Nuclear β^+/EC decays in covariant density functional theory and the impact of isoscalar proton-neutron pairing

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Self-consistent proton-neutron quasiparticle random phase approximation based on the spherical nonlinear point-coupling relativistic Hartree-Bogoliubov theory is established and used to investigate the β^+/electron-capture (EC)-decay half-lives of neutron-deficient Ar, Ca, Ti, Fe, Ni, Zn, Cd, and Sn isotopes. The isoscalar proton-neutron pairing is found to play an important role in reducing the decay half-lives, which is consistent with the same mechanism in the β decays of neutron-rich nuclei. The experimental β^+/EC-decay half-lives can be well reproduced by a universal isoscalar proton-neutron pairing strength.

Nuclear β decays play important roles in many subjects of nuclear physics. Specifically, the investigation of β decay provides information on the spin and isospin dependence of the effective nuclear interaction, as well as on nuclear properties such as masses [1], shapes [2], and energy levels [3]. Moreover, nuclear β decays are also important in nuclear astrophysics, because they set the time scale of the rapid neutron-capture process (r-process) [4–8], which is a major mechanism for producing the elements heavier than iron. In addition, nuclear β decays can provide tests for the electroweak standard model [9–11]. With the development of radioactive ion beam facilities, the measurement of nuclear β-decay half-lives has achieved great progress in recent years [12–15].

On the theoretical side, apart from the macroscopic gross theory [16], two different microscopic approaches have been widely used to describe and predict the nuclear β-decay rates. They are the shell model [5] and the proton-neutron quasiparticle random phase approximation (QRPA) [17–19]. While the shell model takes into account the detailed structure of the β-strength function, the proton-neutron QRPA approach provides a systematic description of β-decay properties of arbitrarily heavy nuclei. In order to reliably predict properties of thousands of unknown nuclei relevant to the r-process, the self-consistent QRPA approach has become a current trend in nuclear structure study, including those based on the Skyrme-Hartree-Fock-Bogoliubov (SHFB) theory [20] and the covariant density functional theory (CDFT) [21–23].

In the CDFT framework, the self-consistent proton-neutron RPA was first developed based on the meson-exchange relativistic Hartree (RH) approach [24]. To describe the spin-isospin excitations in open shell nuclei, it has been extended to the QRPA based on the relativistic Hartree-Bogoliubov (RHB) approach [25] and employed to calculate the β-decay half-lives of neutron-rich nuclei in the $N \approx 50$ and $N \approx 82$ regions [21,22]. In addition, based on the meson-exchange

relativistic Hartree-Fock (RHF) approach [26,27], the self-consistent proton-neutron RPA has been formulated [28] and well reproduces the spin-isospin excitations in doubly magic nuclei, without any readjustment of the parameters of the covariant energy density functional [28,29]. Recently, the self-consistent QRPA based on the relativistic Hartree-Fock-Bogoliubov (RHFB) approach [30,31] was developed and a systematic study on the β-decay half-lives of neutron-rich even-even nuclei with $20 \leqslant Z \leqslant 50$ has been performed [23]. Similar to the nonrelativistic calculations [20], it is found that the isoscalar ($T = 0$) proton-neutron pairing plays a very important role in reducing the decay half-lives. In particular, with an isospin-dependent $T = 0$ proton-neutron pairing interaction as a function of $N - Z$, available data in the whole region of $20 \leqslant Z \leqslant 50$ can be well reproduced [23]. So far, these self-consistent investigations mainly focus on the neutron-rich side.

During the past years, the CDFT framework has been reinterpreted by the relativistic Kohn-Sham scheme, and functionals have been developed based on the zero-range point-coupling interactions [32]. In this framework, the meson exchange in each channel is replaced by the corresponding local four-point contact interaction between nucleons. Such a point-coupling model has attracted more and more attention due to its simplicity and several other advantages [33]. For example, it is even possible to include the effects of Fock terms in a local RHF-equivalent scheme [34,35]. With either nonlinear or density-dependent effective interactions, the point-coupling models have achieved satisfactory descriptions for infinite nuclear matter and finite nuclei on a level of accuracy comparable to that of meson-exchange models [36,37]. Recently, a new nonlinear point-coupling effective interaction PC-PK1 [38] was proposed, which well reproduces the properties of nuclear matter and finite nuclei including the ground state and low-lying excited states [38–40]. In particular, the PC-PK1 provides good isospin dependence of binding energy along either the isotopic or the isotonic chain, which makes it reliable for applications in exotic nuclei [38,39]. Based on the point-coupling effective Lagrangian, the

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spherical (Q)RPA in non-charge-exchange channels has been formulated and well reproduces the excitation energies of giant resonances [37,41,42].

In this work, the self-consistent proton-neutron QRPA based on the spherical nonlinear point-coupling relativistic Hartree-Bogoliubov theory is established. This newly developed approach will be used to investigate the β^+/electron-capture (EC) decays in neutron-deficient isotopes around the proton magic numbers $Z = 20$, 28, and 50 with the PC-PK1 effective interaction. Special attention will be paid to the effects of the $T = 0$ proton-neutron pairing on the decay half-lives.

For a self-consistent QRPA calculation, the particle-hole (p-h) and particle-particle (p-p) residual interactions should be derived from the same energy density functional as the ground state. Here we only collect the essential expressions and refer the readers to Refs. [25,43] for some details of the relativistic proton-neutron QRPA.

For the p-h residual interaction, only the isovector channel of the effective interaction contributes to the charge-exchange excitations. The isovector-vector (TV) interaction in the present relativistic point-coupling model reads

$$V_{\mathrm{TV}}(1, 2) = (\alpha_{\mathrm{TV}} + \delta_{\mathrm{TV}}\Delta)[\gamma_0\gamma^\mu\vec{\tau}]_1[\gamma_0\gamma_\mu\vec{\tau}]_2\delta(\boldsymbol{r}_1 - \boldsymbol{r}_2). \quad (1)$$

Similar to Refs. [21,43], although the direct one-pion contribution is absent in the ground-state description under the Hartree approximation, it has to be included in the calculation of spin-isospin excitations. The corresponding interaction reads

$$V_\pi(1, 2) = -\frac{f_\pi^2}{m_\pi^2}[\vec{\tau}\gamma_0\gamma_5\gamma^k\partial_k]_1[\vec{\tau}\gamma_0\gamma_5\gamma^l\partial_l]_2 D_\pi(1, 2), \quad (2)$$

where $m_\pi = 138.0$ MeV and $f_\pi^2/4\pi = 0.08$, while $D_\pi(1, 2)$ denotes the finite-range Yukawa-type propagator. The derivative type of the pion-nucleon coupling necessitates the inclusion of the zero-range counter term, which accounts for the contact part of the pion-nucleon interaction

$$V_{\delta\pi}(1, 2) = g'\frac{f_\pi^2}{m_\pi^2}[\vec{\tau}\gamma_0\gamma_5\boldsymbol{\gamma}]_1[\vec{\tau}\gamma_0\gamma_5\boldsymbol{\gamma}]_2\delta(\boldsymbol{r}_1 - \boldsymbol{r}_2), \quad (3)$$

where the g' is adjusted to reproduce the excitation energy of the Gamow-Teller (GT) resonances in ^{208}Pb. For the effective interaction PC-PK1, g' is determined to be 0.52.

For the p-p residual interaction, we employ the pairing part of the Gogny force for the isovector ($T = 1$) proton-neutron pairing interaction,

$$V_{T=1}(1, 2) = \sum_{i=1,2} e^{-[(\boldsymbol{r}_1 - \boldsymbol{r}_2)/\mu_i]^2}$$
$$\times (W_i + B_i P^\sigma - H_i P^\tau - M_i P^\sigma P^\tau), \quad (4)$$

with the parameter set D1S [44] for μ_i, W_i, B_i, H_i, and M_i. For the isoscalar ($T = 0$) proton-neutron pairing interaction in the QRPA calculation, we employ a similar interaction as in Refs. [20–23]:

$$V_{T=0}(1, 2) = -V_0 \sum_{i=1,2} g_i e^{-[(\boldsymbol{r}_1 - \boldsymbol{r}_2)/\mu_i]^2} \hat{\prod}_{S=1,T=0}, \quad (5)$$

with $\mu_1 = 1.2$ fm, $\mu_2 = 0.7$ fm, $g_1 = 1$, and $g_2 = -2$. The operator $\hat{\prod}_{S=1,T=0}$ projects onto states with $S = 1$ and $T = 0$. The strength parameter V_0 is determined by fitting to known half-lives.

Similar to that in Refs. [45–47], the β-decay half-life of an even-even nucleus is calculated in the allowed approximation with

$$T_{1/2} = \frac{D}{\sum_\nu[(g_A/g_V)_{\mathrm{eff}}^2 B_{\mathrm{GT}}(E_\nu) + B_{\mathrm{F}}(E_\nu)]f(Z, E_\nu)}, \quad (6)$$

where $D = 6163.4$ s and $(g_A/g_V)_{\mathrm{eff}} = 1$ is the effective ratio of axial and vector coupling constants. $B_{\mathrm{F}}(E_\nu)$ and $B_{\mathrm{GT}}(E_\nu)$ are the transition probabilities for allowed Fermi (F) and GT transitions, which are calculated from the QRPA approach. In the β^+/EC decay of a neutron-deficient nucleus, $f(Z, E_\nu)$ consists of two parts: positron emission (f^{β^+}) and electron capture (f^{EC}). The Fermi integral for positron emission $f^{\beta^+}(Z, E_m)$ is given by

$$f^{\beta^+}(Z, E_m) = \int_{m_e}^{E_m} p_e E_e (E_m - E_e)^2 F_0(Z, E_e) dE_e, \quad (7)$$

where p_e and E_e are the emitted electron momentum and energy, respectively. $F_0(Z, E_e)$ is the Fermi function including Coulomb screening and relativistic nuclear finite-size corrections [5]. In the self-consistent QRPA approach, the β^+-decay energy E_m, i.e., the energy difference between the initial and final states, can be calculated using the QRPA:

$$E_m = -\Delta_{nH} - m_e - E_{\mathrm{QRPA}}, \quad (8)$$

where E_{QRPA} is the QRPA energy with respect to the ground state of the parent nucleus and corrected by the difference of the proton and neutron Fermi energies in the parent nucleus [23] [i.e., $E_{\mathrm{QRPA}} - (\lambda_p - \lambda_n)$ with the definitions in Ref. [20]], m_e and Δ_{nH} are the positron mass and the mass difference between the neutron and the hydrogen atom, respectively. Because the emitted positron energy must be higher than its rest mass, the final states must be those with excitation energies $E_{\mathrm{QRPA}} < -\Delta_{nH} - 2m_e$. Moreover, the decay function f^{EC} for electron capture has also been included following Ref. [46]:

$$f^{\mathrm{EC}} = \frac{\pi}{2} \sum_x q_x^2 g_x^2 B_x, \quad (9)$$

where x denotes the atomic subshell from which the electron is captured, q is the neutrino energy, g is the radial component of the bound-state electron wave function at the nuclear surface, and B stands for other exchange and overlap corrections. The energy threshold for EC is $2m_e$ higher than the β^+ decay, i.e., $E_{\mathrm{QRPA}} < -\Delta_{nH}$.

We first focus on the β^+/EC-decay half-life of ^{100}Cd, and show the corresponding contributions of p-h and p-p interactions in Fig. 1. By comparing the unperturbed results obtained by the RH and RHB approaches, it is clear that the $T = 1$ proton-neutron pairing interaction in Eq. (4) plays an important role in the ground state for the half-life calculation. Note that its corresponding p-p residual interaction is not included in the QRPA for the unnatural parity modes. Then, the TV p-h residual interaction in Eq. (1) is introduced based on the RHB unperturbed result; however, its influence on the half-life calculation is almost negligible. Furthermore, the half-life
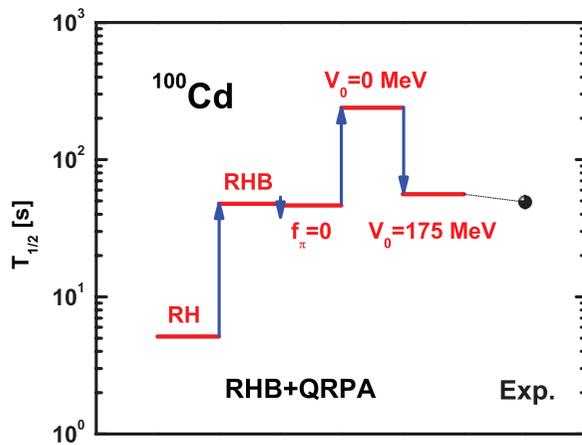
FIG. 1. (Color online) The β^+/EC-decay half-life of ^{100}Cd calculated by the self-consistent RHB + QRPA approach with the effective interaction PC-PK1 [38] without and with the $T = 0$ proton-neutron pairing. The unperturbed results obtained by the RH and RHB approaches, and the QRPA result excluding the pion-nucleon p-h residual interactions are denoted by RH, RHB, and $f_\pi = 0$, respectively. For comparison, the experimental value [15] is also shown.

substantially increases when the pion-nucleon interaction in Eq. (2) and its zero-range counter term in Eq. (3) are included, because their total contributions are repulsive and dominant in p-h residual interactions for the GT excitations. Finally, it is found that the calculated half-lives are very sensitive to the $T = 0$ proton-neutron pairing interaction in Eq. (5) by comparing the results with and without such p-p residual interaction. In previous studies [20–23], the strength V_0 is usually determined by adjusting QRPA results to empirical half-lives. In this work, we take ^{100}Cd as the reference nucleus, and the value of V_0 is determined to be 175 MeV.

As shown in Eq. (6), the nuclear β-decay half-life is determined by the transition strength as well as the transition energy which decides the value of $f(Z, E_\nu)$. In order to illustrate the mechanism of the influence from $T = 0$ pairing on the β^+/EC-decay half-life, the Gamow-Teller transition strength distributions of ^{100}Cd are shown in Fig. 2. It is seen that without $T = 0$ pairing there is mainly one transition with $E_{\mathrm{QRPA}} = -1.879$ MeV contributing to nuclear β^+/EC decay. This transition is dominated by the spin-flip configuration $\pi 1g9/2 \rightarrow \nu 1g7/2$. Because both $\pi 1g9/2$ and $\nu 1g7/2$ orbitals are partially occupied (the occupation probabilities of $\pi 1g9/2$ and $\nu 1g7/2$ orbitals are 0.808 and 0.123, respectively), the $T = 0$ pairing can substantially contribute to the QRPA matrices related to the $\pi 1g9/2 \rightarrow \nu 1g7/2$ pair. When the attractive $T = 0$ pairing is included, the transition built from such a configuration is lowered in energy, and thus the value of function $f(Z, E_\nu)$ increases, while the GT strength decreases slightly. As a result, the β^+/EC-decay half-life is remarkably reduced.

In order to further investigate the impact of $T = 0$ pairing interaction on the β^+/EC decays, the corresponding half-lives for Cd and Sn isotopes calculated by the self-consistent RHB + QRPA approach with and without the $T = 0$ pairing interaction are shown in Fig. 3. It is clear that the calculations
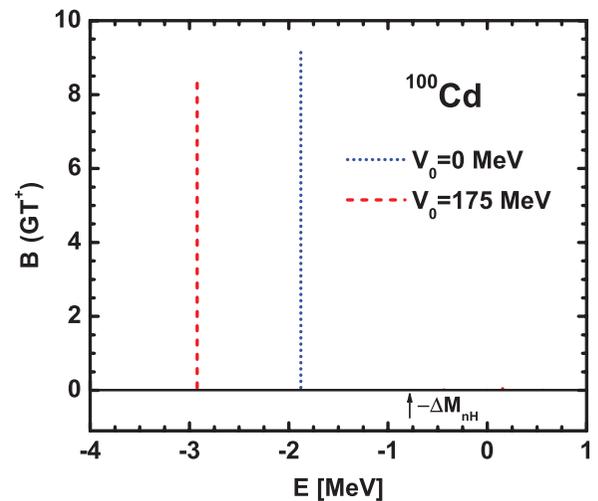

FIG. 2. (Color online) Gamow-Teller transition probabilities of ^{100}Cd calculated by the RHB + QRPA approach with the effective interaction PC-PK1 without and with the $T = 0$ proton-neutron pairing. The threshold for EC decay is shown with an arrow.

without the $T = 0$ pairing interaction generally overestimate experimental values. By including the $T = 0$ pairing interaction, the calculated half-lives are significantly reduced and well reproduce the half-lives of ^{98,100}Cd and ^{100,102,104}Sn. Since ^{96}Cd is a deformed nucleus, the underestimation of half-life may originate from the deformation effect, because the deformation can spread and hinder the low-energy tails of the GT strength distributions [48]. This effect is not included in the present calculations. Therefore, it will be interesting to include deformation degrees of freedom into the self-consistent QRPA calculations and study their effects on β-decay half-lives in the future.

For the QRPA calculations [21,22], the strength of $T = 0$ proton-neutron pairing V_0 was usually determined by adjusting
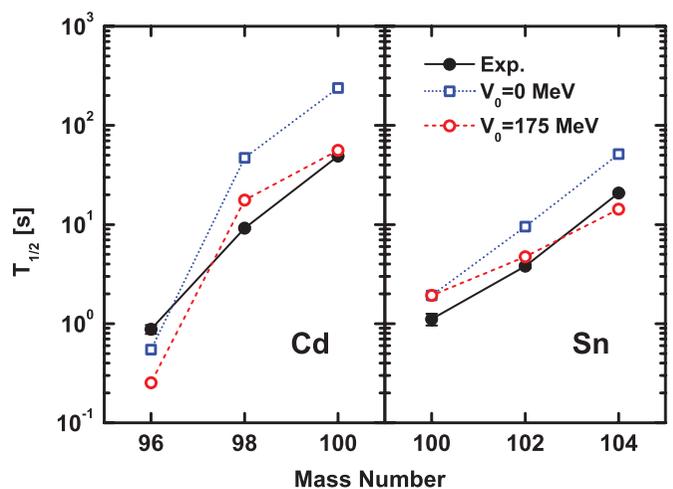

FIG. 3. (Color online) Comparison of the calculated half-lives of Cd and Sn isotopes using the RHB + QRPA approach and the effective interaction PC-PK1 with experimental data [15] (filled circles). The open squares and open circles denote the half-lives with the strength of $T = 0$ pairing $V_0 = 0$ and 175 MeV, respectively.

to the known half-life of selected nucleus in each isotopic chain. However, very different values are found for neutron-rich nuclei of different isotopic chains. Taking the Cd and Fe isotopic chains as examples, the difference between the corresponding V_0 is about 100 MeV [21,22]. This procedure, of course, limits the prediction power of the model.

For improving this dilemma, an isospin-dependent form of V_0 was proposed in Ref. [23] and achieved great success in the description of β-decay half-lives of neutron-rich nuclei with $20 \leqslant Z \leqslant 50$. In this isospin-dependent pairing strength, the values of V_0 are nearly constant for nuclei with $N - Z < 5$, which is exactly the case for the neutron-deficient nuclei in the same region, i.e., $20 \lesssim Z \lesssim 50$. Therefore, we further calculate the half-lives of Fe, Ni, Zn, Ar, Ca, and Ti isotopes with the same V_0 determined by the half-life of ^{100}Cd. The results are shown in Fig. 4. For comparison, the calculated results obtained from the macroscopic-microscopic finite-range droplet model (FRDM) + QRPA [17] and the SHF + BCS + QRPA with separable residual interactions [49] are also shown.

It is found that while these three approaches show similar isotopic trends of nuclear half-lives, the present self-consistent RHB + QRPA calculations reproduce the experimental data remarkably well. In contrast, the SHF + BCS + QRPA approach well reproduces the experimental half-life of ^{54}Ni, but it underestimates the experimental half-lives of ^{50,52}Ni. For the FRDM + QRPA approach, it almost systematically overestimates the experimental half-lives. It has been pointed out that the overestimation of half-lives in the FRDM + QRPA approach can be attributed partially to the neglect of the $T = 0$ pairing [20,23,50]. This is further supported by the present investigation on the β^+/EC decays in neutron-deficient nuclei.

In summary, we have extended the self-consistent QRPA approach to the charge-exchange channel based on the relativistic Hartree-Bogoliubov model for the nonlinear point-coupling effective interaction. This approach is then used to systematically investigate the β^+/EC-decay half-lives of neutron-deficient nuclei around the proton magic numbers $Z = 20, 28$, and 50. It is found that the calculated half-lives are very sensitive to the $T = 0$ proton-neutron pairing interaction. By including the $T = 0$ pairing interaction, the calculated half-lives are remarkably reduced, as the GT transitions are substantially lowered in energy while the transition strengths
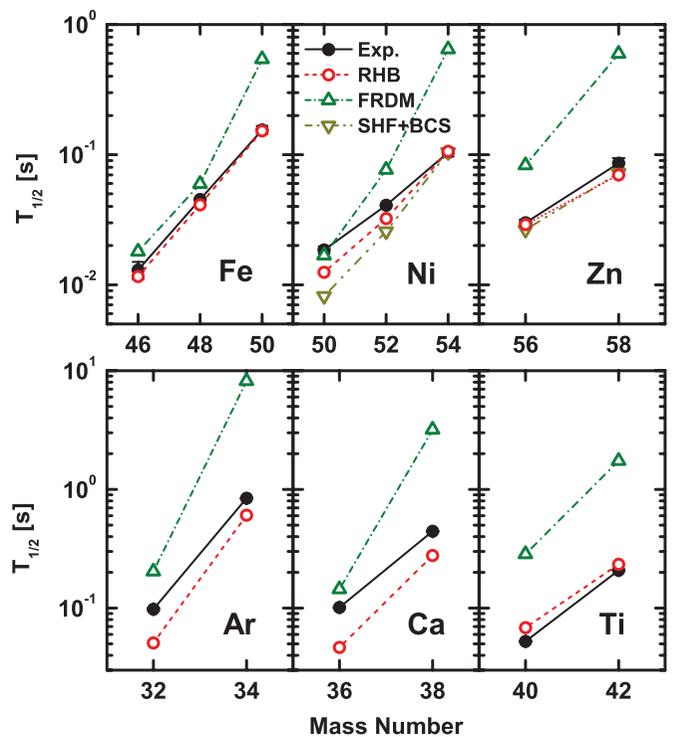

FIG. 4. (Color online) Nuclear β^+/EC-decay half-lives for Fe, Ni, Zn, Ar, Ca, and Ti isotopes calculated by the RHB + QRPA approach with the effective interaction PC-PK1 and $V_0 = 175$ MeV. For comparison, the experimental data [15], as well as theoretical results obtained from FRDM + QRPA [17] and SHF + BCS + QRPA [49] approaches are also shown.

only slightly decrease. The experimental β^+/EC-decay half-lives of Ar, Ca, Ti, Fe, Ni, Zn, Cd, and Sn isotopes can be well reproduced by a universal $T = 0$ pairing strength.

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