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${}^{3}\!PF_{2}$ pairing in high-density neutron matter

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The onset of the superfluid phase in high-density neutron matter is studied within the BCS framework with two- and three-body forces. When including the strong correlation effects in the gap equation, the pairing gap turns out to be nonvanishing in a range of densities about $0.1-0.4 \text{ fm}^{-3}$ with a peak value a bit less than 0.05 MeV. These results could limit the role of the ${}^{3}PF_{2}$ superfluidity in the interpretation of phenomena occurring in the neutron-star core.

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Introduction. So far, neutron stars (NSs) have been considered as rich laboratories of various superfluid phases of nuclear matter [1–11]. Recently interest has been focused on the NS interior, where both the vortex pinning responsible for the observed period glitches [12] and the nucleon superfluidity responsible for the main cooling mechanisms [13] are supposed to be located. In particular, the recent observations of cooling in the NS of Cassiopeia A have been considered to be direct evidence of the anisotropic ${}^{3}\!PF_{2}$ neutron-neutron (nn) pairing in the NS core, and the energy gap needed to explain the data has been estimated to be around 0.1 MeV [13].

From a theoretical viewpoint, the preceding calculations show that ${}^{3}PF_{2}$ pairing might extend to several times the saturation density of nuclear matter in the pure BCS theory without many-body effects. However, in such a high-density regime the short-range correlations are so strong that the momentum distribution around the Fermi level significantly departs from the typical profile of a degenerate Fermi system. This departure is measured by the so-called Z factor (0 < Z <1) [14]. Since the deformation of the Fermi surface hinders particle transitions around the Fermi energy ϵ_F , the pairing gap is expected to get reduced. This effect was investigated in the case of low-density ${}^{1}S_{0}$ pairing channel (see, e.g., Ref. [15]), but it was ignored in the case of high-density nn pairing in the ${}^{3}\!PF_{2}$ channel, owing to the large uncertainty still existing in the strength of the pairing interaction, both two-body-force (2BF) and three-body-force (3BF) components. Many-body effects on the pairing interaction, such as the medium polarization in the uncoupled ${}^{3}P_{2}$ channel [16] and screening in the ${}^{1}S_{0}$ channel [15], also point to gap suppression.

In this Rapid Communication, we present a study of the *Z*-factor effect on the ${}^{3}PF_{2}$ pairing in pure neutron matter. In principle, we should consider asymmetric nuclear matter for application to the NS core, but the small proton fraction is not relevant in this context, as discussed below. The deformation of the Fermi surface and the *Z* factor are studied in the framework of the Brueckner theory with 2BF and 3BF [17]. The energy gap is then calculated within the generalized BCS theory [18], including in the pairing interaction not only 2BF but also 3BF. The latter, in fact, is dominant at high density and therefore is expected to directly influence pairing gap in addition to the Z factor.

Formalism and results: Nucleon propagator in neutron matter. The neutron Green's function is given by

$$G^{-1}(p,\omega) = \omega - \frac{p^2}{2m} - \Sigma(p,\omega) + e_F, \qquad (1)$$

where e_F denotes the Fermi energy and $\Sigma(p, \omega)$ is the self-energy. Expanding the latter in a series of powers of the quasiparticle energy around the Fermi surface, we obtain

$$G^{-1}(p,\omega) \approx Z(p)^{-1}(\omega - \epsilon_p), \qquad (2)$$

where the quasiparticle energy and the quasiparticle strength are

$$\epsilon_p = \frac{p^2}{2m} + \Sigma(p, \epsilon_p) - e_F, \quad Z(p) = \left[1 - \frac{\partial \Sigma(p, \omega)}{\partial \omega}\right]_{\omega = \epsilon_p}^{-1},$$

respectively. The quasiparticle strength Z(p) measures the deviation of a correlated Fermi system from the ideal degenerate Fermi gas. The occupation numbers n(p)and the Z(p) factors have been calculated in the framework of the Brueckner theory [19,20]. Because of the inclusion of the three-body forces, the hole-line expansion can be extended up to high densities. The self-energy, truncated to the second order (see Ref. [21] for details), provides us with a good reproduction of the empirical nuclear mean field [22] and the optical-model potential [23], so that we are quite confident that the next orders are irrelevant for the present calculation. We employ as 2BF the meson exchange Bonn B potential [24], whose meson parameters are constrained by the fit with the experimental phase shifts of NN scattering. The microscopic meson exchange 3BF is the one constructed by Li et al. [17]. It is consistent with the 2BF because it adopts the same meson parameters as Bonn B, so that there are no free parameters in the model.

The Z(p) factor is related to the depletion of the occupation number n(p) around the Fermi surface. According to the Migdal-Luttinger theorem [25] its value $Z = Z(p^F)(Z$ factor) equals the discontinuity of the occupation number at the Fermi surface, i.e.,

$$\lim_{\varepsilon \to 0} [n(p^F - \varepsilon) - n(p^F + \varepsilon)] = Z(p^F),$$
(3)

where p^F is the Fermi momentum. In our approximation $\Sigma(p, \omega) = \Sigma_1(p, \omega) + \Sigma_2(p, \omega)$, where $\Sigma_1(p, \omega)$ determines

the left limit and $\Sigma_2(p, \omega)$ determines the right limit of the preceding equation for $\varepsilon \to 0$.

In Fig. 1(a), we display the calculated occupation probability in pure neutron matter at density $\rho = 0.3$ fm⁻³. One easily observes the remarkable deviation from the ideal Fermi gas (solid line) due to the strong short-range correlations. As expected, the deviation is slightly enhanced by 3BF. In Fig. 1(b) the calculated Z factor is displayed vs density in the two different approximations for the self-energy, i.e., $\Sigma \approx \Sigma_1$ and $\Sigma \approx \Sigma_1 + \Sigma_2$, respectively. The calculation of Z_F from Eq. (3) requires a high numerical accuracy: By increasing the accuracy the calculated Z_F gets lower and until the converging value is reached. It is noticed that without 3BF, the Z factors decrease slowly as a function of density. Adding the contribution of Σ_2 leads to an overall reduction of the Z factor. The 3BF reduces further the Z factor and its effect increases rapidly with density. As a consequence, including 3BF makes the decrease of the Z factor as a function of density much more rapid than that obtained by adopting pure 2BF. Therefore 3BF induces a strong extra deviation from the ideal Fermi gas model.

Gap equation in the ${}^{3}PF_{2}$ channel. The ${}^{3}PF_{2}$ superfluidity in pure neutron matter has been investigated using various theoretical approaches [26–32] with 2BF and extended to microscopic 3BF forces by Zuo *et al.* [18]. In this case, the pairing gaps are determined by the equation:

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FIG. 1. (Color online) Occupation numbers vs momentum at a given density (upper panel) and Z factors vs density (lower panel) in pure neutron matter. The effect of 3BF is shown in both panels. In the lower panel the calculations are reported for two approximations of the self-energy.

$$\begin{pmatrix} \Delta_L(p) \\ \Delta_{L+2}(p) \end{pmatrix} = -\frac{1}{\pi} \int_0^\infty p'^2 dp' \frac{Z(p)Z(p')}{E_{p'}} \begin{pmatrix} V_{L,L}(p,p') & V_{L,L+2}(p,p') \\ V_{L+2,L}(p,p') & V_{L+2,L+2}(p,p') \end{pmatrix} \begin{pmatrix} \Delta_L(p') \\ \Delta_{L+2}(p') \end{pmatrix},$$
(4)

where $E_p^2 = (\epsilon_p - \mu)^2 + \Delta_p^2$ and $\Delta^2 = \Delta_L^2 + \Delta_{L+2}^2$. $V_{L,L'}(p, p')$ are the matrix elements of the realistic *NN* interaction in the coupled ${}^3\!PF_2$ channel. In the gap equation, the *Z* factors and the single-particle energy ϵ_p are calculated from the Brueckner theory. As for the pairing interaction $V_{L,L'}(p, p')$ in the ${}^3\!PF_2$ coupled channel, we adopt the same 2BF and 3BF as in the Brueckner calculation. The 3BF cannot be neglected in the gap calculation because the ${}^3\!PF_2$ pairing is expected to occur in the high-density domain, where 3BF is quite sizable, especially in the ${}^3\!PF_2$ channel.

The results are summarized in Fig. 2. When neglecting the Z factor effect (upper panel), the magnitude of the ${}^{3}PF_{2}$ gap with the new interaction dos not differ from previous calculations with AV18 potential [18]. On the other hand, the 3BF enhances the ${}^{3}PF_{2}$ superfluidity significantly at higher densities. As shown in the lower panel of Fig. 2, the introduction of the Z-factor effect fatally quenches the pairing gaps to a value of less than 0.05 MeV, one order of magnitude smaller than the value with full interaction. The effect of the Z factor appears to be extremely sizable at high densities. It is worth noticing that this effect is opposite the 3BF effect on the ${}^{3}PF_{2}$ pairing in neutron matter: The former turns out to be much stronger than the latter. In conclusion, the departure of the system from the pure degenerate limit drives the pairing attenuation at high density, regardless of whether the 3BF is included.

In Ref. [33], the effect of the Z factor was investigated also for the ${}^{1}S_{0}$ pairing in neutron matter, and it was shown to reduce the energy gap. Compared with the results of Ref. [33],



FIG. 2. (Color online) Effect of the Fermi surface depletion on ${}^{3}\!PF_{2}$ pairing gap in pure neutron matter. Notice the *y*-scale change from panel (a) to panel (b).

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the same effect on the ${}^{3}\!PF_{2}$ superfluidity appears to be much stronger. This is simply because in the density region of the ${}^{3}\!PF_{2}$ superfluidity the deviation of neutron matter from the degenerate Fermi gas limit becomes much larger.

Conclusions. In conclusion, we have studied the anisotropic ${}^{3}PF_{2}$ pairing in pure neutron matter. The effects of the Fermi surface depletion (Z factor) have been included in the calculation of the energy gap. In the pure degenerate limit, the ${}^{3}\!PF_{2}$ superfluid phase extends over a broad density range with a gap peak value of about 0.2 MeV without 3BF and 0.5 MeV with 3BF. The inclusion of the Z factor leads to a rapid decrease of the gap magnitude by one order of magnitude: Its peak value drops to less than 0.05 MeV and the superfluidity domain shrinks to 0.1–0.4 fm⁻³. These results for the ${}^{3}PF_{2}$ superfluidity of high-density neutron matter cannot be directly applied to the NS core, because of the presence of a proton fraction in β equilibrium with neutrons. The appearance of hyperons and/or the transition to deconfined phase can also affect the neutron core structure. Here we briefly discuss the effects of proton and hyperons, leaving for another study the effect of quark phase that can be by itself a source of pairing. The proton fraction in β equilibrium with neutrons, at the

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densities relevant for pairing, is less than 15% of the total density, and recent calculations of the quasiparticle strength show that the enhancement of the neutron Z factor is negligible for such a small proton fraction [34]. As to the hyperon phase, leaving aside the small NS mass predicted by a model of hyperons and baryons interacting via realistic forces [35], its main effect is to reduce the neutron effective mass and then to further suppress any pairing gap. The conclusion is that the present results are expected to have a significant impact on the investigations of the cooling and other transport properties of neutron stars, based on models where the NS core is made of high-density neutron matter (with or without hyperons) in β equilibrium with protons.

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