Symmetry energy and nucleon-nucleon cross sections

M. Veselský[1,*] and Y. G. Ma[2,†]
[1]*Institute of Physics, Slovak Academy of Sciences, Dúbravská cesta 9, 84511 Bratislava, Slovakia*
[2]*Shanghai Institute of Applied Physics, Chinese Academy of Sciences, 2019 Jia-Luo Road, Post Office Box 800-204, Shanghai 201800, China*
(Received 3 December 2012; published 12 March 2013)

The extension of the Boltzmann-Uehling-Uhlenbeck model of nucleus-nucleus collision is presented. The isospin-dependent nucleon-nucleon cross sections are estimated using the proper volume extracted from the equation of state of the nuclear matter transformed into the form of the Van der Waals equation of state. The results of such simulations demonstrate the dependence on symmetry energy, which typically varies strongly from the results obtained using only the isospin-dependent mean field. The evolution of the n/p multiplicity ratio with angle and kinetic energy, in combination with the elliptic flow of neutrons and protons, provides a suitable set of observables for determination of the density dependence of the symmetry energy. The model thus provides an environment for testing of equations of state that are used for various applications in nuclear physics and astrophysics.

DOI: 10.1103/PhysRevC.87.034615 PACS number(s): 24.10.Lx, 21.65.Ef, 25.75.Ld

I. INTRODUCTION

One of the main goals of intermediate-energy heavy-ion collisions (HICs) is to study properties of nuclear matter, especially to determine the nuclear equation of state (EoS). HICs provide a unique possibility to compress nuclear matter to a hot and dense phase within a laboratory environment. The pressures that result from the high densities achieved during such collisions strongly influence the motion of ejected matter and are sensitive to the EoS. Over the past three decades, the EoS of symmetric nuclear matter was studied in detail with the studies of giant dipole resonances, collective flow, and multifragmentation [1–4]. The EoS of isospin asymmetric nuclear matter is under way, particularly for the density dependence of symmetry energy. Considerable progress has been made in determining the sub- and suprasaturation density behavior of the symmetry energy [5–12]. The latter part is still an unanswered question in spite of recent findings in terms of neutron-proton elliptic flow ratio and difference [10,11]. However, the former one is understood to some extent [5–8], although more efforts are needed for precise measurements.

The transport model is very useful to treat HIC dynamics and obtain important information of nuclear matter EoSs as well as the symmetry energy. In intermediate-energy HICs, the Boltzmann-Uehling-Uhlenbeck (BUU) model is an extensively used tool [13,14], which takes both Pauli blocking and the mean field into consideration. The BUU equation reads

$$\begin{aligned}\frac{\partial f}{\partial t} &+ v\ \nabla_r f - \nabla_r U\ \nabla_p f \\ &= \frac{4}{(2\pi)^3}\int d^3p_2 d^3p_3 d\Omega \frac{d\sigma_{NN}}{d\Omega} v_{12} \times [f_3 f_4(1-f) \\ &\quad \times (1-f_2) - f f_2(1-f_3)(1-f_4)]\delta^3(p+p_2-p_3-p_4),\end{aligned} \tag{1}$$

where $f = f(r, p, t)$ is the phase-space distribution function. It is solved with the test particle method of Wong [15], with the collision term as introduced by Cugnon *et al.* [16]. In Eq. (1), $\frac{d\sigma_{NN}}{d\Omega}$ and v_{12} are the in-medium nucleon-nucleon cross section and relative velocity for the colliding nucleons, respectively, and U is the single-particle mean field potential with the addition of the isospin-dependent symmetry energy term:

$$U = a\rho + b\rho^{\kappa} + 2a_s\left(\frac{\rho}{\rho_0}\right)^{\gamma}\tau_z I, \tag{2}$$

where $I = (\rho_n - \rho_p)/\rho$; ρ_0 is the normal nuclear matter density; ρ, ρ_n, and ρ_p are the nucleon, neutron, and proton densities, respectively; τ_z assumes value 1 for neutron and -1 for proton; coefficients a, b, and κ represent properties of the symmetric nuclear matter; and the last term, which describes the influence of the symmetry energy, can be obtained, e.g., from the simple Weizsacker formula, where a_s represents the coefficient of the symmetry energy term and γ is the exponent describing the density dependence. Typical sets of mean field parameters cover a substantial range between the soft EoS with the compressibility K of 200 MeV ($\kappa = 7/6, a\rho_0 = -356$ MeV, $b\rho_0^{\kappa} = 303$ MeV), and the hard EoS with K of 380 MeV ($\kappa = 2$, $a\rho_0 = -124$ MeV, $b\rho_0^{\kappa} = 70.5$ MeV) [13].

It is the aim of the present work to estimate the effect of the symmetry energy parametrization within the equation of the state on the crucial component of the transport simulations, namely the in-medium nucleon-nucleon cross section.

II. ISOSPIN-DEPENDENT NUCLEON-NUCLEON CROSS SECTIONS

When considering influence of the symmetry energy on emission rates of nucleons in nucleus-nucleus collisions, one needs to understand whether and how the medium represented by the equation of state can influence relative probabilities of emission of protons and neutrons. Theoretical investigations of the density dependence of in-medium nucleon-nucleon cross

*martin.veselsky@savba.sk
†ygma@sinap.ac.cn

section were carried out for symmetric nuclear matter [17,18], and significant influence of nuclear density on resulting in-medium cross sections was observed in their density, angular, and energy dependencies. Using momentum-dependent interaction, ratios of in-medium to free nucleon-nucleon cross sections were evaluated via reduced nucleonic masses [19] and used for transport simulations. Still, transport simulations are mostly performed using parametrizations of the free nucleon-nucleon cross sections, eventually scaling them down empirically or using simple prescriptions for density dependence of the scaling factor [20]. In the present work, a prescription for estimation of the density dependence of the in-medium nucleon-nucleon cross sections corresponding to the specific form of phenomenological nuclear equation of state will be presented. The possibility to establish a simple dependence of nucleon-nucleon cross sections on density, temperature, and symmetry energy is potentially important for a wide range of problems in nuclear physics and astrophysics.

A. Equation of state of nucleonic matter

Based on the single-particle potential, shown in Eq. (2), one can construct the corresponding equation of the state. Change of the pressure in the thermodynamical equation of state, which is also a measure of nonideality of a neutron or a proton gas, can be evaluated as

$$\Delta p_{\mathrm{nonideal}} = -\frac{d\mathcal{U}}{dV}|_{T=\mathrm{const}} \tag{3}$$

where $\mathcal{U}$ is the thermodynamic potential, V is the volume, and T is the temperature. When evaluating the thermodynamic potential $\mathcal{U}$ as a sum of single-particle contributions of neutrons and protons, given by Eq. (2), one arrives at the expression

$$\begin{aligned} p = {} & x_n\left[\frac{f_{5/2}(z_n)}{f_{3/2}(z_n)}\rho T + a\rho^2 + b\kappa\rho^{1+\kappa} + 2\gamma a_s\rho_0\left(\frac{\rho}{\rho_0}\right)^{1+\gamma} I\right] \\ & + x_p\left[\frac{f_{5/2}(z_p)}{f_{3/2}(z_p)}\rho T + a\rho^2 + b\kappa\rho^{1+\kappa} \right. \\ & \left. - 2\gamma a_s\rho_0\left(\frac{\rho}{\rho_0}\right)^{1+\gamma} I\right], \end{aligned} \tag{4}$$

where $x_n = \rho_n/\rho$ and $x_p = \rho_p/\rho$ are neutron and proton concentrations and $\frac{f_{5/2}(z)}{f_{3/2}(z)}$ is the factor, a fraction of the Fermi integrals $f_n(z)$, assuring that Fermi statistics are taken into account. The parameters $z_n = \mu_n/T$ and $z_p = \mu_p/T$ are the values of fugacity of neutrons and protons, with μ_n and μ_p being the neutron and proton chemical potentials, respectively. This expression appears to provide separate terms for the pressures of neutrons and protons, which, however, can be combined to obtain the typical quadratic dependence on isospin asymmetry I. The resulting pressure is the weighted average between two terms, which can be, in a similar manner to Eq. (2), summarily expressed as

$$p = \left\langle\frac{f_{5/2}(z)}{f_{3/2}(z)}\right\rangle\rho T + a\rho^2 + b\kappa\rho^{1+\kappa} + 2\gamma a_s\rho_0\left(\frac{\rho}{\rho_0}\right)^{1+\gamma}\tau_z I. \tag{5}$$

These terms can be interpreted as the equation of state of the system of particles with the corresponding single-particle potential, given by Eq. (2).

B. Proper volume in the Van der Waals equation of state

In order to find the relation between the equation of state and nucleon emission rates, one can turn to the Van der Waals equation of state. It can be written, using particle density ρ, as

$$(p + a'\rho^2)(1 - \rho b') = \left\langle\frac{f_{5/2}(z)}{f_{3/2}(z)}\right\rangle\rho T, \tag{6}$$

where the parameter a' is related to attractive interaction among particles and b' represents the proper volume of the constituent particles. In a geometrical picture the proper volume of the particle can be directly related to its cross section for interaction with other particles. It is possible to formally transform the above equations of state for neutrons and protons (5) (and practically any other equation of state) into the form analogous to the Van der Waals equation. Then, by comparison, one obtains the following values of coefficients:

$$a' = -a, \tag{7}$$

and

$$\begin{aligned} b' &= \frac{b\kappa\rho^{\kappa} + 2\gamma a_s\left(\frac{\rho}{\rho_0}\right)^{\gamma}\tau_z I}{p - a\rho^2} \\ &= \frac{b\kappa\rho^{\kappa} + 2\gamma a_s\left(\frac{\rho}{\rho_0}\right)^{\gamma}\tau_z I}{\left\langle\frac{f_{5/2}(z)}{f_{3/2}(z)}\right\rangle\rho T + b\kappa\rho^{1+\kappa} + 2\gamma\rho_0 a_s\left(\frac{\rho}{\rho_0}\right)^{1+\gamma}\tau_z I}, \end{aligned} \tag{8}$$

where the latter provides a measure of the proper volume of the constituents, nucleons in this case, as a measure of deviation from the behavior of the ideal gas. The proper volume of the nucleon can be used to estimate its cross section within the nucleonic medium

$$\sigma = \left(\frac{9\pi}{16}\right)^{1/3} b'^{2/3}, \tag{9}$$

which can be implemented into the collision term of the Boltzmann equation.

Concerning the physical meaning of this procedure, for each point in the ρ-T plane the Van der Waals equation of state is found, which behaves identically to the nuclear equation of state (5) in the vicinity of that point. Thus the dynamics of the system can be described using the two parameters of the Van der Waals EoS, of which one provides a measure of the effective volume of the constituent at a given density and temperature. The variation of the constituent volume reflects the interplay of the long-range attractive interaction, leading to its apparent increase, with the short-range repulsive interaction, leading to its apparent decrease.

The trend of the estimated values of nucleus-nucleus cross sections, obtained using the soft parametrization of symmetry energy with $\rho^{2/3}$ dependence, is shown in Fig. 1 for the temperature 20 MeV. The extracted values of the cross sections are surprisingly close to the expected value of geometric cross sections. With increasing isoscalar density the values of cross sections initially grow until they reach maximum

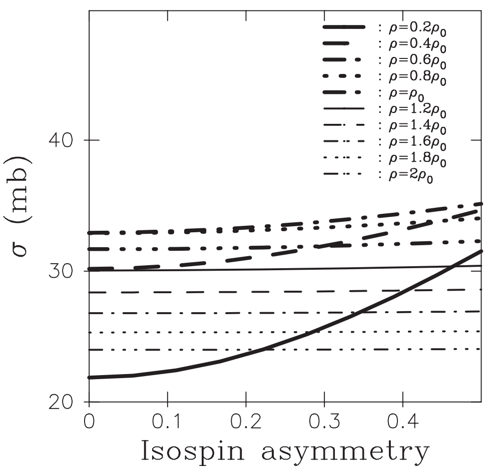

FIG. 1. Extracted isospin dependencies of nucleon-nucleon cross sections for temperature 20 MeV and various densities.

values in the region around half of saturation density and then monotonously decrease. The increase at low densities possibly describes a gradual deviation from the equation of state of ideal gas due to increasing attractive potential, while at higher densities the decreasing trend may indeed represent the properties of short-range repulsive interaction. At high densities, with increasing isoscalar density, the sensitivity to symmetry energy tends to decrease and around the density $2\rho_0$ it is practically lost, which, however, can be preserved using harder parametrizations of the symmetry energy.

It is worth mentioning that a similar rise and fall of nucleon-nucleon cross sections was observed by the Alm *et al.* [18] and explained as a precursor effect of superfluid phase transition. Also in the present case this effect can be related to the phase transition in the nuclear matter, since it is caused by the same interplay of attractive and repulsive interactions which also influence the proper volume and thus extracted cross sections.

C. Implementation into the Boltzmann equation

While the Boltzmann equation (1) is formulated in terms of density, it does not explicitly consider temperature. Therefore temperature T needs to be estimated independently. It is possible to estimate temperature using the Maxwellian momentum distribution of nucleons

$$f(\vec{p}) = \frac{1}{(2\pi m T)^{3/2}} e^{-\frac{p_x^2+p_y^2+p_z^2}{2mT}}, \tag{10}$$

where m is the nucleon mass. Using this formula, local temperature can be estimated from momentum distribution in the c.m. frame by evaluating the momentum variance. At early stages of collision this can be done primarily for transverse momentum since it provides a measure of mutual thermalization of particles from the projectile and target, which proceeds by distant elastic collisions generating the transverse momentum. More violent collisions would lead to emission of colliding nucleons and thus would not contribute to thermalization of the source. This temperature estimate can be done without requiring stopping and formation of the source

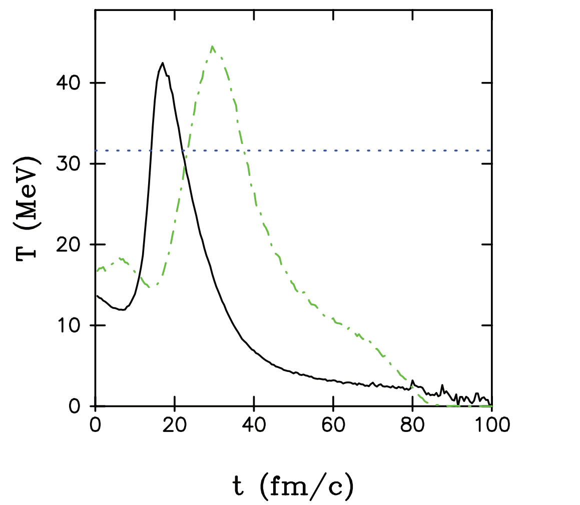

FIG. 2. (Color online) Evolution of average temperature of the fireball (solid line) with the time. The total volume of the fireball is shown on an arbitrary scale as the dash-dotted line. The results were obtained using the BUU in reaction $^{48}Ca + ^{48}Ca$ at 400 AMeV using the impact parameter 1 fm. Dotted straight line shows the fireball temperature estimate for a given beam energy obtained from the systematics of pre-equilibrium spectra [21].

equilibrated in all three dimensions; a closer analog would be the friction of two dilute gas clouds passing through each other.

Evolution of average temperature of the fireball with the time is shown in Fig. 2. The results were obtained using the BUU equation [13,14] in reaction $^{48}Ca + ^{48}Ca$ at 400 AMeV using the impact parameter 1 fm. Temperature was evaluated for each time step in the cubic cells with sides of 1 fm. Average temperature was determined as a mean value of temperature over all cells where number of nucleons was sufficient (corresponding to density of $\rho_0/10$) and temperature thus could be evaluated. The total volume of this fireball is shown in Fig. 2 in an arbitrary scale as a dash-dotted line. The dotted line shows the estimate of fireball temperature for a given beam energy obtained from the systematics of pre-equilibrium spectra [21]. One can see that the average temperature over the fireball at its peak value exceeds the estimate from the systematics, while the value averaged over the lifetime of the hot fireball (between 10 and 30 fm/c) appears to correspond to the value from the systematics. Thus it appears that the procedure introduced here leads to reasonable estimate of local temperature.

Since the temperature, determined using the assumption of Maxwellian distribution, represents the classical Boltzmann statistics, it can be corrected in order to reflect the Fermi statistics, which fermions like nucleons obey. To achieve this, one needs to multiply the classical temperature T_{Boltz}, corresponding to the Boltzmann statistics, by a factor $\langle \frac{f_{5/2}(z)}{f_{3/2}(z)} \rangle^{-1}$ and thus the formula (8) will turn into

$$b' = \frac{b\kappa\rho^{\kappa} + 2\gamma a_s \left(\frac{\rho}{\rho_0}\right)^{\gamma} \tau_z I}{\rho T_{\mathrm{Boltz}} + b\kappa\rho^{1+\kappa} + 2\gamma\rho_0 a_s \left(\frac{\rho}{\rho_0}\right)^{1+\gamma} \tau_z I}, \tag{11}$$

which corresponds to classical case of Boltzmann statistics. Thus, remarkably, this classical expression can be used also

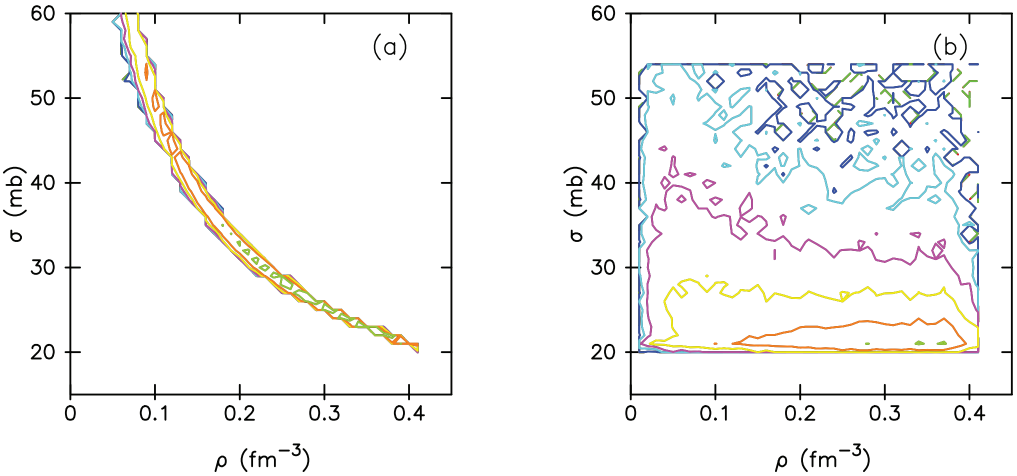

FIG. 3. (Color online) Comparison of the nucleon-nucleon cross sections in two variants of the BUU calculations. On the left panel are the isospin-dependent nucleon-nucleon cross sections, obtained as the proper volume of the Van der Waals form of the equation of state, as a function of density, while on the right panel are shown the corresponding nucleon-nucleon cross sections, obtained using standard energy-dependent parametrization, used in BUU calculations. The results were obtained using the BUU in reaction $^{48}Ca + ^{48}Ca$ at 400 AMeV using the impact parameter 1 fm.

for Fermionic (or even bosonic) particles, obeying their corresponding statistics. From a practical point of view, in this way the nontrivial determination of the Fermionic temperature, depending of fugacity, can be avoided.

Once the local temperature is determined, it is possible to implement the isospin-dependent nucleon-nucleon cross section, obtained using the formulas (11) and (9), to the reaction simulation, which can be used for determination of collision rate. The results for solution of such fully isospin-dependent version of the Boltzmann-Uehling-Uhlenbeck equation are presented in the next section.

III. REACTION SIMULATIONS

The behavior of the in-medium nucleon-nucleon cross sections was investigated using the already mentioned reaction $^{48}Ca + ^{48}Ca$ at 400 AMeV using the impact parameter 1 fm. The soft equation of state was used, leading to incompressibility coefficient $K = 200$ MeV. For the isospin asymmetric part the "asystiff" parametrization was used with two symmetry energy terms, the kinetic term with the parameters $a_{s1} = 12.5$ MeV and $\gamma_1 = 2/3$, resulting from the Pauli principle, and the potential term with the parameters $a_{s2} = 17.5$ MeV and $\gamma_2 = 2$, respectively.

Figure 3 shows a comparison of the nucleon-nucleon cross sections in two variants of the Boltzmann-Uehling-Uhlenbeck simulations. On the left panel are shown, as functions of density, the isospin-dependent nucleon-nucleon cross sections, obtained using the proper volume of the Van der Waals–like equation of state. On the right panel are shown the corresponding free nucleon-nucleon cross sections, obtained as energy-dependent parametrizations of measured nucleon-nucleon cross sections [16]. It is apparent that while the isospin-dependent nucleon-nucleon cross sections essentially follow the $1/\rho^{2/3}$-dependence, the nucleon-nucleon cross-sectional parametrization of Cugnon *et al.* leads to a much larger spread, mostly due to its explicit energy dependence. Nevertheless, one observes that both parametrizations cover essentially the same range of values of the nucleon-nucleon cross sections. Furthermore, from the comparison [22] of the parametrization of Cugnon *et al.* to in-medium cross sections at saturation density, calculated using the G-matrix theory by Cassing *et al.* [23], it can be judged that the in-medium cross sections, obtained using the proper volume of the Van der Waals–like equation of state, are in better agreement with the somewhat higher values of G-matrix in-medium cross sections of Cassing *et al.*, which reflect properly the Fermionic nature of nucleons.

In general, it is remarkable that the in-medium nucleon-nucleon cross sections possibly can be directly related to the equation of state of the isospin asymmetric nuclear matter. This offers a more consistent description of the nuclear reactions and various astrophysical objects and processes in term of properties of nucleonic matter, expressed using the corresponding equation of state. However, one has to take into account that the equation of state of the isospin asymmetric nuclear matter describes isotropic medium and thus the extracted in-medium cross sections represent angle-averaged values. These values are used in this work, and compared to the results obtained with angle-averaged free cross sections of Cugnon *et al.* [16], and thus the effect of the equation of state of the isospin asymmetric nuclear matter on in-medium cross sections is demonstrated. However, one can consider the possibility of implementing angular dependence, either using the compatible microscopic calculations or from the observed experimental free nucleus-nucleus cross sections. This possibility is beyond the scope of the present work and will be investigated in our future work.

The magnitude of the effect of isospin asymmetry on the nucleon-nucleon cross sections can be judged from Fig. 4, which shows the relative difference of the isospin-dependent

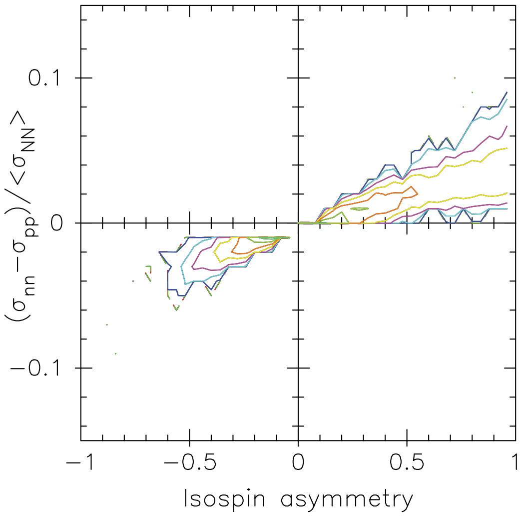

FIG. 4. (Color online) Relative difference of the isospin-dependent neutron-neutron and proton-proton cross sections as a function of isospin asymmetry of the volume cell. The results were obtained using the BUU in reaction ^{48}Ca + ^{48}Ca at 400 AMeV using the impact parameter 1 fm.

neutron-neutron and proton-proton cross sections as a function of isospin asymmetry of the volume cell. The results were again obtained using the BUU in reaction ^{48}Ca + ^{48}Ca at 400 AMeV using the impact parameter 1 fm. One can see that the relative magnitude does not reach very high values even for most isospin-asymmetric cells, and the sensitivity of the Boltzmann-Uehling-Uhlenbeck simulation to the isospin-dependent nucleon-nucleon cross sections will result from the cumulative effect of a large amount of nucleus-nucleus collisions.

The behavior observed in Figs. 3 and 4 appears to be consistent with the behavior shown in Fig. 4 of Ref. [19]. In both cases the absolute values of cross sections decrease monotonously with increasing density while the relative difference of the neutron-neutron and proton-proton cross sections increases with increasing asymmetry. Thus it appears that the procedure used to extract in-medium nucleon-nucleon cross sections by determining the parameters of corresponding Van der Waals EoS reflects the same physics, which is encoded even to a phenomenological equation of state such as Eq. (2).

Figure 5 shows evolution of the difference of n/p multiplicity ratios between the Boltzmann-Uehling-Uhlenbeck simulation with both isospin-dependent mean-field and nucleon-nucleon cross sections, which are correlated to each other by Eq. (9) (based on the analogy with the van der Waals equation of state; hereafter we call this simulation VdWBUU) and Boltzmann-Uehling-Uhlenbeck simulation with isospin-dependent mean-field and free nucleon-nucleon cross sections (hereafter we call it fBUU), for three angular ranges as a function of the kinetic energy in the center-of-mass system. The results were obtained using the BUU simulation in reactions ^{124}Sn + ^{124}Sn and ^{112}Sn + ^{112}Sn at 400 AMeV using the impact parameter 1 fm at the stopping time 200 fm/c. Squares show the result with stiff symmetry energy parametrization (a_{s1} = 12.5 MeV, $\gamma_1 = 2/3$, a_{s2} = 17.5 MeV, and $\gamma_2 = 2$) while triangles show results for with soft symmetry energy parametrization (a_{s1} = 12.5 MeV, $\gamma_1 = 2/3$, a_{s2} = 17.5 MeV, and $\gamma_2 = 1/2$). Soft nuclear equation of state (K = 200 MeV)

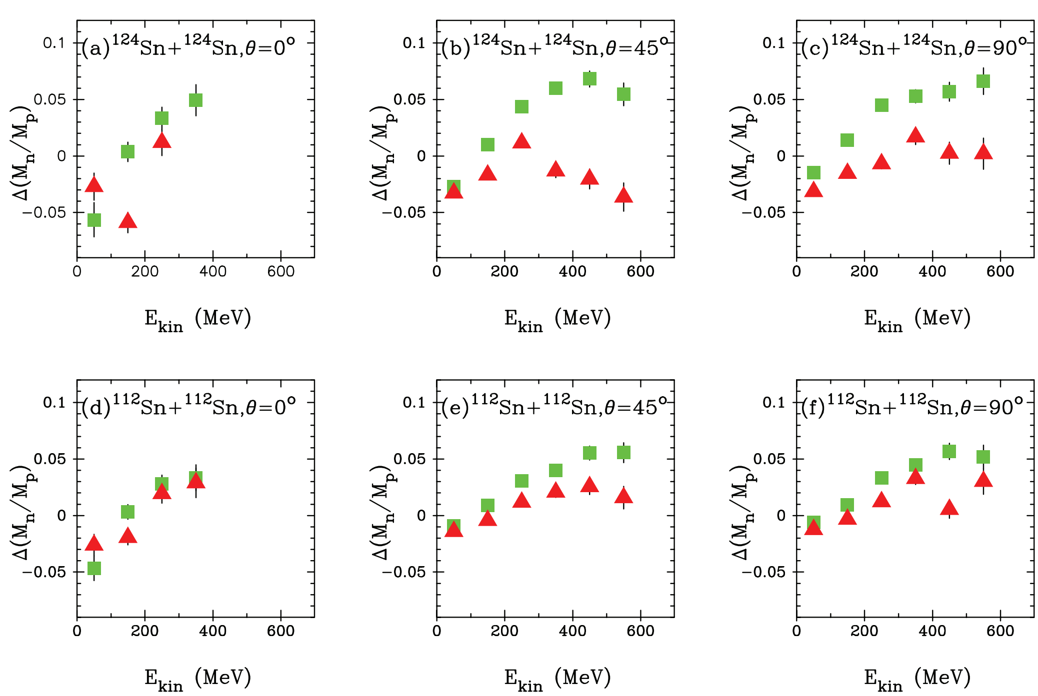

FIG. 5. (Color online) Evolution of the difference of n/p multiplicity ratio between VdWBUU and fBUU calculations for three angular ranges. The results were obtained in reactions ^{124}Sn + ^{124}Sn and ^{112}Sn + ^{112}Sn at 400 AMeV using the impact parameter 1 fm at the time 200 fm/c. Squares show the result with stiff symmetry energy while triangles show results for soft symmetry energy. Soft nuclear equation of state is used.

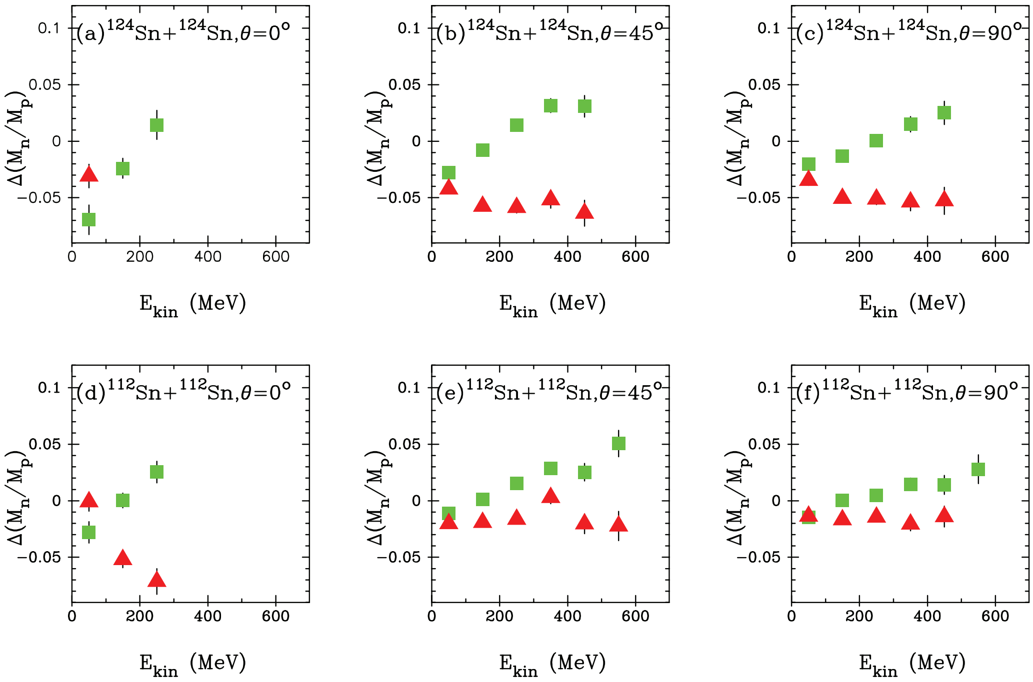

FIG. 6. (Color online) Same as in Fig. 5 but for stiff nuclear equation of state.

is used in this case. Particles are considered as emitted when they are separated in the phase space from any other particle and separation is large enough to assure that two particles are not part of a cluster (a condition $\Delta\vec{p}\Delta\vec{r} > 2h$ is implemented). One can see that implementation of isospin-dependent nucleon-nucleon cross sections leads to significant variation of n/p multiplicity ratio and this effect appears to evolve with both kinetic energy and polar angle. Variation of n/p multiplicity ratio is more significant for the more neutron-rich system, which offers a strong argument for the use

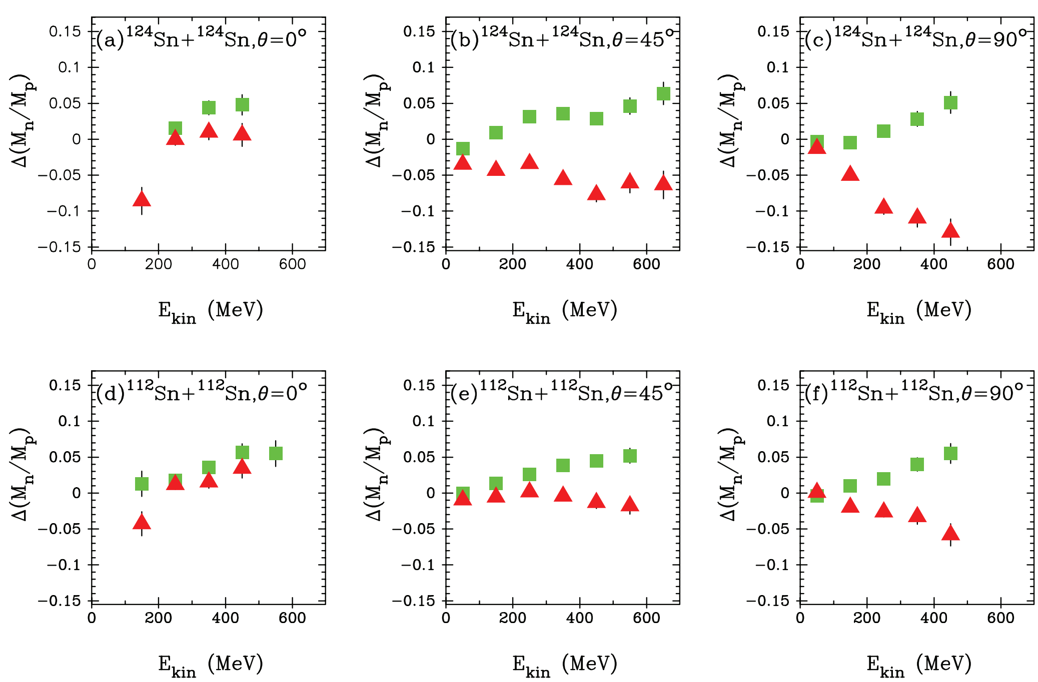

FIG. 7. (Color online) Evolution of the difference of n/p multiplicity ratio between VdWBUU and fBUU calculations for three angular ranges. The results were obtained in reactions $^{124}\mathrm{Sn} + {}^{124}\mathrm{Sn}$ and $^{112}\mathrm{Sn} + {}^{112}\mathrm{Sn}$ at 400 AMeV using the impact parameter 6 fm at the time 200 fm/c. Squares show the results with stiff symmetry energy while triangles show results for soft symmetry energy. The soft nuclear equation of state is used.

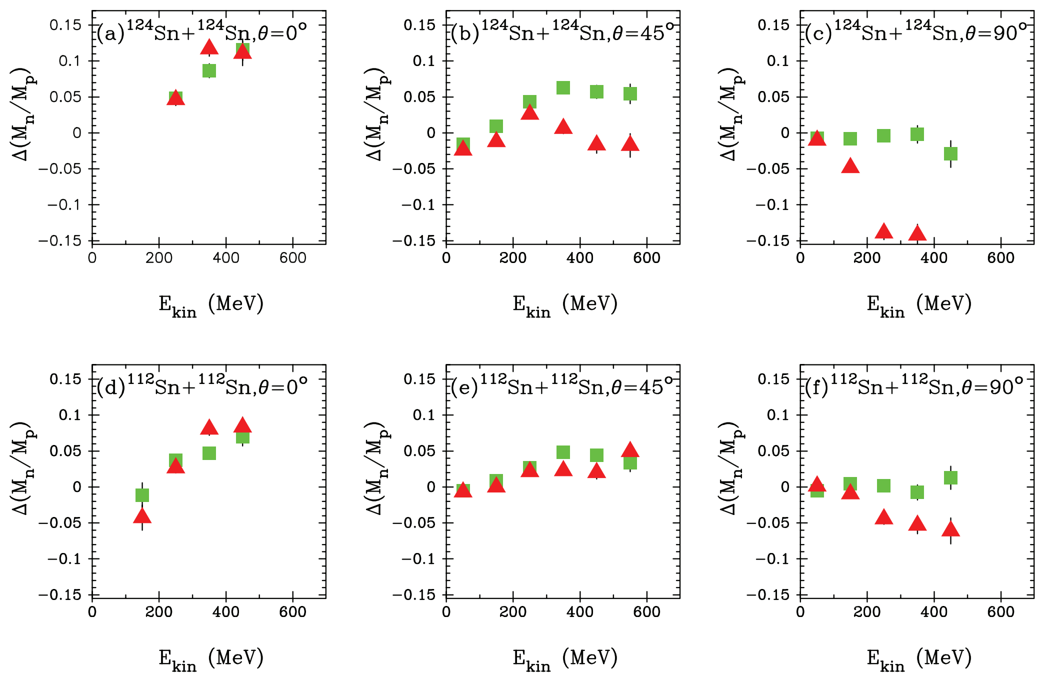

FIG. 8. (Color online) Same as Fig. 7 but for a stiff nuclear equation of state.

of neutron-rich exotic beams for studies of density dependence of the symmetry energy in the future.

Figure 6 again shows evolution of the difference of the n/p multiplicity ratio between the VdWBUU calculation and fBUU calculation, in this case using a stiff nuclear equation of state ($K = 380$ MeV). Also in the case of the stiff nuclear equation of state one can see that implementation of isospin-dependent nucleon-nucleon cross sections leads to considerable variation of the n/p multiplicity ratio, which again evolves with both kinetic energy and polar angle. Also here the variation of the n/p multiplicity ratio is more significant for the more neutron-rich system. It thus appears that variation of the n/p multiplicity ratio between the VdWBUU calculation and fBUU calculation provides a robust signal of the density dependence of nuclear symmetry energy.

Figures 7 and 8 show results for the difference of the n/p multiplicity ratio between the VdWBUU calculation and fBUU calculation, analogous to Figs. 5 and 6, with the impact parameter set to be 6 fm. It can be seen that the effect of isospin-dependent nucleon-nucleon cross sections persists, with comparable magnitude, even in peripheral collisions. This offers the possibility of studying such a signal of the density dependence of the symmetry energy in a wide range of centralities and thus eventually of providing a strong signal of the density dependence of the symmetry energy in a wider range of nuclear density.

The fact, that the isospin-dependent nucleon-nucleon cross sections, obtained using the proper volume of the Van der Waals–like equation of state, need to be introduced in order to fully explore the isospin dependence in the Boltzmann-Uehling-Uhlenbeck simulations, which directly affect the applicability of the symmetry energy parametrizations to the study of astrophysical objects such as neutron stars or supernovae. Due to increased sensitivity to isospin due to isospin-dependent nucleon-nucleon cross sections, the symmetry energy parametrizations may change significantly, and that will affect the extrapolations toward the nuclear densities, typical for neutron stars and similar objects. On the other hand, increased sensitivity may offer more possibilities for the simulations of reactions of exotic nuclear beams, with the possible observation of stronger isospin-dependent signals.

The recently performed simulations based on the ultrarelativistic quantum molecular dynamics model (UrQMD) suggest that one of the most promising probes of the strength of the symmetry energy at suprasaturation densities is the difference of the neutron and proton (or hydrogen) elliptic flows [9,10,24,25]. The simulations were performed using both stiff ($\gamma = 1.5$) and soft ($\gamma = 0.5$) symmetry energy parametrizations. An inversion of the relative strengths of the elliptic flow for neutrons and protons is observed when the symmetry energy parametrization is changed from the stiff behavior to the soft behavior. Neutron- and proton-directed and elliptic flows were measured a decade ago in ^{197}Au + ^{197}Au collisions at beam energies from 400 to 800 AMeV using the Large Area Neutron Detector (LAND) and the FOPI phase 1 forward wall [26,27]. Comparison of predictions of the UrQMD model [28] provided a constraint on symmetry energy at suprasaturation densities from transverse momentum dependence of the neutron's and hydrogen's elliptic flow parameter v_2 measured in the ^{197}Au + ^{197}Au system with FOPI + LAND, suggesting a value of $\gamma = 0.9 \pm 0.4$ [10], in agreement with findings at subsaturation densities. However, the statistics of these data set severe limits on the conclusions that can be drawn by comparison to transport model calculations. The promising results of reanalysis of the FOPI + LAND experiment initiated a proposal for a new experiment [29], which is one of the first dedicated explorations of the symmetry

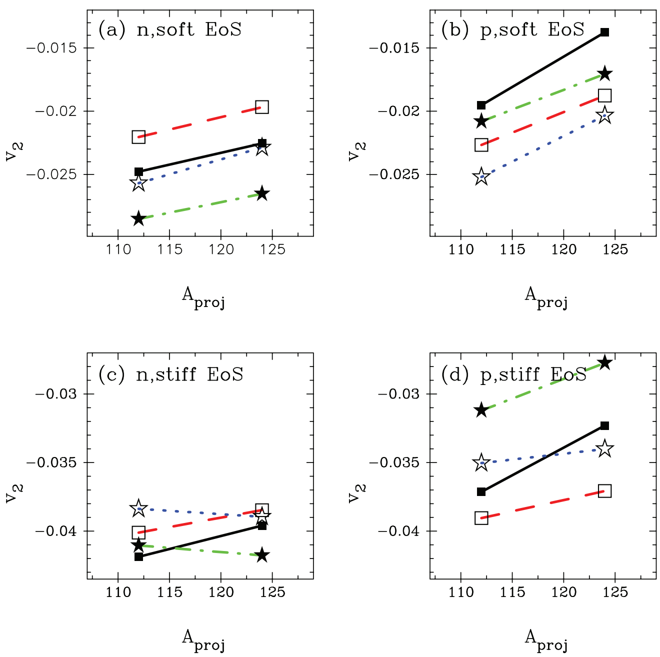

FIG. 9. (Color online) Elliptic flow of neutrons and protons in reactions ^{124}Sn + ^{124}Sn and ^{112}Sn + ^{112}Sn at 400 AMeV at the impact parameter 6 fm. Solid and open squares show results of VdWBUU calculation and fBUU calculation, respectively, with soft symmetry energy. Solid and open asterisks show analogous results with stiff symmetry energy.

energy at high densities. The experiment uses the LAND calorimeter for neutron and charged particle detection, and the impact parameter is determined with a detection system with high effective granularity at forward angles consisting of several CsI rings of the CHarged Ion Mass and Energy Resolving Array (CHIMERA) multi-detector [30] and the A Large Acceptance DIpole magNet (ALADIN) time-of-flight wall [31]. In addition, flow of light fragments are measured with the Krakow telescope [32] array positioned opposite from LAND. Uncertainties are expected to be reduced by a factor of 4 or 5, thus allowing constraint of theoretical calculations. Expected results of this experiment will provide a welcome testing ground for the model calculations introduced in the present work.

It is of interest to estimate what effect the introduction of the isospin-dependent nucleon-nucleon cross sections, obtained as the proper volume of the Van der Waals–like equation of state, will have on the resulting elliptic flow of neutrons and protons. Figure 9 shows calculated values of the elliptic flow of neutrons and protons (determined conventionally as second Fourier coefficient of the invariant triple differential distribution v_2 relative to the reaction plane) in reactions ^{124}Sn + ^{124}Sn and ^{112}Sn + ^{112}Sn at 400 AMeV at the impact parameter 6 fm. Solid and open squares show results of the VdWBUU calculation and fBUU calculation, respectively, with soft symmetry energy parametrization. Solid and open asterisks show analogous results with stiff symmetry energy parametrization. One can see that introduction of isospin-dependent nucleon-nucleon cross sections, with varying ratios of neutron-neutron and proton-proton collision rates and thus in-plane and out-of-plane emission ratios, influences the resulting values of the elliptic flow. Since the effect appears to vary between neutrons and protons, it will strongly influence the differential elliptic flow, thus making it a strong signature of the nuclear equation of state, as suggested in Ref. [10]. However, since the effect of symmetry energy on such differential elliptic flow tends to vary also with the isoscalar part of the nuclear equation of state, it appears necessary to study differential elliptic flow in combination with other observables, such as the evolution of the n/p multiplicity ratio at different polar angles and kinetic energies. Such a study, carried out on a sufficiently neutron-rich system, can provide good sensitivity to both isoscalar and isovector parts of the nuclear equation of state.

IV. CONCLUSIONS

An extension of the Boltzmann-Uehling-Uhlenbeck model of nucleus-nucleus collision is presented. The isospin-dependent nucleon-nucleon cross sections are estimated using the proper volume extracted from the equation of state of the nuclear matter transformed into the form of the Van der Waals equation of state. The results of such simulations demonstrate the dependence on symmetry energy, which typically varies strongly from the results obtained using only the isospin-dependent mean-field. The evolution of the n/p multiplicity ratio with angle and kinetic energy, in combination with the elliptic flow of neutrons and protons, provides a suitable set of observables for determination of the density dependence of the symmetry energy. The model thus provides an environment for testing of equations of state, used for various applications in nuclear physics and astrophysics.

ACKNOWLEDGMENTS

This work is supported in part by the Chinese Academy of Sciences Fellowships for the international scientists under Grant No. 2011T2J13 (M.V.), NSFC of China under Contracts No. 11035009 and No. 10979074, and the Knowledge Innovation Project of CAS under Grant No. KJCX2-EW-N01 (Y.G.M.). M.V. has been also supported by the Slovak Scientific Grant Agency under Contract 2/0105/11 and by the Slovak Research and Development Agency under Contract APVV-0177-11.

[1] D. H. Youngblood, H. L. Clark, and Y.-W. Lui, Phys. Rev. Lett. **82**, 691 (1999).
[2] J. B. Natowitz, K. Hagel, Y. Ma, M. Murray, L. Qin, R. Wada, and J. Wang, Phys. Rev. Lett. **89**, 212701 (2002).
[3] P. Danielewicz, R. Lacey, and W. G. Lynch, Science **298**, 1592 (2002).
[4] W. Reisdorf*el al.*, Nucl. Phys. A **876**, 1 (2012).
[5] M. B. Tsang *et al.*, Phys. Rev. Lett. **92**, 062701 (2004).

[6] M. A. Famiano *et al.*, Phys. Rev. Lett. **97**, 052701 (2006).
[7] M. B. Tsang, Y. Zhang, P. Danielewicz, M. Famiano, Z. Li, W. G. Lynch, and A. W. Steiner, Phys. Rev. Lett. **102**, 122701 (2009).
[8] S. Kumar, Y. G. Ma, G. Q. Zhang, and C. L. Zhou, Phys. Rev. C **84**, 044620 (2011).
[9] B. A. Li, L. W. Chen, and C. M. Ko, Phys. Rep. **464**, 113 (2008).
[10] P. Russotto *et al.*, Phys. Lett. B **697**, 471 (2011).
[11] M. D. Cozma, Phys. Lett. B **700**, 139 (2011).
[12] S. Gautam, A. D. Sood, R. K. Puri, and J. Aichelin, Phys. Rev. C **83**, 034606 (2011).
[13] G. F. Bertsch, H. Kruse, and S. Das Gupta, Phys. Rev. C **29**, 673 (1984).
[14] H. Kruse, B. V. Jacak, and H. Stöcker, Phys. Rev. Lett. **54**, 289 (1985).
[15] C.-Y. Wong, Phys. Rev. C **25**, 1460 (1982).
[16] J. Cugnon, T. Mizutani, and J. Vandermeulen, Nucl. Phys. A **352**, 505 (1981).
[17] G. Q. Li and R. Machleidt, Phys. Rev. C **48**, 1702 (1993); **49**, 566 (1994).
[18] T. Alm, G. Ropke, and M. Schmidt, Phys. Rev. C **50**, 31 (1994); T. Alm, G. Ropke, W. Bauer, F. Daffin, and M. Schmidt, Nucl. Phys. A **587**, 815 (1995).
[19] B. A. Li and L. W. Chen, Phys. Rev. C **72**, 064611 (2005).
[20] D. Klakow, G. Welke, and W. Bauer, Phys. Rev. C **48**, 1982 (1993).
[21] C. K. Gelbke and D. H. Boal, Prog. Part. Nucl. Phys. **19**, 33 (1987).
[22] W. Cassing and U. Mosel, Prog. Part. Nucl. Phys. **25**, 235 (1990).
[23] W. Cassing, K. Niita, and S. J. Wang, Z. Phys. A **331**, 439 (1988).
[24] W. Trautmann *et al.*, Prog. Part. Nucl. Phys **62**, 425 (2009).
[25] W. Trautmann *et al.*, Int. J. Mod. Phys. E **19**, 1653 (2010).
[26] Y. Leifels *et al.*, Phys. Rev. Lett. **71**, 963 (1993).
[27] D. Lambrecht *et al.*, Z. Phys. A **350**, 115 (1994).
[28] Q. Li, Z. Li, and H. Stocker, Phys. Rev. C **73**, 051601(R) (2006).
[29] R. C. Lemmon *et al.*, Constraining the Symmetry Energy at Supra-saturation Densities with Measurements of Neutron and Proton Elliptic Flows, GSI experiment S394 (unpublished).
[30] A. Pagano *et al.*, Nucl. Phys. A **734**, 504 (2004).
[31] A. Schüttauf *et al.*, Nucl. Phys. A **607**, 457 (1996).
[32] J. Łukasik, P. Pawłowski, A. Budzanowski, B. Czech, I. Skwirczyńska, J. Brzychczyk, M. Adamczyk, S. Kupny, P. Lasko, Z. Sosin, A. Wieloch, M. Kiš, Y. Leifels, and W. Trautmann, EPJ Web of Conferences **31**, 00032 (2012).