Effect of spin-spin interactions on nucleon-nucleus scattering

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We report on a model for determining spin-spin terms in the optical potential for elastic scattering of a nucleon from a target with nonzero spin. These terms are constructed microscopically by folding a realistic effective nucleon-nucleon interaction, which includes both direct and exchange parts, over the ground state of the target nucleus wave function to generate a nucleon-nucleus potential. The resulting spin-spin tensors are evaluated within DWBA. This general model has been used for the specific calculation of polarization transfer coefficient D_{NN}, which is unity if there are no spin-spin terms in the optical potential. We have calculated D_{NN} for the elastic scattering of 200 MeV protons from ^{10}B, for which relevant experimental data exist. In our model, the folding formalism leads to both local and nonlocal spin-spin tensor interactions, with higher order couplings in the projectile and target spin that have not been explicitly considered before. We have found that the spin-spin interactions derived from this model have a significant effect on the deviation of D_{NN} from unity, particularly those derived from the tensor exchange term in the NN interaction. The calculation of D_{NN} has also been shown to be particularly sensitive to the parameters of the model used to describe the nuclear structure.

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I. INTRODUCTION

Optical models are extensively used to examine nuclear structure, but terms in the optical potential that depend on the spin of the target nucleus have not been fully explored. As radioactive beam facilities produce more exotic nuclear species, there is a need to determine accurate optical models that describe the interactions between nucleons and target nuclei with nonzero spin. In many scattering experiments random orientations of the target and projectile spin mean spin-spin effects are averaged out. However, experimental evidence for the dependence of proton-nucleus elastic scattering on the spin of the target nucleus has been sought through the measurement of the polarization observable, D_{NN}, [1]. There is a vital need for nuclear reaction theory calculations to make predictions of this and other polarization observables relevant to the proposed experiments in the next generation of radioactive beam facilities. In order to meet this need, we re-examine the microscopic foundation of the nucleon optical model for target nuclei with nonzero spin.

The polarization transfer coefficient, D_{NN} (or D_{yy} as it is sometimes called, where the direction of the y-axis is normal to the scattering plane), measures the extent to which the y-component of the incident proton polarization is transferred to the outgoing proton polarization. It is a very useful observable when investigating the existence of spin-spin interactions. This is because D_{NN} is rigorously unity when the nucleon-nucleus optical potential contains no spin-spin terms. Only terms in the optical potential which depend on the orientation of the nucleus target-spin, $\boldsymbol{I}$, lead to a deviation of D_{NN} from unity. It therefore provides a unique test of the spin dependence in the optical potential. The observable, D_{NN}, is related to the nucleon-nucleus scattering amplitude F and the y-component σ_{0y}, of the projectile nucleon spin operator $\boldsymbol{\sigma}_0$, by the following relation [3]:

$$D_{NN} = \frac{\mathrm{Tr}[F\sigma_{0y}F^{\dagger}\sigma_{0y}]}{\mathrm{Tr}[FF^{\dagger}]}, \tag{1}$$

where the trace is taken over the projectile and target spin projections.

The experimental measurement of D_{NN} analyzed in this work is detailed in Betker *et al.* [2]. They measure D_{NN} for 200 MeV protons elastically scattering from ^{10}B at the Indiana University Cyclotron Facility (IUCF). They observed a significant deviation of D_{NN} from unity at large angles and attempted to explain this result using a coupled-channel distorted wave impulse approximation (DWIA) calculation. Their model does well for cross section and analyzing power data and for D_{NN} at forward angles where its deviation from unity is small. However, at larger angles it deviated from unity by only about 5%—an order of magnitude smaller than the observed data. Here we address this issue through a microscopic folding model to establish if the different types of spin-spin interactions have a significant effect on D_{NN}.

Some aspects of this work have already been reported in [4], which focuses specifically on the role of spin-spin exchange terms arising from the nucleon-nucleon tensor interaction. As we noted in this earlier work, we do not attempt to give a complete microscopic description of the proton optical potential as very extensive fully microscopic calculations have already been performed [5] for proton scattering by ^{10}B at 197 MeV. However, such calculations still fail to give the large deviation of D_{NN} from unity at large angles observed experimentally [2]. Our goal is to explicitly consider the terms in the optical potential known to cause the deviation of D_{NN}

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from unity, spin-spin interactions, and to treat the remaining components of the optical potential nonmicroscopically. In this paper we present our model in more detail and examine our findings in greater depth, including the influence of exchange terms on D_{NN} at large angles and the role of nuclear structure in the calculation. The possibility of using other polarization observables to probe spin-spin interactions is discussed briefly.

In Sec. II we introduce spin-spin interactions and Sec. III describe the calculation of the scattering amplitude. In Sec. IV we describe the nucleon-nucleon interaction, (detailing the direct and exchange components) and in Sec. V the folding model and how it generates the spin-spin interactions used in the final calculations, shown in Sec. VI.

II. SPIN-SPIN INTERACTIONS

A. Previous work

The two-body nucleon-nucleon (NN) interaction contains a component which depends on the relative orientation of the spins of the two nucleons. For a nucleus with spin, a similar component in the nucleon-nucleus interaction may also be expected. Inclusion of terms in the optical potential that depend on the spin operator, $\boldsymbol{I}$, of the target nucleus, were first proposed by Feshbach over 50 years ago [6,7]. Initially, many attempts were made to describe these spin-spin interactions using a central, or so-called 'spherical', spin-spin term of the form, $-V_{SS}F_0(R)\boldsymbol{\sigma}_0 \cdot \boldsymbol{I}$, where R is the projectile target separation, V_{SS} is the strength of the potential, $F_0(R)$ is the form factor, and $\boldsymbol{\sigma}_0$ is the Pauli spin operator for the scattered nucleon. Some early work [8–11] used a phenomenological optical model potential in which the spin-spin interactions were added and determined by a fit to experimental data. Others evaluated the spherical spin-spin potential microscopically from appropriate terms in the NN interaction [1,12–15].

Later works [16–18] included a rank-2 'tensor', spin-spin term of the form, $-V_{ST}F_T(R)[3(\boldsymbol{\sigma}_0 \cdot \hat{\boldsymbol{R}})(\boldsymbol{I} \cdot \hat{\boldsymbol{R}}) - \boldsymbol{\sigma}_0 \cdot \boldsymbol{I}]$, which is analogous to the tensor force in the NN interaction. In each case, the strength of the tensor spin-spin potential was estimated phenomenologically. Most of these works were restricted by the limited amount of experimental data available for observables sensitive to these spin-spin terms and all found the strengths of these potentials to be small.

Two main types of optical model calculations, which include these spin-spin terms, have been carried out. The first of these treats the spin-spin potentials to first order in the framework of distorted wave Born approximation (DWBA) [8,12,16–18] and, more recently, in [19,20]. These take advantage of the supposed weakness of the spin-spin interactions relative to the conventional optical potential terms such as central and spin-orbit. The second type are coupled channel calculations in which the scattering amplitude is handled in the channel spin framework [1,9,11] and, more recently, in [21,22].

A good review of the early work on spin-spin interactions has been written by Sherif [23]. The author ends by stating that "improved microscopic calculations of the spin-spin potentials, particularly the tensor term, will certainly be of great help in tracking down this ever evasive part of the nuclear optical potential".

The work of McAbee [24,25] first attempted to derive a generalised spin-spin operator of the form shown in the next section. Microscopic calculations of the spin-spin potentials were performed using a valence-nucleon model for the target nucleus and an effective NN interaction. The formalism was developed for the folding of the direct central, spin-orbit, and tensor two body interaction terms over the nuclear wave function. A central exchange term was also included. The spin-spin amplitudes were calculated in DWBA, although not all contributions were included in the final calculation of the observables and no attempts were made to calculate exchange terms for the spin-orbit or tensor NN interactions. McAbee found that in many cases the spin-spin interactions should not be neglected, especially for high spin nuclei, as will be produced in the next generation of radioactive beam facilities.

In this work, due to the 3^+ ground state of ^{10}B, new spin-spin interactions that have higher order couplings of the target and projectile spins are included in our calculations. Our spin-spin potentials therefore go beyond the work of McAbee and other authors described here, as we include higher order spin-spin tensors explicitly for the first time.

B. Spin-spin operator

In this work the spin-spin operator will be denoted by $\boldsymbol{S}_{k_I k}$, where k_I is the rank of the spin operator constructed from the components of the target spin $\boldsymbol{I}$, and k is the rank of the spherical harmonic $Y_k(\hat{\boldsymbol{R}})$ of the unit vector of the projectile-target separation. The spherical spin-spin term (discussed in the previous section) contains the operator, $\boldsymbol{S}_{10}$, which is proportional to $\boldsymbol{\sigma}_0 \cdot \boldsymbol{I}$, while the second-rank tensor term, $\boldsymbol{S}_{12}$, is proportional to $[3(\boldsymbol{\sigma}_0 \cdot \hat{\boldsymbol{R}})(\boldsymbol{I} \cdot \hat{\boldsymbol{R}}) - \boldsymbol{\sigma}_0 \cdot \boldsymbol{I}]$. If the projectile and target both have spin-$\frac{1}{2}$ these are the only local spin-spin operators allowed by parity, time reversal and angular momentum conservation. For target nuclei with higher spin, we will show that spin-spin interactions of higher ranks are also allowed.

McAbee [24,25] was the first to discuss a general spin-spin operator and did not limit his calculation of spin-spin interactions to just the spherical and tensor terms. The generalized local spin-spin operator used in this work is given by

$$\boldsymbol{S}_{k_I k}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \hat{\boldsymbol{R}}) = \left[\tau_1(\boldsymbol{\sigma}_0) \times \tau_{k_I}(\boldsymbol{I})\right]_k \cdot Y_k(\hat{\boldsymbol{R}}), \qquad (2)$$

which is analogous to that of Eq. 2.2 in McAbee [24]. The components of Eq. (2) are a spherical harmonic $Y_k(\hat{\boldsymbol{R}})$ as described in [26] and the standard spin operators, $\tau_{1q_0}(\boldsymbol{\sigma}_0)$ and $\tau_{k_I q_I}(\boldsymbol{I})$ [26,27] for the projectile and target spins, respectively. These general spin operators are discussed further in Appendix A.

In Eq. (2), angular momentum coupling requires that for the general spin-spin operator $\boldsymbol{S}_{k_I k}$, $k_I = k, |k \pm 1|$, because the projectile spin operator $\boldsymbol{\sigma}_0$ is rank 1. In order for the generalised spin-spin operator to be invariant under parity and time-reversal, k_I must be odd and k must be even. This is a

useful check for all the local potentials considered in this work. Nonlocal interactions will be discussed later when considering the tensor NN-interaction exchange terms.

The generalized spin-spin operator $\boldsymbol{S}_{k_I k}$, can be reduced to give the spherical spin-spin operator. Taking $k_I = 1$ and $k = 0$ and using standard angular momentum recoupling techniques [26] leads to

$$\boldsymbol{S}_{10}(\boldsymbol{\sigma}_0, \boldsymbol{I}) = \frac{-\boldsymbol{\sigma}_0 \cdot \boldsymbol{I}}{\sqrt{4\pi}\sqrt{I(I+1)}}. \tag{3}$$

Various authors have used different normalizations when dealing with the spin-spin interactions so caution must be taken when comparing the strengths of the potentials.

Using the same techniques, the tensor spin-spin operator can be obtained from Eq. (2) by taking $k_I = 1$ and $k = 2$ to give

$$\boldsymbol{S}_{12}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \hat{\boldsymbol{R}}) = \frac{\sqrt{5}}{\sqrt{8\pi}\sqrt{I(I+1)}}[3(\boldsymbol{\sigma}_0 \cdot \hat{\boldsymbol{R}})(\boldsymbol{I} \cdot \hat{\boldsymbol{R}}) - \boldsymbol{\sigma}_0 \cdot \boldsymbol{I}]. \tag{4}$$

In both cases the transition from the $\tau_{1\mu_0}(\boldsymbol{\sigma}_0)$ and $\tau_{k_I q_I}(\boldsymbol{I})$ to the $\boldsymbol{\sigma}_0$ and $\boldsymbol{I}$ spin operators was made using Wigner-Eckart theorem [26]. The use of the generalized spin-spin operator defined in Eq. (2) will enable the calculation of a general scattering amplitude for each allowed combination of k_I and k. This will be discussed in Sec. III B.

III. SCATTERING AMPLITUDES

A. Total scattering amplitude

The non-spin-spin components of the interaction between the projectile and target are included through a target spin-independent phenomenological optical potential. This potential is taken from [2] and is of the form

$$\begin{aligned} V_0 = {} & V_{\rm Coul}(R) + Vf(R, r_V, a_V) + iWf(R, r_W, a_W) \\ & + V_G g(R, r_G, a_G) + 2\Bigg[V_{\rm SO}\frac{1}{R}\frac{d}{dR}f(R, r_{\rm SOR}, a_{\rm SOR}) \\ & + iW_{\rm SO}\frac{1}{R}\frac{d}{dR}f(R, r_{\rm SOI}, a_{\rm SOI})\Bigg]\boldsymbol{\ell} \cdot \boldsymbol{\sigma}_0, \end{aligned} \tag{5}$$

for 200 MeV protons scattering from ^{10}B. The Woods-Saxon $f(R, r_i, a_i)$, and Gaussian $g(R, r_i, a_i)$ form factors in Eq. (5) are given by

$$f(R, r_i, a_i) = (1 + \exp[(R - r_i A^{1/3})/a_i])^{-1}, \tag{6}$$

$$g(R, r_i, a_i) = \exp\left[-\left(\frac{R - r_i A^{1/3}}{a_i}\right)^2\right], \tag{7}$$

and $V_{\rm Coul}(R)$ is the Coulomb potential generated by a uniformly charged sphere with a Coulomb radius of $R_{\rm Coul} = 1.46\ A^{1/3}$fm [2]. This target spin-independent potential was fitted to differential cross section and analyzing power measurements described in Ref. [2]. These observables are insensitive to the effects of spin-spin interactions and so will not be plotted here. The parameters for this phenomenological potential are given in Table III in Ref. [2] and fit the differential cross section and analyzing power data as well as the microscopic calculations of Ref. [5].

The scattering amplitude from the nucleon-nucleus interactions (not including spin-spin) is calculated exactly (for full details see Appendix A2 of [28]). The total nucleon-nucleus scattering amplitude is calculated using the standard central and spin-orbit amplitudes discussed above, plus the spin-spin amplitudes discussed in the next section.

B. Spin-spin amplitudes

Following on from previous work [23,24] we shall consider spin-spin terms in the optical potential of the form

$$U_{k_I k}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R}) = F_{k_I k}(R)\boldsymbol{S}_{k_I k}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \hat{\boldsymbol{R}}), \tag{8}$$

where $U_{k_I k}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R})$ are the spin-spin optical potentials and $F_{k_I k}(R)$ are the radial from factors. At this point we will assume that the strengths of the spin-spin interactions are small when compared to the conventional $\boldsymbol{I}$-independent terms in the optical potential, so that the former can be treated in the DWBA.

The DWBA is a first-order approximation in which the interaction, $V(a, A)$, between the projectile, a, and target, A, is split into an auxiliary potential, V_0, which includes a large part of the effects of $V(a, A)$, and a residual interaction, U, that is much weaker and can be treated as a perturbation. Previous work indicates that the strength of the central spin-spin interaction $U_{10}(R)$ is of the order 1 MeV [22] and that a large tensor spin-spin potential $U_{12}(\boldsymbol{R})$ has a strength of 2 MeV [29]. The treatment of the spin-spin interactions as a residual interaction in the DWBA therefore seems a sensible starting point for the analysis of these spin-spin terms.

In DWBA, the scattering amplitudes for the nucleon elastic scattering into polar angles (θ, ϕ) caused by the spin-spin potentials can be written as [28]

$$\begin{aligned} & f_{k_I k}(\mu', M_I'; \mu, M_I; \theta, \phi) \\ & \quad = \frac{-\mu_{\rm pt}}{2\pi\hbar^2}\langle \chi^{(-)}_{\mu'}, IM_I'|U_{k_I k}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R})|\chi^{(+)}_{\mu}, IM_I\rangle, \end{aligned} \tag{9}$$

where $\chi^{(-)}_{\mu'}(\boldsymbol{R}, \boldsymbol{k}')$ and $\chi^{(+)}_{\mu}(\boldsymbol{R}, \boldsymbol{k})$ are the final and initial distorted waves, respectively, with a final, μ', and initial, μ, incident nucleon spin projection. The final and initial spin projections of the nucleus are M_I' and M_I, respectively. The scattered and incident wave vectors are $\boldsymbol{k}'$ and $\boldsymbol{k}$ and $\mu_{\rm pt}$ is the reduced mass of the projectile-target system. The distorted wave function $\chi^{(+)}_{\mu}(\boldsymbol{R}, \boldsymbol{k})$ is a solution of the Schrödinger equation for the elastic scattering of the projectile on the target by the auxiliary potential, i.e., they are wave functions distorted from plane waves by the Coulomb and nuclear potentials other than spin-spin.

Following the prescription in Ref. [28] and standard angular momentum recoupling techniques, gives the following

equation for the DWBA spin-spin amplitude:

$$f_{k_I k}(\mu', M_I'; \mu, M_I; \theta, \phi) = \frac{\sqrt{6}}{E_{\text{c.m.}}}(-)^{k_I+k-2\mu+q_I}\hat{k_I}\hat{k}^2 \sum_{\ell,\ell',j,j'} \hat{\ell}^2 \hat{j}_I{}^{\ell-\ell'} e^{\imath(\sigma_\ell+\sigma_{\ell'})}\left(\ell'\mu - q_I - \mu' \tfrac{1}{2}\mu' \middle| j'\mu - q_I\right)$$
$$\times \left(\ell 0 \tfrac{1}{2}\mu \middle| j\mu\right) Y_{\ell',\mu-q_I-\mu'}(\hat{\boldsymbol{k}}')(k0\ell 0|\ell' 0)(I M_I k_I q_I | I M_I') \int_0^\infty u_{\ell',j'}(k_{\text{c.m.}}, R) F_{k_I k}(R)$$
$$\times u_{\ell,j}(k_{\text{c.m.}}, R) dR (j\mu \;\; k_I - q_I | j'\mu - q_I) \begin{Bmatrix} \ell' & \frac{1}{2} & j' \\ \ell & \frac{1}{2} & j \\ k & 1 & k_I \end{Bmatrix}, \tag{10}$$

where $\hat{\ell}$ is used to denote $\sqrt{2\ell+1}$, σ_ℓ are the Coulomb partial-wave phase shifts, $u_{\ell,j}(k_{\text{c.m.}}, R)$ are the distorted radial wave functions, $q_I = M_I' - M_I$, and the center of mass projectile-target energy is $E_{\text{c.m.}} = k_{\text{c.m.}}^2 \hbar^2/2\mu_{\text{pt}}$ with $k_{\text{c.m.}}$ as the magnitude of the wave number in the center of mass system. To simplify the calculation of the scattering amplitude again, the y-axis is chosen to be perpendicular to the scattering plane, along $\boldsymbol{k} \times \boldsymbol{k}'$, so that $\phi = 0$.

In the final calculation for 200 MeV protons elastically scattering from ^{10}B a relativistic energy momentum relation was used. The prescription used to describe potential scattering of relativistic particles is taken from Section 6.8 of [30] and discussed for elastic deuteron-nucleus scattering in [31,32].

Evaluating Eq. (10) when the spin-orbit term in the main optical potential is set to zero, provides a useful check of the calculation of the scattering amplitude. This simplification means there is no interaction in the distorting potential that depends on the projectile spin. In this case, the distorted radial wave functions in Eq. (10) no longer depend on j and the sums over j and j' may be performed using the orthogonality of the Clebsch-Gordon coefficients. For a full discussion see [25,33].

It is also useful to evaluate this amplitude for the central spin-spin potential only, i.e., $k_I = 1$ and $k = 0$. This simplification gives

$$f_{10}^{\text{no LS}}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \theta) = \frac{1}{\sqrt{4\pi}E_{\text{c.m.}}} \sum_\ell \hat{\ell}^2 e^{\imath 2\sigma_\ell} P_\ell(\cos\theta) \int_0^\infty u_\ell(k_{\text{c.m.}}, R)^2 F_{10}(R) dR \frac{\boldsymbol{\sigma}_0 \cdot \boldsymbol{I}}{\sqrt{I(I+1)}}, \tag{11}$$

where θ is the scattering angle between the vectors $\boldsymbol{k}$ and $\boldsymbol{k}'$ and $P_\ell(\cos\theta)$ is a Legendre Polynomial as described in [26]. With this simplified spin-spin amplitude the whole nucleon-nucleus elastic scattering amplitude can now be written as

$$F(\boldsymbol{\sigma}_0, \boldsymbol{I}, \theta) = g(\theta)\mathbf{1} + h_{10}(\theta) \frac{\boldsymbol{\sigma}_0 \cdot \boldsymbol{I}}{\sqrt{I(I+1)}}, \tag{12}$$

where $g(\theta)$ is the scattering amplitude for a spin-zero projectile and target, taken from [28] and $h_{10}(\theta)$ is taken from Eq. (11). This amplitude can be substituted into Eq. (1) for D_{NN} to give

$$D_{NN} = \frac{\text{Tr}[F\sigma_{0y}F^\dagger\sigma_{0y}]}{\text{Tr}[FF^\dagger]} = \frac{|g(\theta)|^2 - \frac{1}{3}|h_{10}(\theta)|^2}{|g(\theta)|^2 + |h_{10}(\theta)|^2},$$
$$= 1 - \frac{\frac{4}{3}|h_{10}(\theta)|^2}{|g(\theta)|^2 + |h_{10}(\theta)|^2}, \tag{13}$$

where σ_{0y} is the y-component of the projectile spin operator $\boldsymbol{\sigma}_0$ and the trace is taken over the projectile and target spin projections.

Expressing D_{NN} in this way shows that, $D_{NN} = 1$, if there is no $\boldsymbol{\sigma}_0 \cdot \boldsymbol{I}$ term in the scattering amplitude in Eq. (12) and that the presence of this spin-spin term means $D_{NN} < 1$. It reveals that not only is the dependence of the spin-spin term on $\boldsymbol{I}$ required for $D_{NN} \neq 1$, but $\boldsymbol{I}$ must be coupled to $\boldsymbol{\sigma}_0$. If there is no dependence on $\boldsymbol{\sigma}_0$ in the scattering amplitude, $\text{Tr}[F\sigma_{0y}F^\dagger\sigma_{0y}] = \text{Tr}[FF^\dagger]$ and $D_{NN} = 1$. Equation (13) also shows that D_{NN} has a second-order dependence on the strength of the central spin-spin interaction. This expression for D_{NN} also provides another useful check: when the $\boldsymbol{I}$-dependent term is much larger than the $\boldsymbol{I}$-independent term, i.e., $|h_{10}(\theta)|^2 \gg |g(\theta)|^2$, the observable $D_{NN} \to -1/3$.

IV. THE NUCLEON-NUCLEON INTERACTION

The effective nucleon-nucleon (NN) interaction $V_{NN}(a, b)$ between nucleon a and nucleon b, with separation, r, can be written as

$$V_{NN}(a, b) = v_{NN}^{\text{cen}}(r) + v_{NN}^{\sigma\sigma}(r)\boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_b + v_{NN}^{\text{ls}}(r)\boldsymbol{\ell}_{ab} \cdot (\boldsymbol{\sigma}_a + \boldsymbol{\sigma}_b)$$
$$+ v_{NN}^{\text{tr}}(r)\boldsymbol{S}_{12}^{NN}(\boldsymbol{\sigma}_a, \boldsymbol{\sigma}_b, \hat{\boldsymbol{r}}), \tag{14}$$

where the relative angular momentum is

$$\boldsymbol{\ell}_{ab} = (2\hbar)^{-1}\boldsymbol{r} \times (\boldsymbol{p}_b - \boldsymbol{p}_b), \tag{15}$$

and $\boldsymbol{p}$ is a nucleon momentum. The NN tensor operator is

$$\boldsymbol{S}_{12}^{NN}(\boldsymbol{\sigma}_a, \boldsymbol{\sigma}_b, \hat{\boldsymbol{r}}) = 3(\boldsymbol{\sigma}_a \cdot \hat{\boldsymbol{r}})(\boldsymbol{\sigma}_b \cdot \hat{\boldsymbol{r}}) - \boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_b. \tag{16}$$

We have added the superscript NN to differentiate this tensor operator from the nucleon-nucleus spin-spin operator, $\boldsymbol{S}_{12}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \hat{\boldsymbol{R}})$. The radial form factors, $v_{NN}^{\rm cen}$, $v_{NN}^{\sigma\sigma}$, $v_{NN}^{\rm ls}$, and $v_{NN}^{\rm tr}$, denote central spin-independent, central spin-spin, spin-orbit, and tensor interactions, respectively.

Spin-spin interactions were first studied within folding models using a single valence-nucleon description of the nucleus by Stamp [1] and then by Nagamine *et al.* [12]. These works only included contributions from the $v_{NN}^{\sigma\sigma}(r)\boldsymbol{\sigma}_0 \cdot \boldsymbol{\sigma}_i$ term in the NN interaction and the central potential $U_{10}(\boldsymbol{\sigma}_0, \boldsymbol{I}, R)$ [of the form in Eq. (8)] was the only spin-spin interaction discussed. Satchler [13,14], went one step further including both $v_{NN}^{\sigma\sigma}(r)\boldsymbol{\sigma}_0 \cdot \boldsymbol{\sigma}_i$ and $v_{NN}^{\rm tr}(r)\boldsymbol{S}_{12}^{NN}(\boldsymbol{\sigma}_a, \boldsymbol{\sigma}_b, \hat{\boldsymbol{r}})$ terms from the NN interaction. The folding of these gave the spin-spin potential, $U_{12}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R})$, in addition to the central term.

A thorough analysis of spin-spin potentials that result from the folding of all the terms in the NN interaction given in Eq. (14) has been given by Petrovich *et al.* [21] and McAbee [24,25]. Both performed a multipole decomposition of an NN interaction containing central, spin-orbit, and tensor parts and the resulting expressions were folded over the target wave functions. In Ref. [21] this was used to derive a general expression for the optical potential for the elastic scattering of two non-spin-zero nuclei and used coupled-equations formalism to obtain the scattering observables. McAbee [24,25] used this to derive the form factors for his generalized spin-spin operators, Eq. (2), which were then treated in the DWBA framework to obtain the spin-spin observables.

We have included spin-spin interactions through the folding of the central $v_{NN}^{\sigma\sigma}$, and tensor $v_{NN}^{\rm tr}$ terms in the NN interaction. These are the same as McAbee [24,25] included in his final calculation where he neglected all momentum dependent nonlocal spin-spin interactions. The folding of these two terms in the NN interaction give all the local spin-spin potentials that can be formed from the generalized spin-spin operator, Eq. (2), by the coupling of $\boldsymbol{\sigma}_0$, $\boldsymbol{I}$, and $\boldsymbol{R}$. The folding of the central spin-independent term, $v_{NN}^{\rm cen}$, does not result in spin-spin interactions. However, Feshbach [6] discussed spin-spin potentials derived from the folding of the spin-orbit term in the NN interaction. These terms have not been thoroughly investigated in relation to spin-spin interactions but some studies of the first order $\boldsymbol{I} \cdot \boldsymbol{\ell}$ term have been performed [34]. In this term, $\boldsymbol{\ell}$ is the relative orbital angular momentum of the projectile and target. This term by itself does not give a deviation of D_{NN} from unity as it is independent of the spin operator for the scattered nucleon $\boldsymbol{\sigma}_0$. When this is the case, Eq. (1) shows that $D_{NN} = 1$. The folding of the spin-orbit term in the NN interaction would lead to higher order spin-spin interactions which do depend on the projectile spin, but it is unclear if such terms would cause a significant deviation of D_{NN} from unity. Their effect on D_{NN} will be the subject of future work.

In this work we use a realistic free-space NN interaction. For 200 MeV protons, the use of a free t matrix is valid, as the difference between cross sections and analyzing power predictions using the free t matrix and the medium-modified G matrix is very small [35]. Our interaction is a version of the Bonn-B potential where the parameters of the original potential have been improved so predictions better agree with later measurements of NN scattering and phase-shift analysis (see references in [36]). Our effective interaction is a superposition of Yukawa terms, the parametrization of which is the same as that detailed in Love and Franey [37]. Our parameters were fitted to a free, complex t matrix and are calculated for an incident proton laboratory energy of 200 MeV, as described in [36].

A. Treatment of exchange

For nucleon-nucleus elastic scattering the antisymmetrization of the wave function yields local direct terms and nonlocal nucleon-exchange terms. It is reasonable, to use a single-nucleon knock-on exchange (SNKE) correction, when considering exchange contributions arising from antisymmetrization. In SNKE the projectile nucleon is exchanged with a single target nucleon [28]. Knock-on exchange can be included in the folding model by replacing the effective NN interaction potential term in Eq. (14), $v_{NN}(r)$, with

$$v_{NN}(r) \rightarrow (1 - P_{NN})v_{NN}(r), \tag{17}$$

where the P_{NN} operator exchanges all coordinates of the two nucleons [38,39]. The exchange operator can be written in terms of the individual exchange operators P^{σ}, P^{τ}, and P_{NN}^{r}, which exchange the spin, isospin, and spatial coordinates, respectively,

$$P_{NN} = P_{NN}^{\sigma} P_{NN}^{\tau} P_{NN}^{r}. \tag{18}$$

The NN potential, $v_{NN}(r)$ can now be written as

$$v_{NN}(r) \rightarrow (1 - P_{NN})v_{NN}(r), \tag{19}$$

$$\rightarrow v_{NN}(r) + \hat{v}_{NN}(r)P_{NN}^{r}, \tag{20}$$

where $\hat{v}_{NN}(r) = -P_{NN}^{\sigma} P_{NN}^{\tau} v_{NN}(r)$. The exchange potential, $\hat{v}_{NN}(r)$, has the same spin-isospin decomposition as $v_{NN}(r)$, except that the sign is changed for the odd relative angular momentum states of the two nucleons.

Exchange effects were included crudely in early folding models for spin-spin interactions [23], as they used effective interactions that were mainly phenomenological. Explicit exchange effects were discussed by Satchler [13], but were not included here. The later works of Petrovich [21] and McAbee [24] used the SNKE correction to approximate exchange for the $v_{NN}^{\sigma\sigma}(r)\boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_b$ term in their effective NN interaction. The spin-orbit exchange contribution was included by Petrovich who just used the odd state spin-orbit components alone, but both Petrovich and McAbee left out the exchange effects for the tensor term in their final calculations. Our work aims to investigate their contributions to the spin-spin interactions by including consistently, for the first time, SNKE terms for both the central $v_{NN}^{\sigma\sigma}(r)\boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_b$ and tensor $v_{NN}^{\rm tr}(r)\boldsymbol{S}_{12}^{NN}(\boldsymbol{\sigma}_a\boldsymbol{\sigma}_b, \hat{\boldsymbol{r}})$ terms in the effective NN interaction.

The exact evaluation of exchange amplitudes is very complicated. Therefore, the treatment of exchange terms in folding models has been performed using zero-range pseudopotentials [40–42]. In the method we adopt for investigating exchange effects on spin-spin interactions, the exchange pseudopotentials are derived to yield the same Born amplitudes as their equivalent exchange terms [28,43]. This is the simplest of these approximate treatments and so the resulting zero-range effective exchange interaction can be used to evaluate the scattering amplitude more readily than the full exchange terms.

B. Approximation for central exchange

From the NN interaction given in Eq. (14) between projectile nucleon a and target nucleon b, the Born amplitude for central spin-spin SNKE term is given by

$$\langle \boldsymbol{k}'|\hat{v}^{\sigma\sigma}_{NN}(r)\boldsymbol{\sigma}_a\cdot\boldsymbol{\sigma}_b P^r_{ab}|\boldsymbol{k}\rangle = \hat{J}_{\sigma\sigma}(Q)\boldsymbol{\sigma}_a\cdot\boldsymbol{\sigma}_b, \tag{21}$$

where the strength $\hat{J}_{\sigma\sigma}(Q)$ is the Fourier transform of $\hat{v}^{\sigma\sigma}_{NN}(r)P_{ab}$

$$\hat{J}_{\sigma\sigma}(Q) = 4\pi\int_0^\infty j_0(Qr)\hat{v}^{\sigma\sigma}_{NN}(r)r^2dr, \tag{22}$$

and $\boldsymbol{Q}$, is the sum of the initial and final wave number of either particle in the center of mass system,

$$\boldsymbol{Q} = \boldsymbol{k}' + \boldsymbol{k}. \tag{23}$$

The value taken for the magnitude of $\boldsymbol{Q}$ is discussed in Sec. IV D. The zero-range pseudopotential used to approximate this exchange term is given by

$$\hat{v}^{\sigma\sigma}_{NN}(r)\boldsymbol{\sigma}_a\cdot\boldsymbol{\sigma}_b P^r_{ab} \rightarrow \hat{J}_{\sigma\sigma}(Q)\delta(\boldsymbol{r})\boldsymbol{\sigma}_a\cdot\boldsymbol{\sigma}_b P^r_{ab}. \tag{24}$$

This pseudopotential has been used by several authors to approximate the central exchange contribution to the NN interaction. For nucleon scattering at energies above 100 MeV it has been shown to be quite accurate (see references in [28,38,39]).

C. Approximation for tensor exchange

In previous works on spin-spin interactions, this contribution has been neglected due to the difficulty in taking the pseudopotential approach for the noncentral tensor potential [25]. In this work the tensor exchange will be included using the pseudopotential approximation consistent with that used for the central exchange.

The Born amplitude for the tensor exchange term is

$$\begin{aligned}&\langle \boldsymbol{k}'|\hat{v}^{\rm tr}_{NN}(r)\boldsymbol{S}^{NN}_{12}(\boldsymbol{\sigma}_a,\boldsymbol{\sigma}_b,\hat{\boldsymbol{r}})P^r_{ab}|\boldsymbol{k}\rangle\\ &\quad = Q^2\hat{J}_{\rm tr}(Q)\boldsymbol{S}^{NN}_{12}(\boldsymbol{\sigma}_a,\boldsymbol{\sigma}_b,\hat{\boldsymbol{Q}}),\end{aligned} \tag{25}$$

where $\boldsymbol{S}^{NN}_{12}(\boldsymbol{\sigma}_a,\boldsymbol{\sigma}_b,\hat{\boldsymbol{Q}}) = 3(\boldsymbol{\sigma}_a\cdot\hat{\boldsymbol{Q}})(\boldsymbol{\sigma}_b\cdot\hat{\boldsymbol{Q}}) - \boldsymbol{\sigma}_a\cdot\boldsymbol{\sigma}_b$, and the strength, $\hat{J}_{\rm tr}(Q)$, is calculated using the Fourier transform

$$\hat{J}_{\rm tr}(Q) = -\frac{4\pi}{Q^2}\int_0^\infty j_2(Qr)\hat{v}^{\rm tr}_{NN}(r)r^2dr. \tag{26}$$

We define $\hat{\boldsymbol{S}}^{NN}_{12}(\boldsymbol{\sigma}_a,\boldsymbol{\sigma}_b,\boldsymbol{Q}) = Q^2\boldsymbol{S}^{NN}_{12}(\boldsymbol{\sigma}_a,\boldsymbol{\sigma}_b,\hat{\boldsymbol{Q}})$ and use

$$\begin{aligned}&\langle \boldsymbol{k}'|\hat{\boldsymbol{S}}^{NN}_{12}(\boldsymbol{\sigma}_a,\boldsymbol{\sigma}_b,\boldsymbol{Q})|\boldsymbol{k}\rangle\\ &\quad = \langle \boldsymbol{k}'|\big[\hat{\boldsymbol{S}}^{NN}_{12}(\boldsymbol{\sigma}_a,\boldsymbol{\sigma}_b,-\imath\nabla_{\boldsymbol{r}})\delta(\boldsymbol{r})\big]\ P^r_{ab}|\boldsymbol{k}\rangle,\end{aligned} \tag{27}$$

where the square brackets mean that the grad operator in $\hat{\boldsymbol{S}}^{NN}_{12}(\boldsymbol{\sigma}_a,\boldsymbol{\sigma}_b,-\imath\nabla_{\boldsymbol{r}})$, acts only on the δ function. Equation (25) can now be rewritten in a form more suitable for manipulations in the folding calculations

$$\begin{aligned}&\langle \boldsymbol{k}'|\hat{v}^{\rm tr}_{NN}(r)\boldsymbol{S}^{NN}_{12}(\boldsymbol{\sigma}_a,\boldsymbol{\sigma}_b,\hat{\boldsymbol{r}})P^r_{ab}|\boldsymbol{k}\rangle\\ &\quad = \hat{J}_{\rm tr}(Q)\langle \boldsymbol{k}'|\big[\hat{\boldsymbol{S}}^{NN}_{12}(\boldsymbol{\sigma}_a,\boldsymbol{\sigma}_b,-\imath\nabla_{\boldsymbol{r}})\delta(\boldsymbol{r})\big]P^r_{ab}|\boldsymbol{k}\rangle.\end{aligned} \tag{28}$$

The zero-range pseudopotential used to approximate this term is

$$\begin{aligned}&\hat{v}^{\rm tr}_{NN}(r)\boldsymbol{S}^{NN}_{12}(\boldsymbol{\sigma}_a,\boldsymbol{\sigma}_b,\hat{\boldsymbol{r}})\ P^r_{ab}\\ &\quad\rightarrow \hat{J}_{\rm tr}(Q)\big[\hat{\boldsymbol{S}}^{NN}_{12}(\boldsymbol{\sigma}_a,\boldsymbol{\sigma}_b,-\imath\nabla_{\boldsymbol{r}})\delta(\boldsymbol{r})\big]P^r_{ab}.\end{aligned} \tag{29}$$

As in our treatment of central exchange, the magnitude of Q on the right-hand side is assumed constant in any particular folding calculation. The form of the pseudopotential Eq. (29) preserves the correct behavior of the exact tensor force under rotations of space and spin coordinates, while the δ function considerably simplifies folding calculations.

D. Approximation for Q

The exchange operator, P^r_{ab}, changes the Born amplitudes discussed in the previous sections from a dependence on the momentum transfer $\boldsymbol{q} = \boldsymbol{k}' - \boldsymbol{k}$ to $\boldsymbol{Q} = \boldsymbol{k}' + \boldsymbol{k}$. The simplest choice for $\boldsymbol{Q}$ is to take it to be the center-of-mass wave vector for the incident particle: $Q = k_{\rm c.m.}$ [28,36]. This is reasonable for scattering in the forward direction, but it is at large scattering angles that the measured deviation of D_{NN} from unity is most significant. In order to perform an improved calculation of these exchange terms at large scattering angles, a more physical approximation for the magnitude of $\boldsymbol{Q}$ is required, one that changes with scattering angle.

In the following discussion we will consider the scattering of the incident nucleon with wave vector $\boldsymbol{k}_a$ by a single valence nucleon with initial wave vector $\boldsymbol{k}_b$, which for simplicity is bound to a spinless core of infinite mass. The Born amplitude of the local NN interaction term, $v_{NN}(r)$, (where $\boldsymbol{r}$ is the relative separation between the two nucleons $\boldsymbol{r} = \boldsymbol{r}_a - \boldsymbol{r}_b$), is given by

$$\begin{aligned}&\langle \boldsymbol{k}'_a,\boldsymbol{k}'_b|v_{NN}|\boldsymbol{k}_a,\boldsymbol{k}_b\rangle\\ &\quad = \int d\boldsymbol{r}\ \ e^{-\imath(\boldsymbol{k}'_a-\boldsymbol{k}'_b)\cdot\boldsymbol{r}/2}v_{NN}(r)e^{\imath(\boldsymbol{k}_a-\boldsymbol{k}_b)\cdot\boldsymbol{r}/2}\\ &\qquad\times\delta(\boldsymbol{k}'_a+\boldsymbol{k}'_b-\boldsymbol{k}_a-\boldsymbol{k}_b)\\ &\quad = \delta(\boldsymbol{k}'_a+\boldsymbol{k}'_b-\boldsymbol{k}_a-\boldsymbol{k}_b)\left\langle\frac{\boldsymbol{k}'_a-\boldsymbol{k}'_b}{2}\middle|v_{NN}\middle|\frac{\boldsymbol{k}_a-\boldsymbol{k}_b}{2}\right\rangle,\end{aligned} \tag{30}$$

where $\boldsymbol{k}'_a$ and $\boldsymbol{k}'_b$ are the final wave numbers of the two nucleons and we have neglected factors of π for this discussion. We have assumed that the potential conserves the total momentum of the system [which is assured by using $\delta(\boldsymbol{k}'_a+\boldsymbol{k}'_b-\boldsymbol{k}_a-\boldsymbol{k}_b)$]. The momentum transfer $\boldsymbol{q}$ and vector $\boldsymbol{Q}$ can be obtained from this Born amplitude by taking

$$\boldsymbol{k}' = \frac{\boldsymbol{k}'_a-\boldsymbol{k}'_b}{2},\quad \boldsymbol{k} = \frac{\boldsymbol{k}_a-\boldsymbol{k}_b}{2}. \tag{31}$$

Using the conservation of momentum $\boldsymbol{k}'_a+\boldsymbol{k}'_b = \boldsymbol{k}_a+\boldsymbol{k}_b$, this gives

$$\boldsymbol{q} = \boldsymbol{k}'-\boldsymbol{k} = \boldsymbol{k}'_a-\boldsymbol{k}_a,\quad \boldsymbol{Q} = \boldsymbol{k}'+\boldsymbol{k} = \boldsymbol{k}'_a-\boldsymbol{k}_b, \tag{32}$$

assuming on-shell scattering, i.e., $k_a = k'_a$. In order to approximate Q, an appropriate value of k_b must be selected.

In order to determine which value of vector $\boldsymbol{Q}$ has the most important contribution to the scattering amplitude, the folded potential will be treated using the Born approximation

in momentum space. This gives the Born amplitude as

$$\langle \boldsymbol{k}'_a, \psi_{A'}|v_{NN}|\boldsymbol{k}_a, \psi_A\rangle = \int d\boldsymbol{k}_b \tilde{\psi}^*_{A'}(\boldsymbol{k}_b - \boldsymbol{q})\langle \boldsymbol{k}'|v_{NN}|\boldsymbol{k}\rangle \tilde{\psi}_A(\boldsymbol{k}_b), \quad (33)$$

where Eqs. (30) and (31) have been used and the ψ_A are the target nucleon wave functions. When v_{NN} is local, $\langle \boldsymbol{k}'|v_{NN}|\boldsymbol{k}\rangle$ is a function of $\boldsymbol{q}$ and can be taken outside of the integral. For nonlocal potentials, the matrix element of v_{NN} depends on $\boldsymbol{Q}$, which in turn is dependent on $\boldsymbol{k}_b$, and cannot be taken out of the $\boldsymbol{k}_b$ integral. Therefore to avoid evaluating the $\boldsymbol{k}_b$ integral exactly, an appropriate value of $\boldsymbol{Q}$ will be chosen at which to evaluate $\langle \boldsymbol{k}'|v_{NN}|\boldsymbol{k}\rangle$.

The most important contribution of $\boldsymbol{k}_b$ to this integral is given by the product of the final and initial target wave functions $\tilde{\psi}^*_{A'}(\boldsymbol{k}_b - \boldsymbol{q})$ and $\tilde{\psi}_A(\boldsymbol{k}_b)$, respectively. The form of the wave function is taken to be harmonic oscillator (HO), $\psi_A(r) = P(r)\exp(-r^2/R^2_{\rm HO})$, where $P(r)$ is a polynomial in r and $R_{\rm HO}$ is the HO parameter. The Fourier transform of this wave function is then $\tilde{\psi}_A(K) = P(K)\exp(-K^2R^2_{\rm HO}/4)$. For large $R_{\rm HO}$, the polynomials can be neglected and the values of $\boldsymbol{k}_b$ that are most important can be found from the maximum of

$$\exp\left[-(\boldsymbol{k}_b - \boldsymbol{q})^2\frac{R^2_{\rm HO}}{4}\right]\exp\left[-k_b^2\frac{R^2_{\rm HO}}{4}\right] = \exp\left[-\left(\frac{q^2}{2} + 2\left[\boldsymbol{k}_b - \frac{\boldsymbol{q}}{2}\right]^2\right)\frac{R^2_{\rm HO}}{4}\right], \quad (34)$$

which occurs when $(q^2/2 + 2[\boldsymbol{k}_b - \boldsymbol{q}/2]^2)R^2_{\rm HO}/4$ is at a minimum, i.e., when $\boldsymbol{k}_b = \boldsymbol{q}/2$. For this approximation of $\boldsymbol{Q}$, the value of $\boldsymbol{k}_b$ is therefore taken to be

$$\boldsymbol{Q} = \boldsymbol{k}'_a - \boldsymbol{k}_b = \boldsymbol{k}'_a - \frac{\boldsymbol{q}}{2} = \frac{\boldsymbol{k}'_a + \boldsymbol{k}_a}{2}. \quad (35)$$

This leads to the approximation for the magnitude of Q used in this work:

$$Q = k_a\cos(\theta/2), \quad (36)$$

assuming $k_a = k'_a$ and θ is the angle between $\boldsymbol{k}_a$ and $\boldsymbol{k}'_a$. This approximation for Q is the same as that obtained from the prescription described in Appendix B of Ref. [36], in the limit $A \to \infty$.

V. FOLDING MODEL

This section describes the formalism developed in order to microscopically derive the spin-spin potentials in terms of the interactions of individual nucleons, rather than using a phenomenological model. McAbee [24,25] evaluated target spin-dependent forces using a valence nucleon+core model. Here we extend his work to the case of two nucleons outside an inert, infinitely massive, core with application to the $I = 3$ ground state of ^{10}B. This is not a very realistic model for this nucleus, but we are concentrating on the calculation of spin dependent forces with $k_I > 0$. Terms in the spin-spin interaction receive no contribution from closed shells in the ground state because they carry no angular momentum. Therefore, only the nonzero rank target spin tensor components of the one-body-density matrix (OBDM) are relevant and deviations of D_{NN} from unity are associated only with these components.

A more realistic, but still simple model, of ^{10}B would be a two-nucleon (neutron and proton) hole in the lowest $p_{3/2}$ shell with a closed $0s_{1/2}$ shell. We show in Appendix B that spin-dependent forces with $k_I > 0$ are trivially related, through a k_I-dependent phase, to the results obtained from the simple two-particle model, for which the folding calculations can be carried through in a relatively straightforward way. This is the analog of a similar theorem for static multiple moments of shell model states discussed in [44].

The two valence particles in our model couple together to give the spin $I = 3$ for the ground state spin of the ^{10}B nucleus and the maximum allowed rank, $k_I = 3$, of the spin operator constructed from the components of the target spin. Therefore, the types of spin-spin interactions allowed are determined by the orbitals occupied by its valence particles as well as the ground state spin of the target nucleus. Using this simple nuclear wave function builds on previous works [23,24], but inclusion of a more realistic model for the ^{10}B nucleus is desirable in future work.

In this calculation the wave function for the valence nucleons is assumed to have the simple form [44]

$$\Psi_{I,M}(\boldsymbol{r}_1, \boldsymbol{r}_2) = [\Phi_{\ell_1 j_1 m_1}(\boldsymbol{r}_1) \times \Phi_{\ell_2 j_2 m_2}(\boldsymbol{r}_2)]_{IM}, \quad (37)$$

where the valence particles, nucleon 1 and nucleon 2 are at a distance of $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ from the core, respectively. The nucleon wave functions $\Phi_{\ell jm}(\boldsymbol{r})$ are defined as

$$\Phi_{\ell jm}(\boldsymbol{r}) = \sum_{\lambda m_s}\left(\ell\lambda\tfrac{1}{2}m_s\middle|jm\right)u_{\ell j}(r)Y_{\ell\lambda}(\hat{\boldsymbol{r}})\chi_{\frac{1}{2}m_s}, \quad (38)$$

where ℓ is the orbital angular momentum, $\frac{1}{2}$ is the intrinsic spin of the nucleon, which couple together to give the total spin j, $u_{\ell j}(r)$ is the real radial wave function, $Y_{\ell\lambda}(\hat{\boldsymbol{r}})$ is a spherical harmonic and $\chi_{\frac{1}{2}m_s}$ is the spin function that describes the intrinsic angular momentum of the valence nucleon.

The total folded spin-spin potential is given by

$$U_{M'_I M_I}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R}) = \sum_{i=1,2} U^i_{M'_I M_I}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R}), \quad (39)$$

where $i = 1$ denotes the proton valence particle and $i = 2$ the neutron valence particle (as labeled in Fig. 1). The folded

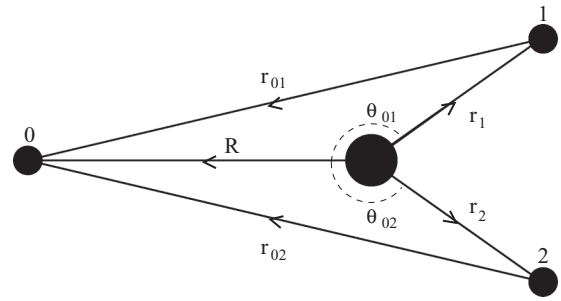

FIG. 1. The folding model coordinate system, particle 0 is the incident proton, particle 1 is the valence proton and particle 2 is the valence neutron. The projectile-valence nucleon separation is $\boldsymbol{r}_{01} = \boldsymbol{R} - \boldsymbol{r}_1$ and $\boldsymbol{r}_{02} = \boldsymbol{R} - \boldsymbol{r}_2$ for the proton and neutron, respectively.

spin-spin potential for nucleon i is therefore

$$U^{i}_{M'_I M_I}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R}) = \int \sum_{m'_1 m'_2 m_1 m_2} \Phi^{*}_{\ell'_1 j'_1 m'_1}(\boldsymbol{r}_1)(j'_1 m'_1 j'_2 m'_2 | I M'_I) \times \Phi^{*}_{\ell'_2 j'_2 m'_2}(\boldsymbol{r}_2) V_{NN}(\boldsymbol{r}_{0i}) \Phi_{\ell_1 j_1 m_1}(\boldsymbol{r}_1) \times (j_1 m_1 j_2 m_2 | I M_I) \Phi_{\ell_2 j_2 m_2}(\boldsymbol{r}_2)\ d\boldsymbol{r}_1 d\boldsymbol{r}_2, \tag{40}$$

where $V_{NN}(\boldsymbol{r}_{0i})$ is the effective NN interaction between the projectile and target valence particle i. As discussed previously, we calculate spin-spin interactions from the folding of the central $v^{\sigma\sigma}_{NN}$, and tensor $v^{\rm tr}_{NN}$ terms in the NN interaction given in Eq. (14). The separation between the projectile and valence nucleon, $\boldsymbol{r}_{0i}$, is defined in Fig. 1. We will now choose which valence nucleon will be examined, the proton ($i = 1$), or the neutron ($i = 2$). The process for calculating the folded potential is exactly the same for each nucleon [the only difference being which $V_{NN}(\boldsymbol{r}_{0i})$ is used] but for the sake of this discussion the interacting nucleon will be taken to be the proton, and hence $\langle \Phi_{\ell'_2 j'_2 m'_2}(\boldsymbol{r}_2) | \Phi_{\ell_2 j_2 m_2}(\boldsymbol{r}_2) \rangle = \delta_{\ell'_2,\ell_2}\delta_{j'_2,j_2}\delta_{m'_2,m_2}$. Equation (40) can now be rearranged using standard angular momentum recoupling techniques and the relationships between Clebsch-Gordon coefficients and Racah coefficients, $W(abcd;ef)$, discussed in [26], to give the general folding equation for the projectile-valence particle 1 (proton) potential

$$U_{M'_I M_I}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R}) = (-)^{1+j_1-j'_1} \hat{I}\hat{j}_1\hat{j}'_1 \int d\boldsymbol{r}_1 \sum_{\lambda'_1\lambda_1 m'_{s1} m_{s1}} Y^{*}_{\ell'_1\lambda'_1}(\hat{\boldsymbol{r}}_1) \chi^{*}_{\frac{1}{2}m'_{s1}} u_{\ell'_1 j'_1}(r_1) V_{NN}(\boldsymbol{R} - \boldsymbol{r}_1) Y_{\ell_1\lambda_1}(\hat{\boldsymbol{r}}_1) \chi_{\frac{1}{2}m_{s1}} u_{\ell_1 j_1}(r_1) (-)^{\frac{1}{2}+m'_{s1}-\ell'_1-\lambda_1} \times \sum_{\substack{k_I q_I \\ k_1 q_1 k_2 q_2}} \hat{k_I}^2 W(k_I j_1 I j_2; j'_1 I)(I M_I k_I q_I | I M'_I)(-)^{k_1-k_2} \hat{k}_1 \left(\tfrac{1}{2} m_{s1} \tfrac{1}{2} - m'_{s1} \middle| k_1 q_1\right) \times (\ell_1 - \lambda_1 \ell'_1 \lambda'_1 | k_2 q_2)(k_1 q_1 k_I q_I | k_2 q_2) \begin{Bmatrix} j_1 & j'_1 & k_I \\ \frac{1}{2} & \frac{1}{2} & k_1 \\ \ell_1 & \ell'_1 & k_2 \end{Bmatrix}. \tag{41}$$

This equation can be used to determine the spin-spin potentials for direct and exchange terms in the NN interaction for both the valence proton and neutron particle.

A. Central NN interaction

Using the general folding expression in Eq. (41), we first evaluate the central term in the NN interaction, $v^{\sigma\sigma}_{NN}(r_{01})\boldsymbol{\sigma}_0 \cdot \boldsymbol{\sigma}_1$. This term can be expressed, using a multipole expansion, as

$$v^{\sigma\sigma}_{NN}(r_{01})\boldsymbol{\sigma}_0 \cdot \boldsymbol{\sigma}_1 = 4\pi \sum_{KQ} \frac{f^{(\sigma\sigma)}_K(R, r_1)}{\hat{K}^2} Y_{KQ}(\hat{\boldsymbol{R}}) Y^{*}_{KQ}(\hat{\boldsymbol{r}}_1) \sum_{\mu_0} (-)^{\mu_0} \sigma^{(0)}_{1\mu_0} \sigma^{(1)}_{1-\mu_0}. \tag{42}$$

The function $f_K(R, r_1)$ is given by

$$f_K(R, r_1) = \frac{\hat{K}^2}{2} \int_{-1}^{1} P_K(\mu) v_{NN}(|\boldsymbol{R} - \boldsymbol{r}_1|) d\mu, \tag{43}$$

where $\mu = \cos\theta_{01}$, θ_{01} is the angle between $\boldsymbol{R}$ and $\boldsymbol{r}_1$ (as shown in Fig. 1) and $P_K(\mu)$ is a Legendre polynomial.

The folding of this central direct term in the NN interaction using Eq. (41) and standard angular momentum recoupling techniques [26], gives the following result:

$$U^{\sigma\sigma}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R}) = (-)^{1+j_1-j'_1} \sqrt{24\pi}\, \hat{I}\hat{j}_1\hat{j}'_1\hat{\ell}'_1 \int_0^{\infty} r_1^2 dr_1 u_{\ell'_1 j'_1}(r_1) \sum_{k_I k_2} (-)^{k_2} \frac{f^{(\sigma\sigma)}_{k_2}(R, r_1)}{\hat{k_2}^2} u_{\ell_1 j_1}(r_1)(\ell'_1 0 k_2 0 | \ell_1 0) \hat{k_I} W(k_I j_1 I j_2; j'_1 I) \times \begin{Bmatrix} j_1 & j'_1 & k_I \\ \frac{1}{2} & \frac{1}{2} & 1 \\ \ell_1 & \ell'_1 & k_2 \end{Bmatrix} \boldsymbol{S}_{k_I k_2}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \hat{\boldsymbol{R}}). \tag{44}$$

Full details of this process are given in Ref. [33]. $\boldsymbol{S}_{k_I k_2}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \hat{\boldsymbol{R}})$ is the generalized spin-spin operator discussed in Sec. II B.

For our model of ^{10}B, summing over k_I and k_2 in Eq. (44) yields three different types of spin-spin interactions, $U^{\sigma\sigma} = U^{\sigma\sigma}_{10} + U^{\sigma\sigma}_{12} + U^{\sigma\sigma}_{32}$, see Appendix C 1. These spin-spin potentials are of the form given in Eq. (8), where the superscript $\sigma\sigma$ indicates that these spin-spin interactions come from the folding of the $v^{\sigma\sigma}_{NN}(r_{01})\boldsymbol{\sigma}_0 \cdot \boldsymbol{\sigma}_1$ term in the NN interaction. The folding of this term has led to the spherical $U_{10}(\boldsymbol{\sigma}_0, \boldsymbol{I}, R)$ and tensor $U_{12}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R})$ spin-spin interactions examined in previous work (see Sec. II A) and also the higher rank term $U_{32}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R})$, first derived by McAbee [24,25].

B. Tensor NN interaction

Using a multipole expansion, the tensor term in the NN interaction, $v^{\rm tr}_{NN}(r_{01})\boldsymbol{S}^{NN}_{12}(\boldsymbol{\sigma}_0,\boldsymbol{\sigma}_1,\hat{\boldsymbol{r}}_{01})$, can be expressed as

$$v^{\rm tr}_{NN}(r_{01})\boldsymbol{S}^{NN}_{12}(\boldsymbol{\sigma}_0,\boldsymbol{\sigma}_1,\hat{\boldsymbol{r}}_{01}) = 4\pi\sum_K \frac{f^{(\rm tr)}_K(R,r_1)}{\hat{K}^2}Y_K(\hat{\boldsymbol{R}})\cdot Y^*_K(\hat{\boldsymbol{r}}_1)2\sqrt{\frac{8\pi}{5}}\tau_2(\boldsymbol{S})\cdot Y_2(\hat{\boldsymbol{r}}_{01}), \tag{45}$$

where we have used the relation

$$\boldsymbol{S}^{NN}_{12}(\boldsymbol{\sigma}_0,\boldsymbol{\sigma}_1,\hat{\boldsymbol{r}}_{01}) = 2\sqrt{\frac{8\pi}{5}}\tau_2(\boldsymbol{S})\cdot Y_2(\hat{\boldsymbol{r}}_{01}), \tag{46}$$

derived using the relationship between the spin operator for the total spin $\boldsymbol{S}$ and the spin operators of the component spins, where $\boldsymbol{S}=\frac{1}{2}(\boldsymbol{\sigma}_0+\boldsymbol{\sigma}_1)$, given by

$$\tau_{2-\mu}(\boldsymbol{S}) = \frac{\sqrt{3}}{2}\sum_{\mu_1\mu_2}(1\mu_1 1\mu_2|2-\mu)\tau_{1\mu_1}(\boldsymbol{\sigma}_0)\tau_{1\mu_2}(\boldsymbol{\sigma}_1), \tag{47}$$

and the relationships between spherical harmonics with different ranks taken from [26]. To aid the folding algebra, the second form of the tensor term in which the spin and radial dependence have been separated, Eq. (45), will be used. This can be further separated [26] into

$$\begin{aligned} r^2_{01}Y_{2\mu}(\hat{\boldsymbol{r}}_{01}) &= |\boldsymbol{R}-\boldsymbol{r}_1|^2Y_{2\mu}(\widehat{\boldsymbol{R}-\boldsymbol{r}_1}) \\ &= R^2Y_{2\mu}(\hat{\boldsymbol{R}}) + r^2_1Y_{2\mu}(\hat{\boldsymbol{r}}_1) \\ &\quad -\sqrt{4\pi}\sqrt{\frac{10}{3}}Rr_1[Y_1(\hat{\boldsymbol{R}})\times Y_1(\hat{\boldsymbol{r}}_1)]_{2\mu}. \end{aligned} \tag{48}$$

Expressing the tensor term in the NN interaction using Eqs. (45), (47), and (48) leads to three terms that can be folded using the general folding expression [Eq. (41)], in the same way as the central term in the NN interaction (for full details see [33]).

For ^{10}B this folding leads to four types of spin-spin interactions, $U^{\rm tr}=U^{\rm tr}_{10}+U^{\rm tr}_{12}+U^{\rm tr}_{32}+U^{\rm tr}_{34}$ (see Appendix C 2). These spin-spin interactions are also of the form given in Eq. (8). To denote that these spin-spin potentials come from the folding of the tensor term, the superscript tr is used.

The folding of the tensor $v^{\rm tr}_{NN}(r_{01})\boldsymbol{S}^{NN}_{12}(\boldsymbol{\sigma}_0,\boldsymbol{\sigma}_1,\hat{\boldsymbol{r}}_{01})$ term in the NN interaction also yields the spherical $U_{10}(\boldsymbol{\sigma}_0,\boldsymbol{I},R)$ and tensor $U_{12}(\boldsymbol{\sigma}_0,\boldsymbol{I},\boldsymbol{R})$ spin-spin interactions examined in previous work. Also obtained is the higher rank term, $U_{32}(\boldsymbol{\sigma}_0,\boldsymbol{I},\boldsymbol{R})$, from McAbee [24,25]. A new spin-spin interaction $U_{34}(\boldsymbol{\sigma}_0,\boldsymbol{I},\boldsymbol{R})$, which goes beyond the work of McAbee, is also derived for the first time.

C. Exchange terms

The general folding equation needs to be slightly modified for exchange terms in the NN interaction. Continuing the discussion for the valence proton (particle 1 in Fig. 1) the potential $V_{NN}(\boldsymbol{r}_{01})$ in Eq. (41) now becomes $\hat{V}_{NN}(\boldsymbol{r}_{01})P^r_{01}$. Substituting this into the general folding equation and acting with the operator P^r_{01}, changes the spatial coordinates of everything to the right of the potential, i.e., $Y_{\ell_1\lambda_1}(\hat{\boldsymbol{r}}_1)u_{\ell_1 j_1}(r_1)$ to $Y_{\ell_1\lambda_1}(\hat{\boldsymbol{R}})u_{\ell_1 j_1}(R)$, in Eq. (41). We also must remember that, in the calculation of the scattering amplitude, this folded potential is placed between two distorted waves and the exchange operator, P^r_{01}, will also act on the wave function to the right.

D. Central exchange NN interaction

The folding of the central SNKE term $\hat{v}^{\sigma\sigma}_{NN}(r_{01})\boldsymbol{\sigma}_0\cdot\boldsymbol{\sigma}_1P^r_{01}$ will be performed using the zero-range pseudopotential from Eq. (24). Inserting this into the modified (as detailed above) general folding equation yields

$$\begin{aligned} &\hat{U}^{\sigma\sigma}(\boldsymbol{\sigma}_0,\boldsymbol{I},\boldsymbol{R}) \\ &= (-)^{3j_1-j'_1-\ell'_1}\hat{J}_{\sigma\sigma}(Q)\sqrt{\frac{6}{4\pi}}\hat{I}\hat{j}_1\hat{j}'_1 \\ &\quad\times\hat{\ell}_1\hat{\ell}'_1u_{\ell'_1j'_1}(R)u_{\ell_1j_1}(R)\sum_{k_Ik}\frac{(-)^k\hat{k}_I}{\hat{k}}(\ell_1 0\ell'_1 0|k0) \\ &\quad\times W(k_Ij_1Ij_2;j'_1I)\begin{Bmatrix} j_1 & j'_1 & k_I \\ \frac{1}{2} & \frac{1}{2} & 1 \\ \ell_1 & \ell'_1 & k\end{Bmatrix}\boldsymbol{S}_{ik}(\boldsymbol{\sigma}_0,\boldsymbol{I},\hat{\boldsymbol{R}}), \end{aligned} \tag{49}$$

where the full details of this folding are given in Ref. [33]. Using our model of ^{10}B gives three different types of spin-spin interaction $\hat{U}^{\sigma\sigma}=\hat{U}^{\sigma\sigma}_{10}+\hat{U}^{\sigma\sigma}_{12}+\hat{U}^{\sigma\sigma}_{32}$ (see Appendix C 3), where the hat denotes that these folded potentials are derived from the central spin-spin *exchange* term in the NN interaction. These three spin-spin interactions are the same type as those obtained from the direct central term.

E. Tensor exchange NN interaction

The contribution of the tensor SNKE term $\hat{v}^{\rm tr}_{NN}(r_{01})\boldsymbol{S}^{NN}_{12}(\boldsymbol{\sigma}_0,\boldsymbol{\sigma}_1,\hat{\boldsymbol{r}}_{01})P^r_{01}$ to the folded spin-spin interactions will be approximated using the zero-range pseudopotential from Eq. (29)

$$\begin{aligned} &\hat{J}_{\rm tr}(Q)\big[\hat{\boldsymbol{S}}^{NN}_{12}(\boldsymbol{\sigma}_0,\boldsymbol{\sigma}_1,-\imath\nabla_{\boldsymbol{r}_{01}})\delta(\boldsymbol{r}_{01})\big]P^r_{01} \\ &\quad = 2\sqrt{\frac{8\pi}{5}}\hat{J}_{\rm tr}(Q)\big[\tau_2(\boldsymbol{S})\cdot\mathscr{Y}_2\big(-\imath\nabla_{\boldsymbol{r}_{01}}\big)\delta(\boldsymbol{r}_{01})\big]P^r_{01}, \end{aligned} \tag{50}$$

where we have defined the operator $\hat{\boldsymbol{S}}^{NN}_{12}(\boldsymbol{\sigma}_0,\boldsymbol{\sigma}_1,\boldsymbol{A}) = A^2\boldsymbol{S}^{NN}_{12}(\boldsymbol{\sigma}_0,\boldsymbol{\sigma}_1,\hat{\boldsymbol{A}})$. The relationship between these two different forms of the pseudopotential is analogous to that derived for the direct tensor term operator. The second form, where the spin and radial dependence of the operator have been separated, is more suitable for manipulation in the folding model and will therefore be used for the evaluation of this term. The solid spherical harmonic $\mathscr{Y}_{2\mu}(-\imath\nabla_{\boldsymbol{r}_{01}})$ described in [45] is related to the regular spherical harmonic by

$$\mathscr{Y}_{2\mu}(\boldsymbol{R}) = R^2Y_{2\mu}(\hat{\boldsymbol{R}}). \tag{51}$$

The solid spherical harmonic $\mathscr{Y}_{2\mu}(-\imath\nabla_{\boldsymbol{r}_{01}})$ can be expressed in terms of the grad operator $\nabla_{\boldsymbol{r}_{01}}$ using the coupling rule for

two spherical harmonics, from [26],

$$\mathscr{Y}_{2\mu}(-\imath\nabla_{\boldsymbol{r}_{01}}) = \frac{-\sqrt{5}}{\sqrt{4\pi}}\sqrt{\frac{3}{2}}\sum_{\mu_2\mu_3}(1\mu_2 1\mu_3|2\mu)\nabla_{\mu_2}\nabla_{\mu_3}. \quad (52)$$

Including Eq. (47), we now have all the components we need to fold the tensor exchange term in the NN interaction using the modified general folding equation. However, we must now also consider that the operation of P^r_{01} is not limited to the initial ground state valence nucleon wave function $\Psi_{I,M_I}(\boldsymbol{r}_1,\boldsymbol{r}_2)$. In the calculation of the scattering amplitude the initial distorted wave is also to the right of this operator (see Sec. III B). If we take all the coordinate-dependent parts of this folding calculation and insert them into Eq. (9) we obtain

$$\hat{f}^{\rm tr}_{\rm co}(\mu',M'_I;\mu,M_I;\theta,\phi) = \frac{-\mu_{\rm pt}}{2\pi\hbar^2}\langle\chi^{(-)}_{\mu'}(\boldsymbol{R},\boldsymbol{k}'),IM'_I|\int d\boldsymbol{r}_1 Y^*_{\ell'_1\lambda'_1}(\hat{\boldsymbol{r}}_1)u_{\ell'_1 j'_1}(r_1)\left[\mathscr{Y}_{2\mu}\left(-\imath\nabla_{\boldsymbol{r}_{01}}\right)\delta(\boldsymbol{r}_{01})\right]Y_{\ell_1\lambda_1}(\hat{\boldsymbol{R}})u_{\ell_1 j_1}(R)|\chi^{(+)}_{\mu}(\boldsymbol{r}_1,\boldsymbol{k}),IM_I\rangle, \quad (53)$$

where $P^r_{01}\ \chi^{(+)}_{\mu}(\boldsymbol{R},\boldsymbol{k}) = \chi^{(+)}_{\mu}(\boldsymbol{r}_1,\boldsymbol{k})$. For the central exchange term this consideration was not required. In this case the properties of the δ function changes $\chi^{(+)}_{\mu}(\boldsymbol{r}_1,\boldsymbol{k})$ back to $\chi^{(+)}_{\mu}(\boldsymbol{R},\boldsymbol{k})$. Equation (53), gives the scattering amplitude for the tensor (denoted by the superscript tr) exchange (denoted by the hat $\hat{f}$) spin-spin interactions. We must remember however, that Eq. (53) does not give the full scattering amplitude for the tensor exchange spin-spin interactions. For the purposes of this discussion we have neglected all but the coordinate-dependent parts of the modified general folding equation. This is denoted by the subscript co.

In order to take advantage of the integral properties of the δ function the coordinates in Eq. (53) can be changed to $\boldsymbol{r}_1 = \boldsymbol{R} - \boldsymbol{r}_{01}$. Integrating by parts and using $\nabla^2_{\boldsymbol{r}_{01}} f(\boldsymbol{R} - \boldsymbol{r}_{01}) = \nabla^2_{\boldsymbol{R}} f(\boldsymbol{R} - \boldsymbol{r}_{01})$, finally allows us to integrate using the δ function resulting in

$$\hat{f}^{\rm tr}_{\rm co}(\mu',M'_I;\mu,M_I;\theta,\phi) = \frac{-\mu_{\rm pt}}{2\pi\hbar^2}\langle\chi^{(-)}_{\mu'}(\boldsymbol{R},\boldsymbol{k}'),IM'_I|Y_{\ell_1\lambda_1}(\hat{\boldsymbol{R}})u_{\ell_1 j_1}(R)\mathscr{Y}_{2\mu}(-\imath\nabla_{\boldsymbol{R}})Y^*_{\ell'_1\lambda'_1}(\hat{\boldsymbol{R}})u_{\ell'_1 j'_1}(R)|\chi^{(+)}_{\mu}(\boldsymbol{R},\boldsymbol{k}),IM_I\rangle, \quad (54)$$

in a symmetry with the direct tensor folded potential, this can be split into three terms: Term A, denoted by $^{\rm trA}$, where the solid spherical harmonic acts on the components of the valence nucleon wave function $[Y^*_{\ell'_1\lambda'_1}(\hat{\boldsymbol{R}})u_{\ell'_1 j'_1}(R)]$; Term B, denoted by $^{\rm trB}$, where it acts on the initial distorted wave $[\chi^{(+)}_{\mu}(\boldsymbol{R},\boldsymbol{k})]$; and Term C, denoted by $^{\rm trC}$, where a rank-one grad operator acts on both using Eq. (52). The total tensor exchange spin-spin interaction is the combination of these three terms. For full details of this folding see Ref. [33].

The folding of Term A, where the solid harmonic acts only on the components of the final valence nucleon wave function, $[\mathscr{Y}_{2\mu}(-\imath\nabla_{\boldsymbol{R}})Y^*_{\ell'_1\lambda'_1}(\hat{\boldsymbol{R}})u_{\ell'_1 j'_1}(R)]$, and not on the distorted waves, leads to spin-spin interactions of the same form, $U_{k_I k}(\boldsymbol{\sigma}_0,\boldsymbol{I},\boldsymbol{R}) = F_{k_I k}(R)\boldsymbol{S}_{k_I k}(\boldsymbol{\sigma}_0,\boldsymbol{I},\hat{\boldsymbol{R}})$, as all the other folded potentials derived previously. The spin-spin amplitude can therefore be calculated using the method described in Sec. III B.

The spin-spin interactions derived from this term are $\hat{U}^{\rm trA} = \hat{U}^{\rm trA}_{10} + \hat{U}^{\rm trA}_{12} + \hat{U}^{\rm trA}_{22} + \hat{U}^{\rm trA}_{32} + \hat{U}^{\rm trA}_{34}$, see Appendix C 4. The hat denotes that these spin-spin potentials are from the folding of an *exchange* term.

For the first time, a spin-spin interaction is obtained that has even rank in $\boldsymbol{I}$ ($k_I = 2$). The nonlocal nature of the tensor exchange term in the NN interaction means it is no longer straightforward to check the parity and time-reversal invariance of the resulting folded potentials. While the whole term (including parts A, B, and C) must be invariant, the individual terms need not be. This was fully investigated in Section 5.52 of [33] and the whole term was proven to be invariant under parity and time reversal.

Term B, in which the solid spherical harmonic acts on the incident distorted wave alone $[\mathscr{Y}_{2\mu}(-\imath\nabla_{\boldsymbol{R}})\chi^{(+)}_{\mu}(\boldsymbol{R},\boldsymbol{k})]$, with the valence nucleon wave function unaffected, yields a folded potential of the form

$$\hat{U}^{\rm trB}_{k_I k}(\boldsymbol{\sigma}_0,\boldsymbol{I},\boldsymbol{R})\chi^{(+)}_{\mu}(\boldsymbol{R},\boldsymbol{k}) = \sum_{k_I k}\hat{F}^{\rm trB}_{k_I k}(R)[Y_k(\hat{\boldsymbol{R}})\times\tau_{k_I}(\boldsymbol{I})]\cdot[\tau_1(\boldsymbol{\sigma}_0)\times\mathscr{Y}_2(-\imath\nabla_{\boldsymbol{R}})]\chi^{(+)}_{\mu}(\boldsymbol{R},\boldsymbol{k}), \quad (55)$$

where the folded potentials from this tensor exchange term will be denoted by $\hat{U}^{\rm trB}_{k_I k}(\boldsymbol{\sigma}_0,\boldsymbol{I},\boldsymbol{R})$. They are different from those derived previously because, instead of the operator $\boldsymbol{S}_{k_I k}(\boldsymbol{\sigma}_0,\boldsymbol{I},\hat{\boldsymbol{R}})$, these spin-spin interactions are proportional to $[Y_k(\hat{\boldsymbol{R}})\times\tau_{k_I}(\boldsymbol{I})]\cdot[\tau_1(\boldsymbol{\sigma}_0)\times\mathscr{Y}_2(-\imath\nabla_{\boldsymbol{R}})]$. The spin-spin potentials from this term $\hat{U}^{\rm trB} = \hat{U}^{\rm trB}_{10} + \hat{U}^{\rm trB}_{12} + \hat{U}^{\rm trB}_{32}$ are described in Appendix C 5. The solid spherical harmonics in these equations act on the initial distorted wave function [which has been left in Eq. (55) for clarity]. This means the spin-spin amplitude given in Eq. (10) cannot be used for this interaction. The modification of the DWBA scattering amplitude for these potential terms will be discussed in the next subsection.

The folding of the third tensor exchange term, which acts one grad on the components of the valence nucleon wave function and one grad on the distorted wave gives a spin-spin potential of the form

$$\hat{U}^{\mathrm{trC}}_{k_I k E}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R})\chi^{(+)}_{\mu}(\boldsymbol{R}, \boldsymbol{k}) = \sum_{k_I k E} \hat{F}^{\mathrm{trC}}_{k_I k E}(R)[Y_k(\hat{\boldsymbol{R}}) \times \tau_1(\boldsymbol{\sigma}_0)]_{E\epsilon}\left[\tau_{k_I}(\boldsymbol{I}) \times \nabla\right]_{E-\epsilon}\chi^{(+)}_{\mu}(\boldsymbol{R}, \boldsymbol{k}). \tag{56}$$

The folded potentials from this tensor exchange term will be denoted by $\hat{U}^{\mathrm{trC}}_{k_I k E}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R})$ where the extra subscript E represents the coupling between the operators $[Y_k(\hat{\boldsymbol{R}}) \times \tau_1(\boldsymbol{\sigma}_0)]$ and $[\tau_{k_I}(\boldsymbol{I}) \times \nabla]$. Performing the sums in Eq. (56) with the angular momentum values from the simplified model of ^{10}B leads to eleven different types of spin-spin interactions, which are detailed in Appendix C 6. These folded potentials also have a grad operator acting on the initial distorted wave function. The modification of the spin-spin amplitude formalism for these spin-spin interactions will be discussed next.

F. Modified scattering amplitudes

The folded spin-spin potentials derived from two of the three tensor exchange terms in the NN interaction have differential operators which act on the initial distorted wave. This means that the DWBA scattering amplitude for the local spin-spin interactions, given by Eq. (9), can no longer be used. The modified scattering amplitude for tensor exchange term B and C are given by

$$\hat{f}^{\mathrm{trB}}_{k_I k}(\mu', M'_I; \mu, M_I; \theta, \phi) = \frac{-\mu_{\mathrm{c.m.}}}{2\pi\hbar^2}\langle\chi^{(-)}_{\mu'}(\boldsymbol{R}, \boldsymbol{k}'), I M'_I|\hat{F}^{\mathrm{trB}}_{k_I k}(R)[Y_k(\hat{\boldsymbol{R}}) \times \tau_{k_I}(\boldsymbol{I})] \cdot [\tau_1(\boldsymbol{\sigma}_0) \times \mathscr{Y}_2(-\imath\nabla_{\boldsymbol{R}})]|\chi^{(+)}_{\mu}(\boldsymbol{R}, \boldsymbol{k}), I M_I\rangle, \tag{57}$$

$$\hat{f}^{\mathrm{trC}}_{k_I k}(\mu', M'_I; \mu, M_I; \theta, \phi) = \frac{-\mu_{\mathrm{c.m.}}}{2\pi\hbar^2}\langle\chi^{(-)}_{\mu'}(\boldsymbol{R}, \boldsymbol{k}'), I M'_I|\sum_E \hat{F}^{\mathrm{trC}}_{k_I k E}(R)[Y_k(\hat{\boldsymbol{R}})\tau_1(\boldsymbol{\sigma}_0)]_{E\epsilon}[\tau_{k_I}(\boldsymbol{I}) \times \nabla]_{E-\epsilon}|\chi^{(+)}_{\mu}(\boldsymbol{R}, \boldsymbol{k}), I M_I\rangle. \tag{58}$$

In order to evaluate these amplitudes the effect of $\mathscr{Y}_{2\mu}(-\imath\nabla_{\boldsymbol{R}})$ and ∇ on the initial distorted wave $\chi^{(+)}_{\mu}(\boldsymbol{R}, \boldsymbol{k})$ must be determined. This can be done using partial wave expansions of the distorted waves and results from [26,28,46] to evaluate the required derivatives. The complete results for the partial wave expansion of the amplitudes $\hat{f}^{\mathrm{trB}}_{k_I k}$ and $\hat{f}^{\mathrm{trC}}_{k_I k}$ are given in [33]. Choosing the incident beam direction to be along the z-axis with the y-axis perpendicular to the scattering plane (along $\boldsymbol{k} \times \boldsymbol{k}'$) yields this equation for the modified scattering amplitude for tensor exchange term B

$$\begin{aligned}
&f^{\mathrm{trB}}_{k_I k}(\mu', M'_I; \mu, M_I; \theta) \\
&= \frac{3\sqrt{30}}{E_{\mathrm{c.m.}}\sqrt{4\pi}}\hat{k}_I\hat{k}\sum_{\ell', \ell, j', j}(-)^{j-\ell'+1/2-2\mu-q_I}\hat{\ell}\hat{j}\imath^{\ell-\ell'}e^{\imath(\sigma_\ell+\sigma_{\ell'})}Y_{\ell', \mu-q_I-\mu'}(\hat{\boldsymbol{k}}')\left(\ell 0\tfrac{1}{2}\mu\middle|j\mu\right) \\
&\times\left(\ell'\mu-q_I-\mu'\ \tfrac{1}{2}\mu'\middle|j'\mu-q_I\right)(I M_I k_I q_I|I M'_I)(k_I-q_I j\mu|j'\mu-q_I)\int_0^\infty R dR u_{\ell', j'}(k_{\mathrm{c.m.}}, R)\hat{F}^{\mathrm{trB}}_{k_I k}(R) \\
&\times\sum_{LE}\left[\hat{\mathscr{O}}_{L\ell}\ \frac{u_{\ell, j}(k_{\mathrm{c.m.}}, R)}{R}\right]\hat{L}^2(20L0|\ell 0)(k0L0|\ell' 0)\hat{E}^2 W\left(k_I E j'\tfrac{1}{2}; \ell' j\right)\begin{Bmatrix} k_I & E & \ell' \\ L & k & 1\end{Bmatrix}\begin{Bmatrix} j & E & \frac{1}{2} \\ 1 & \frac{1}{2} & \ell\end{Bmatrix}\begin{Bmatrix} 1 & E & \ell \\ L & 2 & 1\end{Bmatrix},
\end{aligned} \tag{59}$$

where the operator $\hat{\mathscr{O}}_{L\ell}$ is taken from [46,47] and is defined in Appendix C 4. Substituting the distorted wave function and folded potential into Eq. (58) and using the same choice of axes as term B yields

$$\begin{aligned}
&f^{\mathrm{trC}}_{k_I k}(\mu', M'_I; \mu, M_I; \theta) \\
&= \frac{-\sqrt{6}}{E_{\mathrm{c.m.}}}\hat{k}_I\hat{k}\sum_{\ell', \ell, j', j}\hat{\ell}^2\hat{j}\imath^{\ell-\ell'}(-)^{\ell+j-j'+2\mu-k_I-q_I}e^{\imath(\sigma_\ell+\sigma_{\ell'})}Y_{\ell', \mu-q_I-\mu'}(\hat{\boldsymbol{k}}')\left(\ell 0\tfrac{1}{2}\mu\middle|j\mu\right) \\
&\times\left(\ell'\mu-q_I-\mu'\ \tfrac{1}{2}\mu'\middle|j'\mu-q_I\right)(I M_I k_I q_I|I M'_I)(k_I-q_I j\mu|j'\mu-q_I)\int_0^\infty R dR u_{\ell', j'}(k_{\mathrm{c.m.}}, R)\sum_{EL}\hat{F}^{\mathrm{trC}}_{k_I k E}(R) \\
&\times\left[\hat{g}_{L\ell}\ \frac{u_{\ell, j}(k_{\mathrm{c.m.}}, R)}{R}\right](-)^{-L}\hat{L}\hat{E}^2(\ell 010|L0)(k0L0|\ell' 0)\sum_{E'}\hat{E'}^2 W\left(j\tfrac{1}{2}E'1; \ell\tfrac{1}{2}\right)W\left(j\tfrac{1}{2}k_I\ell'; E' j'\right)\begin{Bmatrix} 1 & \ell & E' \\ E & 1 & k_I \\ k & L & \ell'\end{Bmatrix},
\end{aligned} \tag{60}$$

where the operator $\hat{g}_{L\ell}$ is also taken from [46] and is defined in Appendix C 4.

Sections V A–V F have described all the formalism developed in order to perform a microscopic calculation of our spin-spin interactions. These components have been used to calculate the spin-spin potentials for 200 MeV protons elastically scattered from ^{10}B. The choice of single-particle wave function used in these calculations is given in the next subsection.

G. Single-particle wave function

The single particle wave functions for the valence proton and neutron in the $1p_{\frac{3}{2}}$ shell of ^{10}B are calculated from a Woods-Saxon (WS) potential and will be referred to here as a 'WS wave function'. This wave function is calculated using a binding energy of 7.0 MeV and a WS potential with parameters $R_{\rm WS} = (0.88 \pm 0.13)(A-1)^{1/3}$ fm and $a = 0.81 \pm 0.08$ fm for the diffuseness, taken from Ref. [48] who obtain the $1p_{\frac{3}{2}}$ wave function from the transform of the electron scattering form factor of the 1.740 MeV excited state in ^{10}B. The single-particle wave function of the valence proton incorporates the effect of the Coulomb interaction with the core, making it slightly different from the neutron wave function.

VI. RESULTS FOR D_{NN}

Our folding model has been used to construct new spin-spin tensor interactions within a simple two valence-particle model for the target nucleus, using a realistic nucleon-nucleon interaction that includes direct and exchange terms. This has led to higher order couplings of the target and projectile spins being incorporated into this calculation. In addition, spin-spin interactions from nonlocal tensor exchange contributions to the NN interaction have also been included. We will now show how our folding model and new spin-spin tensor interactions effect D_{NN}.

A. Effects of spin-spin interactions on D_{NN}

All our results are collected in Fig. 2. The separate contributions from terms of different physical origin are described in the following subsections.

1. Contributions from direct terms

The dashed curve shows the calculation of D_{NN} for all the spin-spin interactions obtained from the folding of the 'direct' terms in the NN interaction. It therefore includes contributions to the folded potentials from both the central $v^{\sigma\sigma}_{NN}(r)\boldsymbol{\sigma}_0\cdot\boldsymbol{\sigma}_1 + v^{\sigma\sigma}_{NN}(r)\boldsymbol{\sigma}_0\cdot\boldsymbol{\sigma}_2$ and tensor $v^{\rm tr}_{NN}(r)\boldsymbol{S}^{NN}_{12}(\boldsymbol{\sigma}_0,\boldsymbol{\sigma}_1,\hat{\boldsymbol{r}}) + v^{\rm tr}_{NN}(r)\boldsymbol{S}^{NN}_{12}(\boldsymbol{\sigma}_0,\boldsymbol{\sigma}_2,\hat{\boldsymbol{r}})$ terms. The spin-spin potentials for each k_I and k are therefore given by $U_{k_Ik} = U^{\sigma\sigma}_{k_Ik} + U^{\rm tr}_{k_Ik}$. These interactions are given in Eqs. (C1) to (C7). The D_{NN} from the $U^{\rm tr}_{k_Ik}(\boldsymbol{\sigma}_0,\boldsymbol{I},\boldsymbol{R})$ potentials is not plotted separately as their contribution is negligible compared to the $U^{\sigma\sigma}_{k_Ik}(\boldsymbol{\sigma}_0,\boldsymbol{I},\boldsymbol{R})$ potentials.

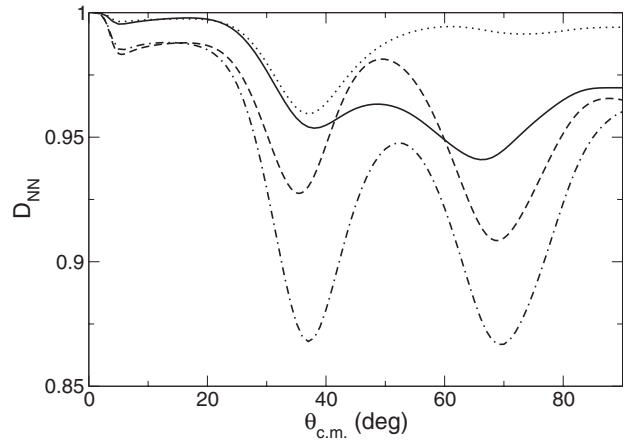

FIG. 2. D_{NN} for elastic proton scattering from ^{10}B, calculated using all spin-spin interactions (solid curve). The individual contributions for the direct terms (dashed curve), central exchange (dot-dash curve), and tensor exchange (dotted curve) are also shown.

2. Contributions from central exchange terms

The dot-dash curve in Fig. 2 shows D_{NN} for all the spin-spin interactions derived from the folding of the central single-nucleon knock-on 'exchange' (SNKE) term, $\hat{v}^{\sigma\sigma}_{NN}(r_{01})\boldsymbol{\sigma}_0\cdot\boldsymbol{\sigma}_1 P^r_{01} + \hat{v}^{\sigma\sigma}_{NN}(r_{02})\boldsymbol{\sigma}_0\cdot\boldsymbol{\sigma}_2 P^r_{02}$, using the zero-range pseudopotential from Eq. (24). This folding gives spin-spin interactions of the form $U_{k_Ik}(\boldsymbol{\sigma}_0,\boldsymbol{I},\boldsymbol{R}) = F_{k_Ik}(R)\boldsymbol{S}_{k_Ik}(\boldsymbol{\sigma}_0,\boldsymbol{I},\hat{\boldsymbol{R}})$, the same form as the direct terms. The form factors from this folding, $\hat{F}^{\sigma\sigma}_{k_Ik}(R)$, are given in Eqs. (C8) to (C10) for protons elastically scattered from ^{10}B and are simply proportional to the square of the single particle wave function $u_{1\frac{3}{2}}(r)$.

These results use $\hat{J}_{\sigma\sigma}(Q)$, which is evaluated for the simple case of $Q = k_{\rm c.m.}$. This gives $\hat{J}^{pp}_{\sigma\sigma}(Q) = -134.775 + 77.363\imath$ MeV fm^3 for like particles and $\hat{J}^{pn}_{\sigma\sigma}(Q) = -75.395 - 5.501\imath$ MeV fm^3 for unlike particles. The effect of a θ-dependent Q on $\hat{J}_{\sigma\sigma}(Q)$ is discussed in Sec. VI B.

3. Cancellation of direct and exchange central terms

The D_{NN} from the folding of the central exchange term has the most significant deviation from unity. However, the opposite signs of the spin-spin potentials from the folding of the direct $v^{\sigma\sigma}_{NN}(r)\boldsymbol{\sigma}_0\cdot\boldsymbol{\sigma}_1 + v^{\sigma\sigma}_{NN}(r)\boldsymbol{\sigma}_0\cdot\boldsymbol{\sigma}_2$ and exchange $\hat{v}^{\sigma\sigma}_{NN}(r)\boldsymbol{\sigma}_0\cdot\boldsymbol{\sigma}_1 P^r_{01} + \hat{v}^{\sigma\sigma}_{NN}(r)\boldsymbol{\sigma}_0\cdot\boldsymbol{\sigma}_2 P^r_{02}$ terms in the NN interaction leads to considerable cancellation. The cause of this can be found by examining how SNKE is formally included in the folding model using Eqs. (19) and (20). The direct interaction $v^{\sigma\sigma}_{NN}(r)$ equals the exchange potential $\hat{v}^{\sigma\sigma}_{NN}(r)$ except that in the latter the sign of the odd relative angular momentum state terms is changed, this leads to significant cancellation between the central direct and exchange terms especially for the odd-state terms. This is why the D_{NN} calculated using all the spin-spin interactions (solid curve in Fig. 2) deviates less from unity than that including each of the central direct and central exchange contributions alone. If the potentials for the direct and exchange odd state terms were replaced with a zero range δ function they would cancel completely [28].

4. Contributions from tensor exchange terms

The dotted curve in Fig. 2 shows the calculation of D_{NN} for all the spin-spin interactions obtained from the folding of the tensor SNKE term, $\hat{v}^{\rm tr}_{NN}(r_{01})\boldsymbol{S}^{NN}_{12}(\boldsymbol{\sigma}_0,\boldsymbol{\sigma}_1,\hat{\boldsymbol{r}}_{01})P^r_{01} + \hat{v}^{\rm tr}_{NN}(r_{02})\boldsymbol{S}^{NN}_{12}(\boldsymbol{\sigma}_0,\boldsymbol{\sigma}_2,\hat{\boldsymbol{r}}_{02})P^r_{02}$, and has the smallest deviation from unity. Each of the spin-spin interactions derived in Sec. V E depends on the coefficient $\hat{J}_{\rm tr}(Q)$. The results in Fig. 2 use $\hat{J}_{\rm tr}(Q)$ evaluated for $Q = k_{\rm c.m.}$. This gives, $\hat{J}^{pp}_{\rm tr}(Q) = 3.541 - 2.267\imath$ MeV fm^5 for like particles and $\hat{J}^{pn}_{\rm tr}(Q) = 3.373 - 1.563\imath$ MeV fm^5 for unlike particles. The effect of a θ-dependent Q on $\hat{J}_{\rm tr}(Q)$ will be examined next.

B. Improved treatment of exchange: The choice of Q

So far in this chapter the coefficients $\hat{J}_{\sigma\sigma}(Q)$ and $\hat{J}_{\rm tr}(Q)$ from the zero-range pseudopotentials used to approximate the SKNE terms in the NN interaction have been calculated using the approximation $Q = k_{\rm c.m.}$. This is a reasonable choice for scattering in the forward direction. However the measurement of D_{NN} for 200 MeV protons elastically scattering from ^{10}B detailed in [2] shows that the deviation of D_{NN} from unity is most significant at large angles. Therefore in Sec. IV D the details of an approximation for the magnitude of $\boldsymbol{Q}$ was given that does better at large angles, $Q = k_{\rm c.m.}\cos(\theta/2)$.

Figure 3 shows as example of the $\hat{J}_{\rm tr}(Q)$ coefficient for the interaction between unlike nucleons 'pn', for the two different approximations of Q. The introduction of a θ-dependent Q changes the values of $\hat{J}_{\sigma\sigma}(Q)$ and $\hat{J}_{\rm tr}(Q)$ significantly as shown in Table I for $\theta = 0$ and $\theta = 90$.

The calculation of the polarization transfer coefficient D_{NN} is plotted in Fig. 4 for all the spin-spin interactions derived in this work. The solid curve (is the same as the solid curve in Fig. 2) shows D_{NN} evaluated using $Q = k_{\rm c.m.}$ to calculate the spin-spin interactions from the folding of the central and tensor exchange terms in the NN interaction. The dashed curve in Fig. 4 shows the same calculation, but using $Q = k_{\rm c.m.}\cos(\theta/2)$.

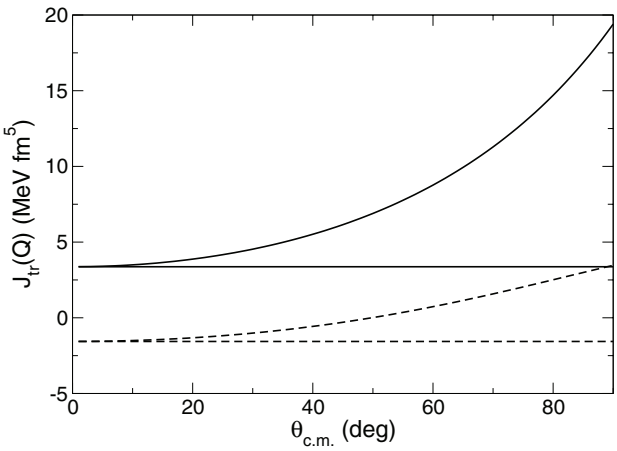

FIG. 3. The exchange coefficient $\hat{J}_{\rm tr}(Q)$ for $Q = k_{\rm c.m.}$ (straight lines) and $Q = k_{\rm c.m.}\cos(\theta/2)$ (curved lines) for unlike 'pn' NN interactions, real (solid lines) and imaginary (dashed lines) parts of the coefficient.

TABLE I. Exchange coefficients $\hat{J}_{\sigma\sigma}(Q)$ and $\hat{J}_{\rm tr}(Q)$ at $\theta = 0$ and $\theta = 90$ for like (pp) and unlike (pn) particles.

		$\theta = 0$	$\theta = 90$
$\hat{J}_{\sigma\sigma}(Q)$	pp	$-134.775 + 77.363\imath$	$-85.900 + 101.759\imath$
MeV fm^3	pn	$-75.395 - 5.501\imath$	$-53.469 - 22.163\imath$
$\hat{J}_{\rm tr}(Q)$	pp	$3.541 - 2.267\imath$	$9.011 - 0.643\imath$
MeV fm^5	pn	$3.373 - 1.563\imath$	$19.401 + 3.463\imath$

Changing the approximation for Q from $Q = k_{\rm c.m.}$ to $Q = k_{\rm c.m.}\cos(\theta/2)$ has a significant effect at large angles. Figure 4 shows that the solid and dashed curves begin to deviate from one another at around 20° which is the angle where the exchange coefficients $\hat{J}_{\sigma\sigma}(Q)$ and $\hat{J}_{\rm tr}(Q)$ for the two different approximations of Q also begin to deviate from one another (see example in Fig. 3). The shape of the solid and dashed curves in Fig. 4 are very similar until around 60° when the solid curve ($Q = k_{\rm c.m.}$) move to smaller $1 - D_{NN}$ and the dashed curve [$Q = k_{\rm c.m.}\cos(\theta/2)$] move to larger $1 - D_{NN}$. At 90°, in Fig. 4, $D_{NN}(Q = k_{\rm c.m.}) = 0.970$ and $D_{NN}(Q = k_{\rm c.m.}\cos(\theta/2)) = 0.904$.

Figure 4 also shows the calculation of D_{NN} using the $Q = k_{\rm c.m.}\cos(\theta/2)$ approximation, but with no tensor exchange contributions (dot-dash curve). Without tensor exchange the calculation of D_{NN} is much closer to the solid curve ($Q = k_{\rm c.m.}$) and has a very similar shape. Comparing the solid curve and dashed curve indicates that using the $Q = k_{\rm c.m.}\cos(\theta/2)$ approximation leads to a significant increase in the deviation of D_{NN} from unity at large angles. The dot-dashed curve shows that it is through the tensor exchange term that this increase is achieved. The curves in Fig. 4 indicate that the effect of the Q approximation is greatest on the spin-spin interactions derived from the tensor exchange term in the NN interaction. It also signifies the importance of the approximation for Q on the calculation of an accurate D_{NN} at large angles.

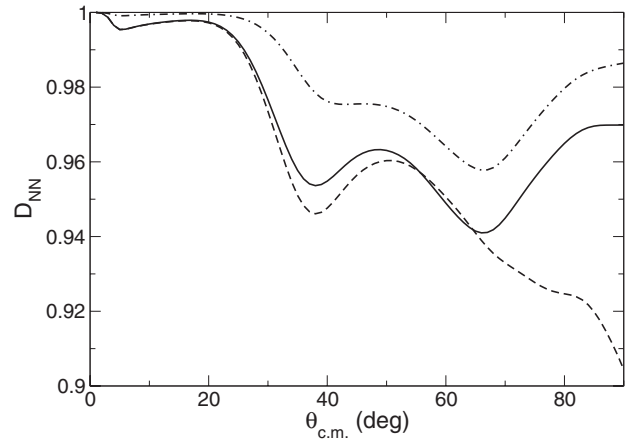

FIG. 4. D_{NN} for elastic proton scattering from ^{10}B, calculated using all spin-spin interactions. The two different approximations for Q are shown as $Q = k_{\rm c.m.}$ (solid curve) and $Q = k_{\rm c.m.}\cos(\theta/2)$ (dashed curve), also shown is the calculation without tensor exchange contributions and $Q = k_{\rm c.m.}\cos(\theta/2)$ (dot-dashed curve).

C. Effect of even k_I

The model of ^{10}B used in this work is a valence proton and neutron hole in the $1p_{\frac{3}{2}}$ shell. Appendix B shows that the only difference this makes to the folding formalism for valence proton and neutron particles occurs when the rank of the spin operator $\tau_{k_I q_I}(\boldsymbol{I})$ is even. When k_I is greater than zero the reduced matrix element $\tau^{M_I' M_I}_{k_I q_I}(\boldsymbol{I})$ must be multiplied by $(-)^{(k_I+1)}$ to change from a particle to a hole description [44].

The total spin-spin amplitude, $f(\mu', M_I'; \mu, M_I; \theta)$, can be written as

$$f(\mu', M_I'; \mu, M_I; \theta) = \sum_{k_I k} f_{k_I k}(\mu', M_I'; \mu, M_I; \theta), \tag{61}$$

where all the individual spin-spin amplitudes, $f_{k_I k}(\mu', M_I'; \mu, M_I; \theta, \phi)$, including those for the nonlocal tensor exchange terms, can be written as

$$f_{k_I k}(\mu', M_I'; \mu, M_I; \theta, \phi) = \sum_{q_I} \tau^{M_I' M_I}_{k_I q_I}(\boldsymbol{I}) \tilde{f}_{k_I q_I, k}(\mu'; \mu; \theta, \phi). \tag{62}$$

Here, $\tilde{f}_{k_I q_I, k}(\mu'; \mu; \theta, \phi)$ contains the partial wave sums, the integration over R and the spherical harmonic $Y_{\ell', \mu - q_I - \mu'}(\hat{\boldsymbol{k}}')$. Using the total spin-spin amplitude in Eq. (1) for D_{NN} gives

$$D_{NN} = \frac{\mathrm{Tr}\left[\sum_{k_I q_I, k, k_I' q_I', k'} \tau^{M_I' M_I}_{k_I q_I}(\boldsymbol{I}) \tilde{f}_{k_I q_I, k}(\mu'; \mu; \theta, \phi) \sigma_y \tau^{\dagger\, M_I' M_I}_{k_I' q_I'}(\boldsymbol{I}) \tilde{f}^{\dagger}_{k_I' q_I', k'}(\mu'; \mu; \theta, \phi) \sigma_y\right]}{\mathrm{Tr}\left[\sum_{k_I q_I, k, k_I' q_I', k'} \tau^{M_I' M_I}_{k_I q_I}(\boldsymbol{I}) \tilde{f}_{k_I q_I, k}(\mu'; \mu; \theta, \phi) \tau^{\dagger\, M_I' M_I}_{k_I' q_I'}(\boldsymbol{I}) \tilde{f}^{\dagger}_{k_I' q_I', k'}(\mu'; \mu; \theta, \phi)\right]}. \tag{63}$$

For Eq. (63) to describe holes rather than particles, the functions in the trace must be multiplied by $(-)^{1+k_I+1+k_I'}$ (top and bottom) for $k_I > 0$. This modification has no effect on the calculation of D_{NN} because of the following property of the spin operators

$$\mathrm{Tr}[\tau_{kq}(\boldsymbol{I}) \tau^{\dagger}_{k'q'}(\boldsymbol{I})] = (2I+1) \delta_{kk'} \delta_{qq'}. \tag{64}$$

Taking the trace over the spin projection of the target gives

$$D_{NN} = \frac{\mathrm{Tr}'\left[\sum_{k_I q_I, k, k_I' q_I', k'} \delta_{k_I k_I'} \delta_{q_I q_I'} \tilde{f}_{k_I q_I, k}(\mu'; \mu; \theta, \phi) \sigma_y \tilde{f}^{\dagger}_{k_I' q_I', k'}(\mu'; \mu; \theta, \phi) \sigma_y\right]}{\mathrm{Tr}'\left[\sum_{k_I q_I, k, k_I' q_I', k'} \delta_{k_I k_I'} \delta_{q_I q_I'} \tilde{f}_{k_I q_I, k}(\mu'; \mu; \theta, \phi) \tilde{f}^{\dagger}_{k_I' q_I', k'}(\mu'; \mu; \theta, \phi)\right]}, \tag{65}$$

where Tr′ is taken over the spin projections of the projectile alone. The phase change now becomes $(-)^{1+k_I+1+k_I'} \rightarrow (-)^{1+k_I+1+k_I} = 1$. So the phase change that results from transferring from the particle to hole model of ^{10}B does not change the polarization transfer coefficient D_{NN} or the cross section.

More generally, this result means that in the calculation of D_{NN} only cross terms in the scattering amplitude with the same rank in the target spin survive the trace over the spin projection of the target. Cross terms with different k_I will give zero.

To see how the deviation of D_{NN} from unity is related to the strengths of all the spin-spin interactions, the ratio of $1 - D_{NN}$ for $F_{k_I k}(R)$ and $0.5 \times F_{k_I k}(R)$ is plotted in Fig. 5. When the strength of the spin-spin potential changes by a factor of 2 the deviation of D_{NN} from 1 changes by a factor of 4. The dependence of $1 - D_{NN}$ on the strength of the spin-spin potentials is a second order one because only terms with the same rank in $\boldsymbol{I}$ can interfere with each other. This brings into question whether it is appropriate to use first order DWBA to calculated the spin-spin amplitudes.

Stamp [1] first presented the polarization transfer coefficient D_{NN} as the simplest way of experimentally confirming the existence of spin-spin interactions, a better approach may be to study first order effects of spin-spin interactions using another observable. Since D_{NN} is only dependent on the polarization of the projectile nucleon, an observable that is dependent on the polarization of the target would lead to a first order dependence on the strength of the spin-spin interactions. Such an observable would be a more sensitive probe of spin-spin interactions.

D. Sensitivity of D_{NN} to $u_{\ell j}(r)$

The potential parameters (from [48]) used to calculate the WS single-particle wave function discussed in Sec. V G have significant error bars, $R_{\mathrm{WS}} = (0.88 \pm 0.13)(A-1)^{1/3}$ fm and

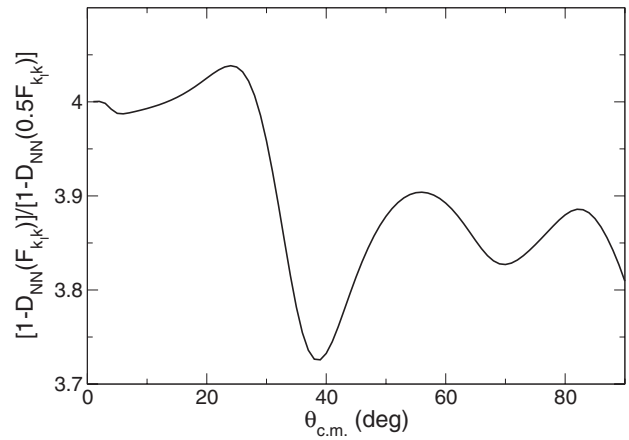

FIG. 5. Ratio of the deviation of D_{NN} from unity, including all the spin-spin interactions $F_{k_I k}(R)$ and $0.5 F_{k_I k}(R)$, direct and exchange, for elastic $p + {}^{10}$B scattering.

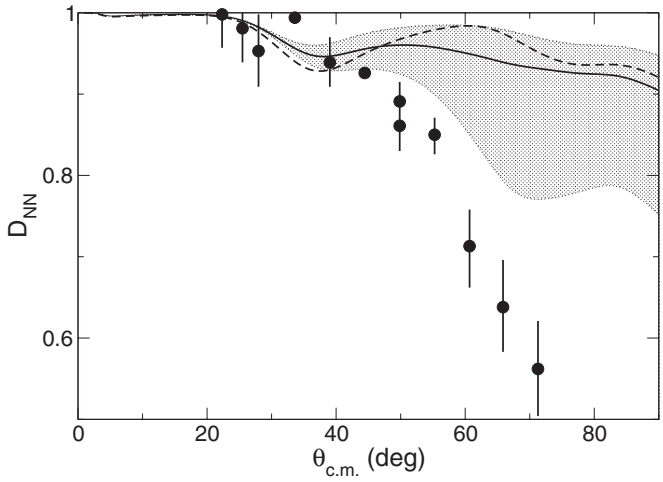

FIG. 6. Calculation of D_{NN}, for elastic proton scattering from ^{10}B using a WS (solid line) and HO (dashed line) single-particle wave function, compared to the measurement (symbols) of D_{NN} from [2]. The grey area shows the dependence of the calculation on the uncertainties in the WS parameters from [48] used to calculate the single particle wave function.

$a = 0.81 \pm 0.08$ fm. To examine the sensitivity of D_{NN} to the wave functions they were calculated using the maximum and minimum values for $R_{\rm WS}$ and a. The corresponding angular distributions of D_{NN} are shown by the shaded area in Fig. 6, compared to the experimentally measured values of D_{NN} from [2], denoted by the circular points.

The solid line in Fig. 6 shows the calculation of D_{NN} using the WS parameters $R_{\rm WS} = 0.88(A-1)^{1/3}$ fm and $a = 0.81$ fm to calculate the single-particle wave function and is the same as the dashed line in Fig. 4. The dashed line in Fig. 6 uses a harmonic oscillator (HO) wave function. The value of $\nu = 0.1966$ fm^{-2} used in this work is taken from Ref. [49] in which the harmonic oscillator constant, ν, was calculated for different p shell nuclei using Coulomb energy data. HO wave functions can be a reasonable approximation for low lying bound states and have the advantage that they can be integrated and differentiated analytically. However, in this case the D_{NN} calculated using the HO wave function varies quite significantly from the WS calculations at large angles. In our work, D_{NN} depends not only on the single-particle wave function, but also on the wave functions' first and second derivatives through the tensor exchange terms (also see Fig. 1 in [4]). This makes this polarization observable very sensitive to changes in the shape of the wave function $u_{1\frac{3}{2}}(r)$. This sensitivity may make D_{NN} a potentially useful probe of nuclear structure.

VII. CONCLUSIONS

We have developed a general model for spin-spin terms in the elastic nucleon-nucleus optical potential. The spin-spin potentials were generated by folding a realistic effective NN interaction over the nuclear wave function. The target is assumed to have a two valence-particle structure with the total spins of the two valence nucleons coupling to give the total spin of the nucleus. This general model has been used for the specific calculation of the elastic scattering of protons from ^{10}B.

Which spin-spin terms are present and how strong they are depends directly on the structure of the target. In the simple model of ^{10}B, the valence proton and neutron (both in $1p_{\frac{3}{2}}$ shells) couple together to give the total spin of the nucleus. In our folding model the total angular momenta of the valence nucleons j_1 and j_2, couple to give the maximum allowed rank, $k_I = 3$, of the target spin $\boldsymbol{I}$. The rank k of the spherical harmonic, $Y_k(\hat{\boldsymbol{R}})$, in the generalized local spin-spin operator is dependent on the orbital angular momentum of the levels occupied by the valence nucleons. This means we have included terms that go beyond previous work on spin-spin interactions and therefore, for valence nucleons in shells with larger orbital angular momentum, ℓ, and total spin, j, higher order spin-spin operators would be allowed. Our calculation of D_{NN} is not only sensitive to the occupation of different orbitals, but also to their radial wave functions. This sensitivity means polarization observables could be used as a probe of nuclear structure.

Comparing the deviations of D_{NN} from unity caused by spin-spin interactions derived from the direct and exchange terms in the NN interaction, show that they both contribute significantly. The approximation for the magnitude of $\boldsymbol{Q} = \boldsymbol{k}' + \boldsymbol{k}$ also has a considerable effect on the calculation of D_{NN} through the exchange terms. In particular, the tensor exchange contribution has been show to be particularly important at large angles, where the largest deviation of D_{NN} from unity is measured. However, while our calculations do lead to a significant deviation of D_{NN} from unity, they still do not match the experimental data from [2] at large angles. In future work we hope to examine the effect of spin-spin interactions from the folding of the spin-orbit term in the NN interaction and the small angle approximations used in the calculations of the exchange terms, to see if we can reproduce the experimental data more closely.

While Stamp [1] identified D_{NN} as the simplest way of experimentally verifying that spin-spin interactions exist, we have determined that this observable has a second-order dependence on the strength of the spin-spin potentials. This brings into question the validity of using the first order DWBA model to calculate the spin-spin amplitude as this approximation discards other second-order terms. Alternatively, to study first order effects of spin-spin interactions another observable could be investigated. D_{NN} is only dependent on the polarization of the projectile nucleon. An observable that is dependent on the polarization of the target would lead to a first order dependence on the strength of the spin-spin interactions. Such an observable would be a more sensitive probe of spin-spin interactions and will be the focus of future work [50].

ACKNOWLEDGMENTS

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APPENDIX A: GENERAL SPIN OPERATORS $\tau_{kq}(S)$

The operators $\tau_{1q_0}(\sigma_0)$ and $\tau_{k_I q_I}(I)$ are constructed out of the x, y and z components of their spin operators σ_0 and I, respectively. For a general spin, S, the relationships between the spin operator $\tau_{kq}(S)$ and the components S_x, S_y, and S_z can be derived using the following expression derived from Eqs. (1d) and (1e) in Hooton and Johnson [27]:

$$\hat{k}_1\hat{k}_2\hat{S}W(Ik_2Ik_1;Ik_3)\tau_{k_3q_3}(S) = \sum_{q_1q_2}(k_2q_2k_1q_1|k_3q_3)\tau_{k_1q_1}(S)\tau_{k_2q_2}(S), \quad \text{(A1)}$$

and the relations between components of vector tensors in the spherical and Cartesian coordinate systems

$$\tau_{1\pm1}(S) = \mp\frac{1}{\sqrt{2}}C(S)[S_x \pm \imath S_y], \quad \text{(A2)}$$

$$\tau_{10}(S) = C(S)S_z. \quad \text{(A3)}$$

The spin dependent coefficient $C(S)$ is the same for all $\tau_{1q}(S)$ because they rotate as spherical harmonics (Eq. (1c) [27]). Thus, $C(S)$ can be determined using the matrix element of $\tau_{10}(S)$

$$\begin{aligned}\langle SS|\tau_{10}(S)|SS\rangle &= \sqrt{3}(SS10|SS) = C(S)\langle SS|S_Z|SS\rangle \\ &= \sqrt{3}\sqrt{\frac{S}{S+1}} = C(S)S, \\ \rightarrow C(S) &= \sqrt{\frac{3}{S(S+1)}}. \end{aligned} \quad \text{(A4)}$$

Therefore

$$\begin{aligned}\tau_{1\pm1}(S) &= \mp\frac{1}{\sqrt{2}}\sqrt{\frac{3}{S(S+1)}}[S_x \pm \imath S_y]; \\ \tau_{10}(S) &= \sqrt{\frac{3}{S(S+1)}}S_z. \end{aligned} \quad \text{(A5)}$$

Using Eq. (A1) with $k_3 = 2$ and $k_1 = k_2 = 1$ leads to

$$\begin{aligned}\tau_{2\pm2}(S) &= \frac{3}{S(S+1)}\frac{1}{6\hat{S}W(S1S1;S2)}[S_x \pm \imath S_y]^2, \\ \tau_{2\pm1}(S) &= \mp\frac{3}{S(S+1)}\frac{1}{6\hat{S}W(S1S1;S2)} \\ &\quad \times [S_z(S_x \pm \imath S_y) + (S_x \pm \imath S_y)S_z], \\ \tau_{20}(S) &= \frac{3}{S(S+1)}\frac{1}{3\sqrt{6}\hat{S}W(S1S1;S2)}\left[3S_z^2 - S(S+1)\right]. \end{aligned} \quad \text{(A6)}$$

This method can be used to calculate all $\tau_{kq}(S)$ in terms of the x, y, and z components of their spin S. However, for this work it is far more convenient to use the spin operator $\tau_{kq}(S)$. Discussion of the spin-spin operator $S_{k_I k}(\sigma_0, I, \hat{R})$ in terms of the projectile and target spin operators σ_0 and I will be limited the spherical and tensor spin-spin terms to allow comparison with previous work.

APPENDIX B: TWO-PARTICLE–TWO-HOLE EQUIVALENCE

In this appendix we show that, for the purpose of calculating terms in the folding model interaction that are tensors of nonzero rank in the target spin, a two-particle or a two-hole model for ^{10}B give results that differ by a phase factor depending only on the rank of the tensor.

Our starting point is a general expression for the folding model interaction, $V^A(0)$ of projectile nucleon 0 with an A-nucleon target of spin I:

$$V^A_{I\,M'_I,I\,M_I}(0) = \int d1\,d1'\rho^A_{I\,M'_I,I\,M_I}(1',\,1)\langle\,1|v(0,1)|1'\rangle, \quad \text{(B1)}$$

where the one-nucleon density matrix, $\rho^A_{I\,M'_I,I\,M_I}(1',\,1)$, can be expanded as

$$\rho^A_I(1',1) = \sum_{k_I\,q_I}\frac{1}{\hat{k}_I}\tau_{k_I,q_I}(I)\sum_{b,b'}(-)^{j'_b+m'_b}(j_b\,m_b,\,j'_b\,-m'_b|k_I\,q_I)\phi^*_{b,j_b,m_b}(1)\phi_{b',j'_b,m'_b}(1')(I\,\|\rho_{k_I}(b,\,j_b,\,,b',\,j'_b)\|\,I). \quad \text{(B2)}$$

The quantities $(I\,\|\rho_{k_I}(b,\,j_b,\,,b',\,j'_b)\|\,I)$ are reduced matrix elements between target states $|\Psi_{I,M_I}\rangle$ of the tensor operators

$$\rho_{k_I,q_I}(b,\,j_b,b',\,j'_b) = \sum_{m_b,m'_b}(-)^{j'_b-m'_b}(j_b\,m_b,\,j'_b\,-m'_b|k_I\,q_I)a^\dagger_{b,j_b,m_b}a_{b',j'_b,m'_b}, \quad \text{(B3)}$$

where $a^\dagger_{b,j_b,m_b}$ and a_{b,j_b,m_b} are creation and destruction operators for a set of single particle states, $\phi_{b,j_b,m_b}(1)$, with definite angular momentum j_b, and b denotes other necessary quantum numbers, including isospin. Nuclear structure considerations enter through the choice of the ϕ_{b,j_b,m_b} and the value of the reduced matrix elements.

We first assume that the ground state of ^{10}B can be approximated as a one-neutron–one-proton hole in a filled $p_{3/2}$ shell that we label j:

$$|\Psi_{I,M_I}) = \sum_{m,m'}(-)^{j+m}(-)^{j+m'}(j\,m,\,j\,m'|I,\,M_I)a_{n,j,-m}a_{p,j,-m'}|\phi_0), \quad \text{(B4)}$$

where $|\phi_0)$ is the state vector describing the j shell filled with neutrons and protons and $I = 3$ in the ground state of ^{10}B. For this state we find

$$(I\,\|\rho_{k_I}(b,\,j,\,,b,\,j)\|\,I) = [\delta_{k_I,0}\delta_{q_I,0}\hat{j} - (-)^I\hat{I}\hat{k}_I W(I\,I\,j\,j;k_I\,j)]. \quad \text{(B5)}$$

An even simpler model, that is convenient in the detailed evaluation of the folding model and is used in the main text, is to assume that the ground state of ^{10}B consists of one neutron and one proton in a $p_{3/2}$ shell, $j = 3/2$, outside a state $|\phi_1)$ having a completely empty $p_{3/2}$ shell and completely filled other shells:

$$|\Psi_{I,M_I}, 1) = \sum_{m,m'} (j\, m,\, j\, m'|I,\, M_I) a^\dagger_{n,j,m} a^\dagger_{p,j,m'} |\phi_1). \tag{B6}$$

In this case the reduced matrix element is

$$(I, 1\|\rho_{k_I}(b, j, , b, j)\|I, 1) = [(-)^{k_I}(-)^I \hat{I}\hat{k}_I W(I\ I\ j\ j; k_I\ j)]. \tag{B7}$$

In both cases we have ignored any contributions from closed shells that do not contribute to the target spin and only contribute to tensors with $k_I = 0$. We see that for $k_I > 0$ the two models give the same reduced matrix elements and hence the same one-nucleon density matrix components apart from a phase factor $(-)^{(k_I+1)}$.

APPENDIX C: SPIN-SPIN POTENTIALS FOR ^{10}B

For the model of ^{10}B used in this work the valence nucleons remain in the $1p_{\frac{3}{2}}$ shell and therefore have initial and final angular momenta of $\ell_1 = \ell'_1 = 1$. These couple with the intrinsic $\frac{1}{2}$-spin of the nucleon to give total angular momenta of $j_1 = j'_1 = j_2 = \frac{3}{2}$ and $I = 3$. These values are used to calculate the following spin-spin potential terms.

1. Central direct term

Using our model of ^{10}B, the folding of the central direct term in the NN interaction [given in Eq. (44)] yields three different types of spin-spin interactions

$$U^{\sigma\sigma}_{10}(\boldsymbol{\sigma}_0, \boldsymbol{I}, R) = -2\sqrt{\frac{\pi}{3}} \int_0^\infty r_1^2 dr_1 u_{1\frac{3}{2}}(r_1)^2 \int_{-1}^1 d\mu P_0(\mu) v^{\sigma\sigma}_{NN}(r_{01}) \boldsymbol{S}_{10}(\boldsymbol{\sigma}_0, \boldsymbol{I}), \tag{C1}$$

$$U^{\sigma\sigma}_{12}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R}) = -\frac{2}{5}\sqrt{\frac{2\pi}{15}} \int_0^\infty r_1^2 dr_1 u_{1\frac{3}{2}}(r_1)^2 \int_{-1}^1 d\mu P_2(\mu) v^{\sigma\sigma}_{NN}(r_{01}) \boldsymbol{S}_{12}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \hat{\boldsymbol{R}}), \tag{C2}$$

$$U^{\sigma\sigma}_{32}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R}) = \frac{3}{5}\sqrt{\frac{2\pi}{5}} \int_0^\infty r_1^2 dr_1 u_{1\frac{3}{2}}(r_1)^2 \int_{-1}^1 d\mu P_2(\mu) v^{\sigma\sigma}_{NN}(r_{01}) \boldsymbol{S}_{32}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \hat{\boldsymbol{R}}), \tag{C3}$$

where $r_{01} = |\boldsymbol{R} - \boldsymbol{r}_1|$.

2. Tensor direct all terms

Summing the potentials from all three tensor terms yields the spin-spin interactions from the folding of the whole tensor term in the NN interaction for protons elastically scattering from ^{10}B:

$$U^{\rm tr}_{10}(\boldsymbol{\sigma}_0, \boldsymbol{I}, R) = \frac{4}{15}\sqrt{3\pi} \int_0^\infty r_1^2 dr_1 u_{1\frac{3}{2}}(r_1)^2 \int_{-1}^1 d\mu \big[r_1^2 P_0(\mu) + R^2 P_2(\mu) - 2Rr_1 P_1(\mu)\big] \frac{v^{\rm tr}_{NN}(r_{01})}{r_{01}^2} \boldsymbol{S}_{10}(\boldsymbol{\sigma}_0, \boldsymbol{I}), \tag{C4}$$

$$U^{\rm tr}_{12}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R}) = \frac{2}{15}\sqrt{\frac{6\pi}{5}} \int_0^\infty r_1^2 dr_1 u_{1\frac{3}{2}}(r_1)^2 \int_{-1}^1 d\mu \left[5R^2 P_0(\mu) + \big(4r_1^2 - R^2\big) P_2(\mu) - \frac{43}{5} Rr_1 P_1(\mu) + \frac{3}{5} Rr_1 P_3(\mu)\right] \times \frac{v^{\rm tr}_{NN}(r_{01})}{r_{01}^2} \boldsymbol{S}_{12}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \hat{\boldsymbol{R}}), \tag{C5}$$

$$U^{\rm tr}_{32}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R}) = \frac{6}{35}\sqrt{\frac{2\pi}{5}} \int_0^\infty r_1^2 dr_1 u_{1\frac{3}{2}}(r_1)^2 \int_{-1}^1 d\mu \left[\big(r_1^2 + R^2\big) P_2(\mu) - \frac{7}{5} Rr_1 P_1(\mu) - \frac{3}{5} Rr_1 P_3(\mu)\right] \frac{v^{\rm tr}_{NN}(r_{01})}{r_{01}^2} \boldsymbol{S}_{32}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \hat{\boldsymbol{R}}), \tag{C6}$$

$$U^{\rm tr}_{34}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R}) = -\frac{6}{35}\sqrt{6\pi} \int_0^\infty r_1^2 dr_1 u_{1\frac{3}{2}}(r_1)^2 \int_{-1}^1 d\mu \big[R^2 P_2(\mu) + r_1^2 P_4(\mu) - 2Rr_1 P_3(\mu)\big] \frac{v^{\rm tr}_{NN}(r_{01})}{r_{01}^2} \boldsymbol{S}_{34}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \hat{\boldsymbol{R}}). \tag{C7}$$

Combining the spin-spin potentials from the three tensor terms in Eqs. (C4)–(C7) enables a simple check of the folding of the direct tensor NN interaction term to be performed. When $\boldsymbol{r}_{01} = 0$ (i.e., $\boldsymbol{R} = \boldsymbol{r}_1$ and $\mu = 1$) the factor $1/r_{01}^2 \to \infty$ in the equations above. For these potentials to remain physical for all values of $\boldsymbol{r}_{01}$, the terms in the square brackets must be zero when $\boldsymbol{r}_{01} = 0$. Expressing the Legendre polynomials in the equations above in terms of μ shows that the terms in the square brackets do indeed sum to zero for $\boldsymbol{r}_{01} = 0$.

3. Central exchange term

The spin-spin potentials allowed for the model of ^{10}B used in this work are determined in the same way for the central exchange term, as they were with the direct terms in the NN interaction. This gives three different types of spin-spin interaction

$$\hat{U}_{10}^{\sigma\sigma}(\boldsymbol{\sigma}_0, \boldsymbol{I}, R) = -\frac{\hat{J}_{\sigma\sigma}(Q)}{\sqrt{3\pi}} u_{1\frac{3}{2}}(R)^2 \boldsymbol{S}_{10}(\boldsymbol{\sigma}_0, \boldsymbol{I}), \tag{C8}$$

$$\hat{U}_{12}^{\sigma\sigma}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R}) = -\frac{\hat{J}_{\sigma\sigma}(Q)}{5}\sqrt{\frac{2}{15\pi}} u_{1\frac{3}{2}}(R)^2 \boldsymbol{S}_{12}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \hat{\boldsymbol{R}}), \tag{C9}$$

$$\hat{U}_{32}^{\sigma\sigma}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R}) = \frac{3}{5}\frac{\hat{J}_{\sigma\sigma}(Q)}{\sqrt{10\pi}} u_{1\frac{3}{2}}(R)^2 \boldsymbol{S}_{32}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \hat{\boldsymbol{R}}). \tag{C10}$$

4. Tensor exchange term A

Using the angular momentum values from the simplified model of ^{10}B the folding of tensor exchange term A yields the allowed spin-spin terms

$$\hat{U}_{10}^{\mathrm{trA}}(\boldsymbol{\sigma}_0, \boldsymbol{I}, R) = \frac{-2\hat{J}_{\mathrm{tr}}(Q)}{5\sqrt{3\pi}} u_{1\frac{3}{2}}(R)\big[\hat{\mathscr{O}}_{11} u_{1\frac{3}{2}}(R)\big] \boldsymbol{S}_{10}(\boldsymbol{\sigma}_0, \boldsymbol{I}), \tag{C11}$$

$$\hat{U}_{12}^{\mathrm{trA}}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R}) = \frac{-\hat{J}_{\mathrm{tr}}(Q)}{25\sqrt{30\pi}} u_{1\frac{3}{2}}(R)\big[13\hat{\mathscr{O}}_{11} + 27\hat{\mathscr{O}}_{31} u_{1\frac{3}{2}}(R)\big] \boldsymbol{S}_{12}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \hat{\boldsymbol{R}}), \tag{C12}$$

$$\hat{U}_{22}^{\mathrm{trA}}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R}) = \frac{9\hat{J}_{\mathrm{tr}}(Q)}{25\sqrt{10\pi}} u_{1\frac{3}{2}}(R)\big[\hat{\mathscr{O}}_{11} - \hat{\mathscr{O}}_{31} u_{1\frac{3}{2}}(R)\big] \boldsymbol{S}_{22}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \hat{\boldsymbol{R}}), \tag{C13}$$

$$\hat{U}_{32}^{\mathrm{trA}}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R}) = \frac{-3\hat{J}_{\mathrm{tr}}(Q)}{175\sqrt{10\pi}} u_{1\frac{3}{2}}(R)\big[7\hat{\mathscr{O}}_{11} + 3\hat{\mathscr{O}}_{31} u_{1\frac{3}{2}}(R)\big] \boldsymbol{S}_{32}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \hat{\boldsymbol{R}}), \tag{C14}$$

$$\hat{U}_{34}^{\mathrm{trA}}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R}) = \frac{3\sqrt{6}\hat{J}_{\mathrm{tr}}(Q)}{35\sqrt{\pi}} u_{1\frac{3}{2}}(R)\big[\hat{\mathscr{O}}_{31} u_{1\frac{3}{2}}(R)\big] \boldsymbol{S}_{34}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \hat{\boldsymbol{R}}). \tag{C15}$$

The differential operator $\hat{\mathscr{O}}_{\ell'\ell}$ is taken from [46,47]. The allowed combinations of differential operators are

$$\hat{\mathscr{O}}_{\ell-2,\ell} = \hat{g}_{\ell-2,\ell-1}\ \hat{g}_{\ell-1,\ell} = \frac{d^2}{dr^2} + \frac{2\ell+1}{r}\frac{d}{dr} + \frac{\ell^2-1}{r^2}, \tag{C16}$$

$$\hat{\mathscr{O}}_{\ell,\ell} = \hat{g}_{\ell,\ell-1}\ \hat{g}_{\ell-1,\ell} = \hat{g}_{\ell,\ell+1}\ \hat{g}_{\ell+1,\ell} = \frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{\ell(\ell+1)}{r^2}, \tag{C17}$$

$$\hat{\mathscr{O}}_{\ell+2,\ell} = \hat{g}_{\ell+2,\ell+1}\ \hat{g}_{\ell+1,\ell} = \frac{d^2}{dr^2} - \frac{2\ell+1}{r}\frac{d}{dr} + \frac{\ell(\ell+2)}{r^2}. \tag{C18}$$

The differential operator $\hat{g}_{\ell'\ell}$, is also taken from [46] and is given by

$$\hat{g}_{\ell+1,\ell} = \frac{d}{dr} - \frac{\ell}{r}, \quad \hat{g}_{\ell-1,\ell} = \frac{d}{dr} + \frac{\ell+1}{r}. \tag{C19}$$

5. Tensor exchange term B

The folding of tensor exchange term B, using the angular momentum values from the simplified model of ^{10}B gives the allowed spin-spin terms

$$\hat{U}_{10}^{\mathrm{trB}}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R}) = -\frac{2\sqrt{2}}{3}\hat{J}_{\mathrm{tr}}(Q) u_{1\frac{3}{2}}(R)^2 [Y_0(\hat{\boldsymbol{R}}) \times \tau_1(\boldsymbol{I})] \cdot [\tau_1(\boldsymbol{\sigma}_0) \times \mathscr{Y}_{2\mu}(-\imath\nabla_{\boldsymbol{R}})], \tag{C20}$$

$$\hat{U}_{12}^{\mathrm{trB}}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R}) = -\frac{4}{15}\hat{J}_{\mathrm{tr}}(Q) u_{1\frac{3}{2}}(R)^2 [Y_2(\hat{\boldsymbol{R}}) \times \tau_1(\boldsymbol{I})] \cdot [\tau_1(\boldsymbol{\sigma}_0) \times \mathscr{Y}_{2\mu}(-\imath\nabla_{\boldsymbol{R}})], \tag{C21}$$

$$\hat{U}_{32}^{\mathrm{trB}}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R}) = \frac{\sqrt{12}}{5}\hat{J}_{\mathrm{tr}}(Q) u_{1\frac{3}{2}}(R)^2 [Y_2(\hat{\boldsymbol{R}}) \times \tau_3(\boldsymbol{I})] \cdot [\tau_1(\boldsymbol{\sigma}_0) \times \mathscr{Y}_{2\mu}(-\imath\nabla_{\boldsymbol{R}})]. \tag{C22}$$

6. Tensor exchange term C

Folding this tensor exchange term yields the following allowed spin-spin terms four our model of ^{10}B:

$$\hat{U}^{\mathrm{trC}}_{011}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R}) = \frac{5\hat{J}_{\mathrm{tr}}(Q)}{6\sqrt{6\pi}} u_{1\frac{3}{2}}(R)[\hat{g}_{01} - \hat{g}_{21}] u_{1\frac{3}{2}}(R)[Y_1(\hat{\boldsymbol{R}}) \times \tau_1(\boldsymbol{\sigma}_0)]_{1\epsilon}[\tau_0(\boldsymbol{I}) \times \nabla]_{1-\epsilon}, \tag{C23}$$

$$\hat{U}^{\mathrm{trC}}_{110}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R}) = \frac{-\hat{J}_{\mathrm{tr}}(Q)}{5\sqrt{3\pi}} u_{1\frac{3}{2}}(R)[5\hat{g}_{01} + 11\hat{g}_{21}] u_{1\frac{3}{2}}(R)[Y_1(\hat{\boldsymbol{R}}) \times \tau_1(\boldsymbol{\sigma}_0)]_0[\tau_1(\boldsymbol{I}) \times \nabla]_0, \tag{C24}$$

$$\hat{U}^{\mathrm{trC}}_{111}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R}) = \frac{\hat{J}_{\mathrm{tr}}(Q)}{15\sqrt{3\pi}} u_{1\frac{3}{2}}(R)[5\hat{g}_{01} + 16\hat{g}_{21}] u_{1\frac{3}{2}}(R)[Y_1(\hat{\boldsymbol{R}}) \times \tau_1(\boldsymbol{\sigma}_0)]_{1\epsilon}[\tau_1(\boldsymbol{I}) \times \nabla)]_{1-\epsilon}, \tag{C25}$$

$$\hat{U}^{\mathrm{trC}}_{112}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R}) = \frac{\hat{J}_{\mathrm{tr}}(Q)}{25\sqrt{3\pi}} u_{1\frac{3}{2}}(R)[10\hat{g}_{01} - 3\hat{g}_{21}] u_{1\frac{3}{2}}(R)[Y_1(\hat{\boldsymbol{R}}) \times \tau_1(\boldsymbol{\sigma}_0)]_{2\epsilon}[\tau_1(\boldsymbol{I}) \times \nabla]_{2-\epsilon}, \tag{C26}$$

$$\hat{U}^{\mathrm{trC}}_{132}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R}) = \frac{-\sqrt{2}\hat{J}_{\mathrm{tr}}(Q)}{25\sqrt{\pi}} u_{1\frac{3}{2}}(R)\big[\hat{g}_{21} u_{1\frac{3}{2}}(R)\big][Y_3(\hat{\boldsymbol{R}}) \times \tau_1(\boldsymbol{\sigma}_0)]_{2\epsilon}[\tau_1(\boldsymbol{I}) \times \nabla]_{2-\epsilon}, \tag{C27}$$

$$\hat{U}^{\mathrm{trC}}_{211}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R}) = \frac{-2\hat{J}_{\mathrm{tr}}(Q)}{3\sqrt{5\pi}} u_{1\frac{3}{2}}(R)[\hat{g}_{01} - \hat{g}_{21}] u_{1\frac{3}{2}}(R)[Y_1(\hat{\boldsymbol{R}}) \times \tau_1(\boldsymbol{\sigma}_0)]_{1\epsilon}[\tau_2(\boldsymbol{I}) \times \nabla)]_{1-\epsilon}, \tag{C28}$$

$$\hat{U}^{\mathrm{trC}}_{212}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R}) = \frac{\hat{J}_{\mathrm{tr}}(Q)}{5\sqrt{\pi}} u_{1\frac{3}{2}}(R)[\hat{g}_{01} - \hat{g}_{21}] u_{1\frac{3}{2}}(R)[Y_1(\hat{\boldsymbol{R}}) \times \tau_1(\boldsymbol{\sigma}_0)]_{2\epsilon}[\tau_2(\boldsymbol{I}) \times \nabla)]_{2-\epsilon}, \tag{C29}$$

$$\hat{U}^{\mathrm{trC}}_{312}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R}) = \frac{-\hat{J}_{\mathrm{tr}}(Q)}{50\sqrt{\pi}} u_{1\frac{3}{2}}(R)[5\hat{g}_{01} + \hat{g}_{21}] u_{1\frac{3}{2}}(R)[Y_1(\hat{\boldsymbol{R}}) \times \tau_1(\boldsymbol{\sigma}_0)]_{2\epsilon}[\tau_3(\boldsymbol{I}) \times \nabla]_{2-\epsilon}, \tag{C30}$$

$$\hat{U}^{\mathrm{trC}}_{332}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R}) = \frac{53\sqrt{6}\hat{J}_{\mathrm{tr}}(Q)}{175\sqrt{\pi}} u_{1\frac{3}{2}}(R)\big[\hat{g}_{21} u_{1\frac{3}{2}}(R)\big][Y_3(\hat{\boldsymbol{R}}) \times \tau_1(\boldsymbol{\sigma}_0)]_{2\epsilon}[\tau_3(\boldsymbol{I}) \times \nabla]_{2-\epsilon}, \tag{C31}$$

$$\hat{U}^{\mathrm{trC}}_{333}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R}) = \frac{-11\sqrt{3}\hat{J}_{\mathrm{tr}}(Q)}{35\sqrt{2\pi}} u_{1\frac{3}{2}}(R)\big[\hat{g}_{21} u_{1\frac{3}{2}}(R)\big][Y_3(\hat{\boldsymbol{R}}) \times \tau_1(\boldsymbol{\sigma}_0)]_{3\epsilon}[\tau_3(\boldsymbol{I}) \times \nabla]_{3-\epsilon}, \tag{C32}$$

$$\hat{U}^{\mathrm{trC}}_{334}(\boldsymbol{\sigma}_0, \boldsymbol{I}, \boldsymbol{R}) = \frac{3\sqrt{3}\hat{J}_{\mathrm{tr}}(Q)}{35\sqrt{2\pi}} u_{1\frac{3}{2}}(R)\big[\hat{g}_{21} u_{1\frac{3}{2}}(R)\big][Y_3(\hat{\boldsymbol{R}}) \times \tau_1(\boldsymbol{\sigma}_0)]_{4\epsilon}[\tau_3(\boldsymbol{I}) \times \nabla]_{4-\epsilon}. \tag{C33}$$

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