PHYSICAL REVIEW C **87**, 061602(R) (2013)

Role of structural effects on the collective transverse flow and the energy of vanishing flow in nuclear collisions

Rajni Bansal,[1] Sakshi Gautam,[2] Rajeev K. Puri,[1,*] and J. Aichelin[3]
[1]*Department of Physics, Panjab University, Chandigarh 160 014, India*
[2]*Dev Samaj College for Women, Sector 45 B, Chandigarh 160047, India*
[3]*SUBATECH, Laboratoire de Physique Subatomique et des Technologies Associées, Université de Nantes - IN2P3/CNRS - EMN 4 rue Alfred Kastler, F-44072 Nantes, France*
(Received 22 October 2012; revised manuscript received 12 February 2013; published 18 June 2013)

We address the question of why so far most of the simulation approaches to find the energy of vanishing flow (EVF) in light systems have failed to reproduce the experimental data. By investigating systematically the dependence of the EVF on the initial setup of the nuclei in these approaches we find out that for light systems a small variation of this setup can create large differences in the EVF whereas for large systems the dependence is weak. These studies have been done with the isospin-dependent quantum molecular dynamics model.

DOI: 10.1103/PhysRevC.87.061602 PACS number(s): 25.70.Pq, 02.70.Ns, 24.10.Lx

During the last three decades, most of the features of heavy-ion reactions in the energy interval of 50 AMeV $\leqslant E_{\rm beam} \leqslant$ 2 AGeV have been quantitatively understood although in these reactions projectile and target do not come to equilibrium. This success has to be credited to dynamical models which simulate the heavy-ion reaction from the initial setup to the final single-particle and fragment distribution which can be experimentally observed [1–9]. These models can be divided into three categories: (i) those based on the one-body density matrix such as Boltzmann-Uehling-Uhlenbeck (BUU) models [3], (ii) those based on the n-body molecular dynamics approach using product wave functions [1,4], and (iii) those based on the n-body molecular dynamics approach using antisymmetric wave functions [6,7]. These models are not only able to reproduce the single-particle spectra but have also been used successfully to identify the nuclear equation of state [10], to study strangeness production [11] in nuclei, and to predict collective phenomena such as the energy dependence of the radial flow [12].

There are few observables where the models have failed and among those the most important is the in-plane flow of light systems and especially the energy of vanishing flow (EVF). This came as a surprise and therefore it has been argued that the difference between calculations and experimental results may be due to yet unknown physical processes.

Good progress has been made on the dynamical part as well as on the initialization of nuclei in a transport model [1,13]; however, no attempt has been made to look into the structural effects via nuclear radius that can affect the reaction dynamics throughout the periodic table. Here we shall discuss how different choices of nuclear radius parameter influence the results. Westfall *et al.* [14] expressed the need for a density-dependent parametrization of the cross section to reproduce the EVF for low-density matter. For larger systems a constant reduction in the cross section was enough to reproduce experimental energy of vanishing flow. Another study by Klakow *et al.* [15] has shown that a proper choice of the surface thickness is necessary for calculating the energy of vanishing flow of the $^{12}C + ^{12}C$ system using the BUU model. One of us and collaborators [16] felt the need of momentum-dependent interactions within the quantum molecular dynamics (QMD) model to reproduce the $^{12}C + ^{12}C$ data. De la Mota *et al.* [17] used the Landau-Vlasov approach to calculate the EVF of the $^{12}C + ^{12}C$ reaction. Similar assumptions were also made in the antisymmetrized molecular dynamics (AMD) model [18].

Unfortunately, none of these studies paid any serious attention to the fact that in these semiclassical simulations, the density of the nuclei is not exactly reproduced but approximated in different forms which are specific for each of the approaches. It has been assumed without proof that the small difference in the density distribution cannot explain the fact that the models fail to reproduce the EVF.

Most of the simulation models at intermediate energies use the liquid drop formula for calculating the radius, i.e., $R = r_0 A^{1/3}$, but different values of r_0 are used in the different approaches ($r_0 = 1.12$, 1.142, and 1.18 in the isospin-dependent quantum molecular dynamics (IQMD) model [1] and Monte Carlo method (MC) [5], in QMD [4], and in internuclear cascade (INC) [2], respectively). Other models made adjustments for the ^{12}C nucleus. For example $r_0 = 1.3$ was taken in the Monte Carlo method when considering the case of ^{12}C [5]. Also the root-mean-square (rms) values used for a ^{12}C nucleus within the fermionic molecular dynamics (FMD) [6], AMD [7], and nucleus-nucleus optical potential (op. pot.) [8] are different (2.79 fm, 2.49 fm, and 2.355 fm, respectively). In the literature, many well-established potentials use radius of the form $aA^{1/3} + bA^{-1/3}$ [19]. Different values of a and b were chosen to give different parametrizations. The proximity potential adds also a constant term in the above formula [19]. Many attempts reported in the literature tried to incorporate isospin effects in the radius and modified the parametrization as $a'A^{1/3} + b'A^{-2/3} - c'I$ where $I = \frac{A-2Z}{A}$, where a', b', and c' are again some constants [20,21]. The range of radii of ^{12}C using these different parametrizations lies between 2.05 fm (due to Bass [19]) and 2.66 fm (due to Aage Winther (AW) [19]). Note that the experimental value is 2.3 fm due to Elton [22]. At the same time, the radius used

*rkpuri@pu.ac.in

in different models at intermediate energies ranges between 2.15 fm (UrQMD) and 2.98 fm (by MC). The standard liquid drop formula $\propto 1.12A^{1/3}$ leads to radius = 2.56 fm. In contrast, the variation of the radius for a ^{197}Au nucleus is much smaller and lies in between 6.5 fm (due to Bass [19]) and 7.05 fm (due to Royer [21]) whereas the liquid drop model gives 6.52 fm. From the above discussion it is clear that the radius of the ^{12}C system can vary considerably when different models are employed.

It is the purpose of this Rapid Communication to show that small differences of the initial density profile lead to by 40%−50% difference in the EVF for small systems such as $^{12}\mathrm{C}+{}^{12}\mathrm{C}$ whereas for large systems such as $^{197}\mathrm{Au}+{}^{197}\mathrm{Au}$, the EVF depends weakly on the initial density profile. Therefore, the disagreement between simulations and experiments is most probably due to an insufficient reproduction of the density profile in small systems and no new physics has to be evoked to explain the experimental data.

Some studies, however not systematic, have been done. Yong *et al.* [23] studied initialization effects by using different parametrizations of the Skyrme forces within the framework of the isospin-dependent BUU model on the symmetry energy sensitive observables such as free neutron to proton ratio (n/p), π^+/π^- ratio, and neutron to proton differential flow, F_x^{n-p}. Earlier, Hartnack *et al.* [1] showed the sensitivity of collective transverse flow for different initializations.

For our first exploratory study, we analyze here the effect of different radii on the collective flow for the reactions of $^{12}\mathrm{C}+{}^{12}\mathrm{C}$ and $^{197}\mathrm{Au}+{}^{197}\mathrm{Au}$. Our study is performed with the isospin-dependent quantum molecular dynamics (IQMD) model, the details of which are given in Ref. [1]. For the present study, we use a soft momentum-dependent (SM) equation of state with $\sigma = 0.8\sigma_{\mathrm{free}}$ and the value of 32 MeV for the strength of symmetry potential. We simulated thousands of events for the reactions of $^{12}\mathrm{C}+{}^{12}\mathrm{C}$ and $^{197}\mathrm{Au}+{}^{197}\mathrm{Au}$ at an impact parameter of $b/b_{\mathrm{max}} = 0.4$ as well as for the impact parameter range $b = 0$–4 fm (as guided by the experimental findings [14,24]), at incident energies between 40 MeV/nucleon and 250 MeV/nucleon in steps of 10 MeV/nucleon. These calculations have been repeated for different values of the radius parameter by keeping Fermi momentum constant. From the excitation function, the energy of vanishing flow (EVF) has been determined.

In Fig. 1, we display the time evolution of the averaged directed transverse momentum $\langle p_x^{\mathrm{dir}}\rangle = \sum_i \mathrm{sgn}(y^i_{c.m.})p^i_x$ with $y_{c.m.}$ being the rapidity of the nucleon in the center-of-mass system, calculated for the reactions of $^{12}\mathrm{C}+{}^{12}\mathrm{C}$ and $^{197}\mathrm{Au}+{}^{197}\mathrm{Au}$ at their corresponding experimental energies of vanishing flow, which reads as 122 MeV/nucleon [14] and 54 MeV/nucleon [24], using different parametrizations of the radius parameter. The dotted, dash-dotted, dash-dot-dotted, short-dashed, short-dotted, dashed, short-dash-dotted, and solid lines represent the calculations using the radius parametrization due to Bass, Elton, Brogila and Winther (BW), Christensen and Winther (CW), Blocki, IQMD, Ngô, and AW, respectively. From the figure, we notice that the directed transverse flow decreases if the size of the ^{12}C increases. We also see that the $\langle p_x^{\mathrm{dir}}\rangle$ is negative for all parametrizations (due to the mean field) during the initial phase of the reaction.

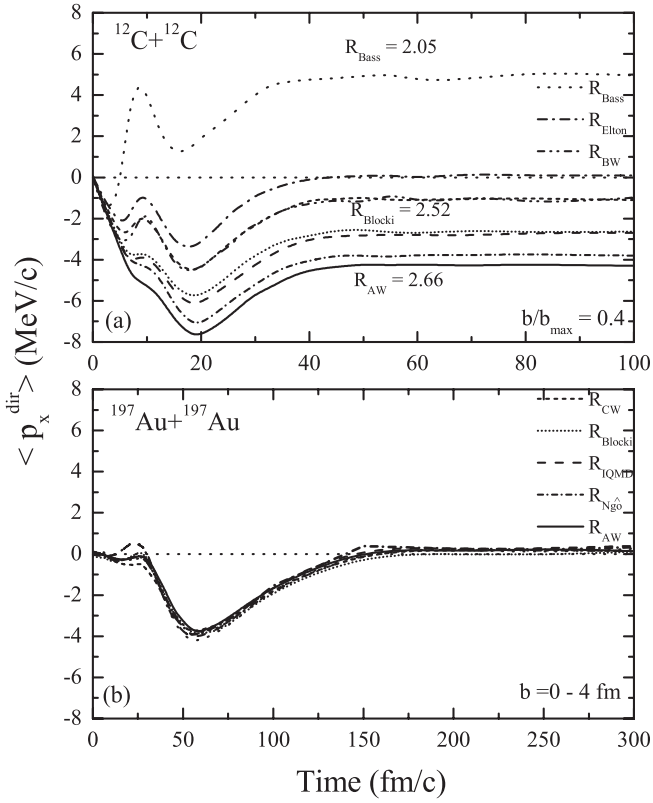

FIG. 1. The time evolution of $\langle p_x^{\mathrm{dir}}\rangle$ for the reactions of $^{12}\mathrm{C}+{}^{12}\mathrm{C}$ and $^{197}\mathrm{Au}+{}^{197}\mathrm{Au}$ using different parametrization of radius at their corresponding measured energies of vanishing flow. The lines are explained in the text.

As the reaction proceeds, binary nucleon-nucleon collisions contribute to the positive collective flow and therefore the collective flow changes. Though the flow decreases with increasing radius for $^{12}\mathrm{C}+{}^{12}\mathrm{C}$, it remains nearly the same for the reaction of $^{197}\mathrm{Au}+{}^{197}\mathrm{Au}$. The decrease in the flow with the increase in the radius is due to the decrease in the density gradient at the nuclear surface. Moreover, in the light systems, the ratio of the surface diffuseness and the radius is larger than in heavy systems. Therefore, if light nuclei collide, repulsive forces $[\propto(\frac{\rho}{\rho_0})^{\gamma}]$ get weakened due to the decrease in the density gradient and lower the momentum transfer into the transverse direction. Also, the number of the binary collisions decreases in lighter systems as radius increases.

In Fig. 2, we display the energy of vanishing flow calculated using different parametrizations of the radius for the reaction of $^{12}\mathrm{C}+{}^{12}\mathrm{C}$ (upper panel). A straight line fit is used to calculate the energy of vanishing flow in each case. The solid squares represent the EVF by varying the radius systematically between 80% to 120% in the IQMD model. This covers the radius range (2.15 to 2.98 fm) used by the different dynamical models at intermediate energies. The open circles represent the calculated energies of vanishing flow using different radius parametrizations and also include the measured one from Elton [22] (as has been labeled). We see that the energy of vanishing flow increases with increasing radius. This is due to the decrease in the strength of the repulsive forces with increasing

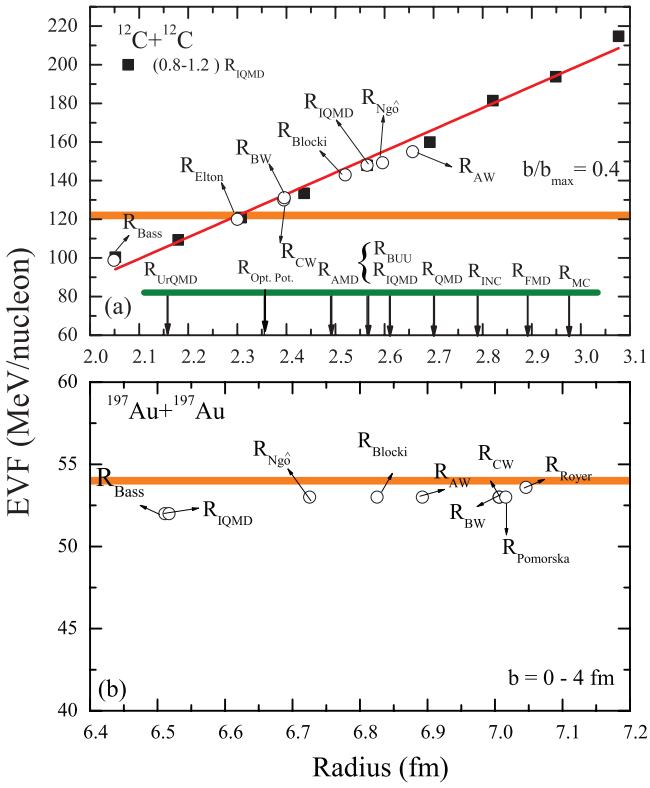

FIG. 2. (Color online) The energy of vanishing flow (EVF) for the reactions of $^{12}C+^{12}C$ (upper panel) and $^{197}Au+^{197}Au$ (lower panel) as a function of radius of colliding nuclei. The symbols are explained in the text. The thick solid horizontal line represents the measured EVF. The radius value of ^{12}C used in the different transport models is shown by vertical arrows. The solid line is a linear fit to the EVF calculated in the IQMD model, for various radii.

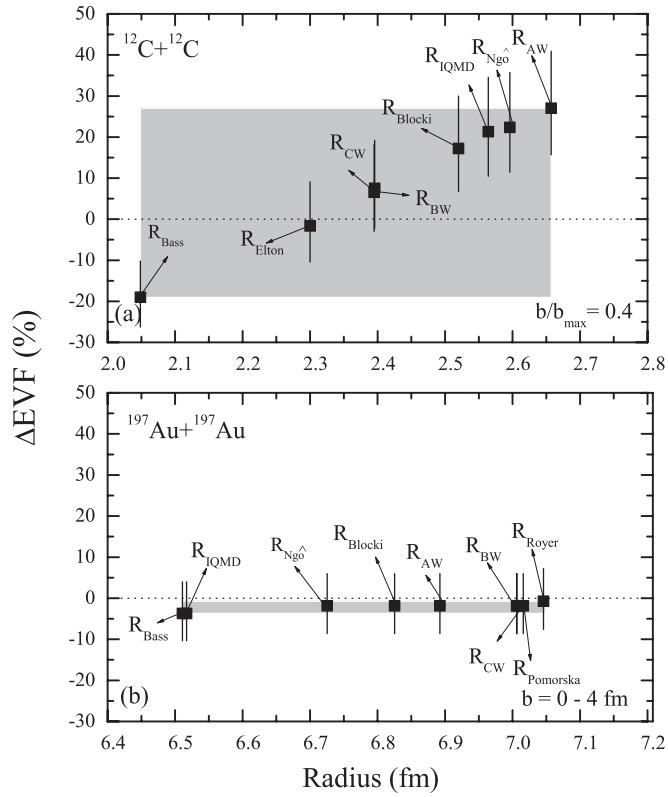

FIG. 3. The percentage deviation ΔEVF (%) as a function of the radii of colliding nuclei for the reactions of $^{12}C+^{12}C$ and $^{197}Au+^{197}Au$. The shaded area represents the region of percentage deviation of EVF calculated using different parametrizations of the radius from the experimental measured EVF.

radius, which causes a lowering of the collective flow and therefore pushes the energy of vanishing flow towards higher values. To see the influence of the radius parameter on the heavier system, we also display the energy of vanishing flow for the reaction of $^{197}Au+^{197}Au$ (lower panel) using different parametrizations of the radius. In contrast to the ^{12}C case, the EVF shows far less dependence for heavy system. The thick solid horizontal line represents the experimentally measured EVF for the reactions of $^{12}C+^{12}C$ and $^{197}Au+^{197}Au$. From the figure, it is evident that the energy of vanishing flow is significantly affected by the radii of colliding nuclei for the reaction $^{12}C+^{12}C$ but only little for $^{197}Au+^{197}Au$. Different parametrizations of radius yield energies of vanishing flow between 50–54 MeV/nucleon and 98–155 MeV/nucleon for the reactions of $^{197}Au+^{197}Au$ and $^{12}C+^{12}C$, respectively. Note that an increase of ∼30% in radius results in a ∼58% change in the EVF for the $^{12}C+^{12}C$ system, whereas a ∼8% change of the radius for $^{197}Au+^{197}Au$ system results in a ∼3% change in the energy of vanishing flow. It is evident that the light system $^{12}C+^{12}C$ is very sensitive to the choice of radius, whereas a heavy nucleus such as ^{197}Au is far less sensitive.

In Fig. 3, we display the percentage of the deviation [ΔEVF(%)] of the calculated energies for vanishing flow from the experimental value as a function of radius for the reactions $^{12}C+^{12}C$ (upper panel) and $^{197}Au+^{197}Au$ (lower panel). Here ΔEVF(%) is given by

$$\Delta\mathrm{EVF}(\%) = \left(\frac{\mathrm{EVF}^{\mathrm{theor}} - \mathrm{EVF}^{\mathrm{expt}}}{\mathrm{EVF}^{\mathrm{expt}}}\right) \times 100. \qquad (1)$$

The corresponding error bars are also displayed. The shaded area represents the region covered by different parametrizations of ^{12}C radius. For the $^{197}Au+^{197}Au$ reaction, all parametrizations yield similar results.

From the above discussion, it is evident that the flow physics at intermediate energies is dominated entirely by the surface effect, whereas they are bulk-dominated for the heavy systems. Further investigations were conducted by reducing the global Fermi momentum by 30% which yields more stable nuclei and choosing SM as well as soft EOS with full and reduced cross sections. Though the absolute value of the flow (and hence EVF) changes, the above conclusions regarding the sensitivity of flow towards different choices of nuclear radius remains unaltered. In some cases, the sensitivity towards different choices of radius has even increased. It is worth mentioning that the present analysis is conducted within the IQMD model and the result may depend on the model one is using.

In summary, employing the IQMD model, we demonstrated that for the light ^{12}C system the collective transverse flow shows a strong dependence on the nuclear radius whereas

a heavy system, such as $^{197}\mathrm{Au} + {}^{197}\mathrm{Au}$, is less affected. Our study indicates that the radii of colliding nuclei must be adjusted carefully if one wants to extract the energy of vanishing flow for lighter systems.

This work is supported by a grant from the University Grant Commission (UGC), Government of India vide Grant No. 42-811/2013 (SR). We are also thankful to Dr. C. Hartnack for fruitful discussions.

[1] C. Hartnack, R. K. Puri, J. Aichelin, J. Konopka, S. A. Bass, H. Stöcker, and W. Greiner, Eur. Phys. J. A **1**, 151 (1998).
[2] Y. Yariv and Z. Fraenkel, Phys. Rev. C **20**, 2227 (1979); Y. Kitazoe, M. Sano, Y. Yamamura, H. Furutani, and K. Yamamoto, *ibid.* **29**, 828 (1984).
[3] W. Bauer, Phys. Rev. Lett. **61**, 2534 (1988).
[4] J. Aichelin *et al.*, Phys. Rev. C **37**, 2451 (1988).
[5] J. Cugnon, Phys. Rev. C **22**, 1885 (1980).
[6] H. Feldmeier and J. Schnack, Prog. Part. Nucl. Phys. **39**, 393 (1997).
[7] A. Ono, H. Horiuchi, T. Maruyama, and A. Ohnishi, Phys. Rev. Lett. **68**, 2898 (1992).
[8] N. Ohtsuka, R. Linden, and A. Faessler, Nucl. Phys. A **465**, 550 (1987).
[9] K. Abdel-Waged, Phys. Rev. C **70**, 014605 (2004).
[10] C. Hartnack, H. Oeschler, and J. Aichelin, Phys. Rev. Lett. **96**, 012302 (2006).
[11] C. Hartnack, H. Oeschler, Y. Leifels, E. L. Bratkovskaya, and J. Aichelin, Phys. Rep. **510**, 119 (2012).
[12] C. Hartnack and J. Aichelin, Phys. Lett. B **506**, 261 (2001).
[13] W. Cassing, V. Metag, U. Mosel, and K. Niita, Phys. Rep. **188**, 363 (1990); A. Bonasera, F. Gulminelli, and J. Molitoris, *ibid.* **243**, 1 (1994); T. Gaitanos, A. B. Larionov, H. Lenske, and U. Mosel, Phys. Rev. C **81**, 054316 (2010); K. Weber *et al.*, Nucl. Phys. A **515**, 747 (1990); P. Danielewicz, *ibid.* **673**, 375 (2000).
[14] G. D. Westfall *et al.*, Phys. Rev. Lett. **71**, 1986 (1993).
[15] D. Klakow, G. Welke, and W. Bauer, Phys. Rev. C **48**, 1982 (1993).
[16] A. D. Sood and R. K. Puri, Eur. Phys. J. A **30**, 571 (2006).
[17] V. de la Mota, F. Sebille, M. Farine, B. Remaud, and P. Schuck, Phys. Rev. C **46**, 677 (1992).
[18] A. Ono, H. Horiuchi, and T. Maruyama, Phys. Rev. C **48**, 2946 (1993).
[19] I. Dutt and R. K. Puri, Phys. Rev. C **81**, 064609 (2010), and references therein.
[20] B. N. Pomorska and K. Pomorski, Z. Phys. A **348**, 169 (1994).
[21] G. Royer, Nucl. Phys. A **807**, 105 (2008); G. Royer and R. Rousseau, Eur. Phys. J. A **42**, 541 (2009).
[22] L. R. B. Elton, *Nuclear Sizes* (Oxford University Press, London, 1961).
[23] G. C. Yong, Y. Gao, W. Zuo, and X. C. Zhang, Phys. Rev. C **84**, 034609 (2011).
[24] J. Łukasik *et al.*, Phys. Lett. B **608**, 223 (2005).