PHYSICAL REVIEW C **87**, 045209 (2013)

Two-photon exchange in the $\ell + p \to \ell + p$ process with a massive lepton

Dian-Yong Chen[1,2,*] and Yu-Bing Dong[3,4]

[1]*Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China*
[2]*Research Center for Hadron and CSR Physics, Lanzhou University & Institute of Modern Physics of CAS, Lanzhou 730000, China*
[3]*Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China*
[4]*Theoretical Physics Center for Science Facilities, Chinese Academy of Sciences, Beijing 100049, China*

(Received 28 December 2012; revised manuscript received 7 April 2013; published 29 April 2013)

Stimulated by the proton size puzzle and the proposal of μp scattering at the Paul Scherrer Institute (PSI), we study the two-photon exchange corrections to the massive-lepton–proton scattering. The leptonic helicity broken caused by lepton mass introduces new form factors. We estimate the two-photon exchange corrections to these form factors as well as to the unpolarized differential cross sections.

DOI: 10.1103/PhysRevC.87.045209 PACS number(s): 14.60.Ef, 13.40.Gp, 24.85.+p, 25.30.Mr

I. INTRODUCTION

Lepton scattering is an ideal probe of electromagnetic structures of materials due to the perturbative properties of quantum electrodynamics. The small coupling constants permit us to evaluate electromagnetic interactions to any increase of accuracy, and the point-like lepton can honestly reflect the information on the initial structures of the target.

As the start point of revealing subnuclear structure, the properties of nucleons are fundamental and significant. Two different methods have been employed to measure the electromagnetic form factors of nucleons. One is the Rosenbluth separation method or longitudinal-transverse separation method [1], which utilized the linear relationship between the unpolarized differential cross section and the kinematic factor ϵ or polarization of the virtual photon. The other is the polarization separation method [2,3], which measured the polarization transfer in the polarized electron and nucleon scattering process. The ratio of proton electric form factor to the magnetic form factor obtained from the former separation method is very close to 1 [4–7], which indicates similar spatial distributions of electric and magnetic fields in the nucleon. However, the polarization separation method points out that the electromagnetic form factor ratio decreases rapidly as Q^2 increases [8,9]. Both separation methods are based on the one-photon exchange approximation, and the discrepancy of the form factor ratio caused by different experimental techniques simulates the theorist to investigate the mechanism beyond the one-photon exchange approximation.

A rough frame of the radiative corrections to the electron-nucleon scattering had been built by Mo and Tsai [10,11] and updated by Maximon and Tjon [12]. A more precise calculation including both infinite and finite part of the two-photon exchange (TPE) diagrams indicates that the TPE correction to the differential cross section were dependent on the kinematic factor ϵ, which would break the linear relationship between differential cross section and kinematic factor ϵ under the one-photon exchange approximation [13]. The TPE process can reconcile the discrepancy of the form factor ratio of protons caused by different separation methods [14]. Many different approaches have been developed to evaluate the TPE corrections to elastic electron-nucleon scattering, such as the hadronic model and quark model. In the hadronic model, the intermediate states between two photons are taken as nucleons or nucleon resonances [10–13,15–18] and all the couplings are treated in the hadron level. In the quark model, the TPE corrections are first evaluated in quark level and then connect the quarks or parton to the nucleon by a set of relations, such as parton distribution functions [19,20] or wave functions in the light cone quark model [21].

The nucleon charge root-mean-square (rms) radius can be determined by the slope of electric form factor at $Q^2 = 0$. With the world data on *ep* scattering before 2003, the extracted rms radius is 0.895 ± 0.018 fm [22]. In Ref. [23], the TPE corrections have been included in the rms radius extractions from the electric form factors, and the obtained radius is 0.897 ± 0.018 fm. In 2010, the value of proton charge radius was updated based on measurements of elastic *ep* scattering cross section performed at the Mainz Microton MAMI. The charge radius of the proton is determined to be $0.879(5)_{\text{stat.}}(4)_{\text{syst.}}(2)_{\text{model}}(4)_{\text{group}}$ fm [24]. In addition, the proton charge radius can also be extracted from the hyperfine structure of the hydrogen atom, since the proton size can modify the hydrogen energy levels. With the $2p$-$1s$ transition energy and the $1s$ hyperfine structure of hydrogen, which is well measured with a high degree of accuracy, the extracted rms radius is 0.8768 ± 0.0069 fm [25]. The proton rms radius from hydrogen spectroscopy is consistent with the one from electron-proton scattering data within the measurement uncertainties.

Similar to hydrogen, the energy levels and hyperfine splitting of muonic hydrogen can also be used to obtain the proton rms radius. Moreover, the short lifetime and heavy mass of muons make the muonic hydrogen energy levels more sensitive to proton size than those of hydrogen. The extracted rms radius of the proton is 0.84184 ± 0.00067 fm [26], which is 5σ smaller than the one based on hydrogen spectroscopy and electron-proton scattering data. The new data from muonic hydrogen spectroscopy make the proton size unintelligible. The μp scattering, which is similar to *ep* scattering, may provide us more information on the

*chendy@impcas.ac.cn

proton size. Studying the proton radius puzzle with μp scattering has been proposed at the Paul Scherrer Institute (PSI) [27]. As in the ep scattering process, the TPE corrections will also be important in accurately extracting the proton form factor from the differential cross sections. Particularly, because the muon is about 200 times heavier than the electron, the lepton helicity conservation will be broken to some extent. This stimulates us to study how the TPE correction behaves in the massive- lepton–proton scattering process.

This paper is organized as follows. After the introduction, the TPE corrections to $\ell + p \to \ell + p$ will be considered in the hadronic model in Sec. II. Our results of TPE corrections to the form factors and differential cross sections are presented in Sec. III, and Sec. IV is devoted to the summary.

II. TPE CORRECTIONS IN HADRONIC MODEL

Generally, under the one-photon exchange approximation, the nucleon electromagnetic vertex, which includes two terms and two form factors as well, is in the form

$$\Gamma_\mu(q) = F_1(Q^2)\gamma_\mu + i\frac{F_2(Q^2)}{2m_N}\sigma_{\mu\nu}q^\nu, \tag{1}$$

where q is the momentum transfer to the nucleon and m_N is the mass of the nucleon. F_1 and F_2 are Dirac and Pauli form factors, respectively, which are only dependent on $Q^2 = -q^2$ under the one-photon exchange approximation. With the electromagnetic vertex in Eq. (1), one can obtain the amplitude of $\ell(k_1) + p(p_1) \to \ell(k_2) + p(p_2)$,

$$\mathcal{M}_{1\gamma} = e^2\bar{u}(k_2)\gamma^\mu u(k_1)\frac{g_{\mu\nu}}{q^2}\bar{U}(p_2)\Gamma^\nu(q^2)U(p_1), \tag{2}$$

where $u(k)$ and $U(p)$ are the Dirac spinors of lepton and nucleon, respectively. Then, the unpolarized differential cross section is in the form

$$\frac{d\sigma}{d\Omega} = \mathcal{C}(Q^2,\epsilon)\left[G_M^2(Q^2) + \frac{\epsilon}{\tau}G_E^2(Q^2)\right], \tag{3}$$

where the kinematics factor ϵ can relate to Mandelstam variables by $\epsilon = [(s-u)^2 + t(4m_N^2 - t)]/[(s-u)^2 - (4m_l^2 + t)(4m_N^2 - t)]$. The Mandelstam variables s, t, u are defined as $s = (p_1 + k_1)^2$, $t = q^2 = -Q^2$ and $u = (p_1 - k_2)^2$ and they satisfy $s + t + u = 2m_\ell^2 + 2m_N^2$. The Sachs electric and magnetic form factors $G_E(Q^2)$ and $G_M(Q^2)$ are the linear combinations of Dirac and Pauli form factors, which are $G_E(Q^2) = F_1(Q^2) + \tau F_2(Q^2)$ and $G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$, respectively.

Considering a general case, a massive lepton scatters on a nucleon, including a multiphoton exchange process. The spins of lepton and nucleon are both 1/2 and thus the independent helicity amplitudes of the $\mathcal{T}$ matrix of $\ell(k_1, h_1) + p(p_1, \lambda_1) \to \ell(k_2, h_2) + p(p_2, \lambda_2)$ should be $2^4 = 16$. The number will be reduced from 16 to 8 due to parity invariance. Time-reversal invariance further reduces the number of the independent helicity amplitudes to 6. Alternatively, we can expand the elastic scattering $\mathcal{T}$ matrix in terms of a set of six independent Lorentz structures multiplied by six generalized form factors in the form

$$\begin{aligned}\mathcal{T}^{h_2,h_1}_{\lambda_2,\lambda_1} = \frac{e^2}{q^2}\Big[&\tilde{F}_1\bar{u}(k_2,h_2)\gamma^\mu u(k_1,h_1)\bar{U}(p_2,\lambda_2)\gamma_\mu U(p_1,\lambda_1)\\ &+\tilde{F}_2\bar{u}(k_2,h_2)\gamma^\mu u(k_1,h_1)\bar{U}(p_2\lambda_2)\frac{i\sigma_{\mu\nu}q^\nu}{2m_N}U(p_1,\lambda_1)\\ &+\tilde{F}_3\bar{u}(k_2,h_2)\gamma^\mu u(k_1,h_1)\bar{U}(p_2,\lambda_2)\frac{\hat{K}P^\mu}{4m_N}U(p_1,\lambda_1)\\ &+\tilde{F}_4\bar{u}(k_2,h_2)\gamma_5 u(k_1,h_1)\bar{U}(p_2,\lambda_2)\gamma_5 U(p_1\lambda_1)\\ &+\tilde{F}_5\bar{u}(k_2,h_2)\frac{i\sigma_{\mu\nu}q^\nu}{2m_N}u(k_1,h_1)\bar{U}(p_2,\lambda_2)\gamma_\mu U(p_1\lambda_1)\\ &+\tilde{F}_6\bar{u}(k_2,h_2)u(k_1,h_1)\bar{U}(p_2,\lambda_2)U(p_1,\lambda_1)\Big],\end{aligned} \tag{4}$$

with $P = (p_1 + p_2)/2$, $K = (k_1 + k_2)/2$, $\hat{p} = \gamma^\mu p_\mu$, and $q = p_2 - p_1 = k_1 - k_2$. The form factors $\tilde{F}_i$ ($i = 1$–6) are not only dependent on Q^2 but also on the kinematics factor ϵ. An alternative expression of $\bar{u}(k_2,h_2)\gamma^\mu u(k_1,h_1)\bar{U}(p_2,\lambda_2)(\hat{K}P^\mu/4m_N)U(p_1,\lambda_1)$ in the above $\mathcal{T}$ matrix is the axial-vector-like term $\bar{u}(k_2,h_2)\gamma^\mu\gamma_5 u(k_1,h_1)\bar{U}(p_2,\lambda_2)\gamma_\mu\gamma_5 U(p_1,\lambda_1)$, and these two expression can be related by the identity

$$\begin{aligned}&\bar{u}(k_2,h_2)\gamma^\mu\gamma_5 u(k_1,h_1)\bar{U}(p_2,\lambda_2)\gamma_\mu\gamma_5 U(p_1,\lambda_1)\\ &\quad=\frac{u-s}{t}\bar{u}(k_2,h_2)\gamma^\mu u(k_1,h_1)\bar{U}(p_2,\lambda_2)\gamma_\mu U(p_1,\lambda_1)\\ &\qquad+\frac{1}{t}\bar{u}(k_2,h_2)\gamma^\mu u(k_1,h_1)\bar{U}(p_2,\lambda_2)\hat{K}P^\mu U(p_1,\lambda_1)\\ &\qquad-\frac{4m_\ell m_N}{t}\bar{u}(k_2,h_2)\gamma_5 u(k_1,h_1)\bar{U}(p_2,\lambda_2)\gamma_5 U(p_1\lambda_1).\end{aligned} \tag{5}$$

If one ignores the lepton mass, then the helicity of the lepton will be conserved, and the independent helicity amplitudes reduce to 3. Only the first three terms survive in the massless lepton limit since the terms related to $\tilde{F}_4$, $\tilde{F}_5$, and $\tilde{F}_6$ change the helicity of the lepton. In the massless lepton limit, Eq. (4) will reduce to the form of Eq. (8) in Ref. [14], and Eq. (5) will be the same as Eq. (11) in Ref. [20]. In the one-photon approximation, Eq. (4) reduces to Eq. (2), with Dirac and Pauli form factors.

In the hadronic model, the intermediate states of the TPE process are treated as nucleons or nucleon resonances. In present work, we are mainly concerned with the TPE correction in the small Q^2 region; thus, only the processes where the nucleon as the intermediate state are considered. The amplitude corresponding to the diagrams in Fig. 1 is

$$\mathcal{M}_{2\gamma} = e^4\int\frac{d^4k}{(2\pi)^4}\left[\frac{N_a(k)}{D_a(k)} + \frac{N_b(k)}{D_b(k)}\right], \tag{6}$$

where the numerators are in the forms

$$\begin{aligned}N_a(k) &= \bar{u}(k_2)\gamma_\mu(\hat{k}_1 - \hat{k} + m_\ell)\gamma_\nu u(k_1)\bar{U}(p_2)\\ &\quad\times\Gamma^\mu(p_2 - p_1 - k)(\hat{p}_1 + \hat{k} + m_N)\Gamma^\nu(k)U(p_1),\\ N_b(k) &= \bar{u}(k_2)\gamma_\mu(\hat{k}_1 - \hat{k} + m_\ell)\gamma_\nu u(k_1)\bar{U}(p_2)\Gamma^\nu(k)\\ &\quad\times(\hat{p}_2 - \hat{k} + m_N)\Gamma^\mu(p_2 - p_1 - k)U(p_1).\end{aligned} \tag{7}$$

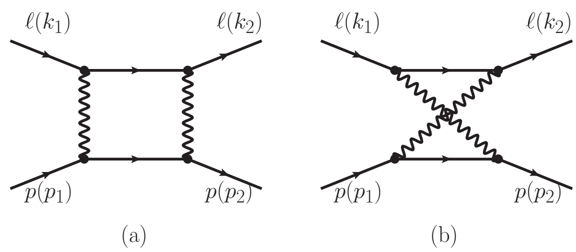

FIG. 1. The TPE process with proton as intermediate state in $\ell + p \to \ell + p$ scattering.

The electromagnetic vertexes of the nucleon are in the form defined in Eq. (1). The numerators in Eq. (6) are the product of the scalar propagators,

$$\begin{aligned} D_a(k) &= \left[(k_1 - k)^2 - m_\ell^2\right][k^2 - \lambda^2] \\ &\quad \times [(p_2 - p_1 - k)^2 - \lambda^2]\left[(p_1 + k)^2 - m_N^2\right], \\ D_b(k) &= \left[(k_1 - k)^2 - m_\ell^2\right][k^2 - \lambda^2] \\ &\quad \times [(p_2 - p_1 - k)^2 - \lambda^2]\left[(p_2 - k)^2 - m_N^2\right], \end{aligned} \quad (8)$$

where an infinitesimal photon mass λ has been introduced in the photon propagator to regulate the infrared (IR) divergences of the loop integrals.

In the previous treatment established by Mo and Tsai (MT) [10,11], the loop integral in Eq. (6) is treated approximately by taking one of the four-momenta of the photons to be zero in the numerator and in the other photon propagator. Neglecting the noninfrared terms in the TPE amplitudes, it can be simplified to be

$$\mathcal{M}_{2\gamma}(\text{MT}) = \delta(\text{MT})\mathcal{M}_{1\gamma}, \quad (9)$$

with $\delta(\text{MT}) = -\alpha/\pi[K(p_1, k_1) + K(p_2, k_2)]$ and

$$\begin{aligned} K(p_i, p_j) &= (p_i \cdot p_j) \int_0^1 \frac{dy}{p_y^2} \ln \frac{p_y^2}{\lambda^2}, \\ p_y &= p_i y + p_j(1 - y). \end{aligned} \quad (10)$$

The MT correction treated the IR divergent part correctly and had usually been considered in the experimental analysis.

The IR divergent part of TPE corrections will be canceled by the IR divergence in other radiative corrections, which include the bremsstrahlung process and vertex corrections. The radiative corrections to the μp elastic process are less well known at present. In Ref. [28], the purely leptonic contributions to the radiative corrections have been investigated considering the rather small energy of the recoil proton.

In present work we are mainly concerned with the effects of lepton mass on the TPE corrections, including the effects on the Lorentz structures and on the corresponding form factors. To compare with the TPE correction in the ep scattering process, the IR part of the TPE corrections in μp scattering is treated in a very similar way. In the present calculations, we keep the mass of the lepton in p_y^2 in Eq. (10), which is consistent with the full calculation for TPE amplitudes in Eq. (6). We only consider the finite or noninfrared part of the TPE amplitudes, which is

$$\mathcal{M}_{2\gamma}^{\text{NonIR}} = \mathcal{M}_{2\gamma} - \mathcal{M}_{2\gamma}(\text{MT}). \quad (11)$$

This amplitude is independent of the infinitesimal photon mass λ.

III. TPE CORRECTIONS TO FORM FACTORS AND DIFFERENTIAL CROSS SECTIONS

In present work, the intermediate states of the TPE process are treated as nucleons. The electromagnetic vertexes involved in the TPE process are the ones under one-photon approximation, which is presented in Eq. (1). We use the form in which the Dirac and Pauli form factors are parameterized directly in terms of sums of monopoles,

$$F_{1,2}(Q^2) = \sum_{i=1}^{3} \frac{n_i}{d_i + Q^2}. \quad (12)$$

The form factors in the above expression are more realistic and reasonable than the simple monopole or dipole form. The parameters are the same as those in Ref. [15]. Similar to the treatment in Ref. [29,30] and using Feynman parametrization, we can obtain the TPE corrections to the form factors.

The TPE corrections to the form factors estimated from ep scattering at $Q^2 = 1$ GeV^2 are presented in Fig. 2. The left panel shows the TPE corrections to $\delta G_{E,M}$ and $Y_{2\gamma}$, which are the combination of $\tilde{F}_1(Q^2, \epsilon)$, $\tilde{F}_2(Q^2, \epsilon)$, and $\tilde{F}_3(Q^2, \epsilon)$,

$$\begin{aligned} \tilde{G}_M(Q^2, \epsilon) &= \tilde{F}_1 + \tilde{F}_2 = \delta\tilde{G}_M(Q^2, \epsilon) + G_M(Q^2), \\ \tilde{G}_E(Q^2, \epsilon) &= \tilde{F}_1 + \tau\tilde{F}_2 = \delta\tilde{G}_E(Q^2, \epsilon) + G_E(Q^2), \\ Y_{2\gamma} &= \text{Re}\left(\frac{\nu\tilde{F}_3}{m_N^2 G_M^2}\right), \end{aligned} \quad (13)$$

with $\nu = P \cdot K = (s - u)/4$. The TPE corrections to $\delta G_{E,M}/G_{E,M}$ are of order 1% and are strongly dependent on the kinematic factor ϵ, while the TPE corrections to $Y_{2\gamma}$ is weakly sensitive to ϵ and very small: nearly one order smaller than $\delta\tilde{G}_{E,M}/G_{E,M}$. The right panel presents the TPE corrections to $\tilde{F}_i/G_M$ $(i = 4, 5, 6)$ depending on the kinematic factor ϵ, which result from the leptonic helicity breaking caused by lepton mass. For the ep scattering process, the mass of the lepton is approximately 1/2000 of proton mass, thus, the TPE corrections to $\tilde{F}_i/G_M$ $(i = 4, 5, 6)$ are ignorable; they are at least one order smaller than $\delta\tilde{G}_{E,M}/G_{E,M}$.

The TPE corrections to the form factors at $Q^2 = 1$ GeV^2 estimated from the μp scattering process are presented in Fig. 3. Compare to the case of ep scattering, the TPE corrections to $\delta\tilde{G}_{E,M}/G_{E,M}$ and $Y_{2\gamma}$ are almost the same. However, the TPE corrections to $\tilde{F}_i/G_M$ $(i = 4, 5, 6)$ are much larger than those obtained from ep scattering, which

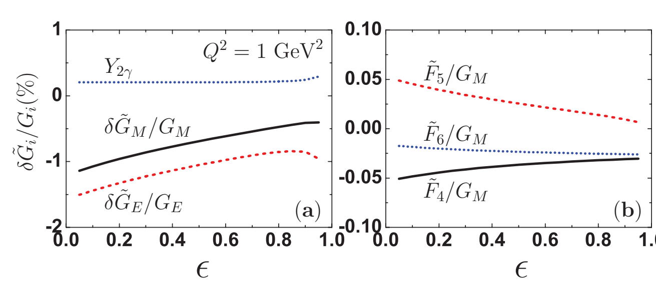

FIG. 2. (Color online) Two-photon exchange corrections to the form factors obtained in the $e^- + p \to e^- + p$ process depending on kinematic factor ϵ. The TPE corrections to the form factors are in units of percent.

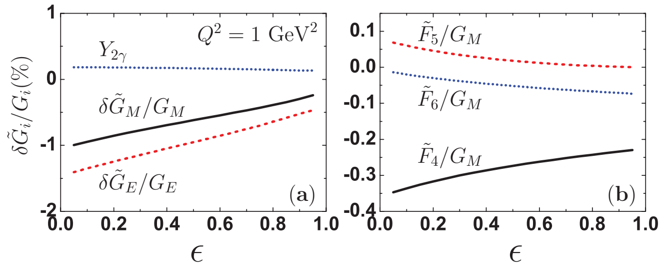

FIG. 3. (Color online) Two-photon exchange corrections to the form factors obtained in the $\mu^- + p \rightarrow \mu^- + p$ process depending on kinematic factor ϵ. The TPE corrections to the form factors are in units of percent.

is understandable since the muon is much heavier than the electron and a larger lepton mass will introduce a larger leptonic helicity broken.

The TPE corrections to the form factors from ep and μp scattering at $Q^2 = 2$ GeV^2 are presented in Figs. 4 and 5, respectively. The TPE corrections to the form factors are much larger than those corresponding corrections at $Q^2 = 1$ GeV^2. For $\delta\tilde{G}_{E,M}/G_{E,M}$ and $Y_{2\gamma}$, visible discrepancies appear between those evaluated from ep and μp process. $\tilde{F}_i/G_M$ $(i = 4, 5, 6)$ resulting from TPE corrections of the μp process are larger than those of the ep process, which is similar to the case of $Q^2 = 1$ GeV^2.

TPE corrections to the differential cross section come from the interferences between TPE amplitude and the one under the one-photon exchange approximation, which is

$$\delta(Q^2, \epsilon) = 2\frac{\mathrm{Re}\big[\mathcal{M}_{2\gamma}^{\mathrm{NonIR}}\mathcal{M}_{1\gamma}^{\dagger}\big]}{|\mathcal{M}_{1\gamma}|^2}. \tag{14}$$

Otherwise, the TPE corrections to the differential cross section can also be evaluated by Eq. (4) together with the TPE corrections to the form factors. With the amplitudes given in Eq. (4), we can obtain the TPE corrections to the reduced differential cross sections of the ℓp scattering process, which is in the form

$$\delta\tilde{\sigma}_R = 2\bigg(G_M\delta\tilde{G}_M + \frac{\epsilon}{\tau}G_E\delta\tilde{G}_E\bigg) + \mathcal{C}_{M3}G_M\tilde{F}_3 + \mathcal{C}_{E3}G_E\tilde{F}_3 + \mathcal{C}_{M5}G_M\tilde{F}_5 + \mathcal{C}_{E5}G_E\tilde{F}_5 + \mathcal{C}_{E6}G_E\tilde{F}_6 + \mathcal{O}\big(\tilde{F}_i^2\big). \tag{15}$$

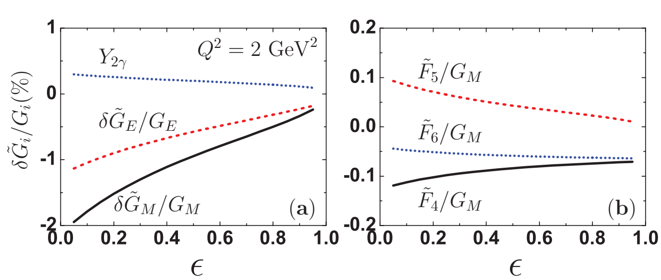

FIG. 4. (Color online) The same as Fig. 2 but for $Q^2 = 2$ GeV^2.

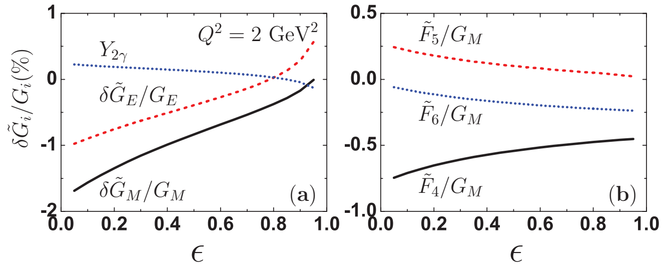

FIG. 5. (Color online) The same as Fig. 3 but for $Q^2 = 2$ GeV^2.

The ignored terms $\mathcal{O}(F_i^2)$ are proportional to $\tilde{F}_i\tilde{F}_j$ $(i, j = 4, 5, 6)$, which are next leading order corrections and of order $\alpha^2 m_\ell^2$, while $G_{E,M}\delta\tilde{G}_{E,M}$ and $G_{E,M}\tilde{F}_3$ are of order α, and the terms proportional to $G_{E,M}\tilde{F}_i$ $(i = 4, 5, 6)$ are of order αm_ℓ. As shown in Eq. (A1) in Appendix, the coefficients $\mathcal{C}_{Ei}$ and $\mathcal{C}_{Mi}$ $(i = 4, 5, 6)$ are proportional to m_ℓ. In the massless lepton limit, these terms vanish and the TPE corrections to the differential cross section reduce to the one in Eq. (34) in Ref. [15].

The differences of the TPE corrections to the differential cross section caused by including or ignoring the electron mass are very small and nearly invisible. In Fig. 6, we present the TPE corrections to differential cross sections for ep and μp including the masses of leptons. For comparison, the corrections to the μp scattering process evaluated by ignoring the muon mass in the numerators are also presented. The TPE corrections to the differential cross sections of massive or massless muon-proton scattering are very similar, but the differences are visible, especially in the small ϵ region. The corrections to μp scattering behave similarly to the corrections to electron-proton scattering. The TPE corrections rise with increasing Q^2. For a fixed Q^2, the corrections are strongly dependent on the kinematic factor ϵ as shown in Fig. 6, which indicates that the TPE corrections must be included when extracting the proton electromagnetic form factors from the unpolarized differential cross section of μp scattering. At $\epsilon = 0$, the TPE corrections reach the maxima, which are about 2% for $Q^2 = 1$ GeV^2 and 3.5% for $Q^2 = 2$ GeV^2. With ϵ increasing, the TPE corrections decrease and nearly vanish at $\epsilon = 1$. Moreover, the TPE corrections for μp scattering are a little bit smaller than those for ep scattering. At $\epsilon = 0$, the

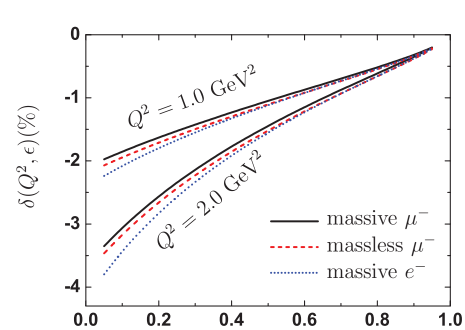

FIG. 6. (Color online) TPE corrections to the differential cross sections of the $\ell + p \rightarrow \ell + p$ process depending on the kinematic factor ϵ at $Q^2 = 1.0$ GeV^2 and $Q^2 = 2.0$ GeV^2.

discrepancies are about 0.3% and 0.5% for $Q^2 = 1.0$ GeV^2 and $Q^2 = 2.0$ GeV^2, respectively.

IV. SUMMARY

TPE corrections are important for eliminating the discrepancy of proton electromagnetic form factor ratios measured by different experimental techniques. However, the size of proton is still not exactly clear due to the disagreements of the data from muonic hydrogen and ep scattering or hydrogen. To clarify the proton size puzzle, the study of proton radius with μp scattering had been proposed at PSI.

The mass of the muon is two orders larger than that of the electron and comparable to the mass of the proton; thus, it is necessary to study the TPE corrections to the μp scattering process including the mass of the muon. In the present work, we present the general form of the amplitudes for the massive-lepton–nucleon scattering process. Three new Lorentz structures as well as form factors, which are leptonic helicity broken, are introduced due to the mass of the lepton. We estimate the TPE corrections to these form factors in the small Q^2 region. We find that the TPE corrections to these new form factors for the ep scattering process are smaller than those for the μp scattering process, which are consistent with expectations.

The TPE corrections to the differential cross sections are also evaluated. Similar to the case of ep scattering, the TPE corrections rise with increasing Q^2. The differences caused by the mass of the muon are visible. Moreover, the TPE corrections are strongly dependent on the kinematic factor ϵ, which means the TPE corrections are crucial in exactly extracting the form factors from μp scattering.

The other radiative corrections, including vertex corrections, vacuum polarization, proton and lepton self-energy process, and bremsstrahlung process, are also important to exactly extract the form factors of the proton from the experimental measurements. In addition, the TPE process and the bremsstrahlung process are opposite for $\ell^+ p$ and $\ell^- p$ scattering. Systematically estimations of the radiative corrections for massive lepton scattering on protons are necessary [31].

ACKNOWLEDGMENTS

This work is supported by the National Sciences Foundations of China under Grants No. 11005129, No. 10975146, No. 11035006, No. 11261130311, by the DFG and the NSFC through funds provided to the Sino-German CRC 110 “Symmetries and the Emergence of Structure in QCD”, and by the West Doctoral Project of the Chinese Academy of Sciences.

APPENDIX: COEFFICIENTS IN TPE CORRECTIONS TO THE DIFFERENTIAL CROSS SECTIONS

With the amplitudes in Eq. (4), we can obtain the TPE corrections to the differential cross sections as shown in Eq. (15). The coefficients before the form factors are the functions of Mandelstam variables s, t and the masses of the proton and lepton. The involved coefficients are

$$\begin{aligned}
\mathcal{C}_{M3} &= -\big(2\big(m_\ell^2+m_N^2-s\big)-t\big)\big(m_\ell^4+\big(m_N^2-s\big)^2 \\
&\quad -2m_\ell^2\big(m_N^2+s\big)+st\big)\big/\big(m_N^2 D_M\big),\\
\mathcal{C}_{E3} &= 4\big(2\big(m_\ell^2+m_N^2-s\big)-t\big)\big(\big(m_\ell^2+m_N^2-s\big)^2 \\
&\quad +\big(-m_\ell^2+s\big)t\big)\big/(tD_M),\\
\mathcal{C}_{M5} &= 2m_\ell\big(4m_N^2-t\big)t\big/(m_N D_M),\\
\mathcal{C}_{E5} &= 4m_\ell m_N\big(4m_N^2-t\big)\big/D_M,\\
\mathcal{C}_{E6} &= 4m_\ell m_N\big(4m_N^2-t\big)\big(2\big(m_\ell^2+m_N^2-s\big)-t\big)/(tD_M),
\end{aligned} \tag{A1}$$

with $D_M = [2m_\ell^4 + 2m_N^4 + 2s^2 - 4m_\ell^2(m_N^2+s) + 2st + t^2 - 4m_N^2(s+t)]$.

[1] M. N. Rosenbluth, Phys. Rev. **79**, 615 (1950).
[2] A. I. Akhiezer and M. P. Rekalo, Dokl. Akad. Nauk SSSR **180**, 1081 (1968) [Sov. Phys. Dokl. **13**, 572 (1968)].
[3] A. I. Akhiezer and M. P. Rekalo, Fiz. Elem. Chast. Atom. Yadra **4**, 662 (1973) [Sov. J. Part. Nucl. **4**, 277 (1974)].
[4] L. Andivahis, P. E. Bosted, A. Lung, L. M. Stuart, J. Alster, R. G. Arnold, C. C. Chang, F. S. Dietrich *et al.*, Phys. Rev. D **50**, 5491 (1994).
[5] R. C. Walker, B. Filippone, J. Jourdan, R. Milner, R. McKeown, D. H. Potterveld, L. Andivahis, R. Arnold *et al.*, Phys. Rev. D **49**, 5671 (1994).
[6] J. Arrington, Phys. Rev. C **68**, 034325 (2003).
[7] M. E. Christy *et al.* (E94110 Collaboration), Phys. Rev. C **70**, 015206 (2004).
[8] M. K. Jones *et al.* (Jefferson Lab Hall A Collaboration), Phys. Rev. Lett. **84**, 1398 (2000).
[9] O. Gayou *et al.* (Jefferson Lab Hall A Collaboration), Phys. Rev. Lett. **88**, 092301 (2002).
[10] L. W. Mo and Y.-S. Tsai, Rev. Mod. Phys. **41**, 205 (1969).
[11] Y.-S. Tsai, Phys. Rev. **122**, 1898 (1961).
[12] L. C. Maximon and J. A. Tjon, Phys. Rev. C **62**, 054320 (2000).
[13] P. G. Blunden, W. Melnitchouk, and J. A. Tjon, Phys. Rev. Lett. **91**, 142304 (2003).
[14] P. A. M. Guichon and M. Vanderhaeghen, Phys. Rev. Lett. **91**, 142303 (2003).
[15] P. G. Blunden, W. Melnitchouk, and J. A. Tjon, Phys. Rev. C **72**, 034612 (2005).
[16] S. Kondratyuk, P. G. Blunden, W. Melnitchouk, and J. A. Tjon, Phys. Rev. Lett. **95**, 172503 (2005).
[17] S. Kondratyuk and P. G. Blunden, Phys. Rev. C **75**, 038201 (2007).
[18] D. Borisyuk and A. Kobushkin, Phys. Rev. C **74**, 065203 (2006).
[19] Y. C. Chen, A. Afanasev, S. J. Brodsky, C. E. Carlson, and M. Vanderhaeghen, Phys. Rev. Lett. **93**, 122301 (2004).
[20] A. V. Afanasev, S. J. Brodsky, C. E. Carlson, Y.-C. Chen, and M. Vanderhaeghen, Phys. Rev. D **72**, 013008 (2005).
[21] D.-Y. Chen and Y.-B. Dong, Nucl. Phys. A **832**, 209 (2010).
[22] I. Sick, Phys. Lett. B **576**, 62 (2003).
[23] P. G. Blunden and I. Sick, Phys. Rev. C **72**, 057601 (2005).

[24] J. C. Bernauer *et al*. (A1 Collaboration), Phys. Rev. Lett. **105**, 242001 (2010).
[25] P. J. Mohr, B. N. Taylor, and D. B. Newell, Rev. Mod. Phys. **80**, 633 (2008).
[26] R. Pohl, A. Antognini, F. Nez, F. D. Amaro, F. Biraben, J. M. R. Cardoso, D. S. Covita, A. Dax *et al*., Nature (London) **466**, 213 (2010).
[27] R. Gilman *et al*. (MUSE Collaboration), arXiv:1303.2160.
[28] E. Borie, arXiv:1207.6651.
[29] P. Van Nieuwenhuizen, Nucl. Phys. B **28**, 429 (1971).
[30] G. 't Hooft and M. J. G. Veltman, Nucl. Phys. B **153**, 365 (1979).
[31] D.-Y. Chen and Y.-B. Dong (unpublished).