Influence of electron screening on α decay

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(Received 1 February 2013; revised manuscript received 4 April 2013; published 16 May 2013)

The effect of electron screening on the α decay rate of typical nuclei is considered. To this end, the adiabatical approach is exploited, which consecutively takes into account the adiabaticity of the motion of the α particle through the shells. The effect is found to be of the order of 0.1% to 0.01% for the considered representative nuclei. The prospects of an experimental study of the effect aimed at a search for the dynamics of α decay are discussed. It is suggested that the presence of a muon in the orbit in muonic atoms should have an effect on the α decay rate that is much stronger than the effect of removing the electron shell.

DOI: 10.1103/PhysRevC.87.054319 PACS number(s): 23.60.+e, 25.55.Ci

I. INTRODUCTION

The influence of the electron shell on nuclear processes is well studied. It may be huge in the case of electromagnetic or weak interactions. In the former case, the electron shell has a great effect on the nuclear lifetime by means of internal conversion. Thus, the internal conversion coefficients reach eight and more orders of magnitude, especially for low transition energies and high multipole orders (e.g., [1] and references therein), thus reducing the lifetime of the excited nuclear states. Moreover, an illustrious example is given by the resonance conversion in the case of the 45-fold ions of ^{125}Te, which takes the place of the conventional internal conversion in the subthreshold region [2]. Analogously, in the latter case, a new β mode is set when the β electron can occupy a discrete atomic state (e.g., [3] and references therein). In the case of nuclear K capture in hydrogen-like ions, a tremendous difference may arise in the capture rates for various components of the hyperfine structure [4]. The situation changes in the case of α decay in view of a high α-to-electron mass ratio, $m_\alpha/m_e \sim 10^4 \ggg 1$. The space scale of the α process is also completely different, being predetermined by the short-range character of the nuclear forces and the Coulomb barrier. This reduces the area of the formation of the process to the vicinity of the nucleus, which is small compared to the size of the atom. As a result, one should not expect any noticeable effects from the electron shell at energies of $\sim$1 MeV [5]. This does not exclude dynamical effects, which can arise owing to the internal or resonance conversion. The effects may manifest themselves, e.g., in the form of violation of parity [6]. We also note the similarity between α decay and fission. The fission probability is predetermined by the probability of ascending the barrier, the succeeding scenario being of no importance for the final fission probability. This argument holds in the case of α decay: why the shape of the α-nucleus potential, after an α particle has passed the barrier, can influence the final decay probability. There is only the outgoing component in the wave function in this region, without any chance of reflection back for the α particle. From this viewpoint, the influence of electron screening on penetration of the barrier was analyzed in [7]. It was demonstrated that the atomic effects are small and actually of no importance for α decay. Also, Patyk *et al.*, in their systematic study [8], showed that the expected effects are at the level of percent.

On the contrary, opposite conclusions can be still noted in the literature (e.g., [9] and [10]). On the other hand, the Q_α value in bare nuclei of heavy elements is about 40 keV higher than the effective values in neutral atoms, which is quite a significant value from the viewpoint of passing the barrier. All this makes revisiting the subject worthwhile. To be specific, we consider the α decay of a sole atom. Furthermore, we want to stress an analogy to a well-studied effect of barrier augmentation in prompt nuclear fission induced by nonradiative transitions in muonic atoms ([11,12], and references therein). In the latter case, the augmentation is caused by a decrease in the binding energy of the muon, which adiabatically moves in quasimolecular orbits in the field of the elongating nucleus and, later on, of separating fragments. The augmentation suppresses the fission probability by an order of magnitude. This adiabatical picture is different from the conventional model of a "frozen" electronic potential used for estimation of the screening effects in nuclear reactions. Muons are 207 times as heavy as electrons, and their 1s orbit is inside the fissile nuclei and their fragments. Let us examine the effect of adiabaticity in usual atoms.

II. THE ADIABATICAL MODEL OF ELECTRON SCREENING

Let us consider a typical isolated atom undergoing α decay. The adiabadicity condition reads

$$V_\alpha \ll v_e, \tag{1}$$

with V_α and v_e being the α and electron velocities, respectively. Owing to the large mass, at typical energies of α decay $E_\alpha \gtrsim$ 1 MeV, condition (1) is fulfilled for inner electrons [13]. It was a surprise when it turned out that emission of α particles is accompanied by radiation [14], though the probabilities of ionization of inner shells in α decay were calculated earlier (A. B. Migdal (1941) and J. Levinger (1953), as cited in [13]). The adiabaticity condition, (1), is only violated for the outermost electrons. At least two of them are ejected to the continuum, leaving the daughter atom neutral. Otherwise, in

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the case of fully adiabatic motion of a very slow α particle, they would be entrained on the α, "dressing" it up to a neutral atom of He. We suppose that all other electrons remain in their orbits, adiabatically adjusting their orbits to the passing α particle. This picture is enough for the present purposes, as the main contribution to the effect comes from the inner s shells.

Consider the α-nucleus potential in the form

$$V(R) = V_N(R) + V_{\rm Coul}(R) + \mathcal{E}(R). \quad (2)$$

In (2), $V_N(R)$ and $V_{\rm Coul}(R)$ are the strong [15] and the Coulomb parts of the potential, respectively, and $\mathcal{E}(R)$ is the electron energy of the term. Actually, in order to calculate the electron energy of the term, one has to solve a two-center problem in the adiabatic quasimolecular basis. Consider two limiting cases, which are important in our picture. At short α-nucleus distances $R \ll a_K$, with a_K being the radius of the K shell, the electrostatic potential produced by the α particle at a point $\mathbf{r}$, in comparison with the potential of the mother nucleus, can be expressed as a multipole series:

$$V_R'(\mathbf{r}) = ze^2\left(\frac{1}{|\mathbf{r}-\mathbf{R}|} - \frac{1}{r}\right) = ze^2\left[\sum_l \frac{r_<^l}{r_>^{l+1}} P_l(\cos\theta) - \frac{1}{r}\right]. \quad (3)$$

In (3) $z = 2$ is the charge of the α particle. Keeping the monopole term in (3), which gives a predominant contribution in the first order of the perturbation series, we arrive at the perturbation potential $V_R'(r)$. It reads

$$V_R'(r) = ze^2\left(\frac{1}{R} - \frac{1}{r}\right) \quad (4)$$

for $R_0 \leqslant r \leqslant R$, R_0 being the nuclear radius, and 0 otherwise. The resulting shift in the electron energy of the term reads

$$\Delta\mathcal{E}(R) \equiv \mathcal{E}(R) - \mathcal{E}(0) = \sum_i N_i \int_{R_0}^{R} \left[G_i^2(r') + F_i^2(r')\right] V_R'(r')\, dr', \quad (5)$$

where $G_i(r)$ and $F_i(r)$ are the radial Dirac wave functions. Summation in (5) is performed over shells i, with N_i the occupation number. Note that potential (5) is far from being constant in the nuclear vicinity, increasing approximately as R^2. This is in contrast with the picture in interstellar plasma [5]. The constant component, as we will see, exactly cancels in our case.

On the contrary, at large α-nucleus distances $R \to \infty$, the interaction is a Coulomb plus polarization one. In the asymptotic region, the solution can be expressed in analytical form by making use of the asymptotics of the radial wave functions [18,19]. This results in

$$\Delta\mathcal{E} = \Delta I - \frac{(Z-z)ze^2}{R} - \frac{\beta}{R^4}, \quad (6)$$

where ΔI is the difference in the total binding energies in the mother and daughter atoms, Z is the atomic number of the mother nucleus, and β is the polarizability. By definition,

$$\mathcal{E}(0) \equiv -\Delta I. \quad (7)$$

We neglect a possible additional loss of energy by α's for the kinetic energy of the two ejected electrons. We also neglect possible energy losses for an extra ionization, as mentioned previously. Equation (7) plays a similar role in our approach to the Hellmann-Feynman equation, (9), in Ref. [8] within the framework of the frozen electron shell model.

Within the framework of the Gamow theory, a conventional expression for the α decay probability is essentially given by the product of two factors: the cluster preformation and the penetration probabilities ([15]). The former is not affected by the shell. The second factor P is determined by action S. For a bare nucleus it reads as follows:

$$P = \exp(-2S), \quad (8)$$

$$S = \int_{\rm R_{in}}^{\rm R_{out}} \sqrt{2\mu(V_N(R) + V_{\rm Coul}(R) - E_\alpha)}\,dR, \quad (9)$$

where $\rm R_{in}$ and $\rm R_{out}$ are the classical inner and outer turning points.

Adding the electron energy $\mathcal{E}(R)$ to the potential energy in (9) and allowing for the reduction of the asymptotic energy of the α particle by an amount of ΔI, one arrives at the expression for the action of a fully dressed atom:

$$S_{\rm at} = \int_{\rm R_{in}}^{\rm R_{out}} \sqrt{2\mu(V_N(R) + V_{\rm Coul}(R) + \mathcal{E}(R) - (E_\alpha - \Delta I))}\,dR \equiv \int_{\rm R_{in}}^{\rm R_{out}} \sqrt{2\mu(V_N(R) + V_{\rm Coul}(R) + \Delta\mathcal{E}(R) - E_\alpha)}\,dR. \quad (10)$$

The reduction in the Q value for an atom compared to the bare nucleus is thus canceled in (10), owing to (7), and does not affect the penetration probability of the barrier. Comparing (8) with (10), one arrives at the expression for the relative change of the α decay rate in bare nuclei compared to atoms:

$$Y = (P/P_{\rm at}) - 1. \quad (11)$$

III. RESULTS OF THE CALCULATION

Results of the calculation for some representative nuclei undergoing α decay are presented in Table I. Wave functions and energies of atoms were calculated by means of the package of computer codes RAINE [20]. In the case of a superheavy element $Z = 118$, $A = 294$, the Q value and half-period $T_{1/2}$ taken from Ref. [16] have been used. For other nuclei, these values were taken from Ref. [17]. Two of the listed

TABLE I. Results for the relative change in half-periods in bare nuclides (last column).

Nuclide	Q (MeV)	$T_{1/2}$	Y (%)
$^{144}_{60}$Nd	1.905	2.29×10^{15} yr	0.24
$^{214}_{86}$Rn	9.208	0.27 μs	0.02
$^{226}_{88}$Ra	4.871	1600 yr	0.23
$^{252}_{98}$Cf	6.217	2.645 yr	0.28
$^{241}_{99}$Es	8.320	9 s	0.12
^{294}118	11.81	0.89 ms	0.27

nuclei, those of $^{226}_{88}$Ra and $^{214}_{86}$Rn, have close Z values but very different Q values and half-periods. Comparison of the Y values calculated for these nuclei with one another shows that the effect of screening strongly decreases with increasing Q and, correspondingly, decreasing $T_{1/2}$. This is because for larger Q, the barrier traversed by the α particle approaches the nucleus, where the electron energy of the term $\Delta\mathcal{E}(R)$ is lower. The same conclusion can be drawn from comparison of the results listed for another, heavier couple of nuclei, $^{252}_{98}$Cf and $^{241}_{99}$Es. For these nuclides, the difference in the Q values is smaller, nevertheless, the half-periods differ by six orders of magnitude. As a consequence, the effect of electron screening Y differs by 2.5 times. In contrast, in the case of $^{144}_{60}$Nd, the calculated effect remains approximately the same, in spite of a much smaller Q value and larger half-period. This comparison shows that the effect also strongly depends on the atomic number of the element, decreasing for smaller Z. This also can be seen in the case of superheavy nuclei of ^{294}118: despite their having the highest Q value, the effect turns out to be approximately like that for the much lighter nuclide of $^{252}_{98}$Cf. As one can see in Table I, for the latter two nuclides, the effect is maximal: 0.27% and 0.28%, respectively. The effect turns out to be minimal in the case of $^{214}_{86}$Rn, where it decreases to 0.02%; that is, the effect still becomes lower by an order of magnitude.

The numerical value of the effect obtained is in qualitative agreement with Refs. [7] and [8]. It is remarkable that the agreement exists despite the opposite characters of the underlying physical models used: the "frozen" electron screening potential in the cited papers and the adiabatic model in our approach. Actually, this stresses the fact that electron screening generally has a minor effect on the α decay rate.

IV. SUMMARY AND DISCUSSION

The probability of α decay is considered with and without taking into account the presence of the electron shell. An adiabatic quasimolecular model is developed, properly allowing for the electron energy of the term. Taking the electron energy into account considerably reduces the effective Q value.

On the other hand, this reduction occurs at a large distance in comparison with the nuclear scale. This can be very distinctively seen in the adiabatical picture of the α motion. Over nuclear distances, the reduction practically does not affect the penetration probability of the barrier. Qualitatively, comparing the adiabatical picture with the model of a "frozen" electron shell, one should expect a lower effect in the adiabatical case. This can be understood in terms of the polarization of the electron shell during the α passage through. Owing to the polarization, atomic electrons concentrate around the α particle, thus additionally lowering the local electrostatic potential. In other words, the polarization increases the probability of finding a piece of the electron cloud between the α particle and the nucleus. This acts as an additional shield between the α particle and the nucleus, partly compensating their mutual repulsion and hence diminishing the Coulomb barrier.

As a result, the final value of the difference in natural α decay turns out to be of the order of a tenth to a hundredth of a percent. Quantitatively, comparing the results listed in Table I with the results for the same nuclei which are presented in Fig. 2 of Ref. [8], one can see that the present results are lower by one and a half to three times. Furthermore, they are closer to those given by the Rubinson and Perlman formula.

Within the lines of the previous discussion, it is noteworthy that in muonic atoms, the effect on the α decay rate of the presence of muons in the K orbits is expected to be much greater than the effect of removing the electron shell in usual atoms. In this relation, we would like to call attention to the fact that the partial lifetime of the α decay of ^{214}Rn exceeds the anticipated lifetime of the muons in orbits by only an order of magnitude.

Concluding the discussion, we note that testing the theoretical results in experiments is of great interest from the viewpoint of studying the dynamics of α decay. As a matter of fact, expression (8) with action (9) for the penetration probability is valid in the quasiclassical approximation within the framework of the Gamow theory of α decay of quasistationary states. A general solution of the Schrödinger equation in the region under the barrier consists of a linear combination of the two fundamental coupled, linearly independent solutions [21]. One of them is exponentially decreasing; the other, exponentially increasing. Actually, their applicability is based on the condition that the linear combination is reduced to only one of them, the exponentially decreasing one. This means that the energy of the α particle coincides with the eigenvalue of the mean α-nucleus potential field (cf. discussion in [21]). In turn, as the basis for such a statement, one can refer, e.g., to Koopmans theorem as well. In the nuclei, the theorem works with an accuracy of $\sim$100 keV [22]. Strictly speaking, one could suppose that the 37-keV shift of the α line (in the case of ^{226}Ra) could lead to a considerable related change in the half-period, up to two orders of magnitude according to the results of [21], [23], and [24]. Fine details of the theory may hence be verified by comparison of the intensities of the α lines in bare nuclei with those in neutral atoms, shifted by the ΔI value. This will answer the quesion whether the α line in both processes, α decay as well as the related reverse α-nucleus merging cross section, is caused by either an α-nucleus eigenstate or a mere coincidence with a real nuclear state [21,25].

ACKNOWLEDGMENTS

The author is indebted to M. B. Trzhaskovskaya for invaluable help in the atomic calculations. He is also grateful to Prof. Yu. Oganessian for fruitful discussions of superheavies.

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