Energies within the $A = 10$ isospin quintet

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We have used a potential model to compute energies of the lowest $T = 2$ states in $A = 10$ nuclei. For ^{10}N, we obtain $E_p = 1.81$ to 1.94 MeV.

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I. INTRODUCTION

Advances in experimental techniques have made possible the measurement of energies of nuclear states that were previously inaccessible. These states are produced in breakup or nucleon removal experiments, and their energies are determined by detecting all the decay products in coincidence and measuring their momenta. For example, Charity *et al.* [1] recently reported the energy of the $T = 5/2$ double analog in ^{11}B. In the hope that these techniques may soon be applied to $A = 10$ nuclei with $T = 2$, we have computed the energies to be expected for members of this isospin quintet.

All the states of ^{10}Li are unbound [2]. From a wide variety of reactions [3–6] the ground state (g.s.) has been established as an s-wave structure at $E_n = 26(13)$ keV. With a simple model of nuclear plus Coulomb potentials (plus angular momentum, as appropriate), we have computed the energies of the lowest $T = 2$ state in the $A = 10$ nuclei. Our procedure is straightforward. It has been used to compute energies of analog and mirror states in several light nuclei. A recent paper on ^{20}F and ^{20}Mg [7] contained some of the references and a description of the model. In normal cases, in which the nuclear structure and excitation energies of the core states are reliably known, our method has been shown to produce results that agreed with experimental values to within about 30 to 40 keV. For example, we found [8] that our model reproduced the energies of eight positive-parity states in ^{18}Ne to this precision. Perhaps the most striking example of the success of our approach was the prediction [9] of the g.s. energy of ^{19}Mg: $E_{2p} = 0.87(7)$ MeV. We assigned an uncertainty of 70 keV to those calculations, because the energies of the relevant core states in ^{18}Na were not known, and it was necessary to also compute them. A later experiment [10] found $E_{2p} = 0.75(5)$ MeV, just at the limit of our uncertainty. Still later, we investigated the sensitivity of the ^{19}Mg (g.s.) predictions to various properties of the model input: a different geometry of the potential well, different spectroscopic factors, and a different source of ^{18}Na core energies. We found [11] that these variations were all within our estimated uncertainty of 70 keV.

We solve the one-body Schrödinger equation, using a Woods-Saxon shape for the nuclear potential. Parameters are r_0, $a = 1.25$, and 0.65 fm. The well depth is adjusted to reproduce a specific energy in the core + n system. An appropriate Coulomb potential is then added and the same nuclear potential is used to compute the energy of the core $+p$ system. The $A = 10$, $T = 2$ isospin quintet runs from $T_z = 2$ at ^{10}Li to $T_z = -2$ at ^{10}N. In a standard description, ^{10}Li is pure ^{9}Li x neutron, ^{10}N is pure ^{9}C x proton, and the interior nuclei are linear combinations of $A = 9$ and $T = 3/2$ cores coupled to a neutron or proton, weighted by the square of an isospin Clebsch-Gordan coefficient $C(3/2T_{zc}1/2t_z;2T_z)$, where t_z is $+1/2$ for a neutron. For example, the $T = 2$ state of ^{10}B is 50% proton and 50% neutron: $^{10}B(T=2) = (1/2)\,^9B(T = 3/2)x\,n + (1/2)\,^9\mathrm{Be}(T = 3/2)xp$, so that $M[^{10}B(T=2)] = (1/2)(\{M[^9B(T=3/2)] + M(n) + E_n\} + \{M[^9\mathrm{Be}(T = 3/2)] + M(^1\mathrm{H}) + E_p\})$, where M are mass excesses and E_n and E_p are the results of the potential calculation. (Here, $E_{n,p}$ are just the negatives of the separation energies $S_{n,p}$, so that E is negative for a bound state and positive for an unbound state.)

Our procedure is to find the nuclear potential that binds ^{10}Li as ^{9}Li + n and then use this potential plus the appropriate Coulomb potential to compute the other nuclei. For the charge of the cores, the proton calculations have $Z = 3$ (^{10}Be) to $Z = 6$ (^{10}N). Because ^{10}Li is slightly unbound, we did the calculations for the case of a slightly bound ^{10}Li and then extrapolated to an unbound energy of 26 keV. For purposes of the extrapolation, we examined the behavior of E_p–E_n for negative values of E_n and extrapolated smoothly to $E_n = +26$ keV. We estimate that the additional uncertainty introduced by this extrapolation procedure is less than about 30 keV. In other applications in which the nuclear structure is accurately known, our procedure has produced results with typical uncertainties of 30–40 keV. Thus, in the present case, for a specific spectroscopic factor, we expect an uncertainty in our calculated energies of about 45–50 keV. As we see below, however,

TABLE I. Energies (in MeV) of the lowest $T = 3/2$ states in $A = 9$ nuclei.

Nucleus	E_x [a]	ME (g.s.) [b]	ME ($T = 3/2$)
^{9}Li	0	24.955	24.955
^{9}Be	14.392(2)	11.348	25.740(2)
^{9}B	14.6550(25)	12.417	27.0720(25)
^{9}C	0	28.909(2)	28.909(2)

[a] Reference [2].
[b] Reference [12].

TABLE II. Experimental and computed energies (in MeV) of the lowest $T = 2$ states in ^{10}Be.

Source	Quantity	Value
Experimental	ME (g.s.)[a]	12.607
	E_x ($T = 2$)[b]	21.216(23)
	ME ($T = 2$)	33.823(23)
Computed[c]	ME ($S_0 = 1.0$)	33.756
	ME ($S_0 = 0.80$	33.772

[a]Reference [12].
[b]Reference [2].
[c]Using a mass excess of 33.053 MeV [12] for ^{10}Li.

TABLE III. Calculated mass excesses (in MeV) for the lowest $T = 2$ state in $A = 10$ nuclei.

Nucleus	ME	
	$S_0 = 1.0$	$S_0 = 0.80$
^{10}B	34.798	34.842
^{10}C	36.150	36.230
^{10}N	38.010	38.138

uncertainty in the s-wave spectroscopic factor adds an additional uncertainty.

II. CALCULATIONS AND RESULTS

We first list in Table I the energies of the $A = 9$, $T = 3/2$ cores. Excitation energies are from the latest compilation [2], and g.s. mass excesses (MEs) are from the new mass evaluation [12]. This quartet was among the first (perhaps the first) isospin quartet to be completed [13]. For all the $A = 10$ cases, because the ^{10}Li (g.s.) parentage is not accurately known, we have done the computations twice—once with an s-wave spectroscopic factor of 1.0 and then again with $S_0 = 0.80$, with the remainder being a d wave. Our calculations used a mass excess of 33.053 MeV [12] for ^{10}Li. For $T = 2$ excited states in $A = 10$, only ^{10}Be is known [2]. Table II lists the experimental energy of the lowest $T = 2$ state in ^{10}Be and the results of our calculation. The experimental value of ME is 33.823 (23) MeV, compared to calculated values of 33.756 ($S_0 = 1$) and 33.772 ($S_0 = 0.8$) MeV.

Predictions for the other nuclei are listed in Table III. From the ^{10}B(^{14}N, ^{14}B) ^{10}N reaction [14],the lowest known state in ^{10}N has $E_p = 2.6(4)$ MeV [ME = 38.8(4) MeV], with a width of 2.3(16) MeV. As pointed out by the compilers [2], this is almost certainly not the g.s., but rather the mirror of the 1^+ state at $E_n = 0.265$ MeV in ^{10}Li. We agree. Our g.s. in ^{10}N is predicted to have $E_p = 1.81$ or 1.94 MeV, for $S_0 = 1.0$ or 0.8, respectively. The compilation [2] estimated E_p (g.s.) ∼ 1.8 MeV.

Much earlier, Aoyama *et al.* [15] used the complex scaling method in a core + nucleon model of ^{10}Li and ^{10}N. They reproduced p-wave 1^+ and 2^+ resonances observed by Bohlen *et al.* [16] in ^{10}Li and predicted the corresponding resonances in ^{10}N. Their 1^+ resonance was at energy 2.84 MeV, with a width of 1.89 MeV. They did not discuss s-wave resonances in ^{10}Li and only remarked that they would be very broad in ^{10}N, "even if they exist" [15].

Our procedure automatically satisfies the quadratic isobaric multiplet mass equation (IMME), but only if the core masses do. Here, the $A = 9$, $T = 3/2$ energies (Table I) exhibit the need for a small cubic term ($d \neq 0$), and this could cause our $A = 10$ energies to deviate slightly from the quadratic IMME.

III. SUMMARY

We have used a simple potential model to calculate the expected energies of the lowest $T = 2$ state in $A = 10$ nuclei, given its energy of +26 keV in ^{10}Li. This state is known only in ^{10}Be, where its mass excess is 33.823(23) MeV (Table II). Our result for a pure s-wave state is 33.756(45) MeV, while a mixture of 80% s and 20% d gives 33.772(45) MeV, both in reasonable agreement. For the g.s. of ^{10}N, our computed range is $E_p = 1.81$ to 1.94 MeV, depending on the spectroscopic factor (Table III). Predictions for the other two nuclei are also listed in Table III. We expect (hope) these predictions will soon be tested experimentally.

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