Gamow peak approximation near strong resonances

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We discuss the most effective energy range for charged-particle-induced reactions in a plasma environment at a given plasma temperature. The correspondence between the plasma temperature and the most effective energy should be modified from the one given by the Gamow peak energy, in the presence of a significant incident-energy dependence in the astrophysical S factor as in the case of resonant reactions. The suggested modification of the effective energy range is important not only in thermonuclear reactions at high temperature in the stellar environment, e.g., in advanced burning stages of massive stars and in explosive stellar environments, as has been already claimed, but also in the application of nuclear reactions driven by ultra-intense laser-pulse irradiation.

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The nuclear reaction rate in a plasma environment at a certain temperature can be related to an effective energy range [1–5], both in the stellar site [1,4,6] and in the laser-induced plasma site [7–13]. This effective energy range gives us an idea of which energy region one can compare to the cross-section data in conventional beam-target experiments, when one needs to know the reaction rate in a plasma at a given temperature T. Of particular interest is the application of this relation to the nuclear reaction yield in the laser-induced plasma site. The neutron yield through the reaction ^{2}H$(d,n)^3$He driven by laser-pulse irradiation on a deuterium cluster target is well studied [7–11] at various laser parameters. The deuteron acceleration in such an experiment is attributed to the Coulomb explosion of the clusters in the laser-pulse field. By measuring the energies of accelerated deuterons, the laser-induced plasma deuterons are known to have Maxwellian-like energy spectra [9]. Recent experiments on the Texas Petawatt Laser are dedicated to the determination of the deuteron plasma temperature [10,11] by using cryogenically cooled deuterium D_2 (or near-room-temperature deuterated methane CD_4) clusters and ^{3}He mixtures. By taking the ratio of the fusion yields from reactions ^{2}H$(d,n)^3$He, ^{2}H$(d,p)^3$H, and ^{3}He$(d,p)^4$He, the temperature of the deuteron plasma is determined to be from 8 to 30 keV. Other examples are the proton-induced reactions ^{11}B$(p,n)^{11}$C and ^{63}Cu$(p,n)^{63}$Zn using laser-accelerated protons [14] and the reaction ^{11}B$(p,\alpha)^8$Be. For the former two reactions, protons are accelerated from a thin foil target and interact with the secondary solid targets. In such a case the mechanism of ion acceleration is attributed to the target normal sheath acceleration (TNSA) and the spectra of the laser-accelerated protons are, again, known to be a near Maxwellian but with the temperature as high as 5 MeV [15]. For the latter reaction, a yield of 10^3 α particles has been reported by a group in Russia using laser-pulse irradiation of peak intensity 2×10^{18} W/cm^2 in 1.5 ps on a ^{11}B + CH_2 composite target [12]. However, no data are published on the spectra of the accelerated ions from this experiment and, besides, corrections taking into account the particle ranges in matter reveal a higher yield (10^5 α particles) [16]. α-particle yield through the reactions ^{11}B$(p,\alpha)^8$Be and ^{10}B$(p,\alpha)^7$Be are observed, using natural-boron-doped plastic (CH_2) targets [13], at the ABC laser facility, which derivers 50 J in 3 ns, in Frascati in Italy. In this experiment the spectra of the accelerated ions of boron as well as protons are characterized, but the observed fusion yield is not fully consistent with the one expected from the characterized ion spectra. In the above-mentioned experiments knowing the effective energy of the plasma ions which contribute to the nuclear reactions of interest is essential both to understand the acceleration mechanisms of energetic ions generated in the laser-plasma interaction and for the optimization of the laser parameters using the scaling relation [15]. By comparing the ion spectra expected from the reaction yield with the one obtained from the direct measurement of the accelerated ions, one can also determine the energy loss of the ions in the plasma [10], which is not understood completely. In this connection, we mention that in recent experiments using the TRIDENT laser at Los Alamos National Laboratory successful deuteron acceleration as high as 170 MeV from an ultrathin (300 nm) foil target [17] by using the newly proposed break-out afterburner (BOA) mechanism has been reported. The intense deuteron beam generated by this mechanism is used to produce an intense neutron beam by means of the reaction ^{9}Be$(d,n)^9$B. The promising result suggests the possibility of a compact neutron source generator driven by high-intensity laser pulses and opens up various potential applications using deuteron-induced reactions, which have the advantage of positive Q values compared with proton-induced reactions [18,19]. We mention also that this relation between the plasma temperature and the effective energy can be applied the other way around [11]. Through the measurement of fusion yields in a laser-induced plasma, one could determine low-energy cross sections [20], which are of great interest for astrophysical applications. For this purpose, one needs to know the exact relation between the plasma temperature and the most effective energy not only in nonresonant reactions but also in resonant reactions.

When both colliding ion species have thermal distributions, the reaction rate can be obtained by integrating the reaction cross section σ multiplied by the relative velocity v and by the spectrum $\phi(v)$ of the relative velocity over the incident energy E (keV) [2,4,5,21,22]. The relative velocity spectrum is written in a form of a Maxwell-Boltzmann distribution in an equilibrium gas at temperature T. One, thus, obtains the reaction rate per pair of particles as a function of temperature:

$$\langle \sigma v \rangle = \sqrt{\frac{8}{\mu\pi}} \frac{1}{(k_B T)^{3/2}} \int_0^\infty S(E) \exp\left(-\frac{E}{k_B T} - \frac{b}{\sqrt{E}}\right), \tag{1}$$

where μ is the reduced mass of the colliding nuclei and $b = 31.28 Z_1 Z_2 A^{1/2}$ keV$^{1/2}$, denoting the atomic numbers and reduced mass number of colliding nuclei Z_1, Z_2, and A, respectively. We write the cross section in terms of the astrophysical S factor, $S(E)$ [4,5,23]; k_B is the Boltzmann constant. The effective energy at a certain temperature T is determined by means of the Gamow peak. If $S(E)$ is a smooth function of energy, we may approximate $S(E)$ with a constant (S_0) and bring it out of the integral. Then, by means of the saddle-point approximation, we expect that the most important contribution to the integral in Eq. (1) comes from $E = E_0$, which satisfies the following condition:

$$\frac{d}{dE}\left(-\frac{E}{k_B T} - \frac{b}{\sqrt{E}}\right)_{E=E_0} = 0. \tag{2}$$

This condition leads to the most effective energy E_0 (or Gamow energy) at temperature T,

$$k_B T = \frac{2}{b} E_0^{3/2}. \tag{3}$$

The method of the saddle-point approximation is equivalent to the replacement of the peak by a Gaussian with the same peak and with a1/e width of

$$\Delta E_0 = \frac{4}{\sqrt{3}} (E_0 k_B T)^{1/2}. \tag{4}$$

The width is a function of the plasma temperature. In Ref. [1] the correspondence between the T_9 axis and the E axis is derived from this equation. T_9 is the temperature in units of 10^9 K. $E_0 \pm \Delta E_0/2$ represents the effective energy window.

However, if the reaction cross section has a significant energy dependence, the astrophysical S factor cannot be approximated by a constant to evaluate the integral in Eq. (1). One, therefore, has to consider the contribution from this term in addition to the two terms in Eq. (2). In such a case, practically, the condition to get the most effective energy, Eq. (2), becomes

$$\frac{d}{dE}\left(\log S(E) - \frac{E}{k_B T} - \frac{b}{\sqrt{E}}\right)_{E=E_0} = 0. \tag{5}$$

To discuss this more concretely, we consider an example of resonant reactions where the astrophysical S factor is approximated by the following Breit-Wigner form:

$$S(E) = S_0 + \frac{S_r}{(E - E_r)^2 + \Gamma^2/4}, \tag{6}$$

where E_r and Γ are the peak and the width of the resonance, respectively. We assume that both S_0 and S_r are positive. Then Eq. (5) leads to

$$k_B T = \left(\left[\frac{-S_r 2(E - E_r)}{[(E - E_r)^2 + \Gamma^2/4][S_r + S_0(E - E_r)^2 + S_0\Gamma^2/4]} + \frac{b}{2} E^{-3/2}\right]_{E=E_0'}\right)^{-1}. \tag{7}$$

By substituting $S_r = 0$, it can be, easily, derived that this equation recovers the conventional Gamow energy at temperature T [Eq. (3)]. If S_r is not zero, i.e., in the presence of a resonance, Eq. (7) implies that the departure of the most effective energy from the Gamow peak is large around the resonance peak $E = E_r$. Another simple limit is the case where the width of the resonance is zero and S_0 is negligible, i.e., the resonance is approximated by a δ function; then

$$k_B T = \left(\frac{-2}{E_0' - E_r} + \frac{b}{2} E_0'^{-3/2}\right)^{-1}. \tag{8}$$

In this limit the absolute value of the correction term becomes larger than the second term and the resonant peak gives the major contribution. Given that both S_0 and S_r are positive, the first term in this equation changes sign as E passes through E_r; i.e., at a given temperature T the most effective energy, E_0', becomes higher than E_0 in the region $E < E_r$ and E_0', becomes lower than E_0 in the region $E > E_r$. We determine the width of the effective energy window with Eq. (4) by replacing E_0 by E_0', that is, by

$$\Delta E_0' = \frac{4}{\sqrt{3}} (E_0' k_B T)^{1/2}. \tag{9}$$

This definition of the width is different from the one chosen in [4,5], but it shows clearly that the width is consistent with the Gamow window if the S factor does not depend on the incident energy. We evaluate this condition numerically by using the S factors determined experimentally for three selected resonant reactions: ^{11}B(p,α)^{8}Be, ^{10}B(p,α)^{7}Be, and ^{3}H(d,n)^{4}He, besides a nonresonant reaction ^{2}H(d,n)^{3}He. All four reactions are of importance in the application of laser-induced nuclear reactions.

It is worth mentioning that the major contribution to the reaction rates coming from the vicinities of both Gamow energy and the resonant peak in resonant reactions is already known [2,22], and it has been demonstrated that the effective energy window in which the most thermonuclear reactions take place at a given temperature can differ significantly from

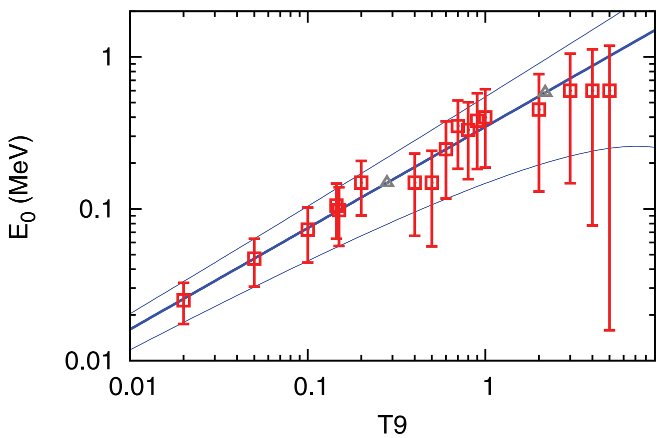

FIG. 1. (Color online) Most effective energy as a function of temperature for the reaction ^{11}B$(p,\alpha)^8$Be, where the abscissa T_9 is the temperature in units of 10^9 K. The thick solid curve shows relation (2) and the effective energy range is the region between two thin curves. The squares with error bars are the most effective energy region at the corresponding temperature. The triangles show the positions of the resonance peaks.

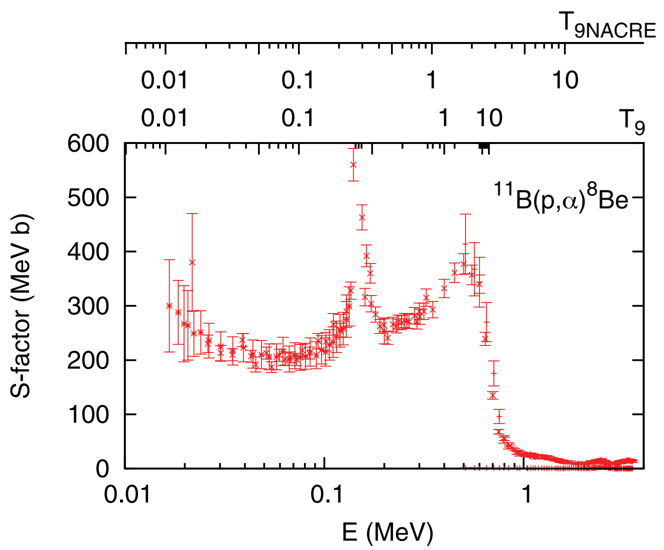

FIG. 2. (Color online) Correspondence between the plasma temperature and the most effective energy for the reaction ^{11}B$(p,\alpha)^8$Be. Experimental S-factor data are retrieved from Refs. [24] (crosses), [25] (asterisks), and [26] (bars).

the Gamow peak [4,5]. Attention was focused on the (p,γ) reactions [4] and proton-, α-, and neutron-induced reactions [5] on targets with $10 \leqslant Z \leqslant 83$ at high temperatures (of the order of 1 GK [21], i.e., $k_BT = 86$ keV), which are relevant in the advanced burning stages of massive stars and in explosive stellar environments. Our aim in this Brief Report is to attract attention on the fact that the effective energy window of the nuclear reaction driven by intense laser-pulse irradiation can deviate from the Gamow peak, because the temperature region of the explosive stellar environment exactly matches the temperatures of the laser-accelerated ions from a thin foil target by the TNSA mechanism and of the BOA mechanism [17] whereas the plasma temperature of laser-cluster fusion [10,11] is lower (30 keV at highest) than this criterion [15].

We begin with the reaction ^{11}B$(p,\alpha)^8$Be, which has two low-energy resonances at $E_r(\Gamma) = 148$ keV (5.2 keV) and at 581.3 keV (300 keV). Figure 1 is a plot of the most effective energies as a function of the plasma temperature for the reaction ^{11}B$(p,\alpha)^8$Be. The most effective energy in the presence of the low-energy resonances evaluated from condition (5) (squares) is compared with relation (2) (solid line). The most effective energy deviates clearly from the solid line at the resonant energies. The $1/e$ width given by the Gamow peak approximation is shown by the region between two thin curves, while the width given by Eq. (9) is indicated as the error bars. The effective energy window deviates clearly from the one given by the Gamow peak approximation especially around the resonances. Another point which should be made is that the effective energy window is widened as the temperature of the plasma rises, as is clearly observed in the figure. With regard to the determination of the low-energy cross section through the measurement of fusion yield in a laser-induced plasma, this means that the approximation of the effective energy by the energy is not adequate, especially in the high-temperature region.

Figure 2 shows the correspondence between T_9 and E, which is derived from relation (5), together with the experimental data of the astrophysical S factor for the reaction ^{11}B$(p,\alpha)^8$Be. For the sake of comparison, the T_9 axis from relation (2) is shown above the figure. Especially in the vicinity of the resonant peaks the change of the T_9 scale is evident.

Next, for the reaction ^{10}B$(p,\alpha)^7$Be the experimental S-factor data are shown in Fig. 3. The S factor increases as the incident energy decreases; this is interpreted as part of a known

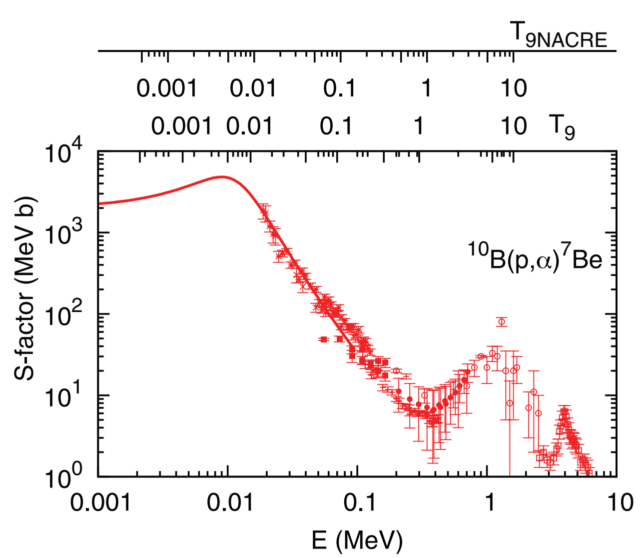

FIG. 3. (Color online) Same as Fig. 2 but for the reaction ^{10}B$(p,\alpha)^7$Be. Experimental S-factor data of have been retrieved from the NACRE compilation database, except the datasets shown by full circles [27] whose cross-section data are retrieved from the EXFOR database and are converted into S-factor data.

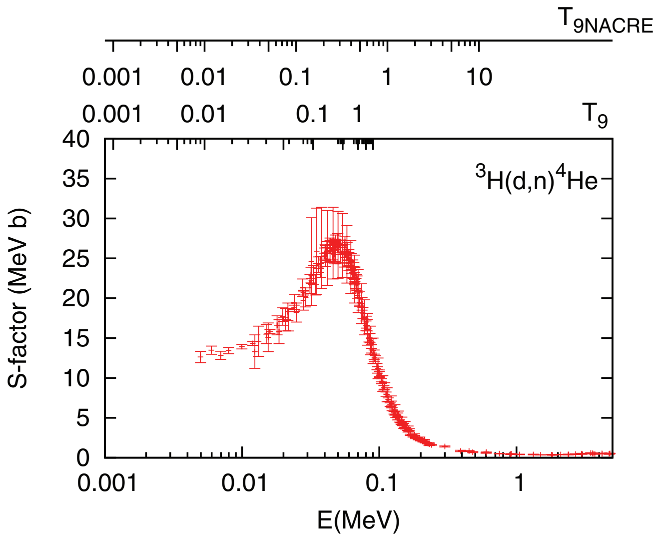

FIG. 4. (Color online) Same as Fig. 2 but for the reaction ^{3}H$(d,n)^4$He. Experimental S-factor data have been retrieved from the NACRE compilation database.

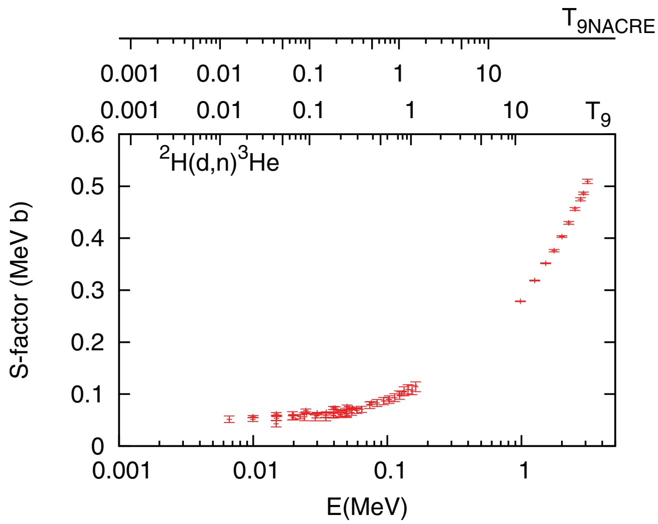

FIG. 5. (Color online) Same as Fig. 2 but for the reaction ^{2}H$(d,n)^3$He. Experimental S-factor data have been retrieved from the NACRE compilation database.

s-wave resonance at an incident energy $E = 9.1$ keV and with a width of $\Gamma = 16.$ keV. We include this resonance by using the Breit-Wigner formula. In Fig. 3 the correspondence between T_9 and E is shown, together with the experimental astrophysical S-factor data for the reaction ^{10}B$(p,\alpha)^7$Be. Compared with the T_9 axis from relation (2), which is shown above the figure, the change of the T_9 scale is evident around the s-wave resonance at $E_r = 9.1$ keV. In the higher temperature region the change of the T_9 scale is less evident, in contrast with the reaction ^{11}B$(p,\alpha)^8$Be. This is because the S factor in the ^{10}B$(p,\alpha)^7$Be reaction is almost constant in the higher temperature region. We note that the S factor is shown on a logarithmic scale only for this reaction. Hereafter the figures for the effective energy range are not shown but the deviation from the Gamow peak approximation is seen in the effective energy range, as well.

The reaction ^{3}H$(d,n)^4$He has a resonance at $E_r(\Gamma) =$ 50 keV(177 keV). In Fig. 4 the correspondence between T_9 and E is shown together with the experimental astrophysical S-factor data for this reaction. Compared with the T_9 axis from relation (2) the change of the T_9 scale is clearly observed at about the low-energy resonance.

The last example is the reaction ^{2}H$(d,n)^3$He, which is nonresonant, but the S factor of this reaction has a slow energy dependence in the energy region above 50 keV, as is shown in Fig. 5. The correspondence between T_9 and E is shown together with the experimental astrophysical S-factor data for this reaction in the same figure. Compared with T_9 axis from relation (2), which is shown above the figure, the T_9 scale shifts moderately toward higher energies, at a temperature T_9 higher than 0.3. This is attributed to the slow rise of the S factor.

In conclusion, we have evaluated the most effective energy region for charged-particle-induced reactions in a plasma environment at a given plasma temperature. The correspondence between the plasma temperature and the most effective energy range is modified, especially where the astrophysical S factor has a significant energy dependence. We have shown this modifications for four selected reactions: ^{11}B$(p,\alpha)^8$Be, ^{10}B$(p,\alpha)^7$Be, ^{3}H$(d,n)^4$He, and ^{2}H$(d,n)^3$He. In the vicinity of the resonant peaks the change of the T_9 scale is remarkable. In the presence of low-energy resonances, the resonances dominate the most effective energy. The moderate change of the T_9 scale is observed also in the nonresonant reaction, in the energy region where the incident-energy dependence of the S factor is significant. The suggested modification of the effective energy range is important not only in thermonuclear reactions at high temperature in advanced burning stages of massive stars and in explosive stellar environments but also in nuclear reactions driven by ultra-intense laser-pulse irradiation.

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