Multiphonon γ-vibrational bands in the γ-soft nucleus ^{138}Nd

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The low spin states of ^{138}Nd have been reinvestigated via the ^{124}Te(^{19}F,4n1p) reaction at a beam energy of 103 MeV. The quasi-one-phonon γ-vibrational band based on the 1013.7 keV level has been expanded and the quasi-two-phonon γ-vibrational band built on the 1842.7 keV level has been proposed. A systematic comparison supports our assignments for multiphonon γ-vibrational bands in ^{138}Nd. The characteristics of γ softness in ^{138}Nd have been discussed. Triaxial projected shell model calculations can well describe the rotational feature of the bands, but they fail to reproduce simultaneously the bandhead energies of the two γ-vibrational bands if a fixed triaxial deformation is assumed.

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The study of multiphonon γ-vibrational bands in deformed nuclei is very important for the understanding of nuclear structure. Until now, one-phonon γ-vibrational bands (1γ bands) have been found in many mass regions. Two-phonon γ-vibrational bands (2γ bands) have been identified also, such as in ^{104,105,106}Mo [1–3] and ^{103}Nb [4] in the $A = 100$ region, and in ^{154}Gd [5] and ^{166,168}Er [6,7] in the $A = 160$ region. For the $A = 130$–140 region, 1γ bands have been reported in many cases, for example, in ^{132,134,136}Nd [8–10]. But so far, no 2γ band has been identified in this mass region. Therefore, the search for multiphonon γ-vibrational bands in the $A = 130$–140 region is important for a systematic understanding of the structure evolution and for testing existing theoretical models.

The ^{138}Nd with $Z = 60$ and $N = 78$ lies in the $A = 140$ transitional region. Nuclei in this region exhibit a small quadrupole deformation with softness in the γ-deformation direction [11]. In addition, the bandhead energy of 2γ bands is comparable to the pairing gap, which may make the collectivity to be mixed easily with the two-quasiparticle states [12]. Therefore, the structural characters of the nuclei are so complex that the observation of multiphonon γ-vibrational bands in the experiment is very difficult. In the previous publications, high spin states of ^{138}Nd have been studied using heavy-ion reactions [13–15]. The low spin states of ^{138}Nd have been investigated by β^+ decay [16]. Using the quasirotational band concept of Sakai [17], the quasi-ground-state band and the quasi-one-phonon γ-vibrational band (quasi-1γ band) have been reported for ^{138}Nd [14–16]. In the present work, we reinvestigate the level structures of the lower spin states in ^{138}Nd. The quasi-1γ band is expanded and the quasi-two-phonon γ-vibrational band (quasi-2γ band) has been proposed. In addition, we show results of theoretical calculations by the triaxial projected shell model (TPSM).

The level structures of ^{138}Nd have been restudied via the ^{124}Te(^{19}F,4n1p) fusion-evaporation reaction at a beam energy of 103 MeV. At this beam energy, the high spin states of ^{138}Pm via the ^{124}Te(^{19}F,5n) and ^{139}Pm via the ^{124}Te(^{19}F,4n) reactions are also well populated. The ^{19}F beam was provided by the HI-13 tandem accelerator at the China Institute of Atomic Energy (CIAE). The enriched ^{124}Te target of 3 mg/cm^2 thick was prepared by evaporating tellurium metal powder on a 4 mg/cm^2 gold foil backing. The detector array consists of nine Compton-suppressed HPGe detectors, two planar HPGe detectors, and one clover detector. The total efficiency as a function of γ-ray energy is calibrated before and after the beam time using a ^{133}Ba and an ^{152}Eu standard source. The total photo peak efficiency of the detector array at 1 MeV γ ray is about 0.3%. The detectors were arranged at 42° (two), 90° (seven), 127° (one), 140° (two), and 150° (three) with respect to the beam direction. A total of 6.8×10^7 coincidence events were collected after the subtraction of background. A γ-γ coincidence symmetry matrix was constructed. Also an asymmetry two-dimensional angular-correlation matrix was constructed to obtain the directional correlation of oriented state (DCO) [18], from which the multipolarities of the observed γ transitions can be obtained. In the asymmetry matrix, the seven 90° detectors except for the clover detector were arranged as an axis, while the eight non-90° detectors were arranged as the other axis. The γ-γ coincidence data were analyzed with the RADWARE software package [19].

Most of the levels and the transitions at high spin states of ^{138}Nd reported in Ref. [14] have been confirmed in the present work. A partial level scheme of ^{138}Nd is shown in Fig. 1. Three bands are labeled on the top of the bands by numbers (1)–(3). The DCO ratios of the γ transitions are listed in Table I. Figure 2 shows a spectrum obtained by gating on

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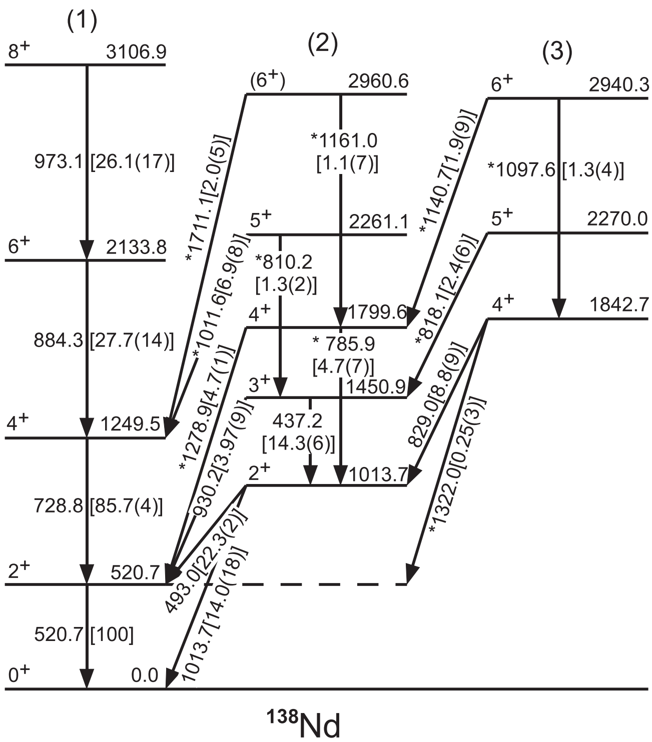

FIG. 1. Partial level scheme of ^{138}Nd deduced from the present work. The $*$ denotes γ transitions reported for the first time here by an in-beam experiment (compare, e.g., to Refs. [14,15]).

the 520.7 keV γ transition in band (1), from which one can see all the γ transitions above the 520.7 keV level in Fig. 1. The 818.1 keV transition from the 2270.0 keV level is newly identified. However, we notice that in Ref. [14], a γ ray at 495 keV is placed in coincidence with a γ ray at 819 keV from the 6568 keV level at high spins. We have checked the level scheme carefully and clearly identified the 819 keV doublet in our data analysis. For example, in Fig. 3(a), the spectrum is generated by summing the gating on 493.0, 930.2, and 437.2 keV γ transitions, and the spectrum in Fig. 3(b) is obtained by summing the gating on 278.6 and 440.1 keV γ transitions at high spin states as seen in Ref. [14]. The newly identified 818.1 keV transition from the 2270 keV level can be seen obviously in Fig. 3(a), while the 818.8 keV transition from the 6568 keV level reported in Ref. [14] is too weak to be seen in this work. Therefore, the 818.1 keV transition from the 2270 keV level can be obviously identified. Figure 3(c) shows the spectrum obtained by gating on 1278.9 keV γ transition, from which one can see 520.7, 1140.7, and 1161.0 keV γ peaks clearly. The 511.0 keV γ peak labeled by (*511.0) is caused by the annihilation of electron and positron. The unmarked

TABLE I. DCO ratios of the γ transitions in ^{138}Nd.

E_γ (keV)	Assignment	R_{DCO}	Multipolarity
437.2	$3^+ \to 2^+$	1.76(10)	$M1/E2$
493.0	$2^+ \to 2^+$	1.16(4)	$M1/E2$
728.8	$4^+ \to 2^+$	0.97(2)	$E2$
785.9	$4^+ \to 2^+$	1.19(11)	$E2$
810.2	$5^+ \to 3^+$	0.89(8)	$E2$
818.1	$5^+ \to 3^+$	1.11(11)	$E2$
829.0	$4^+ \to 2^+$	1.14(10)	$E2$
884.3	$6^+ \to 4^+$	1.08(4)	$E2$
930.2	$3^+ \to 2^+$	1.31(10)	$M1/E2$
973.1	$8^+ \to 6^+$	1.14(4)	$E2$
1011.6	$5^+ \to 4^+$	1.67(16)	$M1/E2$
1097.6	$6^+ \to 4^+$	0.92(13)	$E2$
1278.9	$4^+ \to 2^+$	1.17(7)	$E2$

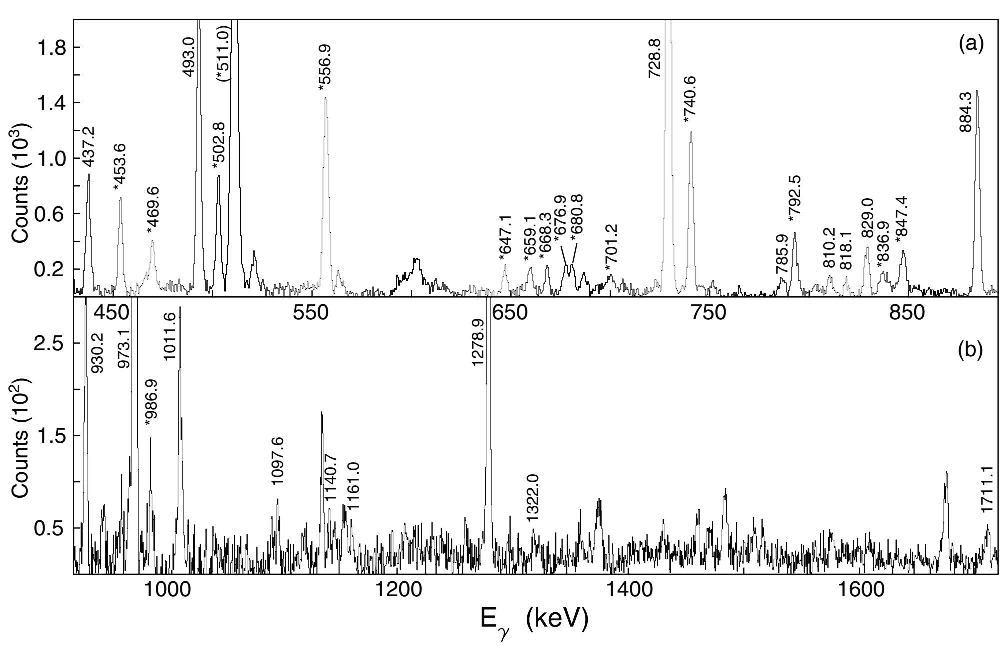

FIG. 2. Portion of γ-ray spectrum obtained by gating on 520.7 keV γ transition in ^{138}Nd. The energy intervals of γ peaks are (a) from 430 to 895 keV and (b) from 920 to 1720 keV. (The peaks marked with $*$ are the transitions at high spins that have been reported in Ref. [14]).

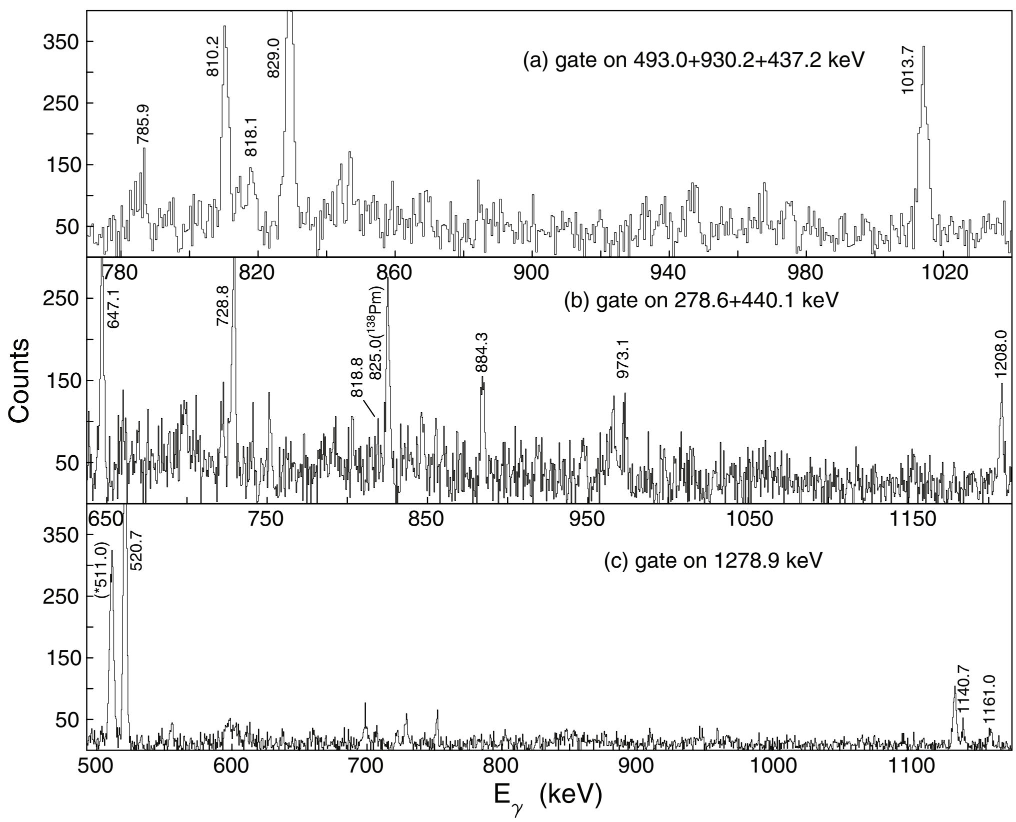

FIG. 3. Portion of γ-ray spectra obtained by (a) summing gating on 493.0, 930.2, and 437.2 keV γ transitions, (b) summing gating on 278.6 and 440.1 keV γ transitions, and (c) gating on 1278.9 keV γ transition in ^{138}Nd.

peaks belong to the disturbing peaks from other products. It is worth mentioning that except for the 818.1 and 1322.0 keV γ transitions, most of the levels and transitions in Fig. 1 were observed in the level scheme from β decay [16]. However, in Ref. [16] the spin-parity for most of the levels was not assigned and those levels were not put in the band structures.

Band (1) is a quasi-ground-state band, which has been reported in Refs. [13–15] and was confirmed in the present work. Band (2) belongs to a quasi-1γ band with the bandhead energy at 1013.7 keV. This band was observed in Ref. [16] only with two levels of 2^+ (1013.7 keV) and 3^+ (1450.9 keV) inside the band. According to the regular level spacings and the transition pattern, we assign other three levels of 4^+ (1799.6 keV), 5^+ (2261.1 keV), and 6^+ (2960.6 keV) as the new members of the quasi-1γ band. Band (3) is newly established in the present work. This band consists of 4^+ (1842.7 keV), 5^+ (2270.0 keV), and 6^+ (2940.3 keV) states. As the bandhead energy at 1842.7 keV of band (3) is less than the lowest two-quasiparticle excited state of 5^- at 1990 keV [14] (which is also observed in the present work, but is not put in Fig. 1), this band is proposed as a quasi-2γ band. In Refs. [14,15], the 4^+ state at 1843 keV was put in band (2). However, according to the systematical comparisons, the observed level energy of 4^+ state in the 2γ band in other nuclei is generally higher than that in the 1γ band, so we assign the 4^+ (1842.7 keV) state as the bandhead of the quasi-2γ band in ^{138}Nd.

The total Routhian surface (TRS) has been calculated in Ref. [14], showing that ^{138}Nd is γ soft with $\beta_2 \approx 0.15$ and $\gamma = -30^\circ$ at low frequency. We also carried out the calculations of the cranked shell model (CSM) [20–22]. The minima in the TRS [23] is obtained at $\beta_2 = 0.156$ and $\gamma = -29.27^\circ$ for $\hbar\omega = 0$ MeV in ^{138}Nd, which is in accord with that in Ref. [14]. It shows that the ^{138}Nd has small quadrupole deformation and the largest triaxial deformation values. However, as discussed for ^{110}Mo in [24], the exact location of the energy minima is not important in characterizing the collective level properties of ^{138}Nd, because an overall profile in the TRS spreads over the γ degrees of freedom. Consequently, the level structure of ^{138}Nd is ascribed to its γ-soft nature rather than the rigid deformation of any kind.

To understand the γ-soft character in ^{138}Nd, the quantity $E_S/E(2_1^+)$ is adopted [24], where $E(2_1^+)$ and E_S denote the 2_1^+ level energy (ground-state band) and the energy difference between the 2_2^+ (1γ band) and 4_1^+ (ground-state band), respectively, to analyze the structural evolution in different nuclei. By using the γ-soft concept defined in Ref. [24], the systematical comparisons for the empirical values of $E_S/E(2_1^+)$ in ^{132,134,136}Nd [8–10] and ^{138}Nd (present work) isotopes with those in ^{128,130,132,134}Ba [25–28] and ^{130,132,134,136}Ce [8,29,30] isotopes are plotted in Fig. 4. It is shown that as the neutron number increases, the $E_S/E(2_1^+)$ ratio decreases in each isotope chain. This shows the structural evolution from a nearly axial rotor with a small-amplitude

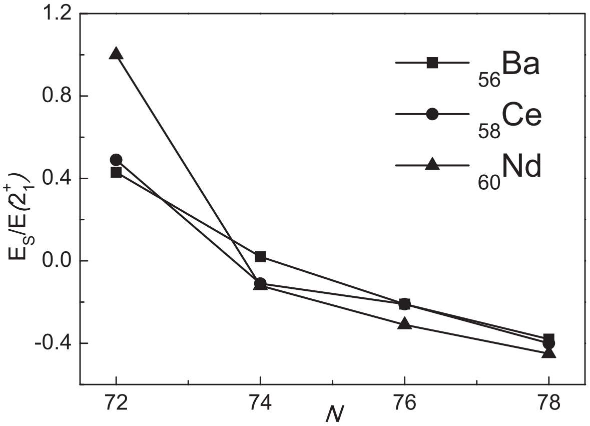

FIG. 4. Empirical values of the $E_S/E(2_1^+)$ in some even-even isotopes of Ba, Ce, and Nd.

γ vibration to a large-amplitude γ-soft dynamics. For the Nd isotopes, the empirical values of $E_S/E(2_1^+)$ are 1.00 in ^{132}Nd, −0.12 in ^{134}Nd, −0.31 in ^{136}Nd, and −0.45 in ^{138}Nd. It indicates that the ^{138}Nd has the maximum γ softness, which agrees with the above TRS calculation.

As a feature of multiphonon γ vibration, the ground-state band and the 1γ- and 2γ bands are expected to exhibit similar inertia parameters. The inertia parameter A can be obtained from the second-order rotational energy formula [3]

$$E(I, K) = E_K + A[I(I+1) - K^2] + B[I(I+1) - K^2]^2. \tag{1}$$

The calculated values of parameter A from the present work are 54.6, 52.3, and 55.9 keV for bands (1)–(3) in ^{138}Nd, respectively. The closeness of these values supports the assignments of the multiphonon quasi-γ-vibrational bands in ^{138}Nd.

To understand the band structure of ^{138}Nd in a more quantitative way, we also perform TPSM [31,32] calculations and compare the theoretical results with data in detail. In the TPSM calculation for ^{138}Nd, the axial deformation parameter is set to be $\varepsilon = 0.17$, which was employed in the previous calculation for this nucleus [33]. The triaxial deformation ε' is an adjustable parameter. As we shall see later, its undefined value reflects the γ-soft nature of this nucleus.

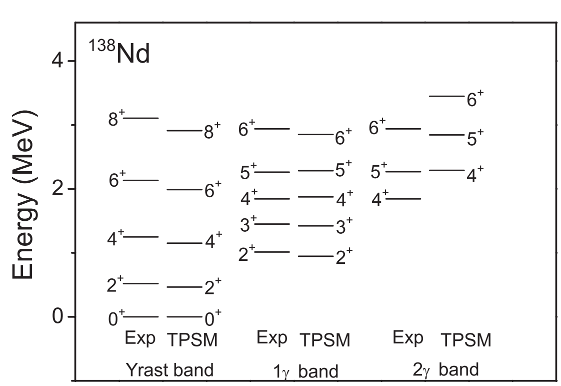

FIG. 5. Comparison of experimental and the calculated band structures for ^{138}Nd.

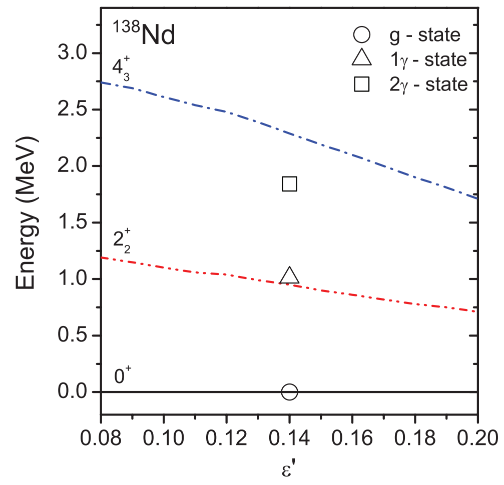

FIG. 6. (Color online) Calculated bandhead energies of the ground, 1γ, and 2γ bands as functions of triaxiality ε'.

We first show the results for the lowest three bands obtained after TPSM diagonalization for each angular momentum in Fig. 5. The calculation is carried out with fixed deformations $\varepsilon = 0.17$ and $\varepsilon' = 0.14$. It was discussed [32] that these bands have the main component from the projected qp vacuum, and therefore, are collective bands in the low spin regime. Thus, the comparison tends to support the conclusion that the experimentally observed band (3) is mainly of a quasi-two-phonon in nature. It can be seen from Fig. 5 that an excellent reproduction of the known data of the yrast and 1γ band has been achieved. The calculation does not give a good agreement with the data for the 2γ bandhead position, although the rotational character (i.e., the moment of inertia) of this band can be described.

This posts a challenge to applying the current theory for calculations with a fixed set of axial and triaxial deformation parameters. For this γ-soft nucleus, it seems difficult for the model to reproduce simultaneously the 1γ and 2γ bands with one fixed ε' parameter. Figure 6 shows the calculated relative positions of the bandheads of the ground, 1γ, and 2γ bands as functions of triaxiality ε'. It is seen that as ε' increases, both 1γ and 2γ states go down in energy. However, it is not possible that the 1γ and 2γ states are reproduced simultaneously with a fixed value of ε'. In an improved description for the present γ-soft nucleus, one should start from a very general ansatz for a trial wave function that is a continuous superposition of states with different ε' parameters. Technically one must be able to compute the overlap matrix elements between different

TABLE II. Systematic comparisons of the bandhead energies of the 2γ and 1γ bands in different mass regions.

Nucleus	$E_{1\gamma}$ (keV)	$E_{2\gamma}$ (keV)	$E_{2\gamma}/E_{1\gamma}$
^{104}Mo	812.2	1583.4	1.95
^{106}Mo	710.4	1434.6	2.02
^{138}Nd	1013.7	1842.7	1.82
^{154}Gd	996.3	1645.8	1.65
^{168}Er	821.2	2055.9	2.50

ε' states. This method is in line with the well-known generator coordinate method [34], and it should be considered for future TPSM extensions.

Through making a systematic comparison for the levels of the ground-state, quasi-1γ, and quasi-2γ bands in ^{138}Nd with the corresponding bands in ^{104,106}Mo, ^{154}Gd, and ^{168}Er, one can see that they have similar structural characters. The comparison of the ratio between bandhead energies of the quasi-2γ and quasi-1γ bands is shown in Table II. The ^{104,106}Mo nuclei exhibit a harmonic characteristic with the ratios near 2.0, whereas ^{138}Nd, ^{154}Gd, and ^{168}Er exhibit an anharmonic characteristic with the ratios of 1.82, 1.65, and 2.50, respectively. The above systematic discussion for the examples taken from different mass regions supports the assignments of multiphonon γ-vibrational bands in ^{138}Nd.

In summary, the level structure at lower spin states in ^{138}Nd has been reinvestigated. The quasi-1γ band has been expanded and the quasi-2γ band has been proposed. Such a 2γ band is found for the first time in the $A = 130$–140 mass region. The systematic analysis supports the assignments for the multiphonon γ-vibrational bands in ^{138}Nd. From the TRS calculations and the analysis for the level structures, we have suggested that ^{138}Nd exhibits a strong γ softness and anharmonicity in the vibrational bands. Triaxial projected shell model calculations can well describe the rotational feature of the bands, but they fail to reproduce the bandhead energies of γ-vibrational bands if a fixed triaxial deformation is assumed. An improved theoretical treatment for γ-soft nuclei is suggested.

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