the four-dimensional YM theory. The critical holographic models such as the SS model, have some extra KK modes too which do not belong to the spectrum of pure YM theory. These undesired KK modes come from the extra internal space over which ten-dimensional string theory is compactified, for example, the S^4 sphere in the SS model. In the noncritical holographic model, which we used here, there is no additional compactified sphere, so there are no such extra KK modes and the QCD spectrum is clear from them. Thus it seems that our model based on the noncritical holography is much more reliable. As we mentioned earlier, we obtained the value of M_{KK} roughly half of its value in the SS model.

(ii) The size of the baryon is proportional to $1/\sqrt{\lambda N_c}$ in the SS model and becomes zero at large 't Hooft coupling while it is of order one in our calculations. So, our model does not suffer from the zero size of the baryon.

(iii) Also, the nucleon mass is obtained at 1.93 M_{KK} in the SS model. The mass scale which can describe the meson spectrum is roughly $M_{KK} = 940$ MeV. So the nucleon mass is about 1.8 GeV in the SS model. It causes some inconsistency in analyzing the baryon, because the scale of the system associated with the baryonic structure is roughly half the one needed to fit to the mesonic data [36]. But in our analyses the mass of the nucleon is obtained at roughly 920 MeV which is very close to the neutron (proton) mass.

(iv) In addition, our noncritical calculations can describe the meson-nucleon couplings successfully at least at the leading order.

This holography potential model can be improved by considering the contributions of derivative couplings and exchange of the other mesons to the NN potential. Also, the couplings can be computed with more accuracy however it seems that the contribution of heavy meson exchange does not play a major role in such calculations. Moreover, we can use the obtained potential here to study the nuclear properties such as the NN scattering data and nuclear binding energies. We leave them here as our future work.

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