Spin-aligned isoscalar pair correlation in ^{96}Cd, ^{94}Ag, and ^{92}Pd

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$T = 0$ states of ^{96}Cd, ^{94}Ag, and ^{92}Pd are studied by using the nucleon-pair approximation and the JUN45 interaction for the $p_{1/2}p_{3/2}f_{5/2}g_{9/2}$ shell. We focus on the approximation of isoscalar spin-aligned pairs in a single-j $1g_{9/2}$ shell. Configurations associated with the $2p_{1/2}$ and $2p_{3/2}$ orbits are investigated in a number of cases. For the yrast 0^+ and 2^+ states in ^{96}Cd and ^{92}Pd, the contribution from S and D nucleon pairs (i.e., $T = 1$ spin-zero and spin-two pairs) is studied. The "level-inversion" isomerism in ^{96}Cd and ^{94}Ag is discussed.

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I. INTRODUCTION

The pairing correlation between like nucleons, i.e., proton-proton or neutron-neutron type, plays an important role in low-lying states of atomic nuclei, and has been studied in many theories, such as the seniority scheme [1], the Bardeen-Cooper-Schrieffer theory [2–5], the interacting bosom model [6], the broken pair model [7,8], and the nucleon-pair approximation of the shell model [9,10]. There are also many efforts [11–23] to study the role played by the $T = 0$ proton-neutron pair correlation in nuclei. Yet there are various challenges in these works. For instance, the particle number is not a good quantum number in the isospin generalized BCS theory [11]; interactions in the SO(5) and SO(8) models [14–17] are different from the shell model Hamiltonian.

Recently Cederwall *et al.* presented the level scheme of 0^+, 2^+, 4^+, and 6^+ states in ^{92}Pd, and thereby stressed the importance of the spin-aligned isoscalar pairs consisting of the $1g_{9/2}$ orbit (i.e., isospin $T = 0$ and spin $J = 9$) [24]. This mechanism was investigated in terms of the $T = 0$, $J = 9$ pairs by Qi and collaborators [25,26] by using the shell model and the multistep shell model [27]. It was also studied via the boson mapping method by Zerguine and Isacker [28].

The purpose of the present work is to study the spin-aligned pair correlation in multi-j shells, for ^{96}Cd, ^{94}Ag, and ^{92}Pd, explicitly in a nucleon-pair basis. The approach that we use is the nucleon-pair approximation (NPA) [9,10] of the shell model (SM), with an extension of isospin symmetry [29]. The configuration space of the NPA is constructed by collective nucleon pairs with given spin and isospin.

This paper is organized as follows. In Sec. II we introduce the NPA with isospin symmetry. In Sec. III we investigate the low-lying states calculated in both the truncated nucleon-pair subspaces and the full SM space, and thereby discuss the contribution from the $T = 0$ spin-aligned pair as well as some other pairs. In Sec. IV we discuss the $J_{\max}$ isomers in ^{96}Cd and ^{94}Ag. Finally we summarize our results in Sec. V.

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II. THE NPA WITH ISOSPIN AND TRUNCATED CONFIGURATION SPACES

The NPA was recently generalized with isospin symmetry [29]. In comparison with previous versions of the NPA [9,10], isospin is considered as a good quantum number in the basis and the Hamiltonian. One considers couplings of both angular momenta and of isospin in each step of the calculations. The only approximation made in the NPA is that one diagonalizes the SM Hamiltonian in truncated nucleon-pair subspace. If all possible nucleon pairs are considered, the NPA calculation becomes the full shell model calculation.

For $2N$ valence nucleons outside a fully occupied closed shell, the configuration space is constructed by N collective nucleon-pairs coupled successively:

$$A^{(J_N)\dagger}(r_1 \cdots r_N, J_1 \cdots J_N) \equiv [\cdots((A^{(r_1)\dagger} \times A^{(r_2)\dagger})^{(J_2)} \times A^{(r_3)\dagger})^{(J_3)} \times \cdots \times A^{(r_N)\dagger}]^{(J_N)},$$

where $A^{(r_i)\dagger} = \sum_{ab} y(abr_i)(a^\dagger \times b^\dagger)^{(r_i)}$ denotes a collective coupled pair with spin J_{ri} and isospin T_{ri}; $a^\dagger$ is the creation operator of a nucleon in the single-particle orbit, with number of nodes n_a, orbit angular momentum l_a, spin j_a, and $b^\dagger$ with n_b, l_b, j_b. (r_i) is short for $(J_{r_i}$ and $T_{r_i})$, and (J_i) for (J_i, T_i). $y(abr_i)$ is called the pair structure coefficient. There is not *a priori* restriction of $y(abr)$. If $y(abr) = \delta_{aj_1}\delta_{bj_2}$, the pair is called the noncollective pair. This flexibility provides us with an opportunity to explore various pairing correlations in nuclei.

We treat ^{96}Cd, ^{94}Ag, and ^{92}Pd to be systems of valence hole states in the $2p_{1/2}2p_{3/2}1f_{5/2}1g_{9/2}$ shell, below the doubly magic core ^{100}Sn, and assume the JUN45 interaction for two-body matrix elements [30]. Although the JUN45 interaction is constructed in the pfg shells and not optimized for the $1g9/2$ or $2p_{1/2}2p_{3/2}1g_{9/2}$ shell, it is generally acceptable for low-lying states of nuclei in this region. When the JUN45 interactions are used for the hole-hole type nucleus, single particle energies are mass dependent. For ^{94}Ag, we take $\epsilon(1g_{9/2}) = 0$, $\epsilon(2p_{1/2}) = 0.153$ MeV, $\epsilon(2p_{3/2}) = 1.456$ MeV, and $\epsilon(1f_{5/2}) = 4.394$ MeV. In order to study the desired nucleon-pair truncation schemes, we study the low-lying states based on the following sets of single-particle orbits: (1) the full SM space $p_{1/2}p_{3/2}f_{5/2}g_{9/2}$, (2) $p_{1/2}p_{3/2}g_{9/2}$, (3) $p_{1/2}g_{9/2}$, and

(4) single-j $g_{9/2}$. For short, below we call the spin-aligned pair with $J = 9$ and $T = 0$ (consisted of one proton and one neutron in the $g_{9/2}$ orbit) the $A^{(9)}$ nucleon pair, the isovector pair with $J = 8$ (consisted with two nucleons in the $g_{9/2}$ orbit) is the K nucleon pair, the isovector pair with $J = 0$ is the S pair and with $J = 2$ the D pair. In order to avoid the structure coefficients $y(abr)$, we consider all possible noncollective S pairs and D pairs in our calculations (namely, four types of S pairs and six types of D from the $p_{1/2}p_{3/2}f_{5/2}g_{9/2}$ orbits).

III. THE VALIDITY OF PAIR APPROXIMATIONS

Because we are interested in the isoscalar $A^{(9)}$ nucleon-pair correlation in low-lying states of ^{96}Cd, ^{94}Ag, and ^{92}Pd, we evaluate the contribution from isoscalar $A^{(9)}$ nucleon pairs in the SM wave functions. This can be done by calculating the overlaps between the wave functions obtained by our truncated nucleon-pair subspace and the SM results. In this paper the nucleon-pair subspaces that we choose include the $A^{(9)}$, $A^{(9)}S$, $A^{(9)}D$, $A^{(9)}K$, SD, SK, $A^{(9)}SD$, $A^{(9)}SK$, $A^{(9)}SDK$ subspaces.

A. ^{96}Cd

^{96}Cd is taken to be a system of two-proton holes and two-neutron holes below the doubly closed-shell core of ^{100}Sn. In Refs. [28,31], a few low-lying states with $T = 0$ were studied by computing the overlaps between wave functions given by the $A^{(9)}$-pair subspace and those of SM calculation with the single-j $g_{9/2}$ shell, which showed that such $A^{(9)}$-pair wave functions overlap with those of the SM calculation by 90% or more for the lowest $T = 0$ states of $J = 0, 2, 4, 12, 14, 16$, and by ∼70% for the lowest $T = 0$ states of $J = 6, 10$, but less than 10% for the lowest $T = 0$ and $J = 8$ state (see Fig. 1 in Ref. [28] and Fig. 2 in Ref. [31]).

Let us first investigate whether it is appropriate to treat the low-lying states of ^{96}Cd by a single-j $g_{9/2}$ shell, and whether or not it is proper to neglect the contribution of the $1f_{5/2}$ orbit (because its single-particle energy is much higher than the other three $p_{1/2}p_{3/2}g_{9/2}$), or to consider only two orbits, $p_{1/2}g_{9/2}$. This is done by finding the squared overlaps, x^2, between the wave functions of low-lying states in the single-particle level truncated subspaces (i.e., single-j $g_{9/2}$ shell, two-j $p_{1/2}g_{9/2}$ shells, and three-j $p_{1/2}p_{3/2}g_{9/2}$ shells) and the full shell model wave functions which consider the four orbits $p_{1/2}p_{3/2}f_{5/2}g_{9/2}$. Our results are as follows. For $T = 0$ states in the $g_{9/2}$ subspace, x^2 is precisely 1 for $J = 16$, larger than 0.99 for the lowest $J = 12$ and 14 states, ∼0.7 for the lowest $J = 0, 2, 4, 6, 8, 10$ states. For $T = 0$ states in the $p_{1/2}g_{9/2}$ subspace, the values of x^2 increase but slightly. For $T = 0$ states in the $p_{1/2}p_{3/2}g_{9/2}$ subspace, x^2 are larger than 0.9 for most of the first three states of each J. These results show that, although the single-hole energy difference between the $2p_{3/2}$ orbit and the $1g_{9/2}$ orbit is ∼1.4 MeV, this gap is not large enough to neglect the contribution from valence holes in the $2p_{3/2}$ orbit for some of these states. According to our calculations by using the JUN45 interaction, the contribution

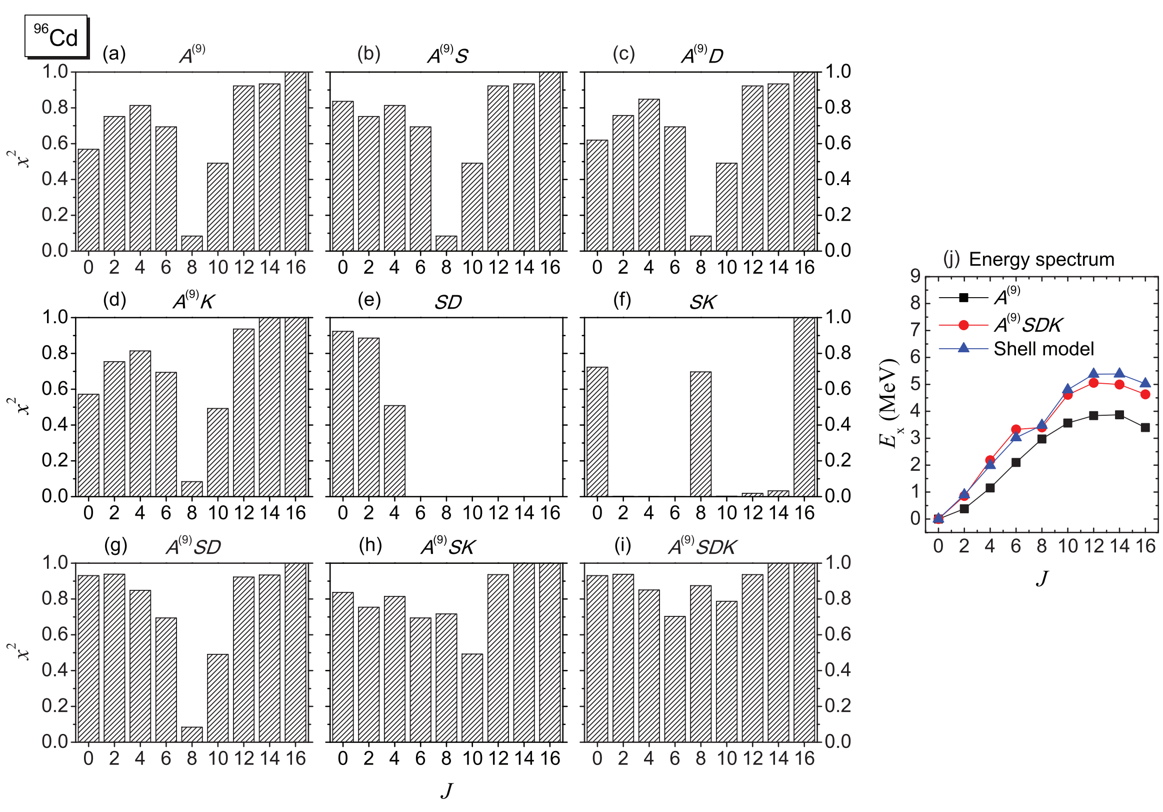

FIG. 1. (Color online) Squared overlaps between wave functions of a few pair-truncated subspaces and those of the full shell model space with four orbits, $p_{1/2}p_{3/2}f_{5/2}g_{9/2}$. Here $A^{(9)}$ denotes the isoscalar spin-aligned pair consisted of the $1g_{9/2}$ orbit (i.e., $T = 0$ and spin $J = 9$). S, D, K denotes $T = 1$ nucleon pairs with spin-0, -2, and -8, respectively.

from the $2p_{1/2}$ orbit and that from the $2p_{3/2}$ orbit are also important in some of the low-lying sates. The calculations also show that the contribution from the $1f_{5/2}$ orbit is very small, and the $p_{1/2}p_{3/2}g_{9/2}$ subspace is an appropriate truncation for low-lying states of this nucleus, assuming the JUN45 interaction both in the pfg shell and truncated subspaces ($g_{9/2}$, $p_{1/2}g_{9/2}$, $p_{1/2}p_{3/2}g_{9/2}$).

Now we calculate the squared overlaps, x^2, between the wave functions of the $A^{(9)}$ nucleon-pair subspace and those of the SM space with four orbits, $p_{1/2}p_{3/2}f_{5/2}g_{9/2}$. Our calculated x^2 is presented in Fig. 1(a). We find that x^2 are $0.5 \sim 0.8$ for the $0^+, 2^+$, and 4^+ states with $T = 0$. These values are smaller than those of the $A^{(9)}$ nucleon-pair subspace in the single-j $g_{9/2}$ SM space studied in Refs. [28,31], by $10 \sim 40\%$. This difference is mainly given by contribution of the $2p_{1/2}$ and $2p_{3/2}$ orbits in these states. We also note that if other interactions were taken for single-j $g_{9/2}$ shell and $A^{(9)}$ nucleon-pair subspace, the calculated results of x^2 would change.

We investigate a few nucleon-pair truncated configurations, including the $A^{(9)}S$, $A^{(9)}D$, $A^{(9)}K$, SD, SK, $A^{(9)}SD$, $A^{(9)}SK$, $A^{(9)}SDK$ subspaces. We note again that in our calculations, all possible noncollective S and D pairs of $T = 1$ are considered. The calculated results of x^2 are plotted in Figs. 1(b)–1(i), respectively. From Fig. 1 one sees that the $A^{(9)}$ nucleon-pair subspace is a reasonably good approximation of the $p_{1/2}p_{3/2}f_{5/2}g_{9/2}$ shell, with the exception of the lowest $T = 0$, $J = 8$ state which is well represented by SK pairs, see Fig. 1(f). In Fig. 1(e), one sees that isovector SD pairs also present reasonably good descriptions of the lowest $J = 0$ and 2 states with $T = 0$ for which $x^2 = 0.92$ and 0.88, respectively. This dual description is a consequence of nonorthogonality feature of nucleon-pair basis.

For the lowest $T = 0$ states with $J = 12$ and 14 states, the contribution of $A^{(9)}$ nucleon pairs is dominant, as shown in Fig. 1(a). The dominance of $A^{(9)}$ nucleon pairs in these states was implied or pointed out in previous studies [24,25,28,31–35]. The lowest $T = 0$, $J = 16$ state can be given either by two $A^{(9)}$ nucleon pairs [see Fig. 1(a)] or equivalently by two K pairs [see Fig. 1(f)], because this state is unique in the shell model here.

In Figs. 1(g)–1(i) one sees that x^2 increases when we successively add more nucleon pairs. The nucleon pair subspace $A^{(9)}SDK$ presents a good description for all the lowest $T = 0$ states with even J values. In Fig. 1(j) one sees that energy levels of these states are also well reproduced in the $A^{(9)}SDK$ subspace.

B. ^{94}Ag

The nucleus ^{94}Ag is assumed to be a system of three-proton holes and three-neutron holes below the ^{100}Sn core. All states discussed in this subsection have $T = 0$. Similarly to the last subsection, we take the JUN45 interaction for both the four-j shell model space and its subspaces, e.g., single-particle truncated subspaces such as single-j ($j = 9/2$) or $p_{1/2}p_{3/2}g_{9/2}$ shells, nucleon-pair truncated subspaces such as the $A^{(9)}$- or $A^{(9)}SD$-pair subspaces.

We first calculate the squared overlaps, x^2, between the wave functions calculated in the single-j $g_{9/2}$ subspace and those in the full SM space of four j orbits, $p_{1/2}p_{3/2}f_{5/2}g_{9/2}$. For the lowest $J = 18, 19, 21$ states with $T = 0$, the values of

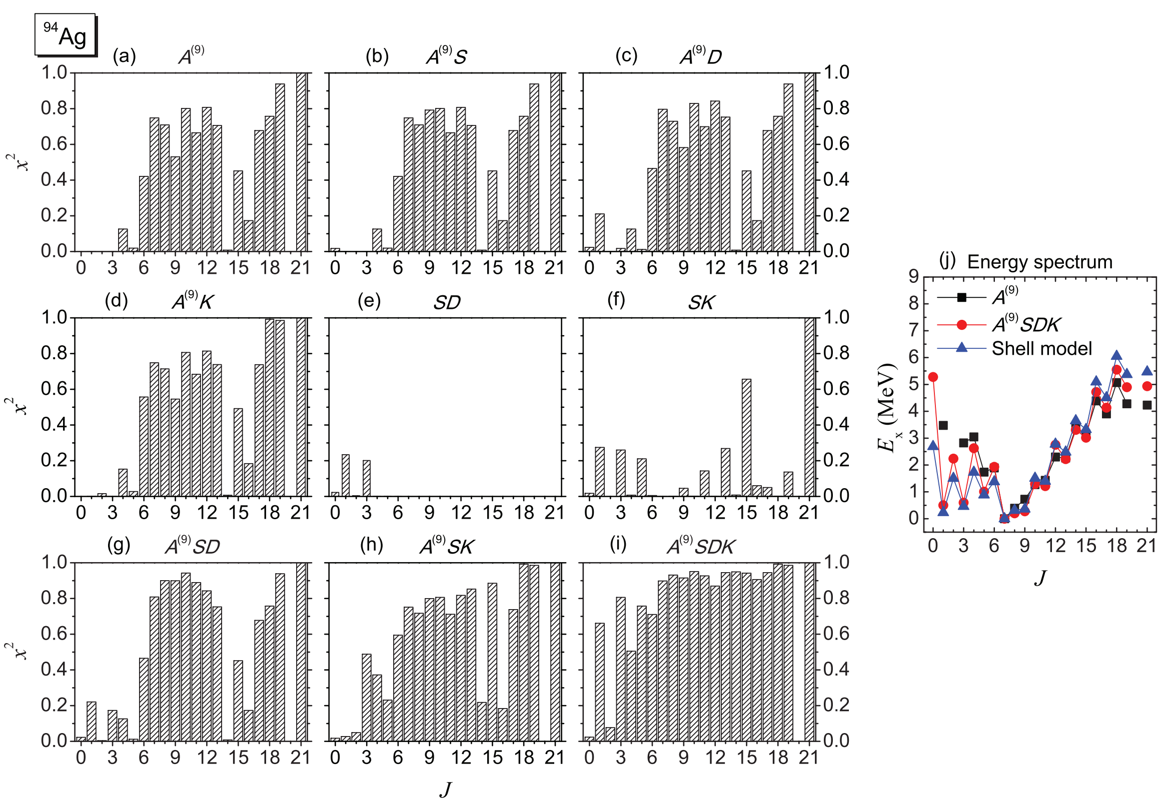

FIG. 2. (Color online) Same as Fig. 1, except for calculated results of ^{94}Ag.

x^2 are larger than 0.98. For the lowest $J = 1$ and $3 \sim 17$ states of $T = 0$, the values of x^2 are larger than 0.5 but below 0.9. The lowest $J = 2$, $T = 0$ state, however, cannot be reasonably reproduced, unless the $2p_{1/2}$ and $2p_{3/2}$ orbits are considered. When three single-particle orbits, $p_{1/2}$, $p_{3/2}$, and $g_{9/2}$, are considered, the lowest $T = 0$ states with $J \leqslant 21$ are well reproduced ($x^2 > 0.8$) with only one exception of $J = 20$.

Figure 2 shows our calculated x^2 between the wave functions calculated in a number of truncated nucleon-pair subspaces (the same as in Fig. 1, i.e., $A^{(9)}$, $A^{(9)}S$, $A^{(9)}D$, $A^{(9)}K$, SD, SK, $A^{(9)}SD$, $A^{(9)}SK$, $A^{(9)}SDK$) and those of the full SM space with four j orbits. First, one sees from Fig. 2(a) that the $A^{(9)}$-pair subspace provides us with a reasonable description for most of the lowest $T = 0$ states ($J =$ 7–9). This subspace also well describes a few low-lying states with $T = 0$ and $J =$ 10–13, 17–19. Because the $T = 0$, $J = 21$ state is unique in the single-j $g_{9/2}$ shell space, the K-pair description of this state in Fig. 2(f) is equivalent to the $A^{(9)}$-pair description in Fig. 2(a). A good agreement between energy levels in the $A^{(9)}$-pair space and the full shell model space is seen in Fig. 2(j). Therefore the $A^{(9)}$-pair correlation plays a dominant role in the few lowest $T = 0$ states ($J =$ 7–9) and some of the excited states of this nucleus.

Unlike the case of ^{96}Cd, the SD-pair approximation does not give a reasonable description of the lowest $T = 0$ states. Yet, further considerations of SD and K pairs successively coupled to the $A^{(9)}$-pair subspace in Figs. 2(b)–2(i) improve the description for the lowest $T = 0$ states. In Fig. 2(i) one sees the $A^{(9)}SDK$-pair subspace well describes all the lowest $T = 0$ states, except the $J = 0, 2$, and 20 states. According to our calculated results, the isoscalar nucleon pairs with spin-2 and spin-1 are responsible for the lowest $T = 0$ states with $J = 0$ and 2, respectively. However, we did not find very simple wave functions for the $T = 0$, $J = 4$ state in terms of nucleon pairs.

The dominant configuration of the 7^+ isomer (the lowest $T = 0$, $J = 7$ state) in ^{94}Ag were investigated in Refs. [26,31]. The half-life time is 0.55 s, according to Ref. [36]. Our numerical calculations here show that the configuration $((A^{(9)\dagger} \times A^{(9)\dagger})^{(J=16,T=0)} \times A^{(9)\dagger})^{(J=7,T=0)}|0\rangle$ is dominant. The squared overlap x^2 between this simple configuration and the full SM wave function is 0.75%. This is consistent with the conclusion made in Refs. [25,26,31].

C. ^{92}Pd

The nucleus ^{92}Pd has four-proton holes and four-neutron holes in the $p_{1/2}p_{3/2}f_{5/2}g_{9/2}$ shell, which is the most complicated nucleus studied in this paper. We first investigate the values of x^2 between the wave functions calculated in the $A^{(9)}$ nucleon-pair subspace and those in a single-j $g_{9/2}$ shell, by using the JUN45 interaction in both configurations. Such calculated x^2 is close to or larger than 0.9, for all even J states with $T = 0$, as shown in Fig. 3, suggesting that the $A^{(9)}$-pair configuration is dominant in the single-j $g_{9/2}$ shell for these two states.

In Table I, we present the squared overlaps, x^2, between wave functions of the lowest $J = 0$ and 2 states of $T = 0$ calculated in the $A^{(9)}$ or SD nucleon-pair subspace and those in the $p_{1/2}p_{3/2}g_{9/2}$ space, by using the JUN45 interaction. According to Table I, the simple $A^{(9)}$ nucleon-pair subspace presents a less satisfactory but relevant description of low-lying states for the ^{92}Pd nucleus than for ^{96}Cd: For the lowest $J = 0$ and 2 states of $T = 0$, the squared overlap x^2 is 0.47 and 0.58. On the other hand, SD nucleon pairs with $T = 1$ present a more relevant description of these two states ($x^2 > 0.8$, see Table I) and other low-lying $J = 0, 2$ states of $T = 0$. This dual description is understood again by the nonorthogonality of the nucleon-pair basis. There are large overlaps between the two sets of basis vectors.

In Table I one sees that $A^{(9)}$-pair subspace is not important (x^2 below 0.01, see Table I) in the second or third $J = 0$, or third $J = 2$ states of $T = 0$. The reason is that for these states neither the single-j shell $g_{9/2}$ nor two-j $p_{1/2}g_{9/2}$ shells are a good approximation of the shell model configurations with the $p_{1/2}p_{3/2}g_{9/2}$ orbits. The corresponding wave functions of these states calculated in such truncated spaces overlaps with the three-j $p_{1/2}p_{3/2}g_{9/2}$ shells by about 0.10. Components connecting with the $p_{3/2}$ orbit is important for these states under the JUN45 interaction.

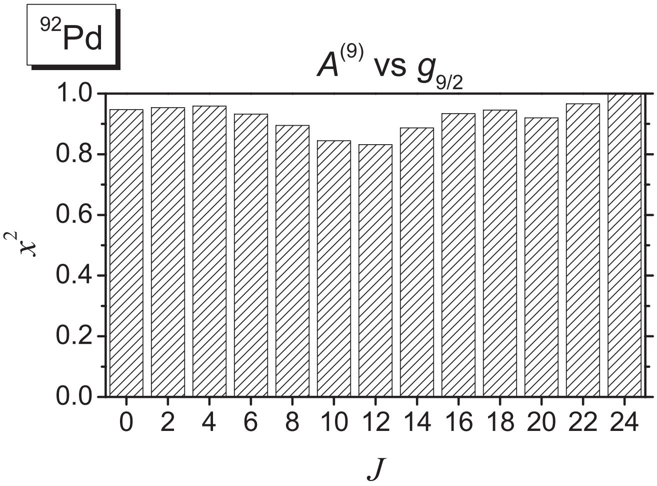

FIG. 3. Same as Fig. 1, except that the results are for $T = 0$ states of ^{92}Pd. We are unable to present squared overlaps between wave functions of the pair approximation and those of the shell model calculations, due to the complexity in this case.

TABLE I. Squared overlaps between wave functions of a few pair-truncated subspaces and those of the shell model space with three orbits, $p_{1/2}p_{3/2}g_{9/2}$. $A^{(9)}$ denotes the isoscalar spin-aligned pair consisted of the $1g_{9/2}$ orbit, as in Fig. 1. SD denotes $T = 1$ nucleon pairs with spin-0 and -2.

$A^{(9)}$&$p_{1/2}p_{3/2}g_{9/2}$	First	Second	Third
0^+	0.47	<0.01	<0.01
2^+	0.58	0.59	<0.01
SD&$p_{1/2}p_{3/2}g_{9/2}$	First	Second	Third
0^+	0.83	0.68	0.35
2^+	0.81	0.76	0.59

TABLE II. Matrix elements of two-body interactions in $1g_{9/2}$ shell for $A = 58$ taken from the JUN45 interaction [30] (in unit of MeV).

Isoscalar	V_1	V_3	V_5	V_7	V_9
	−1.1378	−0.5987	−0.3830	−0.5605	−2.2067
Isovector	V_0	V_2	V_4	V_6	V_8
	−1.6907	−0.9594	−0.0871	0.1515	0.2689

IV. $J_{\max}$ ISOMERS IN ^{96}Cd AND ^{94}Ag

The 16^+ level lies below the 14^+ and 12^+ levels in the SM calculations for ^{96}Cd [35], resulting in a so-called "level-inversion" isomerism. There are attempts to interpret the 21^+ isomer of ^{94}Ag in the same way, unfortunately most SM calculations give the 21^+ level energy above the lowest 19^+ level [26,37]. The 21^+ isomer was discussed in terms of the $A^{(9)}$ nucleon pairs in Refs. [26,38], and the 16^+ isomer was interpreted in Ref. [34] in terms of the same picture. In this section we study these isomeric states.

As demonstrated in the above section, the $A^{(9)}$ nucleon-pair subspace is a very good approximation for these two isomers (see Figs. 1 and 2 in this paper), therefore the calculations here are restricted to the single-j $g_{9/2}$ subspace.

Let us discuss the 21^+ isomer of ^{94}Ag first by this single-j shell. We denote two-body matrix elements $V_J = \langle g_{9/2}g_{9/2}JT|\hat{V}|g_{9/2}g_{9/2}JT\rangle$. Here $J = 1, 3, 5, 7, 9$ for $T = 0$, and 0, 2, 4, 6, 8 for $T = 1$. These V_J are shown in Table II for the reader's convenience (taken from the JUN45 interaction). From numerical calculations we obtain that

$$E(21^+) - E(19^+) = -0.2325V_3 - 1.2932V_4 - 0.1990V_5 + 0.2690V_6 + 0.2132V_7 + 1.0242V_8 + 0.2183V_9. \quad (1)$$

According to this formula, $E(21^+) - E(19^+)$ is very sensitive to V_4 and V_8 ($T = 1$), which have small magnitudes in the JUN45 interaction (see Table II). If one makes slight modifications on them, say, adding 0.05 MeV to V_4 and −0.05 MeV to V_8, $E(21^+) - E(19^+)$ would change from 95 keV to −5 keV (i.e., the sequence would be reversed). In Ref. [37] the particle-hole excitations across the $N = Z = 50$ core were considered to reproduce the "level-inversion" isomerism. In Refs. [26,39] a stronger attractive interaction of V_9 ($T = 0$) was suggested to obtain the 21^+ isomer of ^{94}Ag. In order to achieve this inversion, V_9 should be modified by ∼−0.5 MeV.

Similarly, for the 16^+ "level-inversion" isomer we have

$$E(16^+) - E(14^+) = -0.2758V_5 - 1.4514V_6 - 0.0562V_7 + 1.4514V_8 + 0.3320V_9. \quad (2)$$

In Eqs. (1) and (2) the coefficients of the V_J are negative for smaller J, and positive for larger J. Very clearly, one sees that the "level-inversion" isomers are originated from the strongly attractive V_9.

V. SUMMARY

In this paper, we study the isoscalar spin-aligned pair correlation in the lowest $T = 0$ states of ^{96}Cd, ^{94}Ag, and ^{92}Pd, by using the nucleon-pair approximation (NPA) with isospin symmetry, and using the JUN45 interaction for both the $2p_{1/2}2p_{3/2}1f_{5/2}1g_{9/2}$ shell and nucleon-pair truncated subspaces. We focus on the role played by the isoscalar spin-aligned nucleon pairs (called the $A^{(9)}$ pair) consisting of two particles in the $1g_{9/2}$ orbit. We present squared overlaps between wave functions obtained in our truncated subspaces and the full shell model space explicitly. We also discuss the 21^+ isomer in ^{94}Ag and the 16^+ isomer in ^{96}Cd.

Our NPA calculations demonstrate that, for low-lying states of ^{96}Cd, ^{94}Ag, and ^{92}Pd, $A^{(9)}$ nucleon pairs play an important role, as implied or pointed out in previous studies [24,25,28,31–35]. Yet in the presence of JUN45 interaction the contribution from the $2p_{1/2}2p_{3/2}$ orbits should be further taken into account in order to have a good description for most of the lowest three $J = 0, 2, 4$ states with $T = 0$ of ^{96}Cd, ^{94}Ag, and ^{92}Pd. The calculated overlaps between wave functions of a few nucleon-pair truncated subspaces and exact shell model wave functions suggest that, for 0^+ and 2^+ states in ^{96}Cd and ^{92}Pd, isovector SD nucleon pairs present more relevant components than isoscalar spin-aligned $A^{(9)}$ nucleon pairs, if the JUN45 interaction is taken for both the $p_{1/2}p_{3/2}f_{5/2}g_{9/2}$ shell and its subspaces. For the lowest 0^+ (and 2^+) state with $T = 0$ of ^{94}Ag, isoscalar nucleon pairs with $J = 2$ (1) is very important. We note that although the JUN45 interaction is not optimized for the subspaces (e.g., the single-j $g_{9/2}$ shell, the $p_{1/2}p_{3/2}g_{9/2}$ shell, or nucleon-pair truncated subspaces) of the pfg shell, the overlaps obtained in our calculations are useful and suggestive of future studies.

We investigate the "level-inversion" isomerism of the $J_{\max}$ isomer in ^{96}Cd and ^{94}Ag. The origin of such isomers are investigated. The attractive two-body matrix elements $V_J = \langle g_{9/2}g_{9/2}JT|\hat{V}|g_{9/2}g_{9/2}JT\rangle$ with $T = 0$ and J large lead to a lower value of $E(J_{\max})$ than $E(J_{\max-2})$. We suggest another option to achieve the desired "level-inversion" in ^{94}Ag, i.e., relatively small modifications (50 keV) of V_4 and V_8, instead of adjusting the matrix element $\langle g_{9/2}g_{9/2}J = 9, T = 0|\hat{V}|g_{9/2}g_{9/2}J = 9, T = 0\rangle$ by 0.5 MeV.

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[1] G. Racah, Phys. Rev. **62**, 438 (1942); **63**, 367 (1943).

[2] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **106**, 162 (1957); **108**, 1175 (1957).

[3] A. Bohr, B. R. Mottelson, and D. Pines, Phys. Rev. **110**, 936 (1958).

[4] S. T. Belyaev, Mat. Fys. Medd. Dan. Vid. Selsk. **31**(11) (1959).

[5] A. B. Migdal, Nucl. Phys. **13**, 655 (1959).
[6] A. Arima and F. Iachello, Phys. Rev. Lett. **35**, 1069 (1975); Ann. Phys. (NY) **99**, 253 (1976); **111**, 201 (1978); **123**, 468 (1979); Adv. Nucl. Phys. **13**, 139 (1984).
[7] Y. K. Gambhir, S. Haq, and J. K. Suri, Ann. Phys. (NY) **133**, 154 (1981).
[8] K. Allaart, E. Boeker, G. Bonsignori, M. Savoia, and Y. K. Gambhir, Phys. Rep. **169**, 209 (1988).
[9] J. Q. Chen, Nucl. Phys. A **626**, 686 (1997).
[10] Y. M. Zhao, N. Yoshinaga, S. Yamaji, J. Q. Chen, and A. Arima, Phys. Rev. C **62**, 014304 (2000).
[11] A. L. Goodman, Phys. Rev. C **60**, 014311 (1999).
[12] W. Satula and R. Wyss, Phys. Lett. B **393**, 1 (1997).
[13] W. Satula, D. J. Dean, J. Gary, S. Mizutori, and W. Nazarewicz, Phys. Lett. B **407**, 103 (1997).
[14] J. Engel, S. Pittel, M. Stoitsov, P. Vogel, and J. Dukelsky, Phys. Rev. C **55**, 1781 (1997).
[15] J. Dobeš and S. Pittel, Phys. Rev. C **57**, 688 (1998).
[16] O. Juillet and S. Josse, Eur. Phys. J. A **8**, 291 (2000).
[17] J. Dukelsky, B. Errea, S. Lerma H., D. F., G. G. Dussel, C. Esebbag, and N. Sandulescu, Int. J. Mod. Phys. E **17**, 2155 (2008).
[18] N. Sandulescu, B. Errea, and J. Dukelsky, Phys. Rev. C **80**, 044335 (2009).
[19] N. Zeldes, Phys. Lett. B **455**, 7 (1999).
[20] A. Poves and G. Martinez-Pinedo, Phys. Lett. B **430**, 203 (1998).
[21] G. F. Bertsch and S. Baroni, arXiv:0904.2017v2 (2009).
[22] D. D. Warner, M. A. Bentley, and P. Van Isacker, Nat. Phys. **2**, 311 (2006).
[23] A. V. Afanasjev, arXiv:1205.2134v1 (2012).
[24] B. Cederwall *et al.*, Nature **469**, 68 (2011).
[25] C. Qi, J. Blomqvist, T. Bäck, B. Cederwall, A. Johnson, R. J. Liotta, and R. Wyss, Phys. Rev. C **84**, 021301(R) (2011).
[26] Z. X. Xu, C. Qi, J. Blomqvist, R. J. Liotta, and R. Wyss, Nucl. Phys. A **877**, 51 (2012).
[27] R. J. Liotta and C. Pomar, Phys. Lett. B **105**, 92 (1981); Phys. Rev. Lett. **49**, 1142 (1982); C. Pomar and R. J. Liotta, Phys. Rev. C **25**, 1656 (1982).
[28] S. Zerguine and P. Van Isacker, Phys. Rev. C **83**, 064314 (2011).
[29] G. J. Fu, Y. Lei, Y. M. Zhao, S. Pittel, and A. Arima, Phys. Rev. C **87**, 044310 (2013).
[30] M. Honma, T. Otsuka, T. Mizusaki, and M. Hjorth-Jensen, Phys. Rev. C **80**, 064323 (2009).
[31] C. Qi, J. Blomqvist, T. Bäck, B. Cederwall, A. Johnson, R. J. Liotta, and R. Wyss, Phys. Scr., T **150**, 014031 (2012).
[32] L. Coraggio, A. Covello, A. Gargano, and N. Itaco, Phys. Rev. C **85**, 034335 (2012).
[33] C. Qi, R. J. Liotta, and R. Wyss, J. Phys.: Conf. Ser. **338**, 012027 (2012).
[34] B. S. Nara Singh *et al.*, Phys. Rev. Lett. **107**, 172502 (2011).
[35] K. Ogawa, Phys. Rev. C **28**, 958 (1983).
[36] National Nuclear Data Center, http://www.nndc.bnl.gov/
[37] C. Plettner, H. Grawe, I. Mukha, J. Döring, F. Nowacki, L. Batist, A. Blazhev, C. R. Hoffman, Z. Janas, R. Kirchner, M. La Commara, C. Mazzocchi, E. Roeckl, R. Schwengner, S. L. Tabor, and M. Wiedeking, Nucl. Phys. A **733**, 20 (2004).
[38] P. Van Isacker, Phys. Scr., T **150**, 014042 (2012).
[39] L. Zamick and A. Escuderos, Nucl. Phys. A **889**, 8 (2012).