Effects of the nuclear medium and non-isoscalarity in extracting $\sin^2\theta_W$ using the Paschos-Wolfenstein relation

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We study nuclear medium effects and the non-isoscalarity correction in the extraction of weak mixing angle $\sin^2\theta_W$ using the Paschos-Wolfenstein (PW) relation. The calculations are performed for the iron nucleus. Nuclear medium effects such as Fermi motion, binding, shadowing, and antishadowing corrections and pion and rho meson cloud contributions have been taken into account. Calculations have been performed in the local density approximation by using a relativistic nuclear spectral function which includes nucleon correlations. The results are discussed along with the experimental result inferred by the NuTeV Collaboration. These results may be useful for the ongoing MINERνA experiment as well as for the proposed NuSOnG experiment.

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I. INTRODUCTION

MINERνA [1] is presently taking data using neutrinos from NuMI Lab, and its aim is to perform cross section measurements in the neutrino energy region of 1–20 GeV with different nuclear targets such as helium, carbon, oxygen, iron, and lead. Among the various goals of the MINERνA experiment, one of them is to measure $\frac{d\sigma}{dx dQ^2}$ in the deep-inelastic scattering (DIS) region using these nuclear targets, obtaining the ratio of the structure functions between the different target materials, and also to study structure functions in the DIS and transition region [2,3]. Neutrino Scattering On Glass (NuSOnG) [4] is another experiment proposed at Fermilab to study the neutrino and antineutrino charged current deep-inelastic scattering events to precisely measure the structure functions $F_2^{\nu/\bar{\nu}}$, $xF_3^{\nu/\bar{\nu}}$, etc. Furthermore, experimenters plan to measure $\sin^2\theta_W$ from ν-nucleon scattering using the Paschos-Wolfenstein (PW) relation [5]; a similar type of analysis was performed by the NuTeV group [6–8]. Recently, we studied nuclear medium effects on the electromagnetic structure function $F_2^{\rm EM}(x,Q^2)$ [9] and the weak structure functions $F_2(x,Q^2)$ and $F_3(x,Q^2)$ [10,11]. For $F_2^{\rm EM}(x,Q^2)$ [9], we found that our results are reasonably in good agreement with the recent results from Jefferson Lab (JLab) [12] as well as with some of the older experiments such as those at SLAC [13]. In the case of $\nu(\bar{\nu})$ deep-inelastic-scattering-induced processes, the results were compared with the available data from NuTeV, CDHSW, and CHORUS experiments [14–16] for the weak structure functions $F_2(x,Q^2)$ and $xF_3(x,Q^2)$ in iron [10] and lead [11], as well as with the results of the differential scattering cross section $\frac{1}{E}\frac{d\sigma}{dxdy}$. In this work, we study the effect of the nuclear medium and the non-isoscalarity correction in extracting $\sin^2\theta_W$ using the PW relation by taking iron as the nuclear target.

Paschos and Wolfenstein [5] suggested that the ratio of neutral current to charged current neutrino interaction cross sections on nucleon targets may be used to measure $\sin^2\theta_W$:

$$R_{\rm PW} = \frac{\sigma(\nu_\mu N \to \nu_\mu X) - \sigma(\bar{\nu}_\mu N \to \bar{\nu}_\mu X)}{\sigma(\nu_\mu N \to \mu^- X) - \sigma(\bar{\nu}_\mu N \to \mu^+ X)} = \frac{1}{2} - \sin^2\theta_W. \quad (1)$$

The above relation is valid when there is no contribution from heavy quarks, the charm quark mass is neglected, strange quark and antistrange quark symmetry are assumed, and in the absence of a medium effect and any contributions from outside the standard model. This equation is valid for both the total cross section σ as well as differential cross sections $\frac{d^2\sigma}{dx\,dy}$, because the neutral current (NC) differential cross section can be expressed in terms of the charged current (CC) ones, and these cancel out in the quotient [17]. For the total cross sections, the PW relationship is true under more general assumptions (see for instance [5,17,18]).

The above relation is also valid for an isoscalar nuclear target ($N = Z$) for both the total cross sections and differential cross sections and the above Eq. (1) may be written as

$$R_{\rm PW} = \frac{\sigma(\nu_\mu A \to \nu_\mu X) - \sigma(\bar{\nu}_\mu A \to \bar{\nu}_\mu X)}{\sigma(\nu_\mu A \to \mu^- X) - \sigma(\bar{\nu}_\mu A \to \mu^+ X)}, \quad (2)$$

where $\sigma(\nu_\mu A \to \nu_\mu X)$ and $\sigma(\bar{\nu}_\mu A \to \bar{\nu}_\mu X)$ are the neutral-current-induced neutrino and neutral-current-induced antineutrino cross sections, respectively, and $\sigma(\nu_\mu A \to \mu^- X)$ and $\sigma(\bar{\nu}_\mu A \to \mu^+ X)$ are the charged-current-induced neutrino and charged-current-induced antineutrino cross sections, respectively. The condition of pure isoscalarity includes the requirement of the cancellation of different strong interaction effects which also include the nuclear medium effects in the ratio of the neutral current to the charged current scattering cross sections.

The NuTeV Collaboration [6–8] has measured the ratio R of neutral current to charged current total cross sections in iron, for which they took the ratio of charged current antineutrino to neutrino cross sections, i.e., $r = \frac{\sigma(\bar{\nu}_\mu A \to \mu^+ X)}{\sigma(\nu_\mu A \to \mu^- X)}$, as $\frac{1}{2}$, and obtained the value for the weak mixing angle $\sin^2\theta_W$ using

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Eq. (2) as

$$R_{\rm PW} = \frac{\frac{\sigma(\nu_\mu A\to\nu_\mu X)}{\sigma(\nu_\mu A\to\mu^- X)} - \frac{\sigma(\bar\nu_\mu A\to\bar\nu_\mu X)}{\sigma(\bar\nu_\mu A\to\mu^+ X)}\frac{\sigma(\bar\nu_\mu A\to\mu^+ X)}{\sigma(\nu_\mu A\to\mu^- X)}}{1-\frac{\sigma(\bar\nu_\mu A\to\mu^+ X)}{\sigma(\nu_\mu A\to\mu^- X)}} = \frac{R^\nu - rR^{\bar\nu}}{1-r}, \quad (3)$$

where $R^\nu_{\rm exp} = 0.3916 \pm 0.0007$ and $R^{\bar\nu}_{\rm exp} = 0.4050 \pm 0.0016$ [6,7]. The reported value of $\sin^2\theta_W$ is 0.2277 ± 0.0004 [6–8], which is 3 standard deviations above the global fit of $\sin^2\theta_W = 0.2227 \pm 0.0004$ [19] and this is known as the NuTeV anomaly. To resolve this anomaly, explanations within and outside the standard model of electroweak interactions have been sought [20–36].

The Paschos and Wolfenstein [5] relation is valid for an isoscalar target while iron is a non-isoscalar target ($N = 30$, $Z = 26$); therefore, non-isoscalar corrections are required. Furthermore, nuclear dynamics may also play an important role in the case of neutrino-nucleus scattering. Various corrections made by the NuTeV Collaboration have been discussed in the literature, but still the reported deviation could not be accounted for Ref. [8]. Theoretically, Kulagin [25] has investigated the effect of the nuclear medium on the PW ratio and pointed out that the shadowing effect being a low-x and low-Q^2 phenomenon is small in the Q^2 region of the NuTeV experiment [37] and observed the effects of Fermi motion and binding energy correction to be small but found a significant isoscalar correction. Kumano [21] in a phenomenological analysis pointed out that the difference between the nuclear effects in the valence u and d quark distributions may be a reason for this anomaly. However, this effect is too small to explain the anomaly. Recently, Thomas [36] has discussed various possible corrections and concluded that charge symmetry violation and the isovector EMC effect together may explain this anomaly.

In the present work, we used the results of our earlier study of the nuclear medium and non-isoscalarity correction on the weak structure functions and the differential scattering cross sections [10,11], on the extraction of $\sin^2\theta_W$ using the PW relation. We have obtained the modified PW relation for a non-isoscalar nuclear target. This study has been performed by using a relativistic nucleon spectral function [38,39], which is used to describe the momentum distribution of nucleons in the nucleus. We define everything within a field-theoretical approach where nucleon propagators are written in terms of this spectral function. The spectral function has been calculated by using Lehmann's representation for the relativistic nucleon propagator and nuclear many-body theory is used for calculating it for an interacting Fermi sea of nuclear matter. The local density approximation is then applied to translate these results to finite nuclei [9–11,40,41]. The contributions of the pion and rho meson clouds are taken into account in a many-body field-theoretical approach which is based on Refs. [40,42]. We have taken into account the target mass correction following Ref. [43], which has a significant effect at low Q^2 and at moderate and high Bjorken x. To take into account the shadowing effect, which is important at low Q^2 and low x, and which modulates the contribution of pion and rho cloud contributions, we have followed the works of Kulagin and Petti [44,45]. All the formalism is the same for neutral current scattering as done in the case of charged-current-neutrino-induced and charged current-antineutrino-induced reactions. For the numerical calculations, parton distribution functions for the nucleons have been taken from the parametrization of the CTEQ Collaboration (CTEQ6.6) [46].

The paper is organized as follows. In Sec. II, we present the formalism: we write the expressions for the ν-nucleon and the $\bar\nu$-nucleon differential scattering cross sections in Sec. II A, and we give the expressions for the ν-nucleus and the $\bar\nu$-nucleus differential scattering cross sections for the isoscalar as well as non-isoscalar nuclear targets in Sec. II B. In Sec. II C, we explicitly show the construction of the nuclear hadronic tensor for nonsymmetric nuclear matter and in Sec. II D the nuclear corrections to PW ratio are presented. In Sec. III, we present and discuss the results of our calculations, and finally our conclusions are summarized in Sec. IV.

II. FORMALISM

A. Deep inelastic neutrino nucleon scattering

The cross section for the CC neutrino interaction with a nucleon target, i.e.,

$$\nu_l(k) + N(p) \to l^-(k') + X(p'), \quad l = e, \mu, \quad (4)$$

is given by

$$\sigma = \frac{1}{v_{\rm rel}}\frac{2m_\nu}{2E_\nu(\mathbf{k})}\frac{2M}{2E(\mathbf{p})}\int\frac{d^3k'}{(2\pi)^3}\frac{2m_l}{2E_l(\mathbf{k}')}\prod_{i=1}^{N}\int\frac{d^3p'_i}{(2\pi)^3} \times \prod_{l\epsilon f}\left(\frac{2M'_l}{2E'_l}\right)\prod_{j\epsilon b}\left(\frac{1}{2\omega'_j}\right)\bar{\sum}\sum|T|^2(2\pi)^4 \times \delta^4\left(p+k-k'-\sum_{i=1}^{N}p'_i\right), \quad (5)$$

where f stands for fermions and b for bosons in the final state X. The index i is split into l and j for fermions and bosons, respectively, four-momenta of the particles involved in the process are represented as k (incoming neutrino), k' (outgoing lepton), p (target nucleon), and p' (jet of hadrons).

T is the invariant matrix element for the above reaction and is written as

$$-iT = \left(\frac{iG_F}{\sqrt{2}}\right)\bar{u}_l(k')\gamma^\alpha(1-\gamma_5)u_\nu(k) \times \left(\frac{m_W^2}{q^2-m_W^2}\right)\langle X|J_\alpha|N\rangle. \quad (6)$$

After performing the phase-space integration in Eq. (5), we can express the double differential scattering cross section evaluated for a nucleon target in its rest frame as

$$\frac{d^2\sigma^N_{\nu,\bar\nu}}{d\Omega' dE'} = \frac{{G_F}^2}{(2\pi)^2}\frac{|\mathbf{k}'|}{|\mathbf{k}|}\left(\frac{m_W^2}{q^2-m_W^2}\right)^2 L^{\alpha\beta}_{\nu,\bar\nu}\, W^N_{\alpha\beta}, \quad (7)$$

where G_F is the Fermi coupling constant, m_W is the mass of the W boson, $l(=e,\, \mu)$ is a lepton, $q = k - k'$ is the four-momentum transfer, and Ω' and E' refer to the outgoing lepton. N is a nucleon, X is a jet of n hadrons consisting of fermions (f) and bosons (b) in the final state.

The leptonic tensor $L^{\alpha\beta}$ is given by

$$L^{\alpha\beta} = k^{\alpha}k'^{\beta} + k^{\beta}k'^{\alpha} - k.k' g^{\alpha\beta} \pm i\epsilon^{\alpha\beta\rho\sigma}k_{\rho}k'_{\sigma}, \tag{8}$$

where plus sign is for antineutrino and minus sign is for neutrino, and the hadronic tensor $W^N_{\alpha\beta}$ is defined as

$$W^N_{\alpha\beta} = \frac{1}{2\pi}\bar{\sum_{s_N}}\sum_X\sum_{s_i}\prod_{i=1}^{n}\int\frac{d^3p'_i}{(2\pi)^3}\prod_{l\epsilon f}\left(\frac{2M'_l}{2E'_l}\right) \times\prod_{j\epsilon b}\left(\frac{1}{2\omega'_j}\right)\langle X|J_\alpha|N\rangle\langle X|J_\beta|N\rangle^* \times(2\pi)^4\delta^4\left(p+q-\sum_{i=1}^{n}p'_i\right), \tag{9}$$

where s_N is the spin of the nucleon and s_i is the spin of the fermions in X. In the case of antineutrinos $\langle X|J_\alpha|N\rangle$ is replaced by $\langle X|J^\dagger_\alpha|N\rangle$.

The most general form of the hadronic tensor $W^N_{\alpha\beta}$ is expressed as

$$\begin{aligned} W^N_{\alpha\beta} ={}& \left(\frac{q_\alpha q_\beta}{q^2} - g_{\alpha\beta}\right)W_1^{\nu(\bar\nu)} \\ &+ \frac{1}{M^2}\left(p_\alpha - \frac{p.q}{q^2}q_\alpha\right)\left(p_\beta - \frac{p.q}{q^2}q_\beta\right)W_2^{\nu(\bar\nu)} \\ &- \frac{i}{2M^2}\epsilon_{\alpha\beta\rho\sigma}p^\rho q^\sigma W_3^{\nu(\bar\nu)} \\ &+ \frac{1}{M^2}q_\alpha q_\beta W_4^{\nu(\bar\nu)} + \frac{1}{M^2}(p_\alpha q_\beta + q_\alpha p_\beta)W_5^{\nu(\bar\nu)} \\ &+ \frac{i}{M^2}(p_\alpha q_\beta - q_\alpha p_\beta)W_6^{\nu(\bar\nu)}, \end{aligned} \tag{10}$$

where M is the nucleon mass and W^N_i are the structure functions, which depend on the scalars q^2 and $p.q(=p_0q_0 - \mathbf{p}.\mathbf{q})$. The terms depending on W_4, W_5, and W_6 in Eq. (10) do not contribute to the cross section in Eq. (7) in the limit of lepton mass $m_l \to 0$.

In terms of the Bjorken variables x and y defined as

$$x = \frac{Q^2}{2M\nu}, \quad y = \frac{\nu}{E_\nu}, \quad Q^2 = -q^2, \quad \nu = \frac{p.q}{M}, \tag{11}$$

W^N_i are expressed in terms of dimensionless structure functions $F_i^{\nu,\bar\nu}(x,Q^2)$:

$$\begin{aligned} F_1^{\nu(\bar\nu)}(x,Q^2) &= MW_1^{\nu(\bar\nu)}(\nu,Q^2), \\ F_2^{\nu(\bar\nu)}(x,Q^2) &= \nu W_2^{\nu(\bar\nu)}(\nu,Q^2), \\ F_3^{\nu(\bar\nu)}(x,Q^2) &= \nu W_3^{\nu(\bar\nu)}(\nu,Q^2). \end{aligned} \tag{12}$$

The expression for the differential cross section, for DIS of a neutrino with a nucleon target induced by a charged current reaction, is now given by

$$\begin{aligned} \frac{d^2\sigma^{\nu(\bar\nu)}}{dx\,dy} ={}& \frac{G_F^2 ME_\nu}{\pi\left(1+Q^2/M_W^2\right)^2}\left\{\left(y^2x + \frac{m_l^2y}{2E_\nu M}\right)F_1(x,Q^2)\right. \\ &+\left[\left(1-\frac{m_l^2}{4E_\nu^2}\right)-\left(1+\frac{Mx}{2E_\nu}\right)y\right]F_2(x,Q^2) \\ &\left.\pm\left[xy\left(1-\frac{y}{2}\right)-\frac{m_l^2y}{4E_\nu M}\right]F_3(x,Q^2)\right\}. \end{aligned} \tag{13}$$

In the above equation the plus sign (minus sign) in the coefficient with F_3 is for neutrinos (antineutrinos). F_1 and F_2 are related by the Callan-Gross relation, leading to only two independent structure functions F_2 and F_3. For $l = e, \mu$ we take $m_l = 0$ and assume $Q^2 \ll M_W^2$.

The nucleon structure functions are determined in terms of parton distribution functions for quarks and antiquarks given by

$$\begin{aligned} F_2^{\nu p} &= 2x[d(x)+s(x)+\bar u(x)+\bar c(x)], \\ F_2^{\bar\nu p} &= 2x[u(x)+c(x)+\bar d(x)+\bar s(x)], \\ F_2^{\nu n} &= 2x[u(x)+s(x)+\bar d(x)+\bar c(x)], \\ F_2^{\bar\nu n} &= 2x[d(x)+c(x)+\bar u(x)+\bar s(x)], \\ xF_3^{\nu p} &= 2x[d(x)+s(x)-\bar u(x)-\bar c(x)], \\ xF_3^{\nu n} &= 2x[u(x)+s(x)-\bar d(x)-\bar c(x)], \\ xF_3^{\bar\nu p} &= 2x[u(x)+c(x)-\bar d(x)-\bar s(x)], \\ xF_3^{\bar\nu n} &= 2x[d(x)+c(x)-\bar u(x)-\bar s(x)]. \end{aligned} \tag{14}$$

For the neutral-current-induced reaction

$$\nu_l(\bar\nu_l)(k) + N(p) \to \nu_l(k') + X(p'), \quad l = e,\mu,\tau, \tag{15}$$

the expression of the cross section (13) is modified by changing $M_W \to M_Z$, the mass of the Z^0 boson, and the corresponding NC structure functions are given by

$$\begin{aligned} F_2^{\rm NC}(\nu p,\bar\nu p) &= 2x\left\{\left(u_L^2+u_R^2\right)[u+c+\bar u+\bar c]\right. \\ &\quad\left.+\left(d_L^2+d_R^2\right)[d+s+\bar d+\bar s]\right\}, \\ xF_3^{\rm NC}(\nu p,\bar\nu p) &= 2x\left\{\left(u_L^2-u_R^2\right)[u+c-\bar u-\bar c]\right. \\ &\quad\left.+\left(d_L^2-d_R^2\right)[d+s-\bar d-\bar s]\right\} \end{aligned} \tag{16}$$

for the proton target and

$$\begin{aligned} F_2^{\rm NC}(\nu n,\bar\nu n) &= 2x\left\{\left(u_L^2+u_R^2\right)[d+c+\bar d+\bar c]\right. \\ &\quad\left.+\left(d_L^2+d_R^2\right)[u+s+\bar u+\bar s]\right\}, \\ xF_3^{\rm NC}(\nu n,\bar\nu n) &= 2x\left\{\left(u_L^2-u_R^2\right)[d+c-\bar d-\bar c]\right. \\ &\quad\left.+\left(d_L^2-d_R^2\right)[u+s-\bar u-\bar s]\right\} \end{aligned} \tag{17}$$

for the neutron target. Here $u_L = \frac{1}{2} - \frac{2}{3}\sin^2\theta_W$, $u_R = -\frac{2}{3}\sin^2\theta_W$ and $d_L = -\frac{1}{2} + \frac{1}{3}\sin^2\theta_W$, $d_R = \frac{1}{3}\sin^2\theta_W$.

B. Deep inelastic neutrino nucleus scattering

When the reaction given by Eq. (4) takes place in a nucleus, several nuclear effects have to be considered. One may categorize these medium effects into two parts: a kinematic effect, which arises because the struck nucleon is not at rest but is moving with a Fermi momentum in the rest frame of the nucleus, and a dynamic effect, which arises due to the strong interaction of the initial nucleon in the nuclear medium. For details see the discussion given in Refs. [40,41].

The expression for the differential scattering cross section for a nuclear target A is similar to Eq. (7) and is given by

$$\frac{d\,\sigma_{\rm CC}^{\nu(\bar\nu)A}}{dE'\,d\Omega'} = \frac{G_F^2}{(2\pi)^2}\cdot\frac{|\vec{k}'|}{|\vec{k}|}\cdot\left(\frac{m_W^2}{q^2-m_W^2}\right)^2\cdot L_{\nu,\bar\nu}^{\alpha\beta}W_{\alpha\beta}^{\nu(\bar\nu)A}, \quad (18)$$

where $L_{\nu,\bar\nu}^{\alpha\beta}$ is given by Eq. (8) and $W_{\alpha\beta}^{\nu(\bar\nu)A}$, the nuclear hadronic tensor, is given by

$$\begin{aligned} W_{\alpha\beta}^{\nu(\bar\nu)A} &= \left(\frac{q_\alpha q_\beta}{q^2} - g_{\alpha\beta}\right)W_1^{\nu(\bar\nu)A}(P_A,q) \\ &+ \frac{1}{M_A^2}\left(P_{A\alpha} - \frac{P_A\cdot q}{q^2}q_\alpha\right)\left(P_{A\beta} - \frac{P_A\cdot q}{q^2}q_\beta\right) \\ &\times W_2^{\nu(\bar\nu)A}(P_A,q) - \frac{\imath}{2M_A^2}\,\epsilon_{\alpha\beta\rho\sigma}\,P_A^\rho q^\sigma \\ &\times W_3^{\nu(\bar\nu)A}(P_A,q), \end{aligned} \quad (19)$$

where P_A is the momentum of the nucleus A.

In the local density approximation, the nuclear hadronic tensor $W_{\alpha\beta}^{\nu(\bar\nu)A}$ can be written as a convolution of the nucleonic hadronic tensor with the hole spectral function. For symmetric nuclear matter, this would be [10]

$$\begin{aligned} W_{\alpha\beta}^{\nu(\bar\nu)A} &= 4\int d^3r\int\frac{d^3p}{(2\pi)^3}\frac{M}{E(\mathbf{p})} \\ &\times\int_{-\infty}^{\mu}dp^0\;S_h(p^0,\mathbf{p},k_F(\vec r))W_{\alpha\beta}^{\nu(\bar\nu)N}, \end{aligned} \quad (20)$$

where $k_F(\vec r)$ is the Fermi momentum for symmetric nuclear matter, which depends on the density of nucleons in the nucleus, i.e., $k_F(\vec r) = (\frac{3\pi^2}{2}\rho(\vec r))^{1/3}$. $S_h(p^0,\mathbf{p},k_F(\vec r))$ is the hole spectral function and μ is the chemical potential, both of them have been taken from Ref. [38]. $W^{\nu(\bar\nu)N}$ is the hadronic tensor for the free nucleon target given by Eq. (10). M and $E(\mathbf{p})$ are, respectively, the mass and energy of the nucleon.

The natural extension of the above expression for taking into account the nonsymmetric nature of the target nucleus would be to consider separate distributions of Fermi seas for protons and neutrons, the expression for which is given by Ref. [11]

$$\begin{aligned} W_{\alpha\beta}^{\nu(\bar\nu)A} &= 2\int d^3r\int\frac{d^3p}{(2\pi)^3}\frac{M}{E(\mathbf{p})} \\ &\times\int_{-\infty}^{\mu_p}dp^0\;S_h^{\rm proton}(p^0,\mathbf{p},k_{F,p})\cdot W_{\alpha\beta}^{\nu(\bar\nu)p} \\ &+2\int d^3r\int\frac{d^3p}{(2\pi)^3}\frac{M}{E(\mathbf{p})} \\ &\times\int_{-\infty}^{\mu_n}dp^0\;S_h^{\rm neutron}(p^0,\mathbf{p},k_{F,n})\cdot W_{\alpha\beta}^{\nu(\bar\nu)n}, \end{aligned} \quad (21)$$

where the factor 2 in front of the integral accounts for the two degrees of freedom of the spin of the nucleons. In the above equation, $S_h^{\rm proton}$ and $S_h^{\rm neutron}$ are the two different spectral functions and are normalized, respectively, to the number of protons or neutrons in the nuclear target. $k_{F,p} = (3\pi^2\rho_p)^{1/3}$ [$k_{F,n} = (3\pi^2\rho_n)^{1/3}$] is the Fermi momentum of the proton [neutron]. For the proton and neutron densities in iron, we have used a two-parameter Fermi density distribution and the density parameters are taken from Ref. [47].

C. Construction of the nuclear hadronic tensor for nonsymmetric nuclear matter

The natural invariant quantities for DIS of neutrinos with nuclei are

$$x_A = \frac{Q^2}{2P\cdot q};\quad \nu_A = \frac{P\cdot q}{M_A};\quad y_A = \frac{P\cdot q}{P\cdot k}, \quad (22)$$

where x_A is the Bjorken variable in the nucleus and $x_A\in[0,1]$; y_A is the inelasticity. These two variables are related to the nucleonic ones via

$$x_A = \frac{x}{A};\quad y_A = \frac{q^0}{E_\nu} = y, \quad (23)$$

where x and y are defined in Eq. (11). We can see that $x\in[0,A]$, although for $x>1$ the nuclear structure functions are negligible. The variable y_A varies between the following limits:

$$0\leqslant y_A\leqslant\frac{1}{1+\frac{M_Ax_A}{2E_\nu}}\approx\frac{1}{1+\frac{Mx}{2E_\nu}}, \quad (24)$$

so, for sufficient high neutrino energy we have $0\leqslant y_A\leqslant 1$.

If we express the differential cross section with respect to these variables (x_A, y_A), we obtain the following expression in terms of the nuclear structure functions:

$$\begin{aligned} \frac{d^2\sigma_{\rm CC}^{\nu(\bar\nu)A}}{dx_A\,dy_A} &= \frac{G_F^2M_AE_\nu}{\pi}\left(\frac{m_W^2}{Q^2+m_W^2}\right)^2\Bigg\{y_A^2x_AF_1^{\nu(\bar\nu)A} \\ &+\left[1-y_A-\frac{M_Ax_Ay_A}{2E_\nu}\right]F_2^{\nu(\bar\nu)A} \\ &\pm x_Ay_A\left(1-\frac{y_A}{2}\right)F_3^{\nu(\bar\nu)A}\Bigg\}. \end{aligned} \quad (25)$$

For the neutral-current-induced neutrino interaction, the form of the differential cross section is the same as for the charged-current-induced process but with the following changes:

$$\frac{d^2\sigma_{\rm CC}^{\nu(\bar\nu)A}}{dx_A\,dy_A}\longrightarrow\frac{d^2\sigma_{\rm NC}^{\nu(\bar\nu)A}}{dx_A\,dy_A},\quad m_W\longrightarrow m_Z,\quad F_i^{\rm CC}\longrightarrow F_i^{\rm NC}. \quad (26)$$

First we look at the denominator of the Paschos-Wolfenstein relationship, an expression similar to Eq. (2), but in terms of the differential scattering cross section, for which we subtract the charged current antineutrino-nucleus cross section from the charged current neutrino-nucleus cross section and obtain the expression as

$$\begin{aligned} &\frac{d^2\sigma_{\rm CC}^{\nu A}}{dx_A\,dy_A} - \frac{d^2\sigma_{\rm CC}^{\bar\nu A}}{dx_A\,dy_A} \\ &= \frac{G_F^2M_AE_\nu}{\pi}\Bigg\{y_A^2x_A\left(F_1^{\nu A}-F_1^{\bar\nu A}\right) \\ &+\left[1-y_A-\frac{M_Ax_Ay_A}{2E_\nu}\right]\left(F_2^{\nu A}-F_2^{\bar\nu A}\right) \\ &+x_Ay_A\left(1-\frac{y_A}{2}\right)\left(F_3^{\nu A}+F_3^{\bar\nu A}\right)\Bigg\}. \end{aligned} \quad (27)$$

Since $Q^2 \ll M_W^2$ in the present study, we have neglected the W-boson propagator term.

We need to relate the nuclear structure functions F_i^A to the nucleon ones via an integral with the spectral function. Therefore, we introduce the following notation to avoid writing the integration symbols every time. For example, we may rewrite Eq. (21) with the following notation:

$$W_{\alpha\beta}^{\nu(\bar{\nu})A} = \langle W_{\alpha\beta}^{\nu(\bar{\nu})p} \rangle_{S_h^{\text{proton}}} + \langle W_{\alpha\beta}^{\nu(\bar{\nu})n} \rangle_{S_h^{\text{neutron}}}, \quad (28)$$

where $\langle W_{\alpha\beta}^{\nu(\bar{\nu})p} \rangle_{S_h^{\text{proton}}}$ stands for the proton:

$$\langle W_{\alpha\beta}^{\nu(\bar{\nu})p} \rangle_{S_h^{\text{proton}}} = 2 \int d^3r \int \frac{d^3p}{(2\pi)^3} \frac{M}{E(\mathbf{p})} \int_{-\infty}^{\mu_p} dp^0 \times S_h^{\text{proton}}(p^0, \mathbf{p}, k_{F,p}) W_{\alpha\beta}^{\nu(\bar{\nu})p}, \quad (29)$$

and for the neutron the expression is the same when the indices for the proton are replaced by the neutron indices.

Taking the three-momentum transfer $\vec{q}$ along the z axis, i.e., $q^\mu = (q^0, 0, 0, q_z)$, and writing the xx component of the nuclear hadronic tensor [Eq. (19)], we get it in terms of the nuclear structure function F_1, i.e.,

$$W_{xx}^{\nu(\bar{\nu})A} = W_1^{\nu(\bar{\nu})A} = \frac{F_1^{\nu(\bar{\nu})A}(x_A)}{M_A} = W_{yy}^{\nu(\bar{\nu})A}. \quad (30)$$

Similarly, if we take the xx components of the nucleonic hadronic tensor given by Eq. (10) and recall that nucleons in the nucleus are not at rest, the xx component of the nucleonic hadronic tensor is related only not to just the nucleon structure function F_1, but it is a mixture of F_1 and F_2 components like the following expression:

$$W_{xx}^{\nu(\bar{\nu})N} = W_1^{\nu(\bar{\nu})N} + \frac{p_x^2}{M^2} W_2^{\nu(\bar{\nu})N} = \frac{F_1^{\nu(\bar{\nu})N}(x_N)}{M} + \frac{p_x^2}{M^2} \frac{F_2^{\nu(\bar{\nu})N}(x_N)}{\nu_N}, \quad (31)$$

where $N \equiv p, n$; $x_N \equiv \frac{Q^2}{2p\cdot q} = \frac{Q^2}{2(p^0q^0 - p_zq_z)}$; and $\nu_N = \frac{p\cdot q}{M} = \frac{p^0q^0 - p_zq_z}{M}$.

Using Eqs. (30) and (31) we may write

$$\frac{F_1^{\nu(\bar{\nu})A}(x_A)}{M_A} = \left\langle \frac{F_1^{\nu(\bar{\nu})p}(x_N)}{M} + \frac{p_x^2}{M} \frac{F_2^{\nu(\bar{\nu})p}(x_N)}{p\cdot q} \right\rangle_{S_h^p} + \left\langle \frac{F_1^{\nu(\bar{\nu})n}(x_N)}{M} + \frac{p_x^2}{M} \frac{F_2^{\nu(\bar{\nu})n}(x_N)}{p\cdot q} \right\rangle_{S_h^n}. \quad (32)$$

The difference $F_1^{\nu A} - F_1^{\bar{\nu}A}$ that appears in Eq. (27) may then be written as

$$\frac{F_1^{\nu A}(x_A)}{M_A} - \frac{F_1^{\bar{\nu}A}(x_A)}{M_A} = \left\langle \frac{1}{M}[F_1^{\nu p}(x_N) - F_1^{\bar{\nu}p}(x_N)] + \frac{p_x^2}{M(p\cdot q)}[F_2^{\nu p}(x_N) - F_2^{\bar{\nu}p}(x_N)] \right\rangle_{S_h^p} + \left\langle \frac{1}{M}[F_1^{\nu n}(x_N) - F_1^{\bar{\nu}n}(x_N)] + \frac{p_x^2}{M(p\cdot q)}[F_2^{\nu n}(x_N) - F_2^{\bar{\nu}n}(x_N)] \right\rangle_{S_h^n}, \quad (33)$$

where

$$F_1^{\nu p} - F_1^{\bar{\nu}p} = d_v - u_v,$$
$$F_2^{\nu p} - F_2^{\bar{\nu}p} = 2x_N(d_v - u_v),$$
$$F_1^{\nu n} - F_1^{\bar{\nu}n} = -d_v + u_v = -(F_1^{\nu p} - F_1^{\bar{\nu}p}),$$
$$F_2^{\nu n} - F_2^{\bar{\nu}n} = 2x_N(-d_v + u_v) = -(F_2^{\nu p} - F_2^{\bar{\nu}p}).$$

Here u_v and d_v are the valence parton distribution functions (PDFs) and we are working in the *up and down quark approximation*, where we neglect strange and charm quark contributions.

In the case of symmetric nuclear matter, we may relate the Fermi momentum with the baryon density via $k_F^3 = \frac{3\pi^2}{2}\rho$, where ρ is the baryon density. For nonsymmetric nuclear matter, we have different densities for protons and neutrons and, corresponding to those, we also have different Fermi momenta for protons and neutrons. These are related by

$$k_{F,p}^3 = 3\pi^2\rho_p(r); \quad k_{F,n}^3 = 3\pi^2\rho_n(r). \quad (34)$$

Instead of discussing in terms of neutron number (N) and proton number (Z) as independent variables, we define two independent variables $A = N + Z$ and their difference $\delta = N - Z$ such that

$$N = \frac{A}{2} + \frac{\delta}{2}; \quad Z = \frac{A}{2} - \frac{\delta}{2}. \quad (35)$$

Dividing the above equations by the nuclear volume V, we obtain the densities of neutrons and protons:

$$\rho_n = \frac{\rho}{2} + \frac{\delta}{2V}; \quad \rho_p = \frac{\rho}{2} - \frac{\delta}{2V}, \quad (36)$$

where $\rho = \rho_p + \rho_n$ and $\frac{\delta}{V} = \frac{N-Z}{V} = \rho_n - \rho_p$. Replacing the densities for neutrons and protons by their corresponding Fermi momenta one has $k_{F,p/n}$ in terms of k_F, δ, and V, i.e.,

$$k_{F,p}^3 = k_F^3 - \frac{3\pi^2}{2}\frac{\delta}{V}; \quad k_{F,n}^3 = k_F^3 + \frac{3\pi^2}{2}\frac{\delta}{V}. \quad (37)$$

A nonzero value of δ would imply that we are looking for deviations from isoscalarity. For $k_{F,p}$ and $k_{F,n}$, we are going to perform an expansion in powers of the parameter $\frac{\delta}{V}$, retaining the first-order term only, with the assumption that the higher orders would be negligible. For instance, the expansion for the Fermi momentum of the proton would be

$$k_{F,p} = \left(k_F^3 - \frac{3\pi^2}{2}\frac{\delta}{V}\right)^{1/3} = k_F - \frac{\pi^2}{2k_F^2}\frac{\delta}{V} + O\left(\frac{\delta}{V}\right)^2 \quad (38)$$

and the proton's spectral function may be written as

$$S_h^{\text{proton}}(p^0, \mathbf{p}, k_{F,p}) \simeq S_h^p\left(p^0, \mathbf{p}, k_F - \frac{\pi^2}{2k_F^2}\frac{\delta}{V}\right) \simeq S_h^p(p^0, \mathbf{p}, k_F) + \left.\frac{\partial S_h^p(p^0, \mathbf{p}, k)}{\partial k}\right|_{k=k_F} \times \left(-\frac{\pi^2}{2k_F^2}\frac{\delta}{V}\right). \quad (39)$$

With a change of $\delta \to -\delta$ one gets the neutron spectral function.

Using Eq. (32), one may write $F_1^{\nu A} - F_1^{\bar{\nu} A}$ as

$$F_1^{\nu A} - F_1^{\bar{\nu} A} = M_A \left\langle \frac{F_1^{\nu p} - F_1^{\bar{\nu} p}}{M} + \frac{p_x^2}{M(p\cdot q)}\left(F_2^{\nu p} - F_2^{\bar{\nu} p}\right)\right\rangle_{S_h^p} + M_A \left\langle \frac{F_1^{\nu n} - F_1^{\bar{\nu} n}}{M} + \frac{p_x^2}{M(p\cdot q)}\left(F_2^{\nu n} - F_2^{\bar{\nu} n}\right)\right\rangle_{S_h^n}. \quad (40)$$

If we look at the convolution with the proton spectral function given by Eq. (29), we may write the first term on the right-hand side as

$$2M_A \int d^3r \int \frac{d^3p}{(2\pi)^3}\frac{M}{E(\mathbf{p})}\int_{-\infty}^{\mu_p} dp^0\, S_h^p(p^0,\mathbf{p},k_{F,p}) \times \left(\frac{F_1^{\nu p} - F_1^{\bar{\nu} p}}{M} + \frac{p_x^2}{M(p\cdot q)}\left(F_2^{\nu p} - F_2^{\bar{\nu} p}\right)\right), \quad (41)$$

where $\mu_p = M + \frac{k_{F,p}^2}{2M}$ is the chemical potential, which must be expanded around the isoscalarity condition. μ_p may be written as

$$\mu_p = M + \frac{k_{F,p}^2}{2M} \simeq M + \frac{1}{2M}\left(k_F^2 - \frac{\pi^2}{k_F}\frac{\delta}{V}\right). \quad (42)$$

Inserting Eq. (39) in Eq. (41), we obtain the following expression with the spectral function:

$$2M_A \int d^3r \int \frac{d^3p}{(2\pi)^3}\frac{M}{E(\mathbf{p})} \times \int_{-\infty}^{M+\frac{1}{2M}(k_F^2 - \frac{\pi^2}{k_F}\frac{\delta}{V})} dp^0 \left\{ S_h^p(p^0,\mathbf{p},k_F) + \left.\frac{\partial S_h^p(p^0,\mathbf{p},k)}{\partial k}\right|_{k=k_F}\left(-\frac{\pi^2}{2k_F^2}\frac{\delta}{V}\right)\right\} G^{\nu p-\bar{\nu} p}(\mathbf{p},x_N), \quad (43)$$

where

$$G^{\nu p-\bar{\nu} p}(\mathbf{p},x_N) = \left(\frac{F_1^{\nu p} - F_1^{\bar{\nu} p}}{M} + \frac{p_x^2}{M(p\cdot q)}\left(F_2^{\nu p} - F_2^{\bar{\nu} p}\right)\right). \quad (44)$$

Integrating the first term of Eq. (43), over the variable p^0, gives

$$\begin{aligned}\int_{-\infty}^{M+\frac{k_F^2}{2M}-\frac{\pi^2}{2Mk_F}\frac{\delta}{V}} & dp^0\, S_h^p(p^0,\mathbf{p},k_F) G^{\nu p-\bar{\nu} p}(p^0,\mathbf{p},x_N) \\ &= \int_{-\infty}^{\mu} dp^0 S_h^p(p^0,\mathbf{p},k_F) G^{\nu p-\bar{\nu} p}(p^0,\mathbf{p},x_N) \\ &\quad + \int_{\mu}^{\mu-\frac{\pi^2}{2Mk_F}\frac{\delta}{V}} dp^0\, S_h^p(p^0,\mathbf{p},k_F) G^{\nu p-\bar{\nu} p}(p^0,\mathbf{p},x_N) \\ &\simeq \int_{-\infty}^{\mu} dp^0 S_h^p(p^0,\mathbf{p},k_F) G^{\nu p-\bar{\nu} p}(p^0,\mathbf{p},x_N) \\ &\quad + S_h^p(\mu,\mathbf{p},k_F) G^{\nu p-\bar{\nu} p}(p^0=\mu,\mathbf{p},x_N)\left(-\frac{\pi^2}{2Mk_F}\frac{\delta}{V}\right),\end{aligned} \quad (45)$$

where $\mu = M + \frac{k_F^2}{2M}$ and we have used the following property:

$$\int_{\mu}^{\mu+\delta x} dy\, f(y) \simeq f(\mu) \times \delta x \quad (46)$$

for $\delta x \to 0$.

Let us analyze what we have obtained from the first term of the integral in Eq. (43). We have obtained an isoscalar term [the first term in Eq. (45)], which still has to be integrated over p^0, plus a correction proportional to $\frac{\delta}{V}$. This correction would be zero for an isoscalar target.

In Eq. (43), we still have another integral (the one that goes with the partial derivative), which is already of $O(\frac{\delta}{V})$. Therefore, we will only have to calculate the contribution coming from the limits of integration in p^0 at $O(1)$. Indeed, we have

$$\begin{aligned}\int_{-\infty}^{\mu-\frac{\pi^2}{2Mk_F}\frac{\delta}{V}} & dp^0 \left.\frac{\partial S_h^p(p^0,\mathbf{p},k)}{\partial k}\right|_{k=k_F}\left(-\frac{\pi^2}{2k_F^2}\frac{\delta}{V}\right) G^{\nu p-\bar{\nu} p}(p^0,\mathbf{p},x_N) \\ &= \left(-\frac{\pi^2}{2k_F^2}\frac{\delta}{V}\right)\int_{-\infty}^{\mu} dp^0 \left.\frac{\partial S_h^p(p^0,\mathbf{p},k)}{\partial k}\right|_{k=k_F} G^{\nu p-\bar{\nu} p}(p^0,\mathbf{p},x_N) + O\left(\frac{\delta}{V}\right)^2.\end{aligned} \quad (47)$$

Therefore, Eq. (41) may be written as

$$\begin{aligned}2M_A \int d^3r \int \frac{d^3p}{(2\pi)^3}\frac{M}{E(\mathbf{p})} & \left\{ \int_{-\infty}^{\mu} dp^0\, S_h^p(p^0,\mathbf{p},k_F) G^{\nu p-\bar{\nu} p}(p^0,\mathbf{p},x_N) - \frac{\pi^2}{2Mk_F(r)}\frac{\delta}{V} S_h^p(\mu,\mathbf{p},k_F) G^{\nu p-\bar{\nu} p}(\mu,\mathbf{p},x_N)\right. \\ & \left. -\frac{\pi^2}{2k_F^2}\frac{\delta}{V}\int_{-\infty}^{\mu} dp^0 \left.\frac{\partial S_h^p(p^0,\mathbf{p},k)}{\partial k}\right|_{k=k_F} G^{\nu p-\bar{\nu} p}(p^0,\mathbf{p},x_N)\right\}.\end{aligned} \quad (48)$$

This result is only for the convolution with the proton-hole spectral function. Similarly, there will be a corresponding term for the neutron-hole spectral function. But the changes are minimal. They reduce to change p (protons) by n (neutrons) in the structure functions and $\delta \to -\delta$. With these changes, we may write the equivalent expression to the one above for the convolution with the neutron spectral function as

$$2M_A \int d^3r \int \frac{d^3p}{(2\pi)^3}\frac{M}{E(\mathbf{p})}\left\{\int_{-\infty}^{\mu} dp^0\, S_h^n(p^0,\mathbf{p},k_F)G^{\nu n-\bar{\nu}n}(p^0,\mathbf{p},x_N) + \frac{\pi^2}{2Mk_F(r)}\frac{\delta}{V}S_h^n(\mu,\mathbf{p},k_F)G^{\nu n-\bar{\nu}n}(\mu,\mathbf{p},x_N) + \frac{\pi^2}{2k_F^2}\frac{\delta}{V}\int_{-\infty}^{\mu} dp^0 \frac{\partial S_h^n(p^0,\mathbf{p},k)}{\partial k}\bigg|_{k=k_F} G^{\nu n-\bar{\nu}n}(p^0,\mathbf{p},x_N)\right\}, \tag{49}$$

where $G^{\nu n-\bar{\nu}n}(p^0,\mathbf{p},x_N)$ is the same as Eq. (44) but with the replacement of $p \to n$ in the structure functions.

Equations (48) and (49) are summed, by keeping in mind that now the spectral functions are identical as they are evaluated at $k = k_F$, which results in

$$2M_A \int d^3r \int \frac{d^3p}{(2\pi)^3}\frac{M}{E(\mathbf{p})}\left\{\int_{-\infty}^{\mu} dp^0\, S_h(p^0,\mathbf{p},k_F)[G^{\nu p-\bar{\nu}p}(p^0,\mathbf{p}) + G^{\nu n-\bar{\nu}n}(p^0,\mathbf{p})] + \frac{\pi^2}{2Mk_F(r)}\frac{\delta}{V}S_h(\mu,\mathbf{p},k_F)[G^{\nu n-\bar{\nu}n}(\mu,\mathbf{p}) - G^{\nu p-\bar{\nu}p}(\mu,\mathbf{p})] + \frac{\pi^2}{2k_F^2}\frac{\delta}{V}\int_{-\infty}^{\mu} dp^0\frac{\partial S_h(p^0,\mathbf{p},k)}{\partial k}\bigg|_{k=k_F}[G^{\nu n-\bar{\nu}n}(p^0,\mathbf{p}) - G^{\nu p-\bar{\nu}p}(p^0,\mathbf{p})]\right\}. \tag{50}$$

The sum of the structure functions $G^{\nu n-\bar{\nu}n}(p^0,\mathbf{p})$ and $G^{\nu p-\bar{\nu}p}(p^0,\mathbf{p})$ in the first line of the above expression is zero in the limit of considering only light quarks (up and down quark approximation), and

$$G^{\nu n-\bar{\nu}n}(p^0,\mathbf{p}) - G^{\nu p-\bar{\nu}p}(p^0,\mathbf{p}) = (u_v - d_v)\left[\frac{2}{M} + \frac{4p_x^2}{M(p\cdot q)}x_N\right], \tag{51}$$

where $u_v = u - \bar{u}$ and $d_v = d - \bar{d}$ are the up and down valence distributions, respectively.

Furthermore, in the denominator of the PW relation we have the difference $F_2^{\nu A} - F_2^{\bar{\nu}A}$, which is written as

$$F_2^A = q^0 W_2^A = q^0\frac{\left\langle\left\{q^2\left[\vec{p}^{\,2} + 2(p^0)^2 - p_z^2\right] - 2(q^0)^2\left[(p^0)^2 + p_z^2\right] + 4p^0q^0p_z\sqrt{(q^0)^2 - q^2}\right\}\cdot\frac{F_2^p}{\nu_N}\right\rangle_{S_h^p}}{2M^2(q^2 - (q^0)^2)} + q^0\frac{\left\langle\left\{q^2\left[\vec{p}^{\,2} + 2(p^0)^2 - p_z^2\right] - 2(q^0)^2\left[(p^0)^2 + p_z^2\right] + 4p^0q^0p_z\sqrt{(q^0)^2 - q^2}\right\}\cdot\frac{F_2^n}{\nu_N}\right\rangle_{S_h^n}}{2M^2[q^2 - (q^0)^2]}. \tag{52}$$

Let us define the kinematic factor which goes in front of F_2 as

$$G(p^0,\vec{p}) \equiv \frac{q^0\left\{q^2\left[\vec{p}^{\,2} + 2(p^0)^2 - p_z^2\right] - 2(q^0)^2\left[(p^0)^2 + p_z^2\right] + 4p^0q^0p_z\sqrt{(q^0)^2 - q^2}\right\}}{2M[q^2 - (q^0)^2]\cdot(p\cdot q)}. \tag{53}$$

This expression for F_2 is valid for neutrinos as well as for antineutrino. Therefore, when performing the subtraction, we get the following:

$$F_2^{\nu A} - F_2^{\bar{\nu}A} = \left\langle G(p^0,\vec{p})\left(F_2^{\nu p} - F_2^{\bar{\nu}p}\right)\right\rangle_{S_h^p} + \left\langle G(p^0,\vec{p})\left(F_2^{\nu n} - F_2^{\bar{\nu}n}\right)\right\rangle_{S_h^n}. \tag{54}$$

Defining

$$G_2^{\nu p-\bar{\nu}p}(p^0,\vec{p}) \equiv G(p^0,\vec{p})\left(F_2^{\nu p} - F_2^{\bar{\nu}p}\right), \tag{55}$$

$$G_2^{\nu n-\bar{\nu}n}(p^0,\vec{p}) \equiv G(p^0,\vec{p})\left(F_2^{\nu n} - F_2^{\bar{\nu}n}\right) \tag{56}$$

and using the same procedure as we did for F_1, one gets

$$\left\langle G_2^{\nu p-\bar{\nu}p}(p^0,\vec{p})\right\rangle_{S_h^p} = \left\langle\int_{-\infty}^{\mu} dp^0\, S_h^p(p^0,\vec{p},k_F)G_2^{\nu p-\bar{\nu}p}(p^0,\vec{p})\right\rangle - \left\langle\frac{\delta}{2V}\frac{\pi^2}{Mk_F}S_h^p(\mu,\vec{p},k_F)G_2^{\nu p-\bar{\nu}p}(\mu,\vec{p})\right\rangle - \left\langle\frac{\delta}{2V}\frac{\pi^2}{k_F^2}\int_{-\infty}^{\mu} dp^0\,\frac{\partial S_h^p(p^0,\vec{p},k)}{\partial k}\bigg|_{k=k_F} G_2^{\nu p-\bar{\nu}p}(p^0,\vec{p}) + O\left(\frac{\delta}{V}\right)^2\right\rangle, \tag{57}$$

where the symbols $\langle \cdots \rangle$ indicate the integrals in d^3p and d^3r. Similarly, one obtains the convolution with the hole spectral function for neutrons with the known changes ($p \to n$ and $\delta \to -\delta$) and gets

$$\langle G_2^{\nu n-\bar\nu n}(p^0,\vec p)\rangle_{S_h^n} = \left\langle \int_{-\infty}^{\mu} dp^0\, S_h^n(p^0,\vec p,k_F) G_2^{\nu n-\bar\nu n}(p^0,\vec p)\right\rangle + \left\langle \frac{\delta}{2V}\frac{\pi^2}{Mk_F} S_h^n(\mu,\vec p,k_F) G_2^{\nu n-\bar\nu n}(\mu,\vec p)\right\rangle + \left\langle \frac{\delta}{2V}\frac{\pi^2}{k_F^2}\int_{-\infty}^{\mu} dp^0 \left.\frac{\partial S_h^n(p^0,\vec p,k)}{\partial k}\right|_{k=k_F} G_2^{\nu n-\bar\nu n}(p^0,\vec p) + O\left(\frac{\delta}{V}\right)^2\right\rangle. \tag{58}$$

Therefore, when summing over both the contributions to obtain $F_2^{\nu A} - F_2^{\bar\nu A}$, we get two kinds of contributions: the first-order one and the second-order one (proportional to $\frac{\delta}{V}$):

$$\begin{aligned} F_2^{\nu A} - F_2^{\bar\nu A} &= \langle G_2^{\nu p-\bar\nu p}(p^0,\vec p)\rangle_{S_h^p} + \langle G_2^{\nu n-\bar\nu n}(p^0,\vec p)\rangle_{S_h^n} \\ &= \left\langle \int_{-\infty}^{\mu} dp^0\, S_h(p^0,\vec p,k_F)\big[G_2^{\nu p-\bar\nu p}(p^0,\vec p) + G_2^{\nu n-\bar\nu n}(p^0,\vec p)\big]\right\rangle \\ &\quad + \left\langle \frac{\delta}{2V}\frac{\pi^2}{Mk_F} S_h(\mu,\vec p,k_F)\big[G_2^{\nu n-\bar\nu n}(p^0,\vec p) - G_2^{\nu p-\bar\nu p}(p^0,\vec p)\big]\right\rangle \\ &\quad + \left\langle \frac{\delta}{2V}\frac{\pi^2}{k_F^2}\int_{-\infty}^{\mu} dp^0 \left.\frac{\partial S_h(p^0,\vec p,k)}{\partial k}\right|_{k=k_F}\big[G_2^{\nu n-\bar\nu n}(p^0,\vec p) - G_2^{\nu p-\bar\nu p}(p^0,\vec p)\big] + O\left(\frac{\delta}{V}\right)^2\right\rangle. \end{aligned} \tag{59}$$

Writing the sum and the difference of G_2 for neutrons and protons, one obtains (in the up and down quark approximation)

$$\begin{aligned} G_2^{\nu p-\bar\nu p}(p^0,\vec p) + G_2^{\nu n-\bar\nu n}(p^0,\vec p) &= G(p^0,\vec p)\big(F_2^{\nu p} - F_2^{\bar\nu p} + F_2^{\nu n} - F_2^{\bar\nu n}\big) = 0, \\ G_2^{\nu n-\bar\nu n}(p^0,\vec p) - G_2^{\nu p-\bar\nu p}(p^0,\vec p) &= G(p^0,\vec p)\big(-F_2^{\nu p} + F_2^{\bar\nu p} + F_2^{\nu n} - F_2^{\bar\nu n}\big) = G(p^0,\vec p)\times 4\,x_N(u_v - d_v). \end{aligned} \tag{60}$$

Therefore, in this limit, we obtain a dominant non-isoscalarity correction (because the isoscalar term is zero) proportional to $\frac{\delta}{V}$. This is exactly the same effect as was obtained in the calculation of $F_1^{\nu A} - F_1^{\bar\nu A}$.

For the F_3 structure functions for the CC case we have

$$\begin{aligned} F_3^{\nu A} + F_3^{\bar\nu A} &= 2A\frac{q^0}{q_z}\int d^3r \int \frac{d^3p}{(2\pi)^3}\frac{M}{E(\vec p)}\int_{-\infty}^{\mu_p} dp^0\, S_h^p(p^0,\vec p,k_{F,p})\frac{p^0q_z - q^0p_z}{(p\cdot q)}\big(F_3^{\nu p} + F_3^{\bar\nu p}\big) \\ &\quad + 2A\frac{q^0}{q_z}\int d^3r \int \frac{d^3p}{(2\pi)^3}\frac{M}{E(\vec p)}\int_{-\infty}^{\mu_n} dp^0\, S_h^n(p^0,\vec p,k_{F,n})\frac{p^0q_z - q^0p_z}{(p\cdot q)}\big(F_3^{\nu n} + F_3^{\bar\nu n}\big) \\ &\equiv \langle G_3^{\nu p+\bar\nu p}(p^0,\vec p)\rangle_{S_h^p} + \langle G_3^{\nu n+\bar\nu n}(p^0,\vec p)\rangle_{S_h^n}, \end{aligned} \tag{61}$$

where in the last step we have defined

$$G_3^{\nu p+\bar\nu p}(p^0,\vec p) \equiv 2A\,\frac{q^0}{q_z}\,\frac{p^0q_z - q^0p_z}{(p\cdot q)}\big(F_3^{\nu p} + F_3^{\bar\nu p}\big), \tag{62}$$

$$G_3^{\nu n+\bar\nu n}(p^0,\vec p) \equiv 2A\,\frac{q^0}{q_z}\,\frac{p^0q_z - q^0p_z}{(p\cdot q)}\big(F_3^{\nu n} + F_3^{\bar\nu n}\big). \tag{63}$$

We perform the expansion of the spectral functions around the Fermi momentum:

$$\langle G_3^{\nu p+\bar\nu p}(p^0,\vec p)\rangle_{S_h^p} = \left\langle \int_{-\infty}^{\mu} dp^0\, S_h^p(p^0,\vec p,k_F) G_3^{\nu p+\bar\nu p}(p^0,\vec p)\right\rangle - \left\langle \frac{\delta}{2V}\frac{\pi^2}{Mk_F} S_h^p(\mu,\vec p,k_F) G_3^{\nu p+\bar\nu p}(\mu,\vec p)\right\rangle - \left\langle \frac{\delta}{2V}\frac{\pi^2}{k_F^2}\int_{-\infty}^{\mu} dp^0 \left.\frac{\partial S_h^p(p^0,\vec p,k)}{\partial k}\right|_{k=k_F} G_3^{\nu p+\bar\nu p}(p^0,\vec p) + O\left(\frac{\delta}{V}\right)^2\right\rangle. \tag{64}$$

The changes to obtain the G_3 convoluted structure function for neutrons are the replacements $p \to n$ and $\delta \to -\delta$, which results in

$$\langle G_3^{\nu n+\bar\nu n}(p^0,\vec p)\rangle_{S_h^n} = \left\langle \int_{-\infty}^{\mu} dp^0\, S_h^n(p^0,\vec p,k_F) G_3^{\nu n+\bar\nu n}(p^0,\vec p)\right\rangle + \left\langle \frac{\delta}{2V}\frac{\pi^2}{Mk_F} S_h^n(\mu,\vec p,k_F) G_3^{\nu n+\bar\nu n}(\mu,\vec p)\right\rangle + \left\langle \frac{\delta}{2V}\frac{\pi^2}{k_F^2}\int_{-\infty}^{\mu} dp^0 \left.\frac{\partial S_h^n(p^0,\vec p,k)}{\partial k}\right|_{k=k_F} G_3^{\nu n+\bar\nu n}(p^0,\vec p) + O\left(\frac{\delta}{V}\right)^2\right\rangle. \tag{65}$$

Summing the above two equations for the proton and neutron, we get the following:

$$F_3^{\nu A} + F_3^{\bar\nu A} = \left\langle \int_{-\infty}^{\mu} dp^0\, S_h(p^0, \vec p, k_F)\big[G_3^{\nu p+\bar\nu p}(p^0,\vec p) + G_3^{\nu n+\bar\nu n}(p^0,\vec p)\big]\right\rangle + \left\langle \frac{\delta}{2V}\frac{\pi^2}{Mk_F} S_h(\mu,\vec p,k_F)\big[G_3^{\nu n+\bar\nu n}(\mu,\vec p) - G_3^{\nu p+\bar\nu p}(\mu,\vec p)\big]\right\rangle + \left\langle \frac{\delta}{2V}\frac{\pi^2}{k_F^2}\int_{-\infty}^{\mu} dp^0\, \frac{\partial S_h(p^0,\vec p,k)}{\partial k}\bigg|_{k=k_F}\big[G_3^{\nu n+\bar\nu n}(p^0,\vec p) - G_3^{\nu p+\bar\nu p}(p^0,\vec p)\big] + O\left(\frac{\delta}{V}\right)^2\right\rangle. \tag{66}$$

If we write explicitly the combinations of G_3 functions in terms of PDFs considering only light quarks (up and down), we get

$$G_3^{\nu p+\bar\nu p}(p^0,\vec p) + G_3^{\nu n+\bar\nu n}(p^0,\vec p) = 4G_3(p^0,\vec p)(u_v + d_v), \tag{67}$$

where

$$G_3(p^0,\vec p) \equiv 2A\,\frac{q^0}{q_z}\,\frac{p^0 q_z - q^0 p_z}{(p\cdot q)}, \tag{68}$$

and for the difference we get exactly zero, i.e.,

$$G_3^{\nu n+\bar\nu n}(p^0,\vec p) - G_3^{\nu p+\bar\nu p}(p^0,\vec p) = 0. \tag{69}$$

Therefore, for F_3, the dominant contribution is the isoscalar one (the one which is not proportional to $\frac{\delta}{V}$) and the non-isoscalar contribution (proportional to $\frac{\delta}{V}$) is zero at first order $[O(\frac{\delta}{V})]$. Of course there will be corrections at higher orders, but we are retaining only the first-order non-isoscalar corrections.

For neutral currents, in the numerator of the PW relation, we would only have the structure function F_3^{0A}, which can be written as

$$F_3^{0A}(x_A) = 2A\,\frac{q^0}{q_z}\int d^3r \int \frac{d^3p}{(2\pi)^3}\frac{M}{E(\vec p)} \times \int_{-\infty}^{\mu_p} dp^0\, S_h^p(p^0,\vec p,k_{F,p})\frac{p^0 q_z - p_z q^0}{p\cdot q} F_3^{0p}(x_N) + 2A\,\frac{q^0}{q_z}\int d^3r \int \frac{d^3p}{(2\pi)^3}\frac{M}{E(\vec p)} \times \int_{-\infty}^{\mu_n} dp^0\, S_h^n(p^0,\vec p,k_{F,n})\frac{p^0 q_z - p_z q^0}{p\cdot q} F_3^{0n}(x_N) \tag{70}$$

$$\equiv \big\langle G_3^{0p}(p^0,\vec p)\big\rangle_{S_h^p} + \big\langle G_3^{0n}(p^0,\vec p)\big\rangle_{S_h^n}, \tag{71}$$

where for the neutral current case, we define

$$G_3^{0p}(p^0,\vec p) \equiv G_3(p^0,\vec p)F_3^{0p}(x_N), \tag{72}$$

$$G_3^{0n}(p^0,\vec p) \equiv G_3(p^0,\vec p)F_3^{0n}(x_N). \tag{73}$$

$G_3(p^0,\vec p)$ is defined in Eq. (68).

The expression for F_3^{0A} around the isoscalar condition is

$$F_3^{0A} = \left\langle \int_{-\infty}^{\mu} dp^0\, S_h(p^0,\vec p,k_F)\big[G_3^{0p}(p^0,\vec p) + G_3^{0n}(p^0,\vec p)\big]\right\rangle + \left\langle \frac{\delta}{2V}\frac{\pi^2}{Mk_F}S_h(\mu,\vec p,k_F)\big[G_3^{0n}(\mu,\vec p) - G_3^{0p}(\mu,\vec p)\big]\right\rangle + \left\langle \frac{\delta}{2V}\frac{\pi^2}{k_F^2}\int_{-\infty}^{\mu} dp^0\,\frac{\partial S_h(p^0,\vec p,k)}{\partial k}\bigg|_{k=k_F} \times \big[G_3^{0n}(p^0,\vec p) - G_3^{0p}(p^0,\vec p)\big] + O\left(\frac{\delta}{V}\right)^2\right\rangle. \tag{74}$$

For the neutral current in the parton model

$$F_3^{0p} = 2\big\{\big[\epsilon_L^2(u) - \epsilon_R^2(u)\big](u-\bar u) + \big[\epsilon_L^2(d) - \epsilon_R^2(d)\big](d-\bar d)\big\} \equiv 2[g_-^2(u)(u-\bar u) + g_-^2(d)(d-\bar d)], \tag{75}$$

where

$$g_-^2(u) \equiv \epsilon_L^2(u) - \epsilon_R^2(u) = \tfrac14 - \tfrac23\sin^2\theta_W, \tag{76}$$

$$g_-^2(d) \equiv \epsilon_L^2(d) - \epsilon_R^2(d) = \tfrac14 - \tfrac13\sin^2\theta_W \tag{77}$$

and invoking isospin symmetry, we also have

$$F_3^{0n} = 2[g_-^2(u)\left(d-\bar d\right) + g_-^2(d)\,(u-\bar u)]. \tag{78}$$

With these, we obtain the expressions for the sum and difference of neutral current G_3 functions for protons and neutrons, i.e.,

$$G_3^{0p}(p^0,\vec p) + G_3^{0n}(p^0,\vec p) = 2\,G_3(p^0,\vec p)[g_-^2(u) + g_-^2(d)](u_v + d_v), \tag{79}$$

$$G_3^{0n}(p^0,\vec p) - G_3^{0p}(p^0,\vec p) = 2\,G_3(p^0,\vec p)[g_-^2(d) - g_-^2(u)](u_v - d_v). \tag{80}$$

D. Nuclear corrections to the PW ratio

Using the expressions obtained in the previous section, we may write the ratio of the differential scattering cross sections as

$$R^- = \frac{\frac{d\sigma_{NC}^{\nu A}}{dx\,dy} - \frac{d\sigma_{NC}^{\bar{\nu} A}}{dx\,dy}}{\frac{d\sigma_{CC}^{\nu A}}{dx\,dy} - \frac{d\sigma_{CC}^{\bar{\nu} A}}{dx\,dy}} = \frac{2xy\left(1-\frac{y}{2}\right)\left\{2[g_-^2(u)+g_-^2(d)]\left\langle\int_{-\infty}^{\mu} dp^0\, S_h\, G_3'[u_v(x_N)+d_v(x_N)]\right\rangle + O\left(\epsilon_3^{0A}\right)\right\}}{y^2x\, O\left(\epsilon_1^{\nu A-\bar{\nu}A}\right) + \left\{1-y-\frac{Mxy}{2E_\nu}\right\}O\left(\epsilon_2^{\nu A-\bar{\nu}A}\right) + xy\left(1-\frac{y}{2}\right)\left\langle 4\int_{-\infty}^{\mu} dp^0\, S_h\, G_3'(u_v+d_v)\right\rangle}$$
$$= \frac{4xy\left(1-\frac{y}{2}\right)[g_-^2(u)+g_-^2(d)]\left\langle\int_{-\infty}^{\mu} dp^0\, S_h\, G_3'[u_v(x_N)+d_v(x_N)]\right\rangle + O'\left(\epsilon_3^{0A}\right)}{4xy\left(1-\frac{y}{2}\right)\left\langle\int_{-\infty}^{\mu} dp^0\, S_h\, G_3'[u_v(x_N)+d_v(x_N)]\right\rangle + O'(\epsilon^{\nu A-\bar{\nu}A})}, \tag{81}$$

where

$$G_3' = \frac{G_3(p^0,\vec{p})}{A}, \tag{82}$$

$$O'\left(\epsilon_3^{0A}\right) = 2xy\left(1-\frac{y}{2}\right)O\left(\epsilon_3^{0A}\right), \tag{83}$$

$$O'(\epsilon^{\nu A-\bar{\nu}A}) = y^2x\; O\left(\epsilon_1^{\nu A-\bar{\nu}A}\right) + \left\{1-y-\frac{Mxy}{2E_\nu}\right\} O\left(\epsilon_2^{\nu A-\bar{\nu}A}\right). \tag{84}$$

Here ϵ indicates that the contribution is of $O(\frac{\delta}{V})$.

Expanding the denominator of Eq. (81) using a Taylor series and keeping only the first-order term in $\frac{\delta}{V}$, we may write the above equation as

$$R^- \simeq \left[4xy\left(1-\frac{y}{2}\right)[g_-^2(u)+g_-^2(d)]\left\langle\int_{-\infty}^{\mu} dp^0\, S_h\, G_3'\,(u_v+d_v)\right\rangle + O'\left(\epsilon_3^{0A}\right)\right]$$
$$\times\left[\frac{1}{4xy\left(1-\frac{y}{2}\right)\left\langle\int_{-\infty}^{\mu} dp^0\, S_h\, G_3'\,(u_v+d_v)\right\rangle} - \frac{O'(\epsilon^{\nu A-\bar{\nu}A})}{\left[4xy\left(1-\frac{y}{2}\right)\left\langle\int_{-\infty}^{\mu} dp^0\, S_h\, G_3'\,(u_v+d_v)\right\rangle\right]^2}\right]$$
$$\simeq g_-^2(u)+g_-^2(d) + \frac{O'\left(\epsilon_3^{0A}\right)}{4xy\left(1-\frac{y}{2}\right)\left\langle\int_{-\infty}^{\mu} dp^0\, S_h\, G_3'\,(u_v+d_v)\right\rangle} - \frac{[g_-^2(u)+g_-^2(d)]O'(\epsilon^{\nu A-\bar{\nu}A})}{4xy\left(1-\frac{y}{2}\right)\left\langle\int_{-\infty}^{\mu} dp^0\, S_h\, G_3'\,(u_v+d_v)\right\rangle}$$
$$= \frac{1}{2} - \sin^2\theta_W + \delta R^-, \tag{85}$$

where δR^- is the correction due to the non-isoscalarity of the target and is written as

$$\delta R^- = \delta R_1^- + \delta R_2^-. \tag{86}$$

The first correction δR_1^- is given by

$$\delta R_1^- = \frac{O'\left(\epsilon_3^{0A}\right)}{4xy\left(1-\frac{y}{2}\right)\left\langle\int_{-\infty}^{\mu} dp^0\, S_h\, G_3'\,(u_v+d_v)\right\rangle} = \frac{O\left(\epsilon_3^{0A}\right)}{2\left\langle\int_{-\infty}^{\mu} dp^0\, S_h\, G_3'\,(u_v+d_v)\right\rangle}$$
$$= \frac{\left\langle\frac{\delta}{2V}\frac{\pi^2}{Mk_F}S_h(\mu,\vec{p},k_F)\left[G_3^{\prime 0n}(\mu,\vec{p}) - G_3^{\prime 0p}(\mu,\vec{p})\right] + \frac{\delta}{2V}\frac{\pi^2}{k_F^2}\int_{-\infty}^{\mu} dp^0 \left.\frac{\partial S_h}{\partial k}\right|_{k=k_F}\left[G_3^{\prime 0n}(p^0,\vec{p}) - G_3^{\prime 0p}(p^0,\vec{p})\right]\right\rangle}{2\left\langle\int_{-\infty}^{\mu} dp^0\, S_h\, G_3'\,(u_v+d_v)\right\rangle}$$
$$= \frac{[g_-^2(d)-g_-^2(u)]\left\langle\frac{\delta}{2V}\frac{\pi^2}{Mk_F}\left[S_h\, G_3'(u_v-d_v)\right]\Big|_{p^0=\mu} + \frac{\delta}{2V}\frac{\pi^2}{k_F^2}\int_{-\infty}^{\mu} dp^0 \left.\frac{\partial S_h}{\partial k}\right|_{k=k_F} G_3'(u_v-d_v)\right\rangle}{\left\langle\int_{-\infty}^{\mu} dp^0\, S_h\, G_3'\,(u_v+d_v)\right\rangle}, \tag{87}$$

and the second correction δR_2^- is given by

$$\delta R_2^- = -[g_-^2(u)+g_-^2(d)]\,\frac{O'(\epsilon^{\nu A-\bar{\nu}A})}{4xy\left(1-\frac{y}{2}\right)\left\langle\int_{-\infty}^{\mu} dp^0\, S_h G_3'(u_v+d_v)\right\rangle}$$
$$= -[g_-^2(u)+g_-^2(d)]\,\frac{y^2x\; O\left(\epsilon_1^{\nu A-\bar{\nu}A}\right) + \left\{1-y-\frac{Mxy}{2E_\nu}\right\}O\left(\epsilon_2^{\nu A-\bar{\nu}A}\right)}{4xy\left(1-\frac{y}{2}\right)\left\langle\int_{-\infty}^{\mu} dp^0\, S_h G_3'(u_v+d_v)\right\rangle} = \delta R_{2,1}^- + \delta R_{2,2}^-, \tag{88}$$

where

$$\delta R_{2,1}^- = -[g_-^2(u)+g_-^2(d)]\,\frac{y^2x\; O\left(\epsilon_1^{\nu A-\bar{\nu}A}\right)}{4xy\left(1-\frac{y}{2}\right)\left\langle\int_{-\infty}^{\mu} dp^0\, S_h G_3'(u_v+d_v)\right\rangle}, \tag{89}$$

$$\delta R_{2,2}^{-} = -[g_{-}^{2}(u) + g_{-}^{2}(d)]\,\frac{1 - y - \frac{Mxy}{2E_\nu}}{4xy\left(1 - \frac{y}{2}\right)}\,\frac{O\left(\epsilon_2^{\nu A-\bar\nu A}\right)}{\left\langle \int_{-\infty}^{\mu} dp^0\, S_h G_3'(u_v + d_v)\right\rangle}. \tag{90}$$

Making the substitution of $O(\epsilon_1^{\nu A-\bar\nu A})$ in Eq. (89), we get

$$\begin{aligned}\delta R_{2,1}^{-} = &-[g_{-}^{2}(u) + g_{-}^{2}(d)]\,\frac{y\,2M\left\langle \frac{\delta}{2V}\frac{\pi^2}{Mk_F} S_h(\mu,\vec p,k_F)\left(G_1^{\nu n-\bar\nu n} - G_1^{\nu p-\bar\nu p}\right)_{p^0=\mu}\right\rangle}{4\left(1-\frac{y}{2}\right)\left\langle \int_{-\infty}^{\mu} dp^0\, S_h G_3'(u_v + d_v)\right\rangle}\\ &-[g_{-}^{2}(u) + g_{-}^{2}(d)]\,\frac{y\,2M\left\langle \frac{\delta}{2V}\frac{\pi^2}{k_F^2}\int_{-\infty}^{\mu} dp^0 \left.\frac{\partial S_h}{\partial k}\right|_{k=k_F}\left(G_1^{\nu n-\bar\nu n} - G_1^{\nu p-\bar\nu p}\right)\right\rangle}{4\left(1-\frac{y}{2}\right)\left\langle \int_{-\infty}^{\mu} dp^0\, S_h G_3'(u_v + d_v)\right\rangle}.\end{aligned} \tag{91}$$

In the parton model, we have

$$G_1^{\nu n-\bar\nu n} - G_1^{\nu p-\bar\nu p} = \frac{1}{M}(u_v - d_v)\left(2 + \frac{4p_x^2 x_N}{(p\cdot q)}\right) = \frac{1}{M}(u_v - d_v)\,G_1(p^0,\vec p). \tag{92}$$

With this, we can write

$$\delta R_{2,1}^{-} = -[g_{-}^{2}(u) + g_{-}^{2}(d)]\,\frac{y}{2-y}\,\frac{\left\langle \frac{\delta}{2V}\left[\frac{\pi^2}{Mk_F}\left[S_h G_1(u_v - d_v)\right]\Big|_{p^0=\mu} + \frac{\pi^2}{k_F^2}\int_{-\infty}^{\mu} dp^0 \left.\frac{\partial S_h}{\partial k}\right|_{k=k_F} G_1\,(u_v - d_v)\right]\right\rangle}{\left\langle \int_{-\infty}^{\mu} dp^0\, S_h G_3'(u_v + d_v)\right\rangle}. \tag{93}$$

Similarly, using $G_2^{\nu n-\bar\nu n} - G_2^{\nu p-\bar\nu p} = G_2(p^0,\vec p)\cdot 4x_N(u_v - d_v)$, we may write

$$\delta R_{2,2}^{-} = -[g_{-}^{2}(u) + g_{-}^{2}(d)]\,\frac{1 - y - \frac{Mxy}{2E_\nu}}{xy\left(1-\frac{y}{2}\right)}\,\frac{\left\langle \frac{\delta}{2V}\left[\frac{\pi^2}{Mk_F}\left[S_h G_2\, x_N(u_v - d_v)\right]\Big|_{p^0=\mu} + \frac{\pi^2}{k_F^2}\int_{-\infty}^{\mu} dp^0 \left.\frac{\partial S_h}{\partial k}\right|_{k=k_F} G_2\, x_N(u_v - d_v)\right]\right\rangle}{\left\langle \int_{-\infty}^{\mu} dp^0\, S_h G_3'(u_v + d_v)\right\rangle}. \tag{94}$$

We must point out that when $p^0 = \mu$, $S_h(\mu,\vec p,k_F)$ is zero [38,40] and the imaginary part of the self-energy of the nucleon is also zero, and this has been used in the definition (numerator) of the hole spectral function while performing the numerical calculations.

III. RESULTS AND DISCUSSION

In this section we present and discuss the results of our numerical calculations. For performing the numerical calculations we have used the expressions for the weak structure functions F_2^A and F_3^A for an isoscalar target [10]:

$$F_2^A(x_A,Q^2) = 4\int d^3r\int \frac{d^3p}{(2\pi)^3}\frac{M}{E(\mathbf p)}\int_{-\infty}^{\mu} dp^0\; S_h(p^0,\mathbf p,\rho(\mathbf r))\frac{x}{x_N}\left(1 + \frac{2x_N p_x^2}{M\nu_N}\right)F_2^N(x_N,Q^2), \tag{95}$$

$$F_3^A(x_A,Q^2) = 4\int d^3r\int \frac{d^3p}{(2\pi)^3}\frac{M}{E(\mathbf p)}\int_{-\infty}^{\mu} dp^0 S_h(p^0,\mathbf p,\rho(\mathbf r))\frac{p^0\gamma - p_z}{(p^0 - p_z\gamma)\gamma}F_3^N(x_N,Q^2) \tag{96}$$

and for a non-isoscalar target, the expressions for $F_2^A(x)$ and $F_3^A(x)$ are given by Ref. [11]

$$\begin{aligned}F_2^A(x_A,Q^2) = 2\int d^3r\int \frac{d^3p}{(2\pi)^3}\frac{M}{E(\mathbf p)}\bigg[&\int_{-\infty}^{\mu_p} dp^0\; S_h^{\text{proton}}(p^0,\mathbf p,k_{F,p})F_2^{\text{proton}}(x_N,Q^2)\\ &+\int_{-\infty}^{\mu_n} dp^0\; S_h^{\text{neutron}}(p^0,\mathbf p,k_{F,n})F_2^{\text{neutron}}(x_N,Q^2)\bigg]\frac{x}{x_N}\left(1 + \frac{2x_N p_x^2}{M\nu_N}\right),\end{aligned} \tag{97}$$

$$\begin{aligned}F_3^A(x_A,Q^2) = 2\int d^3r\int \frac{d^3p}{(2\pi)^3}\frac{M}{E(\mathbf p)}\bigg[&\int_{-\infty}^{\mu_p} dp^0\; S_h^{\text{proton}}(p^0,\mathbf p,k_{F,p})F_3^{\text{proton}}(x_N,Q^2)\\ &+\int_{-\infty}^{\mu_n} dp^0\; S_h^{\text{neutron}}(p^0,\mathbf p,k_{F,n})F_3^{\text{neutron}}(x_N,Q^2)\bigg]\frac{p^0\gamma - p_z}{(p^0 - p_z\gamma)\gamma},\end{aligned} \tag{98}$$

where

$$\gamma = \frac{q_z}{q^0} = \left(1 + \frac{4M^2x^2}{Q^2}\right)^{1/2}, \quad x_N = \frac{Q^2}{2(p^0q^0 - p_z q_z)}. \tag{99}$$

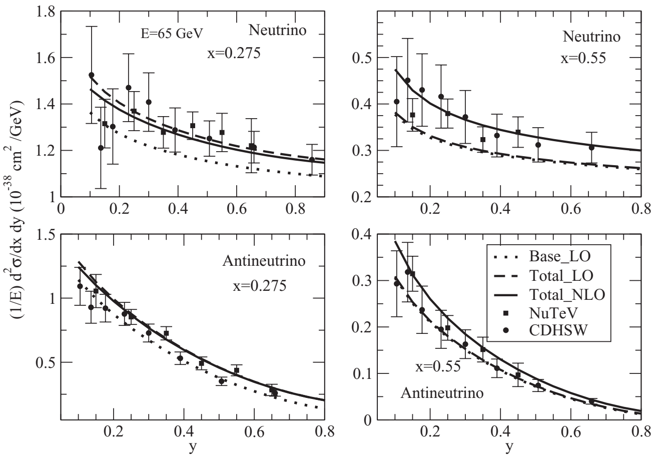

FIG. 1. $\frac{1}{E}\frac{d^2\sigma}{dxdy}$ vs y at different x for charged-current-induced $\nu_\mu(\bar{\nu}_\mu)$ ($E_{\nu_\mu} = 65$ GeV) reactions in ^{56}Fe. The dotted lines are the base results; for numerical calculations CTEQ [46] PDFs at leading order have been used. The dashed lines are the full model at leading order. The solid lines are the full calculation at next to leading order. The experimental points are from CDHSW [14] (solid circles) and NuTeV [15] (solid squares) experiments.

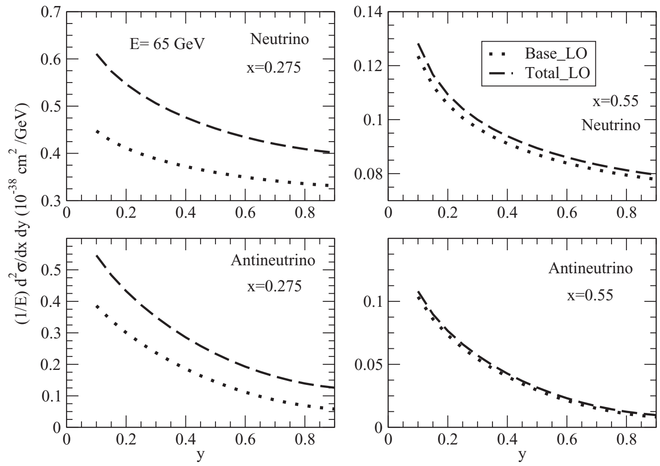

FIG. 2. $\frac{1}{E}\frac{d^2\sigma}{dxdy}$ vs y at different x for neutral-current-induced $\nu_\mu(\bar{\nu}_\mu)$ ($E_{\nu_\mu} = 65$ GeV) reactions in ^{56}Fe. Lines have the same meaning as in Fig. 1.

Here $F_{2,3}^{\text{proton}}$ and $F_{2,3}^{\text{neutron}}$ are the dimensionless structure functions for the free proton and the free neutron, respectively. These structure functions are calculated with the target mass correction (TMC) [43] and CTEQ6.6 PDFs at leading order (LO) [46]. Fermi motion and nucleon binding are implemented through the use of a nucleon spectral function. This is our base result. We also include pion and rho cloud contributions in F_2^A following the model of Ref. [40] and shadowing corrections in F_2^A and F_3^A [44], which is our full calculation (Total). Using them we have obtained the charged current differential scattering cross sections, the expression for which is given by Eq. (25). For the neutral current the expression would remain the same with the changes given in Eq. (26). Using these cross sections, we evaluate the ratio

$$R^- = \frac{\frac{d^2\sigma(\nu_\mu N \to \nu_\mu X)}{dxdy} - \frac{d^2\sigma(\bar{\nu}_\mu N \to \bar{\nu}_\mu X)}{dxdy}}{\frac{d^2\sigma(\nu_\mu N \to \mu^- X)}{dxdy} - \frac{d^2\sigma(\bar{\nu}_\mu N \to \mu^+ X)}{dxdy}}. \tag{100}$$

Results for the charged-current-induced (anti)neutrino-iron differential cross section have been discussed in detail in Ref. [10], where the comparisons have been made with the experimental data of NuTeV and CDHSW, which are corrected for an isoscalar iron target. However, for completeness we are showing here the results for the charged- and neutral-current-induced (anti)neutrino-iron differential cross sections in Figs. 1 and 2, respectively. In the case of neutral current we have performed numerical calculations at LO. We observe that medium effects are important in the study of differential scattering cross sections. For example, when the calculations are performed for the charged-current neutrino-induced process at LO, medium effects such as Pauli blocking, Fermi motion, and nucleon correlations which are taken into account for our base calculations, result in the reduction of the differential cross section, which changes by 3%–4% at low y and increases to 10%–11% at high values of y for the studied region of x from the free case. When pion and rho cloud contributions as well as the shadowing effect are also taken into account there is a further change of about 8%–9% at low y and this change decreases with the increase in y for $x = 0.2$–0.3. This difference becomes smaller with the increase in the value of x; for example, at $x = 0.5$–0.6, it becomes less than 1% for all values of y. Similarly, in the case of the antineutrino-induced charged-current process the change in the base resultings from the free nucleon scattering process is around 10%–12% at $x = 0.2$–0.3 for the studied region of y, which gets further modulated by 12% at low values of y and significantly increases with the increase in the value of y when pion and rho cloud contributions and shadowing effects are incorporated. At higher values of x, the difference in the results obtained using the base calculation and the full calculation is negligibly small and the difference between the base results and the results for the free case is the same as in the case of neutrinos. Furthermore, calculations performed at next to LO leads to better results. In the case of neutral current we have performed calculations at LO and observe that the difference between the base results and the results for the free case (not shown here) is 10%–12% at $x = 0.2$–0.3 for neutrino- as well as antineutrino-induced processes. When pion and rho cloud contributions and shadowing effects are taken into account, results from the base change to 17%–25% for the neutrino case and for the antineutrino-induced process the change is 30%–40% at $x = 0.2$–0.3 for all values of y. At higher x, when Fermi motion, Pauli blocking, and nucleon correlation are taken into account, the results change by 4%–10% from the free results at all values of y for both neutrino- as well as antineutrino-induced processes. When pion and rho cloud contributions as well as shadowing effects are also considered, there is a further reduction of 2%–3% for the neutrino-induced process and 4%–9% for the antineutrino case for all values of y. Therefore, we observe that in the study of charged current and neutral current differential scattering cross sections medium effects are important.

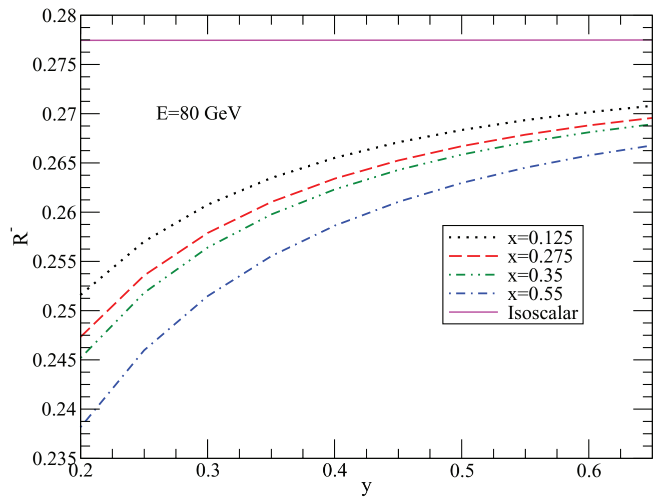

FIG. 3. (Color online) Paschos and Wolfenstein ratio R^- in ^{56}Fe for the non-isoscalar case. R^- is calculated for (anti)neutrino energy (E) of 80 GeV and at different Bjorken x. The solid line is the result when one treats ^{56}Fe as an isoscalar target.

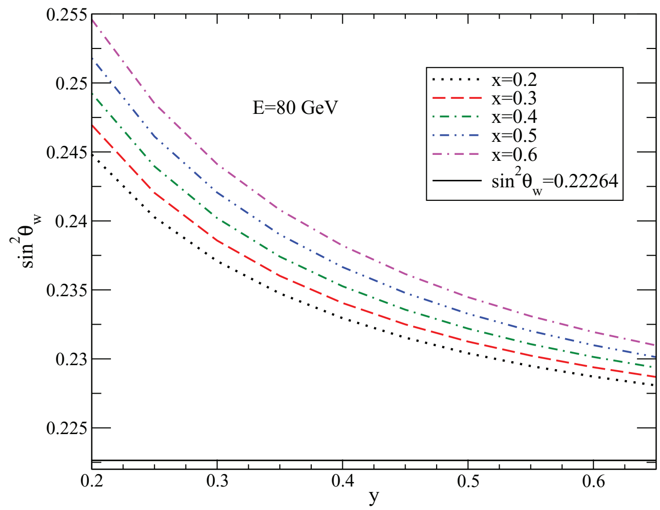

FIG. 5. (Color online) $\sin^2\theta_W$ vs y at different values of x in ^{56}Fe by treating it as a non-isoscalar nuclear target for (anti)neutrino energy (E) of 80 GeV.

We show the PW ratio (R^-), given by Eq. (100), in Fig. 3 using the numerical values from charged and neutral current differential scattering cross sections in iron, treating it to be isoscalar as well as non-isoscalar nuclear targets, at different values of x for a fixed value of neutrino or antineutrino energy $E_{\nu(\bar{\nu})} = 80$ GeV. We have incorporated Fermi motion, Pauli blocking, and nucleon correlation while calculating R^-. We find that R^- is almost independent of x and y for an isoscalar target, while for the non-isoscalar target there is x as well as y dependence. We find that the non-isoscalarity and medium effect increase with the increase in the value of x; for example, in the mid region of x it is about 14%–15% for low values of y and 3%–4% at higher values of y. Therefore, for non-isoscalar targets such as iron, medium effects and an excess of neutrons strongly affect R^- and hence $\sin^2\theta_W$, while for an isoscalar target such as carbon (not shown here) medium effects cancel out and the extracted value of $\sin^2\theta_W$ from the PW ratio is in complete agreement with the global value. When the contributions of pion and rho clouds are also taken into account, we find that these changes do not bring any difference in the results of the PW ratio as obtained with our base calculations. Since shadowing is a low-x phenomenon, we have not considered it in our present study of the PW ratio [8]. To see the effect of non-isoscalarity in the iron target, we have plotted δR^- using Eq. (86) at $E_{\nu(\bar{\nu})} = 80$ GeV in Fig. 4. We find that the effect of non-isoscalarity is large at low y and high values of x; it decreases with an increase in the value of y and this effect is smaller at low values of x. Hence there is a non-isoscalarity dependence in the determination of $\sin^2\theta_W$. Using the results of R^- from Eq. (100) and δR^- from Eq. (86), we have obtained $\sin^2\theta_W$ using Eq. (85) and presented the results in Fig. 5. The above calculated value of the weak mixing angle is now corrected for the isoscalar target where we should also keep in mind that medium effects are still present. We find that due to medium effects $\sin^2\theta_W$ is different from the global fit, and this difference is ≈7% when evaluated for low values of y at $x = 0.2$ and this decreases to 1% at high values of y, while this change is ≈9% when calculated for low y at $x = 0.6$ and this reduces to 2% at high values of y. Thus we observe that both non-isoscalarity and medium effects such as Pauli blocking, binding energy, and Fermi motion are important while extracting $\sin^2\theta_W$. To see the effect of neutrino energy E_ν and Q^2 dependence on $\sin^2\theta_W$, we have plotted in Fig. 6 $\sin^2\theta_W$ as a function of x at various values of E_ν and Q^2. We observe that at $E_\nu =$ 80 GeV and $Q^2 = 25$ GeV^2 it is almost close to the standard value at low values of x, and the value of $\sin^2\theta_W$ changes significantly with $E_{\nu(\bar{\nu})}$, Q^2, and x. Therefore, when one looks at the NuTeV results it is also important to know at what values of x, E_ν and Q^2 the analysis was performed as a wide range of these variables could change considerably the value of $\sin^2\theta_W$.

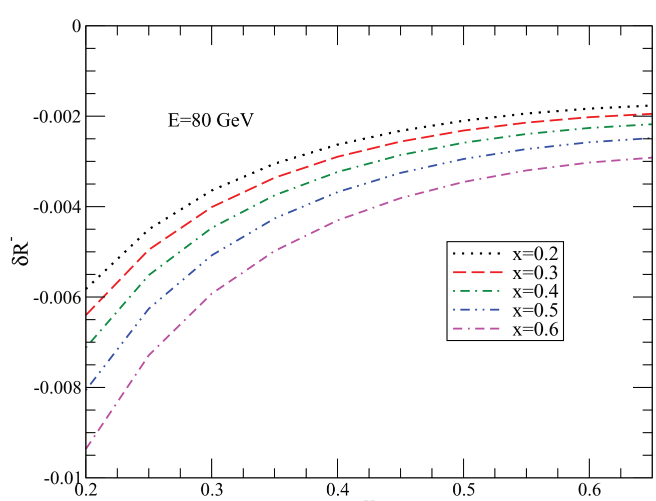

FIG. 4. (Color online) Non-isoscalar correction (δR^-) vs y at different values of x for (anti)neutrino energy (E) of 80 GeV.

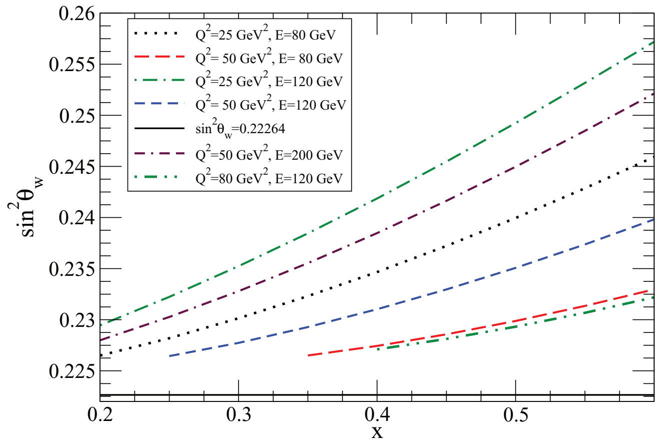

FIG. 6. (Color online) $\sin^2\theta_W$ vs x in ^{56}Fe by treating it as a non-isoscalar nuclear target. The results are shown at different values of Q^2 and (anti)neutrino energy E.

IV. CONCLUSIONS

To summarize our results, in this work we have studied the effects of the nuclear medium on the structure functions $F_2^A(x, Q^2)$ and $F_3^A(x, Q^2)$ in the iron nucleus by treating it to be an isoscalar nuclear target and then we have made non-isoscalarity corrections. We have used many-body theory to describe the spectral function of a nucleon in an infinite nuclear medium. The spectral function takes into account the Fermi motion and the binding energy of the nucleons. The spectral function also includes nucleon correlations. Then to apply it to the case of finite nucleus we have used the local density approximation. Target mass corrections have also been considered. We have used CTEQ [46] PDFs in the numerical evaluation. We have taken into consideration the effects of mesonic degrees of freedom, of shadowing and antishadowing for the calculation of F_2^A, and for F_3^A shadowing and antishadowing effects only.

With these structure functions, we have evaluated numerically the differential scattering cross sections for the charged- and neutral-current-induced (anti)neutrino interactions on the iron target. These differential scattering cross sections are then used to study the nuclear medium effects and non-isoscalarity correction in the extraction of weak mixing angle $\sin^2\theta_W$ using the Paschos-Wolfenstein relation.

Beginning with a model for nonsymmetric nuclear matter, we have expanded the nuclear hadron tensor in an isoscalar part plus nuclear corrections which are, roughly speaking, proportional to the difference between the neutron and the proton density profiles. We have performed this expansion for all the nuclear structure functions which appear in the Paschos-Wolfenstein ratio for differential cross sections and obtained the pure isoscalar Paschos-Wolfenstein result plus nuclear corrections which depend on the phase-space Bjorken variables.

We have also observed that the Paschos-Wolfenstein ratio for isoscalar nuclear structure functions is almost independent of x and y, as it should be. However, when one considers the model for nonsymmetric nuclei, such as ^{56}Fe, one obtains a PW ratio which depends on the phase-space variables. Furthermore, we have evaluated the first-order *nuclear correction* to the isoscalar PW ratio and found it to be important for a wide range of phase space. Whether this *nuclear correction* can explain the NuTeV anomaly is a question that could be answered if one could weight our results for δR^- by the relative amount of events in every x and y bin, under the assumption of assigning all the correction to $\sin^2\theta_W$.

Another important point that may be taken into account is the fact that we have worked in the up and down quark approximation. Therefore, we have neglected the contribution coming from heavier flavors. We leave this subject for future studies.

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[1] D. Schmitz, AIP Conf. Proc. **1405**, 243 (2011); http://minerva.fnal.gov/.

[2] J. Mousseau, talk presented at the XX International Workshop on Deep-Inelastic Scattering and Related Subjects Bonn, Germany, March, 2012.

[3] K. S. McFarland (private communication).

[4] T. Adams *et al.* (NuSOnG Collaboration), Int. J. Mod. Phys. A **24**, 671 (2009); http://www-nusong.fnal.gov.

[5] E. A. Paschos and L. Wolfenstein, Phys. Rev. D **7**, 91 (1973).

[6] G. P. Zeller, Ph.D. thesis, FERMILAB-THESIS-2002-34, UMI-30-50615, 2002.

[7] G. P. Zeller *et al.* (NuTeV Collaboration), Phys. Rev. Lett. **88**, 091802 (2002); **90**, 239902(E) (2003).

[8] K. S. McFarland *et al.*, Nucl. Phys. Proc. Suppl. **112**, 226 (2002).

[9] M. Sajjad Athar, I. Ruiz Simo, and M. J. Vicente Vacas, Nucl. Phys. A **857**, 29 (2011).

[10] H. Haider, I. Ruiz Simo, M. Sajjad Athar, and M. J. Vicente Vacas, Phys. Rev. C **84**, 054610 (2011).

[11] H. Haider, I. Ruiz Simo, and M. Sajjad Athar, Phys. Rev. C **85**, 055201 (2012).

[12] J. Seely *et al.*, Phys. Rev. Lett. **103**, 202301 (2009).

[13] J. Gomez *et al.*, Phys. Rev. D **49**, 4348 (1994).

[14] J. P. Berge *et al.*, Z. Phys. C **49**, 187 (1991).

[15] M. Tzanov *et al.*, Phys. Rev. D **74**, 012008 (2006).

[16] G. Onengut *et al.*, Phys. Lett. B **632**, 65 (2006).

[17] S. M. Bilenky, *Basics of Introduction to Feynman Diagrams and Electroweak Interactions Physics* (Editions Frontières, Gif-sur-Yvette, France, 1994).
[18] Kevin S. McFarland and Sven-Olaf Moch, arXiv:hep-ph/0306052.
[19] D. Abbaneo *et al*. (ALEPH Collaboration and DELPHI Collaboration and L3 Collaboration and OPAL Collaboration and LEP Electroweak Working Group and SLD Heavy Flavor and Electroweak Groups), arXiv:hep-ex/0112021.
[20] S. Davidson, S. Forte, P. Gambino, N. Rius, and A. Strumia, J. High Energy Phys. 02 (2002) 037.
[21] S. Kumano, Phys. Rev. D **66**, 111301 (2002).
[22] J. T. Londergan and A. W. Thomas, Phys. Rev. D **67**, 111901 (2003).
[23] A. Kurylov *et al*., Nucl. Phys. B **667**, 321 (2003).
[24] S. Kretzer, F. Olness, J. Pumplin, D. Stump, W. K. Tung, and M. H. Reno, Phys. Rev. Lett. **93**, 041802 (2004).
[25] S. A. Kulagin, Phys. Rev. D **67**, 091301 (2003).
[26] M. Hirai, S. Kumano, and T. H. Nagai, Phys. Rev. D **71**, 113007 (2005).
[27] A. D. Martin *et al*., Eur. Phys. J. C **35**, 325 (2004).
[28] G. A. Miller and A. W. Thomas, Int. J. Mod. Phys. A **20**, 95 (2005).
[29] A. D. Martin *et al*., Eur. Phys. J. C **39**, 155 (2005).
[30] M. Glück, P. Jimenez-Delgado, and E. Reya, Phys. Rev. Lett. **95**, 022002 (2005).
[31] K. J. Eskola and H. Paukkunen, J. High Energy Phys. 06 (2006) 008.
[32] D. Mason *et al*., Phys. Rev. Lett. **99**, 192001 (2007).
[33] R. D. Ball *et al*., Nucl. Phys. B **823**, 195 (2009).
[34] I. C. Cloet, W. Bentz, and A. W. Thomas, Phys. Rev. Lett. **102**, 252301 (2009).
[35] W. Bentz, I. C. Cloet, J. T. Londergan, and A. W. Thomas, Phys. Lett. B **693**, 462 (2010).
[36] A. W. Thomas, AIP Conf. Proc. **1418**, 147 (2011).
[37] G. P. Zeller *et al*., arXiv:hep-ex/0207052.
[38] P. Fernandez de Cordoba and E. Oset, Phys. Rev. C **46**, 1697 (1992).
[39] P. Fernandez de Cordoba, E. Marco, H. Muther, E. Oset, and A. Faessler, Nucl. Phys. A **611**, 514 (1996).
[40] E. Marco, E. Oset, and P. Fernandez de Cordoba, Nucl. Phys. A **611**, 484 (1996).
[41] M. Sajjad Athar, S. K. Singh, and M. J. Vicente Vacas, Phys. Lett. B **668**, 133 (2008).
[42] C. Garcia-Recio, J. Nieves, and E. Oset, Phys. Rev. C **51**, 237 (1995).
[43] I. Schienbein *et al*., J. Phys. G **35**, 053101 (2008).
[44] S. A. Kulagin and R. Petti, Nucl. Phys. A **765**, 126 (2006).
[45] S. A. Kulagin and R. Petti, Phys. Rev. D **76**, 094023 (2007).
[46] P. M. Nadolsky *et al*., Phys. Rev. D **78**, 013004 (2008); http://hep.pa.msu.edu/cteq/public.
[47] H. de Vries, C. W. de Jager, and C. de Vries, At. Data Nucl. Data Tables **36**, 495 (1987).