Shear and bulk viscosities of strongly interacting "infinite" parton-hadron matter within the parton-hadron-string dynamics transport approach

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We study the shear and bulk viscosities of partonic and hadronic matter as functions of temperature T within the parton-hadron-string dynamics (PHSD) off-shell transport approach. Dynamical hadronic and partonic systems in equilibrium are studied by the PHSD simulations in a finite box with periodic boundary conditions. The ratio of the shear viscosity to entropy density $\eta(T)/s(T)$ from PHSD shows a minimum (with a value of about 0.1) close to the critical temperature T_c, while it approaches the perturbative QCD limit at higher temperatures in line with lattice QCD (lQCD) results. For $T < T_c$, i.e., in the hadronic phase, the ratio η/s rises fast with decreasing temperature due to a strong decrease of the entropy density s in the hadronic phase at decreasing T. Within statistics, we obtain practically the same results in the Kubo formalism and in the relaxation time approximation. The bulk viscosity $\zeta(T)$—evaluated in the relaxation time approach—is found to strongly depend on the effects of mean fields (or potentials) in the partonic phase. We find a significant rise of the ratio $\zeta(T)/s(T)$ in the vicinity of the critical temperature T_c, when consistently including the scalar mean-field from PHSD, which is also in agreement with that from lQCD calculations. Furthermore, we present the results for the ratio $(\eta + 3\zeta/4)/s$, which is found to depend nontrivially on temperature and to generally agree with the lQCD calculations as well. Within the PHSD calculations, the strong maximum of $\zeta(T)/\eta(T)$ close to T_c has to be attributed to mean-field (or potential) effects that in PHSD are encoded in the temperature dependence of the quasiparticle masses, which is related to the infrared enhancement of the resummed (effective) coupling $g(T)$.

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I. INTRODUCTION

High-energy heavy-ion reactions are studied experimentally and theoretically to obtain information about the properties of nuclear matter under the extreme conditions of high baryon density and/or temperature. Ultrarelativistic heavy-ion collisions at the Relativistic Heavy-Ion Collider (RHIC) and the Large Hadron Collider (LHC) at CERN have produced a new state of matter, the quark-gluon plasma (QGP), for a couple of fm/c. The produced QGP shows features of a strongly interacting fluid unlike a weakly interacting parton gas [1–4]. Large values of the observed azimuthal asymmetry of charged particles in momentum space, i.e., the elliptic flow v_2 [5–9], could quantitatively be well described by hydrodynamics up to transverse momenta on the order of 1.5 GeV/c [10–15]. A perfect fluid has been defined as having a zero shear viscosity,

η; yet semiclassical arguments have been given suggesting that the shear viscosity cannot be arbitrarily small [16]. Indeed, the lower bound for the shear viscosity to entropy density ratio $\eta/s = 1/4\pi$ was conjectured by Kovtun-Son-Starinets (KSS) [17] for infinitely coupled supersymmetric Yang-Mills gauge theory based on the anti de Sitter/conformal field theory (AdS/CFT) duality conjecture. On the basis of holographically dual computations [18], also for the bulk viscosity of strongly coupled gauge theory plasmas a lower bound was conjectured: $\zeta/\eta \geqslant 2(1/3 - c_s^2)$, where c_s is the speed of sound. Empirically, relativistic viscous hydrodynamic calculations—using the Israel-Stewart framework—require a very small but finite η/s of 0.08–0.24 in order to reproduce the RHIC elliptic flow v_2 data [19–22]. The main uncertainty in these estimates results from the equation of state and the initial conditions employed in the hydrodynamical calculations as well as in the temperature dependence of $\eta/s(T)$.

Thus not only the absolute values of the shear and bulk viscosities are of great interest but also the temperature

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dependence of these coefficients, which is expected to be quite strong. There is evidence from atomic and molecular systems that η/s should have a minimum in the vicinity of the phase transition or—in case of strong interactions at vanishing chemical potential—of the rapid crossover between hadronic matter and the quark-gluon plasma [23–25]. Furthermore, it is argued that the ratio of the bulk viscosity to entropy density ζ/s should have a maximum close to T_c—as suggested by lattice QCD—and might even diverge in the case of a second-order phase transition [26–31]. Such a peak in the bulk viscosity can lead to instabilities in viscous hydrodynamics simulations for heavy-ion collisions and possibly to clusterization effects [32].

Shear and bulk viscosities of strongly interacting systems have been evaluated within different approaches. Calculations have been performed at high temperatures, where perturbation theory can be applied [33,34], as well as at extremely low temperatures [34–36]. First results for shear and bulk viscosities obtained within lattice QCD (lQCD) simulations above the critical temperature of pure gluon matter have been presented in Refs. [37–40]. There are several methods for the calculation of shear and bulk viscosities for strongly interacting systems: the relaxation time approximation (RTA) [41], the Chapmann-Enskog (CE) method [42], and the Green-Kubo approach [43,44]. The RTA method has been used to calculate the viscosity [16,30,45–50], as well as the Green-Kubo approach [4,46,51–55], for both hadronic and partonic matter providing a rough picture of the transport properties of strongly interacting matter.

In this study we calculate the shear and bulk viscosities as a function of temperature (or energy density) with the parton-hadron-string dynamics (PHSD) transport approach that has provided a good description of collective flow properties and differential particle spectra in nucleus-nucleus collisions from lower CERN Super Proton Synchrotron (SPS) to RHIC energies [56–60]. In this approach the shear and bulk viscosities do not enter as external parameters but are generic properties of the matter under consideration and can be calculated for systems in equilibrium as a function of temperature explicitly without incorporating any additional parameters. Furthermore, the PHSD aproach allows one to evaluate the transport coefficients within the partonic phase as well as within the hadronic phase on the same footing.

The paper is organized as follows. In Sec. II we provide a brief reminder of the off-shell dynamics and the ingredients of the PHSD transport approach. We then first present in Sec. III the actual results for the shear and bulk viscosities in "infinite" parton-hadron matter within the PHSD employing the Green-Kubo formalism and the RTA and compare these results to the available lQCD results. The summary and conclusions are given in Sec. IV.

II. THE PHSD TRANSPORT APPROACH

In this work we extract the shear and bulk viscosities for infinite parton-hadron matter employing different methods within the PHSD transport approach [56,57], which is based on generalized transport equations on the basis of the off-shell Kadanoff-Baym equations [61,62] for Green's functions in phase-space representation (in the first-order

gradient expansion, beyond the quasiparticle approximation). The approach consistently describes the full evolution of a relativistic heavy-ion collision from the initial hard scatterings and string formation through the dynamical deconfinement phase transition to the strongly interacting quark-gluon plasma (sQGP) as well as hadronization and the subsequent interactions in the expanding hadronic phase. In the hadronic sector PHSD is equivalent to the hadron-string dynamics (HSD) transport approach [63,64]—a covariant extension of the Boltzmann-Uehling-Uhlenbeck approach [65]—that has been used for the description of pA and AA collisions from GSI Heavy Ion Synchrotron (SIS) to RHIC energies in the past. In PHSD the partonic dynamics is based on the dynamical quasiparticle model (DQPM) [66–68], which describes QCD properties in terms of single-particle Green's functions (in the sense of a two-particle irreducible approach) and reproduces lattice QCD results—including the partonic equation of state—in thermodynamic equilibrium.

A. Reminder of the DQPM

In the scope of the DQPM the running coupling constant g^2 (squared) for partons is approximated (for $T > T_c$) by

$$g^2(T/T_c) = \frac{48\pi^2}{(11N_c - 2N_f)\ln[\lambda^2(T/T_c - T_s/T_c)^2]}, \quad (1)$$

where the parameters $\lambda = 2.42$ and $T_s/T_c = 0.56$ have been extracted from a fit to the lattice data for purely gluonic systems (N_f=0) as described in Ref. [68]. In Eq. (1), $N_c = 3$ stands for the number of colors, T_c is the critical temperature (=158 MeV), and N_f denotes the number of flavors. In the actual PHSD calculations for $N_f = 3$ we employ a slightly different analytical form for $g^2(T/T_c)$ that has been fitted to the lattice data from Ref. [69]. For the details we refer the reader to Ref. [70].

The functional forms for the dynamical quasiparticle masses (for gluons and quarks) are chosen so that they become identical to the perturbative thermal masses in the asymptotic high-temperature regime; i.e., for gluons

$$M_g^2(T) = \frac{g^2(T/T_c)}{6}\left[\left(N_c + \frac{1}{2}N_f\right)T^2 + \frac{N_c}{2}\sum_q \frac{\mu_q^2}{\pi^2}\right], \quad (2)$$

and for quarks (antiquarks)

$$M_{q(\bar{q})}^2(T) = \frac{N_c^2 - 1}{8N_c}g^2\left(T^2 + \frac{\mu_q^2}{\pi^2}\right), \quad (3)$$

but the running coupling $g(T/T_c)$ is the resummed coupling of Eq. (1). The effective quarks, antiquarks, and gluons in the DQPM have finite widths, which for $\mu_q = 0$ are adopted in the following form [71]:

$$\Gamma_g(T) = \frac{1}{3}N_c\frac{g^2T}{8\pi}\ln\left(\frac{2c}{g^2} + 1\right), \quad (4)$$

$$\Gamma_{q(\bar{q})}(T) = \frac{1}{3}\frac{N_c^2 - 1}{2N_c}\frac{g^2T}{8\pi}\ln\left(\frac{2c}{g^2} + 1\right), \quad (5)$$

where the parameter $c = 14.4$ is related to a magnetic cutoff (see Ref. [4]).

In line with Ref. [68], the parton spectral functions are no longer δ functions in the invariant mass squared but have a Lorentzian form,

$$
\begin{aligned}
\rho_j(\omega, \mathbf{p}) &= \frac{\Gamma_j}{E_j} \left(\frac{1}{(\omega - E_j)^2 + \Gamma_j^2} - \frac{1}{(\omega + E_j)^2 + \Gamma_j^2} \right) \\
&= \frac{4\omega\Gamma_j}{\left(\omega^2 - \mathbf{p}^2 - M_j^2 \right)^2 + 4\Gamma_j^2\omega^2},
\end{aligned}
\tag{6}
$$

with the notation $E_j^2(\mathbf{p}^2) = \mathbf{p}^2 + M_j^2 - \Gamma_j^2$, where the index j stands for quarks, antiquarks, and gluons ($j = q, \bar{q}, g$). The spectral function (6) is antisymmetric in ω and normalized as

$$
\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \, \omega\rho_j(\omega, \mathbf{p}) = \int_0^{\infty} \frac{d\omega}{2\pi} \, 2\omega\rho_j(\omega, \mathbf{p}) = 1.
\tag{7}
$$

The parameters Γ_j and M_j from the DQPM have been defined above. Note, however, that the decomposition of the total width Γ_j into the collisional width (due to elastic and inelastic collisions) and the decay width is not addressed in the DQPM. The effective cross sections for each of the various partonic channels as a function of the energy density ε, which fixes the partial widths of the dynamical quasiparticles as well as the various interaction rates, have been determined in Ref. [72].

B. Hadronization in PHSD

The hadronization, i.e., the transition from partonic to hadronic degrees of freedom and vice versa, is described in PHSD by covariant transition rates for the fusion of quark-antiquark pairs to mesonic resonances or three quarks (antiquarks) to baryonic states [57], e.g., for $q + \bar{q}$ fusion to a meson m of four-momentum $p = (\omega, \mathbf{p})$ at space-time point $x = (t, \mathbf{x})$:

$$
\begin{aligned}
\frac{dN_m(x, p)}{d^4x\, d^4p} &= \mathrm{Tr}_q \mathrm{Tr}_{\bar{q}} \delta^4(p - p_q - p_{\bar{q}}) \delta^4\left(\frac{x_q + x_{\bar{q}}}{2} - x \right) \\
&\times \omega_q \rho_q(p_q) \omega_{\bar{q}} \rho_{\bar{q}}(p_{\bar{q}}) |v_{q\bar{q}}|^2 W_m\left(x_q - x_{\bar{q}}, \frac{p_q - p_{\bar{q}}}{2} \right) \\
&\times N_q(x_q, p_q) N_{\bar{q}}(x_{\bar{q}}, p_{\bar{q}}) \delta(\text{flavor, color}).
\end{aligned}
\tag{8}
$$

In Eq. (8) we have introduced the shorthand notation,

$$
\mathrm{Tr}_j = \sum_j \int d^4x_j \int \frac{d^4p_j}{(2\pi)^4},
\tag{9}
$$

where $\sum_j$ denotes a summation over discrete quantum numbers (spin, flavor, color); $N_j(x, p)$ is the phase-space density of parton j at space-time position x and four-momentum p. In Eq. (8) $\delta(\text{flavor, color})$ stands symbolically for the conservation of flavor quantum numbers as well as color neutrality of the formed hadron m, which can be viewed as a color-dipole or "prehadron." Furthermore, $v_{q\bar{q}}(\rho_p)$ is the effective quark-antiquark interaction from the DQPM (displayed in Fig. 10 of Ref. [67]) as a function of the local parton

($q + \bar{q} + g$) density ρ_p (or energy density). Furthermore, $W_m(x, p)$ is the dimensionless phase-space distribution of the formed prehadron; i.e.,

$$
W_m(\xi, p_\xi) = \exp\left(\frac{\xi^2}{2b^2} \right) \exp\left[2b^2 \left(p_\xi^2 - \frac{(M_q - M_{\bar{q}})^2}{4} \right) \right],
\tag{10}
$$

with $\xi = x_1 - x_2 = x_q - x_{\bar{q}}$ and $p_\xi = (p_1 - p_2)/2 = (p_q - p_{\bar{q}})/2$ (which had been introduced in Ref. [73]). The width parameter b has been fixed by $\sqrt{\langle r^2 \rangle} = b = 0.66$ fm (in the rest frame), which corresponds to an average rms radius of mesons. We note that the expression (10) corresponds to the limit of independent harmonic oscillator states and that the final hadron-formation rates are approximately independent of the parameter b within reasonable variations. By construction the quantity (10) is Lorentz invariant; in the limit of instantaneous "hadron formation," i.e., $\xi^0 = 0$, it provides a Gaussian dropping in the relative distance squared $(\mathbf{r}_1 - \mathbf{r}_2)^2$. The four-momentum dependence reads explicitly (except for a factor $1/2$)

$$
(E_1 - E_2)^2 - (\mathbf{p}_1 - \mathbf{p}_2)^2 - (M_1 - M_2)^2 \leqslant 0
\tag{11}
$$

and leads to a negative argument of the second exponential in Eq. (10) favoring the fusion of partons with low relative momenta $p_q - p_{\bar{q}} = p_1 - p_2$.

Note that, due to the off-shell nature of both partons and hadrons, the hadronization process obeys all conservation laws (i.e., the four-momentum conservation and the flavor current conservation) in each event, the detailed balance relations, and the increase in the total entropy S for rapidly expanding systems. The physics behind Eq. (8) is that the inverse reaction, i.e., the dissolution of hadronic states to quark-antiquark pairs (in the case of mesons), at low energy density is inhibited by the large masses of the partonic quasiparticles according to the DQPM. Vice versa the resonant q–$\bar{q}$ pairs have a large phase-space to decay to several 0^- octet mesons. We recall that the transition matrix element becomes huge below the critical energy density [57]. For further details on the PHSD off-shell transport approach and hadronization we refer the reader to Refs. [56,57,70,72,74].

III. CALCULATION OF SHEAR AND BULK VISCOSITY COEFFICIENTS

In this section we concentrate on the extraction of the shear and bulk viscosities for "infinite" parton-hadron matter employing the Green-Kubo formalism and the RTA. We simulate the "infinite" matter within a cubic box with periodic boundary conditions at various values for the energy density within PHSD. The size of the box is fixed to 9^3 fm^3. The initialization is done by populating the box with light (u, d) and strange (s) quarks, antiquarks, and gluons. If the energy density in the system is below the critical energy density ($\varepsilon_c \approx 0.5$ GeV/fm^3), the evolution proceeds through the dynamical phase transition (as described in Sec. II B) and ends up in an ensemble of

interacting hadrons. The system is initialized slightly out of equilibrium and, at all energy-densities, approaches kinetic and chemical equilibrium during it's evolution within PHSD as was shown in our previous investigations in Ref. [72]. After equilibration, the properties of the system at given temperature T can be studied. For more details we refer the reader to Ref. [72], where the particle abundances, spectra, fluctuations, and spectral functions have been studied. In the present work we extend our investigations to the calculation of transport coefficients.

A. The Kubo formalism

The Kubo formalism relates linear transport coefficients such as heat conductivity and shear and bulk viscosities to nonequilibrium correlations of the corresponding dissipative fluxes and treats dissipative fluxes as perturbations to local thermal equilibrium [43,44]. The Green-Kubo formula for the shear viscosity η is as follows [75]:

$$\eta = \frac{1}{T} \int d^3 r \int_0^\infty dt \langle \pi^{xy}(\mathbf{0}, 0) \pi^{xy}(\mathbf{r}, t) \rangle, \quad (12)$$

where T is the temperature of the system and $\langle \cdots \rangle$ denotes the ensemble average in thermal equilibrium. In Eq. (12), π^{xy} is the shear component (nondiagonal spacial part) of the energy momentum tensor $\pi^{\mu\nu}$:

$$\pi^{xy}(\mathbf{r}, t) \equiv T^{xy}(\mathbf{r}, t) = \int \frac{d^3 p}{(2\pi)^3} \frac{p^x p^y}{E} f(\mathbf{r}, \mathbf{p}; t), \quad (13)$$

where the scalar mean-field U_s (from PHSD) enters in the energy $E = \sqrt{\mathbf{p}^2 + U_s^2}$.

In our numerical simulation—within the test particles representation—the volume averaged shear component of the energy momentum tensor can be written as

$$\pi^{xy}(t) = \frac{1}{V} \sum_{i=1}^N \frac{p_i^x p_i^y}{E_i}, \quad (14)$$

where V is the volume of the system and the sum is over all particles in the box at time t. Note that the scalar mean-field contribution U_s only enters via the energy E. Taking into account that point particles are uniformly distributed in our box [implying $\pi^{xy}(\mathbf{r}, t) = \pi^{xy}(t)$], we can simplify the Kubo formula for the shear viscosity to

$$\eta = \frac{V}{T} \int_0^\infty dt \langle \pi^{xy}(0) \pi^{xy}(t) \rangle. \quad (15)$$

The correlation functions $\langle \pi^{xy}(0) \pi^{xy}(t) \rangle$ are empirically found to decay almost exponentially in time,

$$\langle \pi^{xy}(0) \pi^{xy}(t) \rangle = \langle \pi^{xy}(0) \pi^{xy}(0) \rangle e^{-t/\tau}, \quad (16)$$

as shown in Fig. 1, where τ is the respective relaxation time. Finally, we end up with the Green-Kubo formula for the shear viscosity:

$$\eta = \frac{V}{T} \langle \pi^{xy}(0)^2 \rangle \tau, \quad (17)$$

which we use to extract the shear viscosity from the PHSD simulations in the box at given energy density. Note that the temperature T is uniquely related to the energy density $\varepsilon(T)$ in PHSD (in thermodynamic equilibrium).
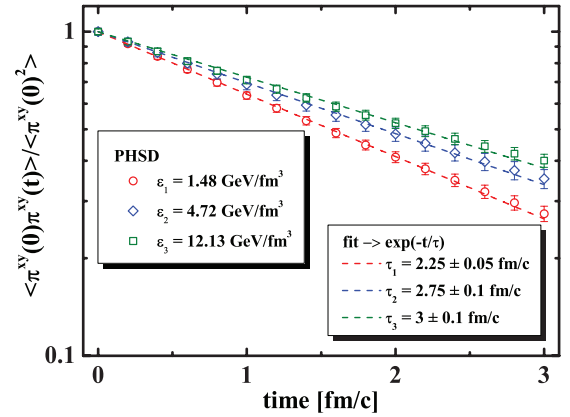

FIG. 1. (Color online) The correlation functions $\langle \pi^{xy}(0) \pi^{xy}(t) \rangle$, which are normalized by $\langle \pi^{xy}(0)^2 \rangle$, as a function of time from the PHSD simulations in the box (open symbols) for systems at different energy densities. The corresponding exponential fits are given by dashed lines; the extracted relaxation times τ are given too.

We check the numeric stability of the method by plotting the respective relaxation times τ, extracted from the PHSD simulations in the box, as a function of the number of test particles in Fig. 2. The results for the relaxation time τ converge for $N_{\text{test}} \geqslant 400$ independent of the energy density. In this study, we use a high amount of microcanonical simulations in the ensemble average ($N_{\text{test}} = 500$), which leads to reliable (within statistical error bars) results.

We also note that our numerical results for η do not depend on the volume V of the box within reasonable variations by factors of 6 as shown in Fig. 3.

B. The relaxation time approximation

The starting hypothesis of the RTA is that the collision integral can be approximated by

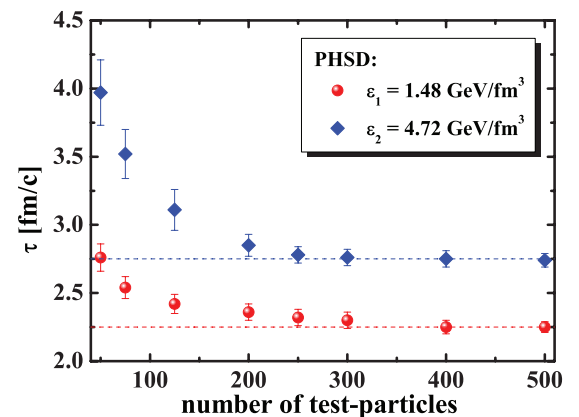$$C[f] = -\frac{f - f^{\text{eq}}}{\tau}, \quad (18)$$

FIG. 2. (Color online) The relaxation time τ as a function of the number of test particles obtained by the PHSD simulations in the box (symbols) for systems at different energy densities. The dashed lines provide the convergent values for the relaxation time τ.
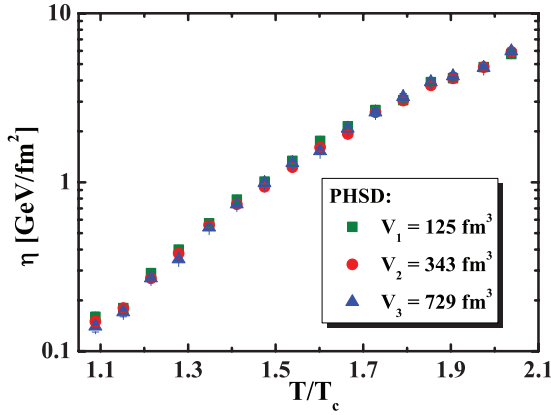
FIG. 3. (Color online) The shear viscosity η as a function of temperature from the PHSD simulations in the box for various volumes of the box: $V = 125$ GeV/fm^3 (green squares), $V = 343$ GeV/fm^3 (red circles), and $V = 729$ GeV/fm^3 (blue triangles).

where τ is the relaxation time. In this approach it has been shown that the shear and bulk viscosities (without mean-field or potential effects) can be written as [76–78]

$$\eta = \frac{1}{15T} \sum_a \int \frac{d^3 p}{(2\pi)^3} \frac{|\mathbf{p}|^4}{E_a^2} \tau_a(E_a) f_a^{eq}(E_a/T), \quad (19)$$

$$\zeta = \frac{1}{9T} \sum_a \int \frac{d^3 p}{(2\pi)^3} \frac{\tau_a(E_a)}{E_a^2} \left[(1-3v_s^2)E_a^2 - m_a^2\right]^2 f_a^{eq}(E_a/T), \quad (20)$$

where the sum is over particles of different type a (in our case, $a = q, \bar{q}, g$). In the PHSD transport approach the relaxation time is given by

$$\tau_a(T) = \Gamma_a^{-1}(T), \quad (21)$$

where $\Gamma_a(T)$ is the width of particles of type $a = q, \bar{q}, g$ as defined by Eqs. (4) and (5). In our numerical simulation—within the test particle representation—the volume averaged shear and bulk viscosities are given by the following expressions:

$$\eta = \frac{1}{15TV} \sum_{i=1}^{N} \frac{|\mathbf{p}_i|^4}{E_i^2} \Gamma_i^{-1}, \quad (22)$$

$$\zeta = \frac{1}{9TV} \sum_{i=1}^{N} \frac{\Gamma_i^{-1}}{E_i^2} \left[(1 - 3v_s^2)E_i^2 - m_i^2\right]^2, \quad (23)$$

where the speed of sound $v_s = v_s(T)$ is taken from lQCD [69] or the DQPM, alternatively. Note that $v_s(T)$ from both approaches is practically identical since it is governed by the DQPM, which reproduces the lQCD results.

C. Results for the shear viscosity

In Fig. 4 we present the shear viscosity to entropy density ratio η/s as a function of temperature T of the system extracted from the PHSD simulations in the box, where the viscosity was extracted employing the RTA (red line + diamonds) and the Kubo formalism (blue line + dots). We find that these approaches give roughly the same η/s as a function of temperature within error bars. For comparison, the results
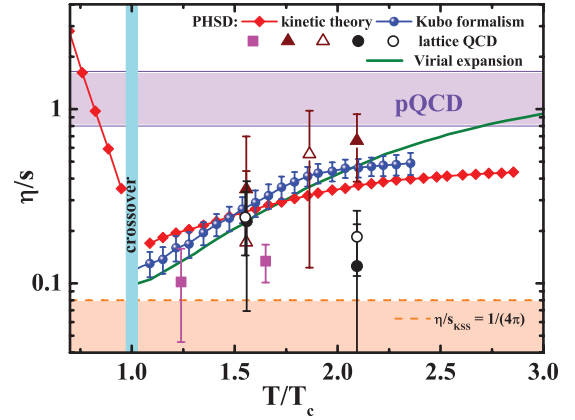

FIG. 4. (Color online) The shear viscosity to entropy density ratio η/s as a function of temperature of the system obtained by the PHSD simulations using different methods: the RTA (red line + diamonds) and the Kubo formalism (blue line + dots). The other symbols denote lQCD data for pure $SU_c(3)$ gauge theory from Ref. [37] (magenta squares), from Ref. [39] (open and solid triangles), and from Ref. [40] (black open and solid circles). The orange dashed line demonstrates the Kovtun-Son-Starinets bound [17] $(\eta/s)_{KSS} = 1/(4\pi)$. For comparison, the results from the virial expansion approach [24] are shown as a function of temperature too.

from the virial expansion approach [24] are given by the green line as well as lQCD data for pure $SU_c(3)$ gauge theory. The results for $T < T_c$ stem from PHSD in the relaxation time framework and rapidly rise with decreasing temperature. This is mainly because of a strong decrease of the entropy density, $s \to 0$ at $T \to 0$ as $e^{-m_\pi/T}$.

The behavior of the specific shear viscosity with temperature in PHSD is in agreement with the results of the scaling hadron masses and couplings and "heavy quark bag" (SHMC-HQB) approach [49,50,79], where the partonic phase is described in the "heavy quark bag" model. However, we obtain considerably lower values for the shear viscosity, in particular, in the partonic phase. The low viscosity of the quark-gluon matter in PHSD is caused by the stronger interaction between the degrees of freedom and is supported by the successful description of experimental data on the collective flow in heavy-ion collisions within PHSD [57,60].

At $T < T_c$, the PHSD results for the viscosity of the hadronic matter at vanishing quark chemical potential $\mu_q = 0$ qualitatively agree with the calculations in Refs. [80–83]. On the other hand, let us note that the results for the hadronic phase here have to be extended to finite μ_q before applications to realistic heavy-ion collisions can be performed. This is the topic of a separate forthcoming study.

D. Mean-field or potential effects

We recall that for vanishing quark chemical potential the partonic mean fields are essentially of scalar type and vector or tensor fields are suppressed, since the average quark current is zero. Furthermore, partonic mean fields affect the bulk viscosity but not the shear viscosity (except for a contribution

in the energy E in the denominator). According to Ref. [78], the expression for the bulk viscosity with potential effects reads

$$\zeta = \frac{1}{T} \sum_a \int \frac{d^3 p}{(2\pi)^3} \frac{\tau_a(E_a)}{E_a^2} f_a^{eq}(E_a/T)$$

$$\times \left[\left(\frac{1}{3} - v_s^2 \right) |\mathbf{p}|^2 - v_s^2 \left(m_a^2 - T^2 \frac{dm_a^2}{dT^2} \right) \right]^2. \quad (24)$$

In the numerical simulation the volume averaged bulk viscosity (including the mean-field effects from PHSD) is evaluated as

$$\zeta = \frac{1}{TV} \sum_{i=1}^{N} \frac{\Gamma_i^{-1}}{E_i^2} \left[\left(\frac{1}{3} - v_s^2 \right) |\mathbf{p}|^2 - v_s^2 \left(m_i^2 - T^2 \frac{dm_i^2}{dT^2} \right) \right]^2. \quad (25)$$

By using the DQPM expressions for the masses of quarks and gluons (for $\mu_q = 0$),

$$m_q^2(T/T_c) = \frac{1}{3} g^2(T/T_c)T^2, \quad m_g^2(T/T_c) = \frac{3}{4} g^2(T/T_c)T^2,$$

we can calculate the derivatives with respect to T^2. Thus all quantities in Eq. (25) are uniquely determined within PHSD. We recall that the DQPM description of thermodynamic properties of lQCD results [69] and its implementation in PHSD give practically the same results [72]. The derivation of partonic mean fields as well as their values can be found in Ref. [66].

E. Results for the bulk viscosity

In Fig. 5 we show the bulk viscosity to entropy density ratio ζ/s as a function of temperature T of the system obtained by the PHSD simulations in the box employing the RTA with mean-field (or potential) effects (red line + diamonds) and without potential effects (blue line + open triangles) for
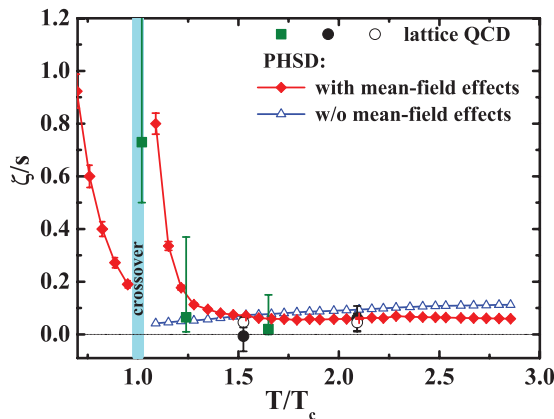
the partons. For comparison, we show in the same figure the available lQCD data [38,40]. Without mean-field effects we find an almost constant ratio $\zeta(T)/\eta(T)$ (see below), which is not in line with the findings from the lattice. Thus the dynamical mean fields (as incorporated in PHSD) play a decisive role in the temperature dependence of the bulk viscosity $\zeta(T)$ of the strongly interacting quark-gluon plasma. The increase of the bulk viscosity per unit entropy at $T \approx T_c$ is generated by the collective interaction of partons via mean fields rather than by their scatterings. At high temperature the mean-field effects are less pronounced and the values for the bulk viscosity of partonic matter from PHSD are approaching those obtained in the scope of the SHMC-HQB model [49,50,79].

On the hadronic side, we observe that ζ/s falls with temperature, which is in agreement with the results of the SHMC-HQB model [49,50,79] and of the chiral model for an interacting pion gas [84,85]. However, we do not see a divergent behavior of the bulk viscosity to entropy density ratio for $T \to 0$ as predicted in Ref. [85].

Further related quantities are of interest, in particular, the specific sound $(\eta + 3\zeta/4)/s$. A sound wave propagation in the z direction with wavelength $\lambda = 2\pi/k$ is damped according to

$$T_{03}(t, k) \propto \exp\left[-\frac{\left(\frac{4}{3}\eta + \zeta\right)k^2 t}{2(\varepsilon + p)} \right], \quad (26)$$

where T_{03} is the momentum density in the z direction, ε is the energy density, and p is the pressure. Thus both the shear η and bulk ζ viscosities contribute to the damping of sound waves in the medium and provide a further constraint on the viscosities. In Fig. 6 we present the specific sound channel $(\eta + 3\zeta/4)/s$ as a function of temperature T of the system obtained by the PHSD simulations in the box using the RTA with mean-field effects (red line + diamonds). It is compared with lQCD results for pure $SU_c(3)$ gauge theory from Ref. [86] (green circles) and from combining results of Refs. [37,38]
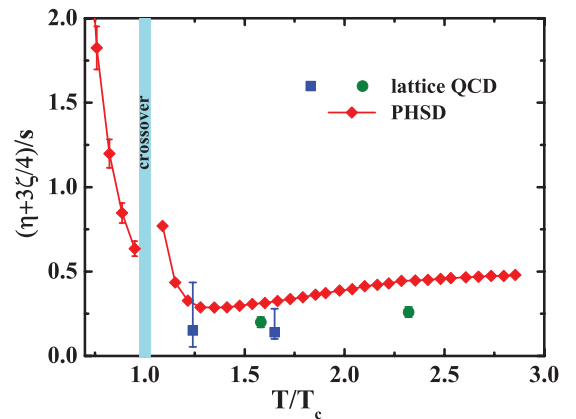

FIG. 5. (Color online) The bulk viscosity to entropy density ratio ζ/s as a function of temperature T extracted from the PHSD simulations in the box using the RTA with mean-field effects (red line + diamonds) and without potential effects (blue line + open triangles). The available lQCD data from Ref. [38] are given by green squares and from Ref. [40] by black open and solid circles, respectively.

FIG. 6. (Color online) The specific sound channel $(\eta + 3\zeta/4)/s$ as a function of temperature T of the system obtained by the PHSD simulations in the box using the RTA with mean-field effects (red line + diamonds). It is compared with lQCD data from Ref. [86] (green circles) and from combining the results of Refs. [38,40] (blue squares).
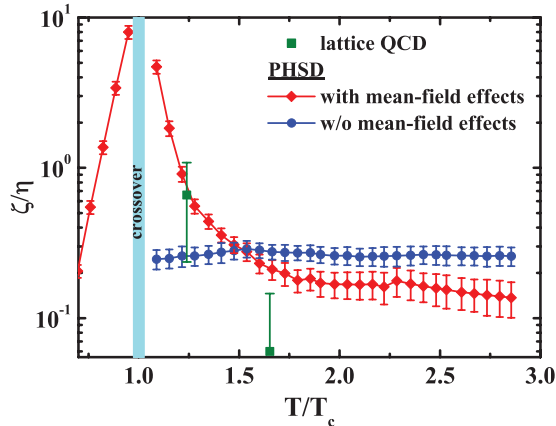
FIG. 7. (Color online) The bulk to shear viscosity ratio ζ/η as a function of temperature of the system obtained by the PHSD simulations in the box employing the RTA with mean-field effects (red line + diamonds) and without potential effects (blue line + circles). It is compared with lQCD data from Refs. [37,38] (green squares). Note the logarithmic scale in ζ/η.

(blue squares). Note that the PHSD calculations correspond to unquenched three-flavor QCD and thus are not expected to match the results for the pure gauge theory exactly.

Finally, in Fig. 7, we show the bulk to shear viscosity ratio ζ/η as a function of temperature of the system extracted from the PHSD simulations in the box using the RTA with mean-field (or potential) effects (red line + diamonds) and without potential effects (blue line + circles). Whereas an almost temperature-independent result is obtained in the partonic phase when discarding mean-field effects, a strong increase close to T_c is found in the PHSD when including the mean fields for the partons. The results for the shear to bulk viscosity ratio in the deconfined phase are in agreement with the lattice data [37,38] and with Ref. [87]. Since the PHSD gives a minimum in the shear viscosity η and a strong maximum in the bulk viscosity ζ close to T_c (note the logarithmic scale), the ratio ζ/η has a sizable maximum in the area of the (crossover) phase transition.

IV. SUMMARY AND CONCLUSIONS

We have employed the off-shell PHSD approach in a finite box with periodic boundary conditions for the study of the shear and bulk viscosities as a function of temperature (or energy density) for dynamical infinite partonic and hadronic systems in equilibrium. The PHSD transport model is based on a lQCD equation of state [69] and well describes the entropy

density $s(T)$, the energy density $\varepsilon(T)$, and the pressure $p(T)$ in thermodynamic equilibrium in comparison to the lQCD results [56,57,72]. We have employed the Kubo formalism as well as the RTA to calculate the shear viscosity $\eta(T)$. We find that both methods provide very similar results for the ratio η/s with a minimum close to the critical temperature T_c while approaching the perturbative QCD limit at higher temperatures. For $T < T_c$, i.e., in the hadronic phase, the ratio η/s rises fast with decreasing temperature due to a lower interaction rate of the hadronic system and a significantly smaller number of degrees of freedom (or entropy density). Our results are, furthermore, also in almost quantitative agreement with the ratio $\eta(T)/s(T)$ from the virial expansion approach in Ref. [24] as well as with lQCD data for the pure gauge sector.

We have, furthermore, evaluated the bulk viscosity $\zeta(T)$ in the RTA and focused on the effects of mean fields (or potentials) in the partonic phase. Here we find a significant rise of the ratio $\zeta(T)/s(T)$ in the vicinity of the critical temperature T_c due to the scalar mean fields from PHSD. The result for this ratio is in line with that from lQCD calculations. Additionally, the specific sound $(\eta + 3\zeta/4)/s(T)$ has been calculated and presents a nontrivial temperature dependence; the absolute value for this combination of the shear and bulk viscosities is in an approximate agreement with the lattice gauge theory. Furthermore, the ratio $\zeta(T)/\eta(T)$ within the PHSD calculations shows a strong maximum close to T_c, which has to be attributed to mean-field (or potential) effects that in PHSD are encoded in the infrared enhancement of the resummed coupling $g(T)$.

Because the PHSD calculations have proven to describe single-particle as well as collective observables from relativistic nucleus-nucleus collisions from lower SPS to top RHIC energies, the extracted transport coefficients $\eta(T)$ and $\zeta(T)$ are compatible with experimental observations in a wide energy/temperature range. Furthermore, the qualitative and partly quantitative agreement with lQCD results is striking.

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