Neutrinoless double-positron decay and positron-emitting electron capture in the interacting boson model

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Neutrinoless double-β decay is of fundamental importance for determining the neutrino mass. Although double electron decay is the most promising mode, in very recent years interest in double positron decay, positron emitting electron capture, and double electron capture has been renewed. We present here results of a calculation of nuclear matrix elements for neutrinoless double-β^+ decay and positron emitting electron capture within the framework of the microscopic interacting boson model (IBM-2) for ^{58}Ni, ^{64}Zn, ^{78}Kr, ^{96}Ru, ^{106}Cd, ^{124}Xe, ^{130}Ba, and ^{136}Ce decay. By combining these with a calculation of phase space factors we calculate expected half-lives.

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Introduction. Double-β decay is a process in which a nucleus (A, Z) decays to another nucleus $(A, Z \pm 2)$ by emitting two electrons or positrons and, usually, other light particles:

$$(A, Z) \rightarrow (A, Z \pm 2) + 2e^{\mp} + \text{anything}. \tag{1}$$

Double-β decay can be classified in various modes according to the various types of particles emitted in the decay. The processes where two neutrinos are emitted are predicted by the standard model, and two neutrino double electron ($2\nu\beta^-\beta^-$) decay has been observed in several nuclei. For processes not allowed by the standard model, i.e., the neutrinoless double-β decay ($0\nu\beta\beta$), neutrinoless positron emitting electron capture ($0\nu EC\beta$), and neutrinoless double electron capture ($0\nu ECEC$), the half-life can be factorized as

$$\left[\tau_{1/2}^{0\nu}\right]^{-1} = G_{0\nu}|M_{0\nu}|^2\,|f(m_i, U_{ei})|^2\,, \tag{2}$$

where $G_{0\nu}$ is a phase space factor, $M_{0\nu}$ is the nuclear matrix element, and $f(m_i, U_{ei})$ contains physics beyond the standard model through the masses m_i and mixing matrix elements U_{ei} of neutrino species. For all processes, two crucial ingredients are the phase space factors (PSFs) and the nuclear matrix elements (NMEs). Recently, we have initiated a program for the evaluation of both quantities and presented results for $\beta^-\beta^-$ decay [1–7]. This is the most promising mode for the possible detection of neutrinoless double-β decay and thus of a measurement of the absolute neutrino mass scale. However, in very recent years, interest in the double positron ($\beta^+\beta^+$) decay, positron emitting electron capture ($EC\beta^+$), and $ECEC$ has been renewed. This is due to the fact that positron emitting processes have interesting signatures that could be detected experimentally [8]. In a previous article [9] we initiated a systematic study of $\beta^+\beta^+$ decay, $EC\beta^+$, and $ECEC$ processes and presented a calculation of PSFs for $2\nu\beta^+\beta^+$, $2\nu EC\beta^+$, $2\nu ECEC$, and $0\nu\beta^+\beta^+$, $0\nu EC\beta^+$. The process $0\nu ECEC$ cannot occur to the order of approximation used in Ref. [9], since the emission of additional particles, $\gamma\gamma$ or others, is needed to conserve energy and momentum. In this Brief Report, we focus on the calculation of neutrinoless decay NMEs, which are common to all three modes, and half-life predictions for $0\nu\beta^+\beta^+$ and $0\nu EC\beta^+$ modes. Results of our calculations are reported for nuclei listed in Table I.

Nuclear matrix elements. The theory of $0\nu\beta\beta$ decay was first formulated by Furry [14] and further developed by Primakoff and Rosen [15], Molina and Pascual [16], Doi *et al.* [17], Haxton and Stephenson [18], and, more recently, by Tomoda [19] and Šimkovic *et al.* [20]. All these formulations often differ by factors of 2, by the number of terms retained in the nonrelativistic expansion of the current, and by their contribution. In order to have a standard set of calculations to be compared with the quasiparticle random-phase

TABLE I. Double-β decays considered in this Brief Report, the mass differences between neutral mother and daughter atoms, $M(A, Z) - M(A, Z - 2)$, and their isotopic abundances.

Transition	$M(A, Z) - M(A, Z - 2)$ (keV)[a]	$P(\%)$
$^{58}_{28}$Ni$_{30}$ $\rightarrow ^{58}_{26}$Fe$_{32}$	1926.3 ± 0.3	68.077 ± 0.009
$^{64}_{30}$Zn$_{34}$ $\rightarrow ^{64}_{28}$Ni$_{36}$	1094.8 ± 0.7	49.17 ± 0.75
$^{78}_{36}$Kr$_{42}$ $\rightarrow ^{78}_{34}$Se$_{44}$	2846.3 ± 0.7	0.355 ± 0.003
$^{96}_{44}$Ru$_{52}$ $\rightarrow ^{96}_{42}$Mo$_{54}$	2714.51 ± 0.13[b]	5.54 ± 0.14
$^{106}_{48}$Cd$_{58}$ $\rightarrow ^{106}_{46}$Pd$_{60}$	2775.39 ± 0.10[c]	1.25 ± 0.06
$^{124}_{54}$Xe$_{70}$ $\rightarrow ^{124}_{52}$Te$_{72}$	2865.4 ± 2.2	0.0952 ± 0.0003
$^{130}_{56}$Ba$_{74}$ $\rightarrow ^{130}_{54}$Xe$_{76}$	2619 ± 3	0.106 ± 0.001
$^{136}_{58}$Ce$_{78}$ $\rightarrow ^{136}_{56}$Ba$_{80}$	2378.53 ± 0.27[d]	0.185 ± 0.002

[a]Reference [10].
[b]Reference [11].
[c]Reference [12].
[d]Reference [13].

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TABLE II. Hamiltonian parameters employed in the IBM-2 calculation of the final wave functions along with their references.

Nucleus	ϵ_{d_ν}	ϵ_{d_π}	κ	χ_ν	χ_π	ξ_1	ξ_2	ξ_3	$c_\nu^{(0)}$	$c_\nu^{(2)}$	$c_\nu^{(4)}$	$c_\pi^{(0)}$	$c_\pi^{(2)}$	$c_\pi^{(4)}$
^{58}Ni[a]	1.454													
^{58}Fe[a]	0.98	0.98	−0.26	0.00	−0.40	0.80	0.80	0.80						
^{64}Zn [30]	1.20	1.20	−0.22	−0.25	−0.75	−0.18	0.24	−0.18	−0.30	−0.50	0.30	−0.30	−0.50	0.30
^{64}Ni[a]	1.346									−0.415	0.082			
^{78}Kr [31]	0.96	0.96	−0.18	−0.495	−1.127	−0.10		−0.10						
^{78}Se [32]	0.99	0.99	−0.21	0.71	−0.90			−0.10						
^{96}Ru[a]	1.08	1.08	−0.21	0.80	0.40	0.25	0.25	0.25	0.30	0.10	−0.50			
^{96}Mo [33]	0.73	1.10	−0.09	−1.20	0.40	−0.10	0.10	−0.10	−0.50	0.10				
^{106}Cd [34]	1.05	1.05	−0.325	1.25	0.00	−0.18	0.24	−0.18	0.20	0.15	0.00			
^{106}Pd [35]	0.760	0.844	−0.160	−0.22	−0.30	0.20	0.05	0.00	−0.45	−0.20	0.01			
^{124}Xe [36]	0.70	0.70	−0.14	0.00	−0.80	−0.18	0.24	−0.18	0.05	−0.16				
^{124}Te [34]	0.82	0.82	−0.15	0.00	−1.20	−0.18	0.24	−0.18	0.10					
^{130}Ba [36]	0.70	0.70	−0.175	0.32	−0.90	−0.18	0.24	−0.18	0.26					
^{130}Xe [36]	0.76	0.76	−0.19	0.50	−0.80	−0.18	0.24	−0.18	0.30	0.22				
^{136}Ce [36]	0.90	0.90	−0.21	0.79	−1.00	−0.18	0.24	−0.18	0.26	−0.11				
^{136}Ba [36]	1.03	1.03	−0.23	1.00	−0.90	−0.18	0.24	−0.18	0.30	0.10				

[a]Parameters fitted to reproduce the spectroscopic data of the low lying energy states.

approximation (QRPA) and the interacting shell model, we adopt in this Brief Report the formulation of Šimkovic *et al.* [20]. A detailed discussion of involved operators can also be found in Ref. [4].

We consider the decay of a nucleus $^{A}_{Z}\mathrm{X}_N$ into a nucleus $^{\;\;A}_{Z-2}\mathrm{Y}_{N+2}$. An example is shown in Fig. 1. If the decay proceeds through an S-wave, with two leptons in the final state, we cannot form an angular momentum greater than 1. We therefore calculate, in this Brief Report, only $0\nu\beta\beta$ matrix elements to final 0^+ states, to the ground state 0_1^+, for which, in a previous article [9] we have calculated the phase space factors, and to the first excited state 0_2^+.

In order to evaluate the matrix elements we make use of the microscopic interacting boson model (IBM-2) [21]. The method of evaluation is discussed in detail in Ref. [1] for $\beta^-\beta^-$ decay. For $\beta^+\beta^+$ decay and $EC\beta^+$ the same method applies except for the interchange $\pi \rightarrow \nu$ in Eq. (5) of Ref. [1]

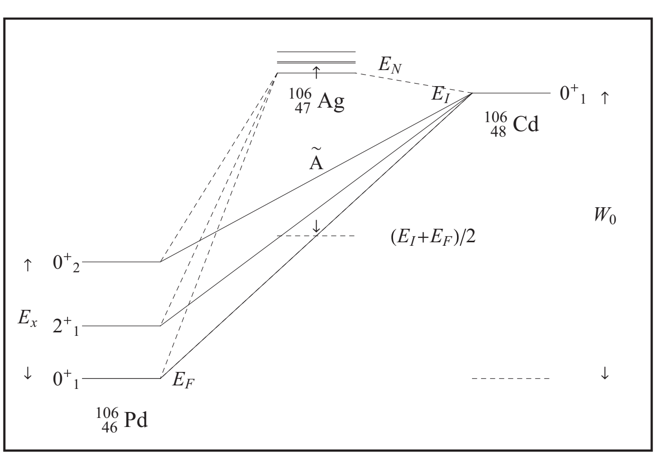

FIG. 1. The decay $^{106}_{48}\mathrm{Cd}_{58} \rightarrow ^{106}_{46}\mathrm{Pd}_{60}$, an example of double-$\beta^+$ decay.

and in the mapped boson operators of Eq. (18) of Ref. [1]. The matrix elements of the mapped operators are evaluated with realistic wave functions, taken either from the literature, when available, or obtained from a fit to the observed energies and other properties like reduced transition probabilities ($B(E2)$ and $B(M1)$ values), quadrupole moments, magnetic moments, etc. A detailed description of the IBM-2 Hamiltonian is given in Refs. [21,29]. For most nuclei, the Hamiltonian parameters are taken from the literature [30–36]. The values of the Hamiltonian parameters, as well as the references from which they were taken, are given in Table II. The quality of the description can be seen from these references and ranges from very good to excellent.

Here, we present our calculated NMEs for the decays of Table I. The NMEs depend on many assumptions, in particular, on the treatment of the short-range correlations (SRCs). In Table III, we show the results of our calculation of the matrix elements to the ground state, 0_1^+, and to the first excited state, 0_2^+, using the Miller-Spencer (MS) parametrization of SRCs, and broken down into Gamow-Teller (GT), Fermi (F), and tensor (T) contributions and their sum as

$$M_{0\nu} = g_A^2 M^{(0\nu)}, \quad M^{(0\nu)} = M_{GT}^{(0\nu)} - \left(\frac{g_V}{g_A}\right)^2 M_F^{(0\nu)} + M_T^{(0\nu)}. \tag{3}$$

We note that we have two classes of nuclei, those in which protons and neutrons occupy the same major shell ($A = 64, 78, 124, 128, 130, 136$) and those in which they occupy different major shells ($A = 58, 96, 106$). The magnitude of the Fermi matrix element, which is related to the overlap of the proton and neutron wave functions, is therefore different in these two classes of nuclei, being large in the former case and small in the latter. This implies a considerable amount of isospin violation for nuclei in the first class. This problem has been discussed in detail in Ref. [4] and will form a subject of

TABLE III. IBM-2 nuclear matrix elements $M^{(0\nu)}$ (dimensionless) for neutrinoless $\beta^+\beta^+$, $EC\beta^+$, and $ECEC$ decays with Jastrow MS SRCs and $g_V/g_A = 1/1.269$.

Nucleus	0_1^+				0_2^+			
	$M_{GT}^{(0\nu)}$	$M_F^{(0\nu)}$	$M_T^{(0\nu)}$	$M^{(0\nu)}$	$M_{GT}^{(0\nu)}$	$M_F^{(0\nu)}$	$M_T^{(0\nu)}$	$M^{(0\nu)}$
^{58}Ni	2.072	−0.152	0.144	2.310	2.042	−0.153	0.101	2.237
^{64}Zn	4.762	−2.449	−0.156	6.127	0.633	−0.360	−0.019	0.837
^{78}Kr	3.384	−2.146	−0.238	4.478	0.771	−0.479	−0.055	1.014
^{96}Ru	2.204	−0.269	0.112	2.483	0.036	−0.012	0.001	0.045
^{106}Cd	2.757	−0.255	0.191	3.106	1.395	−0.110	0.074	1.537
^{124}Xe	3.967	−2.224	−0.192	5.156	0.647	−0.359	−0.032	0.839
^{130}Ba	3.911	−2.108	−0.176	5.043	0.285	−0.152	−0.014	0.366
^{136}Ce	3.815	−2.007	−0.161	4.901	0.318	−0.167	−0.014	0.408

subsequent investigation. It is common to most calculations of NMEs and has been addressed recently within the framework of the QRPA in Refs. [22,23]. Here we take it into account by assigning a large error to the calculation of the Fermi matrix elements. In the same Ref. [4] it is also shown that the NMEs depend on the SRCs, and that use of Argonne and CD-Bonn SRCs increase the NMEs by a factor of 1.1 to 1.2. The same situation occurs for $\beta^+\beta^+$ decay. In order to take into account the sensitivity of the calculation to parameter changes, model assumptions, and operator assumptions [4], we list in Table IV IBM-2 NMEs with an estimate of the error. The values of the 0_1^+ matrix elements vary between 2.3 to 6.1, the matrix element for the ^{64}Zn $\rightarrow$ ^{64}Ni transition being notably the largest. They are therefore of the same order of magnitude as the nuclear matrix elements for $\beta^-\beta^-$ decay, 2.0 to 5.4.

In the same Table IV we also compare our results with the available QRPA calculations from Ref. [24] with the addition of some more recent calculations from Refs. [25,26]. The QRPA [24] NMEs are calculated taking into account GT and F contributions, and using the value $g_A = 1.25$. As in the case of $\beta^-\beta^-$ decay, QRPA tend to give larger values than IBM-2, and these two methods seem to be in a rather good correspondence with each other.

TABLE IV. IBM-2 matrix elements with MS SRCs and error estimates compared with available QRPA calculations.

Decay	0_1^+			0_2^+	
	IBM-2	QRPA[a]		IBM-2	QRPA
^{58}Ni	2.31(37)	1.55		2.24(36)	
^{64}Zn	6.13(116)			0.84(16)	
^{78}Kr	4.48(85)	4.19		1.01(19)	
^{96}Ru	2.48(40)	3.25	3.22–5.83[b]	0.05(1)	1.28–2.26[b]
^{106}Cd	3.11(50)	4.12	5.94–9.08[c]	1.54(25)	0.66–0.91[c]
^{124}Xe	5.16(98)	4.78		0.84(16)	
^{130}Ba	5.04(96)	4.98		0.37(7)	
^{136}Ce	4.90(93)	3.09		0.41(8)	

[a]Reference [24].
[b]Reference [25].
[c]Reference [26].

Predicted half-lives for $0_1^+ \rightarrow 0_1^+$ transitions. The calculation of nuclear matrix elements in IBM-2 can now be combined with the phase space factors calculated in Ref. [9] to produce our final results for half-lives for light neutrino exchange in Table V and Fig. 2. The half-lives are calculated using the formula

$$\left[\tau_{1/2}^{0\nu}\right]^{-1} = G_{0\nu}^{i}|M_{0\nu}|^2\left|\frac{\langle m_\nu\rangle}{m_e}\right|^2, \tag{4}$$

where $i = \beta^+\beta^+, EC\beta^+$. The values in Table V and Fig. 2 are for $\langle m_\nu\rangle = 1$ eV. They scale with $\langle m_\nu\rangle^2$ for other values.

Comparing the half-life predictions listed in Table V to the ones reported in Ref. [4] for $0\nu\beta^-\beta^-$ we can see that values reported here are much larger. This is due to the fact that in the cases studied here the available kinetic energy is much smaller compared to $\beta^-\beta^-$ decay. Furthermore, the Coulomb repulsion on positrons from the nucleus gives a smaller decay rate. As concluded also in Refs. [27,28], the ^{124}Xe $0\nu EC\beta^+$decay is expected to have the shortest half-life.

TABLE V. Calculated half-lives in IBM-2 MS SRCs for neutrinoless double-β^+ decays and positron emitting electron captures for $\langle m_\nu\rangle = 1$ eV and $g_A = 1.269$.

Nucleus	$T_{1/2}(10^{27}$yr)	
	$\beta^+\beta^+$	$EC\beta^+$
^{58}Ni		213
^{64}Zn		52.9
^{78}Kr	2.01	0.79
^{96}Ru	19.3	1.70
^{106}Cd	10.8	0.80
^{124}Xe	3.32	0.19
^{130}Ba	15.4	0.23
^{136}Ce	174	0.27

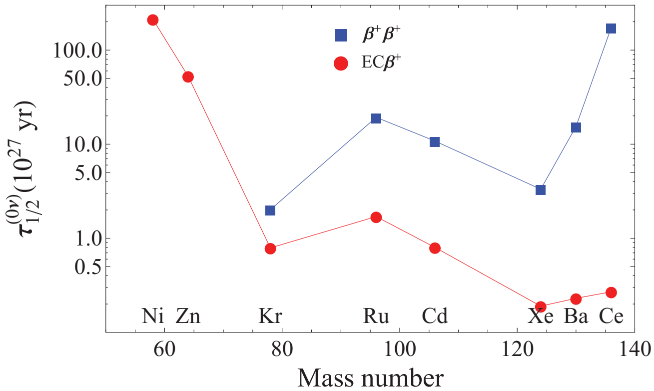

FIG. 2. (Color online) Expected half-lives for $\langle m_\nu \rangle = 1$ eV, $g_A = 1.269$. The figure is in semilogarithmic scale.

In case of the $0\nu ECEC$, the available kinetic energy is larger and Coulomb repulsion does not play a role. However, this decay mode cannot occur to the order of approximation we are considering, since it must be accompanied by the emission of one or two particles in order to conserve energy, momentum, and angular momentum.

Conclusions. In this Brief Report we have presented an evaluation of nuclear matrix elements in $0\nu\beta^+\beta^+$, $0\nu EC\beta^+$, and $0\nu ECEC$ within the framework of IBM-2 in the closure approximation. The closure approximation is expected to be good for these decays since the virtual neutrino momentum is of the order of 100 MeV/c and thus much larger than the scale of nuclear excitations. By using these matrix elements and the phase space factors of Ref. [9], we have calculated the expected $0\nu\beta^+\beta^+$ and $0\nu EC\beta^+$ half-lives in all nuclei of interest with $g_A = 1.269$ and $g_V = 1$, given in Table V and Fig. 2.

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