

**Damping of high-energy particle-hole-type nuclear excitations: A semimicroscopic model**

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A semimicroscopic model (particle-hole dispersive optical model) is formulated to describe the main relaxation modes of high-energy particle-hole-type excitations in medium-heavy mass nuclei. Within this model, Landau damping and the single-particle continuum are considered microscopically, while the spreading effect is treated phenomenologically, employing a statistical assumption. Description of direct nucleon decay of the above-mentioned excitations (including giant resonances) is a unique feature of the proposed model, which in applying to closed-shell nuclei is arranged for practical implementations. The methodical similarity of formulations of the single-quasiparticle and particle-hole optical models is emphasized.

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**I. INTRODUCTION**

High-energy particle-hole-type excitations are the doorway states for many nuclear reactions. To use information deduced from reaction cross sections for an analysis of nuclear structure and reaction mechanisms, it is desirable to develop a model that describes damping of the above-mentioned excitations. A great variety of these excitations, including giant resonances (GRs), is characterized by three main relaxation modes. They are as follows: (i) the particle-hole (p-h) strength distribution, or Landau damping, which is a result of the shell structure of nuclei; (ii) coupling of (p-h)-type states with the single-particle continuum that leads to direct nucleon decay of these states and related phenomena; and (iii) coupling of (p-h)-type states with many-quasiparticle configurations, or chaotic states, that leads to the spreading effect. Actually, the interplay of these relaxation modes varies with increasing excitation energy. Giant resonances correspond to collective (p-h)-type excitations.

As applied to the description of GR damping, we developed a semimicroscopic approach based on the continuum-random-phase-approximation (cRPA) version of Migdal’s finite Fermi-system theory [1]. Within this approach, Landau damping and coupling with the single-particle continuum are described microscopically, using a mean field and p-h interaction, while the spreading effect is phenomenologically taken into account directly in the cRPA equations for energy-averaged quantities in terms of an effective single-particle optical-model potential. Such a method allows one to realize the statistical assumption: After energy averaging, different (p-h)-type states, having the same angular momentum and parity, “decay” into chaotic states independently of one another. In implementations of the approach to describe GR main properties, a phenomenological mean field and the Landau-Migdal p-h interaction bound by some self-consistency conditions are used. The imaginary part of the effective single-particle optical-model potential determines the contribution of the spreading effect to the GR main characteristics together with the corresponding real part found from a proper dispersive relationship. The intensity of the

imaginary part, which is parameterized as a universal function, exhibiting a saturation-like energy dependence, is adjusted to describe the experimental GR strength distribution. In such a description the semimicroscopic approach is intermediate between “fully microscopic” (chaotic states are substituted by a number of 2p-2h configurations) and semiclassical (the shell structure of nuclei is ignored) approaches. The unique feature of the semimicroscopic approach is a possibility to describe direct-nucleon-decay properties of GRs without the use of specific adjustable parameters. Formulation and a diversity of implementations of the semimicroscopic approach, which is valid in the “pole” approximation (i.e., at the close range of the GR energy) are reviewed in Refs. [2,3].

It is essential that the semimicroscopic approach should be extended to an energy region far from the centroid energy of the GR (to describe, for instance, the GR high- and/or low-energy “tails”). Such a need appears in the “Coulomb” description of the isobaric analog resonance properties, which are determined by the low-energy “tail” of the charge-exchange monopole GR [4,5]. Another example is the asymmetry (relative  $90^\circ$ ) of the differential photoneutron and inverse reaction cross sections in the energy region of the isovector  $E2$ -GR [6]. The asymmetry is due to interference of the  $E1$ - and  $E2$ -reaction amplitudes. Recently, a problem with explanation of experimental data on distribution of the isoscalar monopole strength in a wide excitation energy interval has appeared [7]. One more reason for formulation of an extended model is a verification of the method used within the semimicroscopic approach for the phenomenological description of the spreading effect on GR properties.

In the present work we extend the above-outlined semimicroscopic approach to describe damping of (p-h)-type states at arbitrary (but high-enough) excitation energies. The extended version is formulated starting from the many-body nonlocal p-h Green function, which satisfies the Bethe-Goldstone-type equation. We call this version the p-h dispersive optical model (PHDOM), bearing in mind the analogy with a transition to the single-quasiparticle dispersive optical model (SQDOM). The latter is also formulated semimicroscopically, starting from the many-body single-particle Green function, which satisfies the Dyson equation. The proper parametrization of the averaged self-energy operator in this equation leads, finally,

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to the basic equations of the SQDOM—the equations for the optical-model single-particle Green function and nucleon-nucleus scattering wave function [8,9]—and to a dispersive relationship for the real part of the optical-model potential [9]. Direct nucleon decay of sub-barrier single-particle quasistationary states can be also described within the SQDOM [10,11]. As compared with the semimicroscopic description of GR damping [2,3], no new model parameters are used in the formulation of the PHDOM. First attempts to formulate this model have been undertaken in Ref. [12]. Preliminary consideration and a brief description are given in Refs. [3] and [13], respectively.

The paper is organized as follows. In Sec. II the PHDOM is formulated as an extension of the standard cRPA version by taking the spreading effect into account to describe the energy-averaged strength functions and transition densities of high-energy (p-h)-type excitations. Formulation of the PHDOM as an extension of the cRPA nonstandard version for describing direct-nucleon-decay properties of the mentioned excitations is presented in Sec. III. The methodical similarity of formulations of the SQDOM and PHDOM is discussed in Sec. IV, where an example of implementations of the new model and concluding remarks are also given.

## II. BASIC QUANTITIES AND EQUATIONS OF THE PHDOM

### A. Nonlocal p-h Green function

The basic quantity used within the cRPA standard version is the local p-h Green function (GF)  $A(x, x_1; \omega)$  [14]. Here,  $x$  is the set of single-particle coordinates, including the spin and isospin variables, and  $\omega$  is the current excitation energy. To realize the statistical assumption concerned with spreading (p-h)-type excitations at arbitrary energies (Sec. II C), we start the formulation of the PHDOM from the nonlocal p-h GF  $\mathcal{A}(x, x'; x_1, x'_1; \omega)$  satisfying the spectral expansion (see, e.g., Ref. [1]):

$$\mathcal{A}(x, x'; x_1, x'_1; \omega) = \sum_s \left[ \frac{\rho_s^*(x, x') \rho_s(x_1, x'_1)}{\omega - \omega_s + i0} - \frac{\rho_s^*(x'_1, x_1) \rho_s(x, x')}{\omega + \omega_s - i0} \right]. \quad (1)$$

Here  $\omega_s = E_s - E_0$  is the excitation energy of an exact state  $|s\rangle$  of the system,  $\rho_s(x, x') = \langle s | \hat{\Psi}^+(x) \hat{\Psi}(x') | 0 \rangle$  is the transition density matrix, and  $\hat{\Psi}^+(x)$  is the operator of nucleon creation at the point  $x$ . Being a superposition of 1p–1h, 2p–2h, 3p–3h, . . . configurations, states  $|s\rangle$  belong to the continuum if  $\omega_s > B_N$  ( $B_N$  is the nucleon separation energy). In such a case, which is predominantly considered below, wave functions  $|s\rangle$  are normalized to the delta-function of the energy.

The transition density matrix and various strength functions are related to the main characteristics of (p-h)-type excitations. In accordance with the expansion of Eq. (1), the squared transition density matrix in the continuum region is determined by  $\text{Im}\mathcal{A}$  as follows:

$$-\frac{1}{\pi} \text{Im}\mathcal{A}(x, x'; x_1, x'_1; \omega) = \rho^*(x, x'; \omega) \rho(x_1, x'_1; \omega). \quad (2)$$

The strength function  $S_{\mathcal{V}_0}(\omega) = \sum_s |(\hat{\mathcal{V}}_0)_{s0}|^2 \delta(\omega - \omega_s)$ , corresponding to a nonlocal single-particle external field  $\hat{\mathcal{V}}_0 = \int \hat{\Psi}^+(x) \mathcal{V}_0(x, x') \hat{\Psi}(x') dx dx'$  (hereafter the brackets  $[[. . .]]$  mean proper integration over coordinates, the number of the brackets corresponds to the number of integrations), is also determined by  $\text{Im}\mathcal{A}$ :

$$S_{\mathcal{V}_0}(\omega) = -\frac{1}{\pi} \text{Im}[[[\mathcal{V}_0^+ \mathcal{A}(\omega) \mathcal{V}_0]]]. \quad (3)$$

The alternative representation of the strength function  $S_{\mathcal{V}_0}$  in the continuum region follows from Eqs. (1)–(3):

$$S_{\mathcal{V}_0}(\omega) = |[[\rho(\omega) \mathcal{V}_0]]|^2. \quad (4)$$

In accordance with the expansion of Eq. (1), the free p-h GF  $\mathcal{A}_0(x, x'; x_1, x'_1; \omega)$  is determined by a mean field and the ground-state occupation numbers. Let  $H_0(x)$  be a single-particle Hamiltonian:  $[H_0(x) - \varepsilon_\lambda] \phi_\lambda(x) = 0$ , where  $\phi_\lambda$  and  $\varepsilon_\lambda$  are, respectively, the single-particle wave functions and energies. The free p-h transition density matrices  $\rho_s^{(0)}(x, x') = (1 - n_\lambda) n_\mu \phi_\lambda^*(x) \phi_\mu(x')$ , where  $n_{\lambda, \mu}$  are the single-particle occupation numbers, are orthogonal:  $[[\rho_{s'}^{(0)*} \rho_s^{(0)}]] = \delta_{s's}$ . This point allows one to realize the above-mentioned statistical assumption (Sec. II C) and, therefore, makes reasonable the use of the nonlocal p-h GF in the formulation of the PHDOM. Using further the free (p-h)-state energies  $\omega_s^{(0)} = \varepsilon_\lambda - \varepsilon_\mu$ , one gets the expression for  $\mathcal{A}_0$  in the form

$$\mathcal{A}_0(x, x'; x_1, x'_1; \omega) = \sum_{\lambda\mu} A_{\lambda\mu}(\omega) \phi_\lambda(x) \phi_\mu^*(x') \phi_\mu(x'_1) \phi_\lambda^*(x_1);$$

$$A_{\lambda\mu}(\omega) = \frac{n_\lambda - n_\mu}{\varepsilon_\lambda - \varepsilon_\mu - \omega}. \quad (5)$$

Following the standard cRPA version [14], we introduce the GF of the single-particle Schrodinger equation:  $[H_0(x) - \varepsilon] g(x, x'; \varepsilon) = -\delta(x - x')$ . The use of the spectral expansion of this GF allows one to get the alternative expression for  $\mathcal{A}_0$ :

$$\mathcal{A}_0(x, x'; x_1, x'_1; \omega) = \sum_{\mu} n_\mu \phi_\mu^*(x') \phi_\mu(x'_1) g(x, x_1; \varepsilon_\mu + \omega)$$

$$+ \sum_{\lambda} n_\lambda \phi_\lambda^*(x_1) \phi_\lambda(x) g(x'_1, x'; \varepsilon_\lambda - \omega). \quad (6)$$

As is well known, the radial component of  $g(x, x'; \varepsilon)$  can be presented as a product of regular and irregular solutions of the proper radial Schrödinger equation. After separation of the spin-angular and isobaric variables in Eq. (6), the use of the mentioned presentation allows one to take exactly the single-particle continuum into account.

The above-given expressions are also related to the local GF  $A(x, x_1; \omega) = \mathcal{A}(x = x'; x_1 = x'_1; \omega)$ . In such a case, the external field is supposed to be local:  $\mathcal{V}_0(x, x') = V_0(x) \delta(x - x')$ . It is noteworthy that the spectral expansion for  $A(x, x'; \omega)$  contains the transition densities  $\rho_s(x)$ . The free transition densities  $\rho_s^{(0)}(x)$  are obviously not orthogonal.

### B. Interaction in the p-h channel

The p-h interaction  $\mathcal{F}_{l-r}$ , leading to long-range correlations [in particular, to formation of GRs, as collective (p-h)-type

excitations], can be presented in the form

$$\mathcal{F}_{l-r}(x, x'; x_1, x'_1) = F(x, x_1)\delta(x - x')\delta(x_1 - x'_1). \quad (7)$$

Here  $F$  is the p-h interaction near the Fermi surface. This interaction can be calculated “microscopically” (starting, e.g., from the Skyrme-type forces) or taken phenomenologically (using, e.g., the Landau-Migdal forces). The cRPA standard version is formulated in terms of the local p-h GF  $A(x, x_1; \omega)$  and p-h interaction  $F(x, x_1)$  [14]:

$$A(\omega) = A_0(\omega) + [[A_0(\omega)FA(\omega)]]. \quad (8)$$

The alternative (and fully equivalent) cRPA standard version is formulated in terms of the effective external field  $V(x, \omega)$  corresponding to an external field (or to a probing operator)  $V_0(x)$ . The effective-field method (initially proposed in Ref. [1]) can be also based on the definition  $[A(\omega)V_0] = [A_0(\omega)V(\omega)]$ . From this definition and Eq. (8) follows the equation for the effective field:

$$V(\omega) = V_0 + [[FA_0(\omega)V(\omega)]]. \quad (9)$$

The strength function corresponding to the probing operator  $V_0(x)$  is determined by the effective field as follows:

$$S_{V_0}(\omega) = -\frac{1}{\pi} \text{Im}[[V_0^+ A_0(\omega)V(\omega)]]. \quad (10)$$

[compare with Eq. (3)].

For high-enough excitation energies, an additional specific interaction appears in the p-h channel,  $\mathcal{F}_{\text{spr}}(x, x'; x_1, x'_1; \omega)$ , responsible for the spreading effect. This interaction is determined by the p-h polarization operator, or the p-h fragmentation scattering amplitude,  $\pi(x, x'; x_1, x'_1; \omega)$ , describing coupling of (p-h)-type states with many-quasiparticle configurations. Because 2p-2h configurations are the doorway states for the spread of (p-h)-type states, the polarization operator is the convolution of the (local) 2p-2h GF,  $B(x_1, x_2, x_3, x_4; \omega)$ , with a “residual” pair interaction  $H'(x, x')$ :

$$\pi(\omega) = [[H'B(\omega)H']]. \quad (11)$$

The polarization operator is described by a set of diagrams irreducible in the p-h channel and takes into account the interference between particle and hole “decays” into chaotic states. Using the spectral expansion of the 2p-2h GF, one can present the spectral expansion of the polarization operator in the form

$$\pi(x, x'; x_1, x'_1; \omega) = \sum_m \left[ \frac{v_m^*(x, x')v_m(x_1, x'_1)}{\omega - \omega_m + i0} - \frac{v_m^*(x'_1, x_1)v_m(x', x)}{\omega - \omega_m + i0} \right]. \quad (12)$$

Here the chaotic state  $|m\rangle$  is a complicate superposition of 2p-2h, 3p-3h, ..., configurations;  $\omega_m$  and  $v_m$  are, respectively, the chaotic-state energy and transition potential matrix. The latter is the convolution of  $H'(x, x')$  and the transition density matrix  $\rho_m(x, x') = \langle m|\hat{\Psi}^+(x)\hat{\Psi}(x)\hat{\Psi}^+(x')\hat{\Psi}(x')|0\rangle$ ;  $v_m = [H'\rho_m]$ . The chaotic-state level density  $d^{-1}$  is extremely high and can be described by statistical formulas. After proper parametrization the quantity  $\pi(\omega = 0)$  should be included into the interaction  $\mathcal{F}_{l-r}$  (7), so the difference  $\pi(\omega) - \pi(0)$

can be considered as the p-h interaction  $\mathcal{F}_{\text{spr}}(\omega)$  responsible for the spreading effect.

### C. Extension of the cRPA standard version

The Bethe-Goldstone-type equation for the nonlocal p-h GF incorporates the full p-h interaction,

$$A(\omega) = A_0(\omega) + [[[[A_0(\omega)(\mathcal{F}_{l-r} + \mathcal{F}_{\text{spr}}(\omega))A(\omega)]]]]. \quad (13)$$

Being considered at high excitation energies, this equation presents formally the extension of the RPA standard version for taking the spreading effect into account.

In view of the high density of poles in the energy dependence of the polarization operator  $\pi(\omega)$  (12) and, therefore, of the p-h interaction  $\mathcal{F}_{\text{spr}}(\omega)$  in Eq. (13), only the energy-averaged functions  $\bar{\pi}(\omega) = \pi(\omega + iJ)$ ,  $\bar{\mathcal{F}}_{\text{spr}}(\omega) = \mathcal{F}_{\text{spr}}(\omega + iJ)$  ( $J \gg d$ ) can be properly parameterized and then considered as phenomenological quantities:

$$\bar{\mathcal{F}}_{\text{spr}}(x, x'; x_1, x'_1; \omega) = [-i\mathcal{W}(x, x'; \omega) + \mathcal{P}(x, x'; \omega)] \times \delta(x - x_1)\delta(x' - x'_1). \quad (14)$$

The delta-functions in this parametrization appear due to a large momentum transfer (of order of the Fermi momentum) at the “decay” of (p-h)-type states into many-quasiparticle configurations. Appearance of an imaginary part is the specific feature of the interaction  $\bar{\mathcal{F}}_{\text{spr}}$ ,  $\mathcal{W}(x, x'; \omega) = (\pi/d)|v_m(x, x')|^2$ , as it follows from (12) and (14). Supposing the coordinate dependence of  $\mathcal{W}$  and  $\mathcal{P}$  is the same, i.e.,  $\mathcal{W}(x, x'; \omega) = f(x, x')W(\omega)$  and  $\mathcal{P}(x, x'; \omega) = f(x, x')P(\omega)$ , one gets from (12) and (14) the dispersive relationship

$$P(\omega) = \frac{2}{\pi} \text{P.V.} \int_0^\infty W(\omega') \left( \frac{\omega'}{\omega^2 - \omega'^2} + \frac{1}{\omega'} \right) d\omega'. \quad (15)$$

It is reasonable to suppose that the energy dependence of  $W(\omega)$  starts from zero at a gap value  $\Delta$  (that means formally the appearance of chaotic states at a finite excitation energy). In such a case instead of (15) we have

$$P(\omega) = \frac{2}{\pi} \text{P.V.} \int_\Delta^\infty W(\omega') \left( \frac{\omega'}{\omega^2 - \omega'^2} - \frac{\omega'}{\Delta^2 - \omega'^2} \right) d\omega'. \quad (16)$$

This dispersive relationship is exploited in implementations of the semimicroscopic approach [15]. Within this approach the following parametrization of  $W(\omega)$  is used:

$$2W(\omega \geq \Delta) = \alpha \frac{(\omega - \Delta)^2}{1 + (\omega - \Delta)^2/B^2}, \quad 2W(\omega \leq \Delta) = 0, \quad (17)$$

where  $\alpha \simeq 0.1 \text{ MeV}^{-1}$  is the adjustable parameter, while the “gap” parameter  $\Delta = 3 \text{ MeV}$  and the “saturation” parameter  $B = 7 \text{ MeV}$  are taken as the universal phenomenological quantities [2,3]. It is noteworthy that the phenomenological description of the spreading effect in medium-heavy nuclei is justified at high-enough excitation energies (practically at  $\omega \gtrsim B_N$ ), when the statistical formulas for the level density do work. Being obtained with the use of the parametrization (17), the rather cumbersome expression for  $P(\omega)$  (16) is given in

Ref. [15]. In the limit  $\Delta = 0$ , which corresponds to Eq. (15), this expression is simplified as

$$P(\omega) = \frac{\alpha}{\pi} \frac{\omega^2 B^2}{\omega^2 + B^2} \ln \frac{\omega}{B}. \quad (18)$$

Due to the use of the dispersive relationships (15) and (16) the full basis of many-quasiparticle configurations is formally taken into account within the phenomenological description of the spreading effect. This advantage, allowing us to describe correctly the energy shift of (p-h)-type excitations caused by the spreading effect, is lost within the ‘‘fully microscopic’’ approaches which include, as a rule, only a limited number of 2p-2h configurations (see, e.g., Ref. [16]).

In applying to the ‘‘decay’’ of free p-h states  $|s^{(0)}\rangle$  into many-quasiparticle configurations the statistical assumption mentioned in the Introduction means the following [17]:

$$\frac{2\pi}{d} \overline{\langle s_1^{(0)} | \hat{H}' | m \rangle \langle m | \hat{H}' | s^{(0)} \rangle} = \Gamma_{s^{(0)}}^\downarrow \delta_{s^{(0)} s_1^{(0)}}. \quad (19)$$

Here,  $\Gamma_{s^{(0)}}^\downarrow$  is the spreading width of the state  $|s^{(0)}\rangle$ . The condition (19) can be expressed in terms of  $\text{Im}\bar{\mathcal{F}}_{\text{spr}}$  (14) and the free transition density matrices  $\rho_s^{(0)}(x, x')$  considered in Sec. II A:

$$\begin{aligned} \Gamma_{s^{(0)}}^\downarrow \delta_{s^{(0)} s_1^{(0)}} &= \frac{2\pi}{d} \overline{[[\rho_{s_1}^{(0)} v_m]]^* [[v_m \rho_s^{(0)}]]} \\ &= 2W(\omega_s^{(0)}) [[\rho_{s_1}^{(0)*} f \rho_s^{(0)}]]. \end{aligned} \quad (20)$$

Since the different free transition density matrices are orthogonal, the condition (20) is fulfilled provided that the coordinate dependence of  $\mathcal{W}(x, x'; \omega)$  in Eq. (14) is taken to be constant in the nuclear volume. Depending on applications, we take this dependence as  $f(x, x') = f_{\text{WS}}(x) f_{\text{WS}}(x')$ , or  $f(x, x') = f_{\text{WS}}(x)$ , where  $f_{\text{WS}}(x)$  is the well-known Woods-Saxon function. Such a choice leads, for instance, to the expression  $\Gamma_{\lambda\mu}^\downarrow = 2W(\omega = \varepsilon_\lambda - \varepsilon_\mu)(1 - n_\lambda)n_\mu f_\lambda f_\mu$ , where  $f_\lambda = [\phi_\lambda^* f_{\text{WS}} \phi_\lambda]$ .

The initial equation (13) for the nonlocal p-h GF can be practically used only after energy averaging,

$$\bar{\mathcal{A}}(\omega) = \mathcal{A}_0(\omega) + [[[[\mathcal{A}_0(\omega)(\mathcal{F}_{l-r} + \bar{\mathcal{F}}_{\text{spr}})\bar{\mathcal{A}}(\omega)]]]]. \quad (21)$$

Here, the free p-h GF is determined by Eq. (5). To solve Eq. (21), we introduce the energy-averaged free nonlocal p-h GF  $\bar{\mathcal{A}}_0(x, x'; x_1, x'_1; \omega)$  satisfying the equation

$$\bar{\mathcal{A}}_0(\omega) = \mathcal{A}_0(\omega) + [[[[\mathcal{A}_0(\omega)\bar{\mathcal{F}}_{\text{spr}}\bar{\mathcal{A}}_0(\omega)]]]]. \quad (22)$$

The basic Eq. (21) for  $\bar{\mathcal{A}}(\omega)$  can be expressed in terms of  $\bar{\mathcal{A}}_0$  and  $\mathcal{F}_{l-r}$ ,

$$\bar{\mathcal{A}}(\omega) = \bar{\mathcal{A}}_0(\omega) + [[[[\bar{\mathcal{A}}_0(\omega)\mathcal{F}_{l-r}\bar{\mathcal{A}}(\omega)]]]]. \quad (23)$$

The auxiliary Eq. (22), corresponding to the model of non-interacting damping quasiparticles, can be analytically solved with taking the statistical assumption into account in the spirit

of Eq. (20). The result has the form similar to Eq. (5):

$$\begin{aligned} \bar{\mathcal{A}}_0(x, x'; x_1, x'_1; \omega) &= \sum_{\lambda\mu} \bar{A}_{\lambda\mu}(\omega) \phi_\lambda(x) \phi_\mu^*(x') \phi_\mu(x'_1) \phi_\lambda^*(x_1), \\ \bar{A}_{\lambda\mu}(\omega) &= \frac{(n_\lambda - n_\mu)}{\{\varepsilon_\lambda - \varepsilon_\mu - \omega + (n_\lambda - n_\mu)[iW(\omega) - P(\omega)]f_\lambda f_\mu\}}. \end{aligned} \quad (24)$$

Here, the quantities  $P(\omega)$  and  $W(\omega)$  can be taken in accordance with Eqs. (16) and (17), respectively. The solution (24) of Eq. (22) means actually that the free p-h transition density matrices are not changed after taking the spreading effect together with the statistical assumption into account.

#### D. PHDOM equations

The analytical solution of Eq. (22) makes possible the formulation of the PHDOM equations, which can be further used for practical implementations. For this purpose, we turn from Eqs. (23) and (24) to their local limit and, as a result, get direct extension of the RPA equations for taking the spreading effect into account. In particular, the solution of the PHDOM equation

$$\bar{\mathcal{A}}(\omega) = \bar{\mathcal{A}}_0(\omega) + [[\bar{\mathcal{A}}_0(\omega)F\bar{\mathcal{A}}(\omega)]] \quad (25)$$

[compare with Eq. (8)] allows one to get (i) the energy-averaged strength function corresponding to the local probing operator  $V_0(x)$ ,

$$\bar{S}_{V_0}(\omega) = -\frac{1}{\pi} \text{Im}[[V_0^+ \bar{\mathcal{A}}(\omega)V_0(\omega)]], \quad (26)$$

[compare with Eq. (3)] and (ii) the energy-averaged squared transition density and strength function in the continuum region,

$$-\frac{1}{\pi} \text{Im}\bar{\mathcal{A}}(x, x_1; \omega) = \overline{\rho^*(x, \omega)\rho(x_1, \omega)}, \quad (27)$$

$$\bar{S}_{V_0}(\omega) = \overline{[\rho(\omega)V_0]^2} \quad (28)$$

[compare with Eqs. (2) and (4), respectively].

Within the PHDOM the effective-field method can be formulated using the definition  $[\bar{\mathcal{A}}(\omega)V_0] = [\bar{\mathcal{A}}_0(\omega)\bar{V}(\omega)]$ . From this definition and Eqs. (25) and (26) follows (i) the equation for the energy-averaged effective field  $\bar{V}(x, \omega)$ ,

$$\bar{V}(\omega) = V_0 + [[F\bar{\mathcal{A}}_0(\omega)\bar{V}(\omega)]] \quad (29)$$

[compare with Eq. (9)] and (ii) the expression for the energy-averaged strength function via  $\bar{V}(x, \omega)$ ,

$$\bar{S}_{V_0}(\omega) = -\frac{1}{\pi} \text{Im}[[V_0^+ \bar{\mathcal{A}}_0(\omega)\bar{V}(\omega)]], \quad (30)$$

[compare with Eq. (10)]. Extending the Migdal’s terminology of Ref. [1], one can say that Eqs. (25)–(30) correspond to the model of interacting and damping quasiparticles.

Supposing the summation in the expression for  $\bar{\mathcal{A}}_0$

$$\bar{\mathcal{A}}_0(x, ; x_1; \omega) = \sum_{\lambda\mu} \bar{A}_{\lambda\mu}(\omega) \phi_\lambda(x) \phi_\mu^*(x) \phi_\lambda^*(x_1) \phi_\mu(x_1), \quad (31)$$

[with  $\bar{A}_{\lambda\mu}(\omega)$  taken from (24)] is limited by the bound (and quasibound) single-particle states, we get in this approximation Eqs. (25), (26), (29), and (30), which correspond to the extended discrete-RPA (dRPA) version. As an illustrative example of this version, we consider the simplest (“two-level”) model. Within this model the (“single-level”) GR is formed due to p-h transitions from the filled level “1” (simulates the closed shell) to an empty level “2” (simulates the open shell). The energy-averaged strength function corresponding to a low-multipole external field  $V_0(x)$  then can be found in accordance with Eqs. (29), (30), and (31),

$$\bar{S}_{V_0}(\omega) = -\frac{1}{\pi} \text{Im} R_d \left[ \frac{1}{\omega - \omega_d + [iW(\omega) - P(\omega)]f_1 f_2} - \frac{1}{\omega + \omega_d - [iW(\omega) - P(\omega)]f_1 f_2} \right]. \quad (32)$$

Here  $\omega_d$  is the GR energy determined by the intershell distance and p-h interaction and  $R_d$  is the GR strength determined by squared matrix elements of  $V_0$ . Being valid in a wide excitation energy interval, this equation illustrates the following statement: taking the spreading effect into account in the “pole” approximation by the substitution  $\omega \rightarrow \omega + (iW - P)$  directly in RPA equations (this procedure is widely used within the semi-microscopic approach of Refs. [2,3]) leads to a rather incorrect description of the GR low- and high-energy “tails.”

The extended cRPA standard version is based on the representation of Eq. (31), which is obtained with the use of the following approximate equalities:

$$\sum_{\lambda} \frac{\phi_{\lambda}(x)\phi_{\lambda}^*(x_1)}{\varepsilon_{\lambda} - \varepsilon_{\mu} - \omega - [iW(\omega) - P(\omega)]f_{\lambda}f_{\mu}} \simeq -\bar{g}(x, x_1; \varepsilon_{\mu} + \omega), \quad (33)$$

where  $\bar{g}(\varepsilon_{\mu} + \omega)$  is the single-particle optical-model GF, satisfying the equation

$$\{H_0(x) - [\varepsilon_{\mu} + \omega + [iW(\omega) - P(\omega)]f_{\mu}f_{WS}(x)]\} \times \bar{g}(x, x_1; \varepsilon_{\mu} + \omega) = -\delta(x - x_1), \quad (34)$$

$$\sum_{\mu} \frac{\phi_{\mu}(x_1)\phi_{\mu}^*(x)}{\varepsilon_{\lambda} - \varepsilon_{\mu} - \omega + [iW(\omega) - P(\omega)]f_{\lambda}f_{\mu}} \simeq \bar{g}(x_1, x; \varepsilon_{\lambda} - \omega), \quad (35)$$

where the single-particle optical-model GF  $\bar{g}(\varepsilon_{\lambda} - \omega)$  satisfies the equation

$$\{H_0(x_1) - [\varepsilon_{\lambda} - \omega + [iW(\omega) - P(\omega)]f_{\lambda}f_{WS}(x_1)]\} \times \bar{g}(x_1, x; \varepsilon_{\lambda} - \omega) = -\delta(x_1 - x). \quad (36)$$

From the approximate equalities (33) and (35) follows representation of the GF  $\bar{A}_0(x, x_1; \omega)$  in the form, taking formally into account the full basis of single-particle states,

$$\bar{A}_0(x, x_1; \omega) = \sum_{\mu} n_{\mu} \phi_{\mu}^*(x) \phi_{\mu}(x_1) \bar{g}(x, x_1; \varepsilon_{\mu} + \omega) + \sum_{\lambda} n_{\lambda} \phi_{\lambda}^*(x_1) \phi_{\lambda}(x) \bar{g}(x_1, x; \varepsilon_{\lambda} - \omega)$$

$$+ 2 \sum_{\mu\lambda} n_{\mu} n_{\lambda} \phi_{\mu}^*(x) \phi_{\mu}(x_1) \phi_{\lambda}^*(x_1) \phi_{\lambda}(x) \times \frac{[iW(\omega) - P(\omega)]f_{\lambda}f_{\mu}}{(\varepsilon_{\lambda} - \varepsilon_{\mu} + \omega)^2 - [iW(\omega) - P(\omega)]^2 f_{\lambda}^2 f_{\mu}^2}. \quad (37)$$

In the absence of the spreading effect ( $W = P = 0$ ), this representation goes to the local limit of Eq. (6),  $A_0(x, x_1)$ , which is the basic quantity of the cRPA standard version. Within the semimicroscopic approach of Refs. [2,3] the spreading effect is, in particular, taken into account in the “pole” approximation by substitution:  $A_0(\omega) \rightarrow A_0[\omega + [iW(\omega) - P(\omega)]] = \bar{A}_0^{(\text{pole})}(\omega)$ . One can see that the leading (“pole”) terms in the expressions for  $\bar{A}_0^{(\text{pole})}$  and  $\bar{A}_0$  are close, while other (“nonpole”) terms differ. This difference might be noticeable in the description of the GR “tails.”

### III. DIRECT-DECAY PROPERTIES OF (P-H)-TYPE EXCITATIONS WITHIN THE PHDOM

#### A. Extension of the cRPA nonstandard version

As mentioned in the Introduction, description of direct nucleon decay and related phenomena concerned with (p-h)-type excitations is the unique feature of the semimicroscopic approach and an additional motivation to further develop the approach. Within the cRPA standard version discussed in Sec. II only the full basis of single-particle states is taken into account, while direct-decay properties of these states are outside the scope of this version. This weak point has been overcome within the cRPA nonstandard version [2,3,17] used for the continuum region and extended below to formulate the PHDOM in applying to direct-decay properties of (p-h)-type excitations.

Following the methods used in Refs. [2,3], we first derive the alternative expression for the strength function of Eq. (3) in terms of the effective (nonlocal) field  $\mathcal{V}(x, x'; \omega)$ . The latter is defined by the relationship  $[[\mathcal{A}(\omega)\mathcal{V}_0]] = [[\mathcal{A}_0(\omega)\mathcal{V}(\omega)]]$  and satisfies the equation

$$\mathcal{V}(\omega) = \mathcal{V}_0 + [[[\mathcal{F}(\omega)\mathcal{A}_0(\omega)\mathcal{V}(\omega)]]], \quad (38)$$

which follows from the above-given definition and Eq. (13) with  $\mathcal{F}(\omega) = \mathcal{F}_{l-r} + \mathcal{F}_{\text{spr}}(\omega)$ . In accordance with Eqs. (3) and (38) we get the alternative expression for the strength function:

$$S_{\mathcal{V}_0}(\omega) = -\frac{1}{\pi} \text{Im} [[[\mathcal{V}^{\dagger} \mathcal{A}_0(\omega) \mathcal{V}(\omega)]]]. \quad (39)$$

This equation is derived under supposition that (p-h)-type and many-quasiparticle states have no common decay channels (pre-equilibrium decay is neglected). Taking  $\text{Im}\mathcal{A}_0(\omega)$  in the continuum region from Eqs. (5) and (6), we further get from Eq. (39) the following representation of the strength function in this region:

$$S_{\mathcal{V}_0}(\omega) = \sum_c S_{\mathcal{V}_0,c}(\omega), \quad S_{\mathcal{V}_0,c}(\omega) = |\mathcal{M}_{\mathcal{V}_0,c}(\omega)|^2, \quad (40)$$

$$\mathcal{M}_{\mathcal{V}_0,c}(\omega) = [[\Psi_{c,0}^{(+)}(\omega)\mathcal{V}(\omega)]], \quad (41)$$

$$\Psi_{c,0}^{(+)}(x, x'; \omega) = n_\mu \phi_{\varepsilon_c}^{(+)}(x) \phi_\mu^*(x').$$

Here, “c” is a set of the nucleon-decay-channel quantum numbers; the quantities  $S_{\mathcal{V}_0,c}(\omega)$  and  $\mathcal{M}_{\mathcal{V}_0,c}(\omega)$  can be respectively called the decay-channel strength function and the amplitude of the “direct+semi-direct” (DSD) reaction induced by the external field  $\mathcal{V}_0$ ; the quantity  $\Psi_{c,0}^{(+)}(x, x'; \omega)$  can be called the free nonlocal decay-channel wave function, corresponding to one hole [the bound-state wave function is  $\phi_\mu(x)$ ] and one particle in the continuum with the energy  $\varepsilon_c = \varepsilon_\mu + \omega > 0$  [the continuum-state wave function is  $\phi_{\varepsilon_c}^{(+)}(x)$ ]. The partial branching ratio for direct nucleon decay from an excitation energy interval  $\delta$  into channel “c” is defined as follows:

$$\beta_c(\delta) = \int_\delta S_{\mathcal{V}_0,c}(\omega) d\omega / \int_\delta S_{\mathcal{V}_0}(\omega) d\omega. \quad (42)$$

The partial branching ratios satisfy the unitary condition  $\sum_c \beta_c = 1$  independently of  $\delta$ , as it follows from (40) and (42).

To get within the PHDOM an expression for the energy-averaged DSD-reaction amplitude (see below), we derive, first, the alternative expression for the amplitude  $\mathcal{M}_{\mathcal{V}_0,c}$  (41). Let the convolution  $[[\mathcal{F}(\omega)\Psi_{c,0}^{(+)}(\omega)]]$  be the free decay-channel transition potential matrix  $v_{c,0}(x, x'; \omega)$ . The effective matrix  $v_c(\omega) = [[\mathcal{F}(\omega)\Psi_c^{(+)}(\omega)]]$  defined by the equation  $[[\mathcal{A}_0(\omega)v_c(\omega)]] = [[\mathcal{A}(\omega)v_{c,0}(\omega)]]$  satisfies Eq. (38) after substitution  $\mathcal{V}_0 \rightarrow v_{c,0}(\omega)$ . As a result, we get the equation for the effective decay-channel wave function  $\Psi_c^{(+)}(x, x'; \omega)$ :

$$\Psi_c^{(+)}(\omega) = \Psi_{c,0}^{(+)}(\omega) + [[[\mathcal{A}_0(\omega)\mathcal{F}(\omega)\Psi_c^{(+)}(\omega)]]]. \quad (43)$$

From Eqs. (38) and (43) follows the alternative expression for the DSD-reaction amplitude (41):

$$\mathcal{M}_{\mathcal{V}_0,c}(\omega) = [[\Psi_c^{(+)}(\omega)\mathcal{V}_0]]. \quad (44)$$

In the local limit and absence of the spreading effect the corresponding quantities  $S_{\mathcal{V}_0,c}(\omega)$ ,  $M_{\mathcal{V}_0,c}(\omega)$ ,  $\psi_{c,0}^{(+)}(x, \omega)$ ,  $\psi_c^{(+)}(x, \omega)$ ,  $b_c(\delta)$  and the equations for these quantities are related to the cRPA nonstandard version [3,17]. Within this version, nucleon-nucleus scattering accompanied by excitation of (p-h)-type states is described in terms of the p-h scattering amplitude  $\Gamma(x, x_1; \omega)$  [3,17]. The amplitude defined as  $[[FA(\omega)]] = [[\Gamma(\omega)A_0(\omega)]]$  satisfies the equation [1]

$$\Gamma(\omega) = F + [[FA_0(\omega)\Gamma(\omega)]]. \quad (45)$$

Matrix elements of  $\Gamma$ ,  $\Gamma_{c'c}(\omega) = [[\Psi_{c,0}^{(+)}(\omega)\Gamma(\omega)\Psi_{c,0}^{(+)}(\omega)]]$ , determine the S-matrix elements as follows:

$$S_{c'c}(\omega) = S_{cc}^{\text{pot}} \delta_{c'c} - 2\pi i \Gamma_{c'c}(\omega), \quad (46)$$

where  $S_{cc}^{\text{pot}}$  is the potential-scattering matrix and  $\sum_{c'} |S_{c'c}|^2 = 1$ .

## B. Energy averaging and PHDOM equations

Being energy averaged, the relationships given in the preceding section lead to the PHDOM equations, which can be practically implemented. We start from averaging Eq. (43),

which is then identically transformed in the same way as was done for Eq. (21), related to the nonlocal energy-averaged p-h GF. The resulting equation for the nonlocal energy-averaged effective decay-channel wave function  $\bar{\Psi}_c^{(+)}(x, x'; \omega)$  is the following:

$$\bar{\Psi}_c^{(+)}(\omega) = \bar{\Psi}_{c,0}^{(+)}(\omega) + [[[[\bar{\mathcal{A}}_0(\omega)\mathcal{F}_{l-r}\bar{\Psi}_c^{(+)}(\omega)]]]] \quad (47)$$

[compare with Eq. (23)]. Here  $\bar{\mathcal{A}}_0(\omega)$  is the energy-averaged free nonlocal p-h GF of Eq. (22) and  $\bar{\Psi}_{c,0}^{(+)}(x, x'; \omega)$  is the energy-averaged free nonlocal decay-channel wave function. The latter satisfies the auxiliary equation:

$$\bar{\Psi}_{c,0}^{(+)}(\omega) = \Psi_{c,0}^{(+)}(\omega) + [[[[\mathcal{A}_0(\omega)\bar{\mathcal{F}}_{spr}\bar{\Psi}_{c,0}^{(+)}(\omega)]]]] \quad (48)$$

[compare with Eq. (22)]. Being sought in the form  $\bar{\Psi}_{c,0}^{(+)}(x, x'; \omega) = n_\mu \bar{\phi}_{\varepsilon_c}^{(+)}(x) \phi_\mu^*(x')$ , the analytical solution of Eq. (48) can be found with taking the statistical assumption into account. The result consists in the integral equation for the energy-averaged continuum-state wave function  $\bar{\phi}_{\varepsilon_c}^{(+)}(x)$ . This equation is equivalent to the corresponding differential equation:

$$\{H_0(x) - [\varepsilon_c + [iW(\omega) - P(\omega)]f_\mu f_{WS}(x)]\} \bar{\phi}_{\varepsilon_c}^{(+)} = 0 \quad (49)$$

[compare with Eq. (34)].

As follows from Eq. (44), the energy-averaged DSD-reaction amplitude, corresponding to the local external field  $V_0(x)$ , equals the convolution,

$$\bar{M}_{\mathcal{V}_0,c}(\omega) = [\bar{\Psi}_c^{(+)}(\omega)V_0]. \quad (50)$$

The structure of the PHDOM equations for both the energy-averaged effective local decay-channel wave function  $\bar{\psi}_c^{(+)}(x, \omega)$  [follows from Eq. (47)] and the energy-averaged effective local external field  $\bar{V}(x, \omega)$  [Eq. (29)] is the same. This point allows one to get the alternative expression for the amplitude of Eq. (50),

$$\bar{M}_{\mathcal{V}_0,c}(\omega) = [\bar{\psi}_{c,0}^{(+)}(\omega)\bar{V}(\omega)], \quad (51)$$

which is the direct extension of the corresponding expression used within the semimicroscopic approach of Refs. [2,3].

The energy-averaged DSD-reaction amplitude of Eq. (51) is the basic quantity in the PHDOM description of direct-nucleon-decay properties of high-energy (p-h)-type excitations. Apart from the energy-averaged DSD-reaction cross sections, the energy-averaged squared amplitude determines the partial branching ratio for direct nucleon decay  $b_c(\delta)$ :

$$b_c(\delta) = \int_\delta \bar{S}_{\mathcal{V}_0,c}(\omega) d\omega / \int_\delta \bar{S}_{\mathcal{V}_0}(\omega) d\omega \quad (52)$$

[compare with Eq. (42)]. Here  $\bar{S}_{\mathcal{V}_0}(\omega)$  is the strength function of Eq. (30), and  $\bar{S}_{\mathcal{V}_0,c}(\omega) = |\bar{M}_{\mathcal{V}_0,c}(\omega)|^2$  is the energy-averaged decay-channel strength function (the fluctuating part of this strength function is neglected). In view of the spreading effect, the total branching ratio  $b_{\text{tot}}(\delta) = \sum_c b_c(\delta)$  is less than unity. Therefore, the difference  $1 - b_{\text{tot}}$ , which is proportional to  $W(\omega)$ , can be considered as the branching ratio for statistical (mainly, neutron) decay.

Implementations of the PHDOM to describe nucleon-nucleus scattering accompanied by (p-h)-type excitations seem to be inevent (probably with the exception of excitation of the

isobaric analog and Gamow-Teller resonances). For completeness, we give the corresponding PHDOM relationships in the “pole” approximation. Starting from Eqs. (45) and (46), one gets the expression for the energy-averaged  $S$ -matrix elements:

$$\bar{S}_{c'c}^{\text{pole}}(\omega) = S_{cc}^{\text{bkg}}\delta_{c'c} - 2\pi i \left[ [\bar{\psi}_{c',0}^{(+)}(\omega) F \bar{\psi}_c^{+(\text{pole})}(\omega)] \right]. \quad (53)$$

Here  $S_{cc}^{\text{bkg}}$  is the “background”  $S$  matrix usually evaluated within the ordinary single-particle optical model, and  $\bar{\psi}_c^{+(\text{pole})}(x, \omega)$  is the energy-averaged local effective decay-channel wave function, satisfying an equation similar to Eq. (47). The difference  $1 - \sum_{c'} |\bar{S}_{c'c}(\omega)|^2 = T_c(\omega)$  determines the reaction cross section for a given entrance channel “c.”

#### IV. OPTICAL MODELS. IMPLEMENTATIONS OF THE PHDOM

##### A. Single-quasiparticle and particle-hole optical models

Possibilities to use the optical models to describe the simple modes of high-energy [single-quasiparticle and (p-h)-type] nuclear excitations are due to the fact that these modes do not lose their “individuality” because of their damping. There is a methodical similarity in formulations of the PHDOM and SQDOM. In both cases, the equations for the corresponding many-body GFs are used as the starting point. In formulating the SQDOM, the Dyson equation for the many-body single-particle GF  $G(x, x_1; \varepsilon)$  was exploited in Ref. [8] as follows:

$$G(\varepsilon) = G_0(\varepsilon) + [[G_0(\varepsilon)\Sigma(\varepsilon)G(\varepsilon)]]. \quad (54)$$

Here  $G_0(x, x'; \varepsilon)$  is the free single-particle GF determined by the mean-field and occupation numbers and  $\Sigma(x, x'; \varepsilon)$  is the self-energy operator describing coupling of single-quasiparticle states with many-quasiparticle configurations and satisfying the proper spectral expansion [1]. In view of the high density of poles in the energy dependence of  $\Sigma(\varepsilon)$  at high-enough excitation energies  $|\varepsilon - \mu|$  ( $\mu$  is the chemical potential), only the energy-averaged quantity  $\bar{\Sigma}(\varepsilon) = \Sigma[\varepsilon + iJ\text{sgn}(\varepsilon - \mu)]$  can be reasonably parameterized and then considered as a phenomenological quantity:

$$\bar{\Sigma}(x, x'; \varepsilon) = \text{sgn}(\varepsilon - \mu)[-iw(x, \varepsilon) + p(x, \varepsilon)]\delta(x - x'). \quad (55)$$

[compare with Eq. (14)]. Supposing that the imaginary and real parts of  $\bar{\Sigma}$  have the same coordinate dependence, i.e.,  $w(x, \varepsilon) = w(\varepsilon)q(x)$  and  $p(x, \varepsilon) = p(\varepsilon)q(x)$ , one gets the proper dispersive relationship for  $p(\varepsilon)$  [9]. After energy averaging from Eqs. (54) and (55) follows the equation for the single-particle optical-model GF  $\bar{g}(\varepsilon) = G[\varepsilon + iJ\text{sgn}(\varepsilon - \mu)]$ :

$$\begin{aligned} [H_0(x) - \varepsilon + \text{sgn}(\varepsilon - \mu)[-iw(\varepsilon) + p(\varepsilon)]q(x)] \\ \times \bar{g}(x, x'; \varepsilon) = -\delta(x - x'). \end{aligned} \quad (56)$$

[compare with Eqs. (34) and (36)]. The GF of Eq. (56) determines the energy-averaged single-particle (s.p.) strength function as follows:  $\bar{S}_{\text{s.p.}}(\varepsilon) = -\frac{1}{\pi}\text{sgn}(\varepsilon - \mu)\text{Im} \int \bar{g}(x = x'; \varepsilon)dx$ . It should be stressed that the imaginary and dispersive

real parts of the single-particle optical-model potential in Eqs. (34), (36), and (56) differ markedly in magnitude. Being taken at the same excitation energy the quantity  $W(\omega)$  of Eq. (17) is noticeably less than  $w(|\varepsilon - \mu|)$  in view of a (destructive) interference in spreading of particles and holes.

Turning to the description of nucleon-nucleus scattering within the SQDOM, one can define the energy-averaged single-particle continuum-state wave function  $\bar{\phi}_\varepsilon^{(+)}(x)$  in accordance with the Dyson-type equation,

$$\bar{\phi}_\varepsilon^{(+)} = \phi_\varepsilon^{(+)} + [[g(\varepsilon)\bar{\Sigma}(\varepsilon)\bar{\phi}_\varepsilon^{(+)}]]. \quad (57)$$

Here  $\phi_\varepsilon^{(+)}(x)$  is the potential scattering wave function, and the GF  $g(x, x'; \varepsilon)$  is defined before Eq. (6). Instead of this integral equation one can use the equivalent homogeneous differential equation [see Eq. (56)], which is the main SQDOM equation in the continuum region. The relative probability for direct decay of a single-particle sub-barrier quasistationary state is defined similarly to Eq. (52) [10,11]:

$$b_{\text{s.p.}} = \int |f^*\bar{\phi}_\varepsilon^{(+)}|^2 d\varepsilon / \left(-\frac{1}{\pi}\right) \text{Im} \int [[f^*\bar{g}(\varepsilon)f]] d\varepsilon. \quad (58)$$

Here integration is performed over the single-particle-resonance region, so the value of  $b_{\text{s.p.}} \simeq \Gamma_{\text{s.p.}}^\uparrow / (\Gamma_{\text{s.p.}}^\downarrow + \Gamma_{\text{s.p.}}^\uparrow)$  is almost independent of the form factor  $f(x)$  related to excitation of the sub-barrier state. Thus, we see again the similarity of formulations of the particle-hole and single-quasiparticle optical models.

##### B. Implementations of the PHDOM. Concluding remarks

To calculate within the PHDOM the main properties of high-energy (p-h)-type excitations, namely the strength function of Eq. (26), which corresponds to a spin-multipole isospin-dependent single-particle external field, the squared transition density of Eq. (27), the DSD-reaction amplitudes of Eq. (51), and the partial direct-nucleon-decay probabilities of Eq. (52), it is necessary to separate in these relationships and in the basic Eqs. (25), (29), (37), and (49) the spin-angular and isospin variables. This straightforward procedure is described, e.g., in Refs. [1,3]. As a result, the corresponding radial equations, having the same structure as the above-listed equations, should be solved numerically. Apart from a mean field and p-h interaction, leading to long-range correlations, the intensity of the imaginary part of the p-h interaction (14) should be also adopted. The specific parameter  $\alpha$  in parametrization (17) for  $W(\omega)$  can be adjusted to reproduce the experimental total width (or the rms energy dispersion) of a given GR in PHDOM calculations of the strength functions. The above-mentioned properties of the (p-h)-type excitations of a given spin-angular and isospin symmetry then can be described in a wide excitation-energy interval without the use of new parameters.

First implementations of the PHDOM applied to the description of neutron radiative capture accompanied by excitation of the isovector giant dipole and quadrupole resonances in a few medium-heavy mass spherical nuclei are given in Ref. [18]. This description is a direct extension of the semimicroscopic approach to the same problem given in

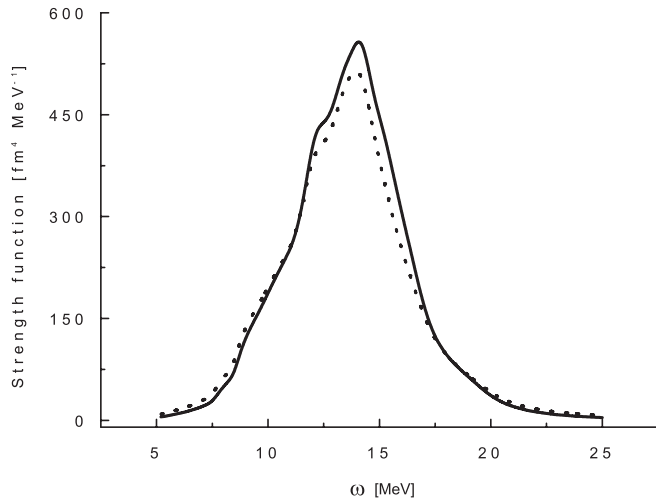


FIG. 1. The isoscalar monopole strength functions calculated for  $^{208}\text{Pb}$  within the PHDOM (full line) and the semimicroscopic approach to the description of GR damping (dotted line).

Ref. [6]. Here we show only one example concerned with calculations of the isoscalar monopole strength functions  $\bar{S}_{V_0}(\omega)$  and  $\bar{S}_{V_0}^{(\text{pole})}(\omega)$  ( $V_0 = r^2 Y_{00}$ ) in a vicinity of the isoscalar monopole GR in  $^{208}\text{Pb}$ . (Fig. 1). (Details of calculations are briefly given in Ref. [18]). The ratio  $R(\omega) = 2[\bar{S}_{V_0}(\omega) - \bar{S}_{V_0}^{(\text{pole})}(\omega)] / [\bar{S}_{V_0}(\omega) + \bar{S}_{V_0}^{(\text{pole})}(\omega)]$  illustrates the difference of the results obtained within the PHDOM and semimicroscopic approach to the description of GR damping (Fig. 2). As expected, the relative difference  $R(\omega)$  is noticeable at the GR “tails” where the considered strength functions are relatively small.

In conclusion, in the present work we formulate a new semimicroscopic model (particle-hole dispersive optical model) to describe the main relaxation modes of high-energy particle-hole-type excitations in medium-heavy mass nuclei. Within this model, which is the extension and verification of the previously developed semimicroscopic approach applied to the description of giant resonance damping, Landau damping and the single-particle continuum are considered

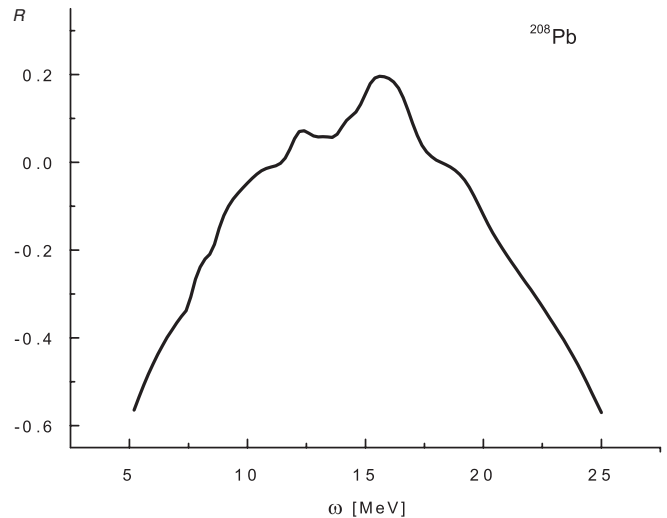


FIG. 2. The relative difference of the isoscalar monopole strength functions calculated for  $^{208}\text{Pb}$  within the PHDOM and the semimicroscopic approach to the description of GR damping (the definition is given in the text).

microscopically, while the spreading effect is treated phenomenologically by employing a statistical assumption. The model is valid at arbitrary (but high-enough) excitation energies. The description of direct-nucleon-decay properties of the above-mentioned excitations (including giant resonances) is a unique feature of the proposed model, which in applying to closed-shell nuclei is formulated for practical implementations.

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