Nuclear symmetry energy from the Fermi-energy difference in nuclei

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The neutron-proton Fermi-energy difference and the correlation to nucleon separation energies for some magic nuclei are investigated with the Skyrme energy density functionals and nuclear masses, with which the nuclear symmetry energy at subsaturation densities is constrained from 54 Skyrme parameter sets. The extracted nuclear symmetry energy at subsaturation density of 0.11 fm^{-3} is 26.2 $\pm$ 1.0 MeV with 1.5σ uncertainty. By further combining the neutron-skin thickness of ^{208}Pb, ten Skyrme forces with slope parameter of $28 \leqslant L \leqslant 65$ are selected for the description of the symmetry energy around saturation densities.

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I. INTRODUCTION

The nuclear symmetry energy, in particular its density dependence, has received considerable attention in recent years [1–12]. As one of the key properties of nuclear matter, the nuclear symmetry energy probes the isospin part of nuclear forces and intimately relates to the structure character of drip-line nuclei and superheavy nuclei, the dynamical process of nuclear reactions, and the behavior of neutron stars. To explore the density dependence of the nuclear symmetry energy from subsaturation to supersaturation densities, various models and experimental observables have been proposed. On the one hand, the constraints on the symmetry energy are investigated from heavy-ion collisions [2,13,14]. Some experimental data for the isospin-sensitive observables are well reproduced by using certain forms of a density-dependent symmetry energy in microscopic dynamics calculations, such as in the improved quantum molecular dynamics [1,2] and the isospin Boltzmann-Uehling-Uhlenbeck [4,5] calculations. In these calculations, the temperature effect and the influence of the isospin-independent terms of nuclear forces are self-consistently involved. It is still difficult to clearly obtain the information of nuclear symmetry energy at zero temperature by removing the influence of the isospin-independent terms.

On the other hand, the symmetry energy is also constrained from the properties of finite nuclei, such as the binding energies [15–20], the neutron-skin thickness [8,21–24], and the pygmy dipole resonance (PDR) [25,26]. By analyzing the more than 2000 measured masses of nuclei with the help of the liquid drop formula, one can obtain the mass dependence of the symmetry energy coefficients of finite nuclei [15,16,20] and the nuclear symmetry energy at subsaturation densities [17,18] based on the relation between the symmetry energy coefficients of finite nuclei and the symmetry energy of infinite nuclear matter [8]. The obtained symmetry energy at the saturation density and its slope parameter [17] are generally close to the results from the PDR [26] and heavy-ion collisions [1]. In addition, the symmetry energy is also constrained from neutron star observations incorporating the microphysics of both the stellar crust and core [9,27,28]. A relatively smaller slope parameter of the symmetry energy at the saturation density $43 < L < 52$ MeV is obtained from the available neutron star mass and radius measurements [9]. It is known that the average density of a finite nucleus is smaller than the saturation density due to the surface diffuseness. The obtained information mainly describes the nuclear symmetry energy at the subsaturation densities rather than the supersaturation densities. The symmetry energies at the saturation and supersaturation densities based on the extrapolation are still very uncertain and more isospin-sensitive observations should be proposed and investigated.

The density dependence of the symmetry energy is also extensively studied with the Skyrme energy density functionals [29–31]. The Hartree-Fock-Bogoliubov approach with the Skyrme force BSk17 [30] can reproduce the 2149 measured masses with a rms deviation of 0.581 MeV, which is comparable to the accuracy of the new finite-range droplet model (FRDM2012) [19]. In the FRDM2012, it is found that a slope parameter of the symmetry energy at normal density $L \approx 70$ MeV can give better results with a rms deviation of 0.570 MeV. However, one should note that the corresponding slope parameter is only $L = 36$ MeV from the BSk17 parameter set which is much smaller than the result of FRDM2012. It is therefore necessary to further investigate the behavior of the symmetry energy at subsaturation densities from the structures of finite nuclei.

In this work, we study the nuclear symmetry energy at subsaturation densities from the Fermi energies of nuclei based on various parametrizations of the Skyrme forces. The Skyrme interaction, originally constructed for finite nuclei and nuclear matter at saturation density, is a low-momentum expansion of the effective two-body nucleon-nucleon (NN) interaction in momentum space. Although all Skyrme forces are usually fit to well reproduce the saturation energy and density of symmetric nuclear matter, they differ significantly in other characteristics of symmetric and pure neutron matter; in particular their density dependence [31]. The Fermi energies of neutrons and protons for some doubly magic nuclei can be measured with a high precision. The neutron-proton Fermi-energy difference and the correlation to nucleon separation energies of magic nuclei are directly related to the symmetry energy of nuclei, with which the isospin-dependent terms of the Skyrme forces and thus the density dependence of the symmetry energy could

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be constrained. The paper is organized as follows: In Sec. II, the correlation between Fermi energies and separation energies of nucleons are introduced. In Sec. III, the nuclear symmetry energy at subsaturation and saturation densities is extracted. Finally, a summary is given in Sec. IV.

II. CORRELATION BETWEEN FERMI ENERGIES AND SEPARATION ENERGIES OF NUCLEONS

Based on the liquid-drop mass formula, the binding energy of a nucleus which is taken as a positive value is expressed as

$$\mathrm{BE}(A,Z) = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_{\mathrm{sym}} I^2 A \quad (1)$$

by neglecting nuclear microscopic corrections. $I = (N - Z)/A$ denotes the isospin asymmetry. The difference between the proton separation energy $[\mathrm{BE}(A,Z) - \mathrm{BE}(A-1, Z-1)]$ and the neutron separation energy $[\mathrm{BE}(A,Z) - \mathrm{BE}(A-1, Z)]$ of a nucleus is written as

$$\Delta S = S_p - S_n \simeq -2a_c \frac{Z}{A^{1/3}} + 4a_{\mathrm{sym}} I. \quad (2)$$

Because the Coulomb energy coefficient $a_c = \frac{3}{5}\frac{e^2}{r_0} \simeq 0.71$ MeV is usually well determined from the masses of mirror nuclei [32,33], the value of ΔS is directly related to the symmetry-energy coefficients of finite nuclei.

On the other hand, the single-particle energies (SPEs) of a nucleus can be uniquely determined by solving the Schrödinger equations or Hartree-Fock equations based on the single-particle potential under the mean-field approximation. In the Hartree-Fock theory for a closed-shell nucleus (A, Z), the single-particle energies for states below the Fermi surface are given by [34,35]

$$\varepsilon_p = \mathrm{BE}^*(A-1, Z-1) - \mathrm{BE}(A,Z) \quad (3)$$

and

$$\varepsilon_n = \mathrm{BE}^*(A-1, Z) - \mathrm{BE}(A,Z). \quad (4)$$

The quantity ε will be negative for bound states. The quantity $(\mathrm{BE}^* = \mathrm{BE} - E_x)$ is the ground-state binding energy minus the excitation energy of the excited states associated with the single-particle states. The difference between the Fermi energy of neutrons and that of protons is

$$\Delta\varepsilon = \varepsilon_n^F - \varepsilon_p^F, \quad (5)$$

is closely related to the proton and neutron separation energies of the nucleus. Here, the Fermi energy ε^F is defined as the energy of the highest occupied quantum state in a system of fermions at absolute zero temperature.

Figure 1 shows the calculated single-particle energies of occupied states for protons and neutrons of ^{132}Sn at its ground state by using the Skyrme Hartree-Fock (SHF) model with the parameter set SLy7 [36]. For this neutron-rich nucleus, the Fermi energy of neutrons is higher than that of protons, and the calculated value of $\Delta\varepsilon$ is 8.1 MeV. The depth of the single-particle potential plays a dominant role for the corresponding Fermi energy of a given nucleus. The difference $\Delta\varepsilon$ closely relates to the difference between the depth of nuclear potentials

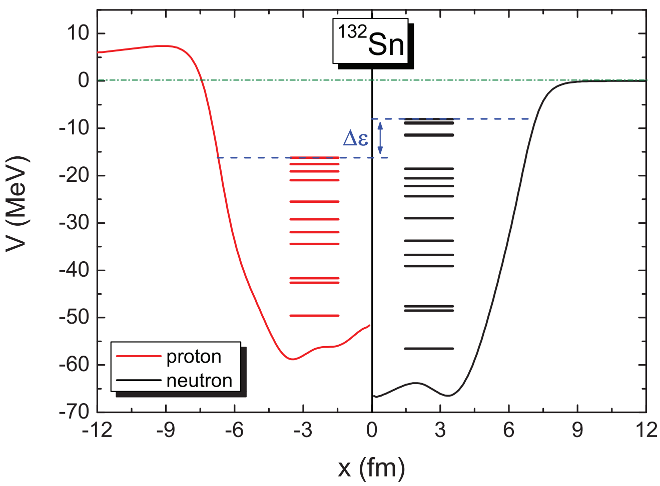

FIG. 1. (Color online) Single-particle potentials and the single-particle energies (SPEs) of bound states for ^{132}Sn with the Skyrme Hartree-Fock calculation by using SLy7 force. The dashed lines denote the highest SPE of the occupied states for nucleons in ^{132}Sn at its ground state.

for neutrons and protons,

$$V_n - (V_p + V_C) \simeq -\frac{3}{2}\frac{e^2}{r_c}\frac{Z}{A^{1/3}} + 2V_{\mathrm{sym}} I. \quad (6)$$

Here, $V_C \simeq \frac{3}{2}\frac{e^2}{r_c}\frac{Z}{A^{1/3}}$ denotes the Coulomb potential of a nucleus at the central position with the potential radius $r_c \approx 1.3$ fm. The information on the symmetry potential V_{sym} from the Fermi energy difference is of great importance for the study of nuclear symmetry energy.

It is known that, under the mean-field approximation, if the single-particle motion plays a dominant role for the behavior of nucleons near the Fermi surface, one expects that the relation $\Delta S \simeq \Delta\varepsilon$ holds for the closed-shell nuclei. The experimental values of $\Delta\varepsilon$ for six doubly magic nuclei ^{16}O, ^{40}Ca, ^{48}Ca, ^{56}Ni, ^{132}Sn, and ^{208}Pb are −3.53, −7.31, 6.10, −9.47, 8.40, and 0.64 MeV, respectively [34,35]. The corresponding separation-energy differences ΔS for these six nuclei are −3.53, −7.31, 5.86, −9.48, 8.39, and 0.64 MeV, respectively. One sees that the relation $\Delta S \simeq \Delta\varepsilon$ does hold very well, as expected for these doubly magic nuclei. To further test the relation, the values of $\Delta\varepsilon$ for 19 doubly magic or semimagic nuclei (^{16,22}O, ^{22,42}Si, ^{40,48,60}Ca, ^{42}Ti, ^{56,68,78}Ni, ^{130}Cd, ^{100,132,134}Sn, ^{134}Te, ^{144}Sm, ^{182,208}Pb) are systematically investigated by using the SHF model together with 54 commonly used Skyrme forces. The calculated values are shown in Fig. 2 as a function of ΔS. Here, the masses of unmeasured nuclei are predicted with the Weizsäcker-Skyrme mass formula combining the radial basis function correction (WS3$^{\mathrm{RBF}}$) [37]. The WS3$^{\mathrm{RBF}}$ model can reproduce the measured 2149 masses in AME2003 with an rms deviation of 184 keV, and the predictive power is also remarkable [38] (the rms deviation with respect to the 154 new masses of extremely neutron-rich and proton-rich nuclei listed in AME2012 [39] is only 397 keV). The red squares in Fig. 2, which denote the experimental data for the six doubly magic nucleus mentioned previously, are quite regularly located along the green line $\Delta S = \Delta\varepsilon$. The solid

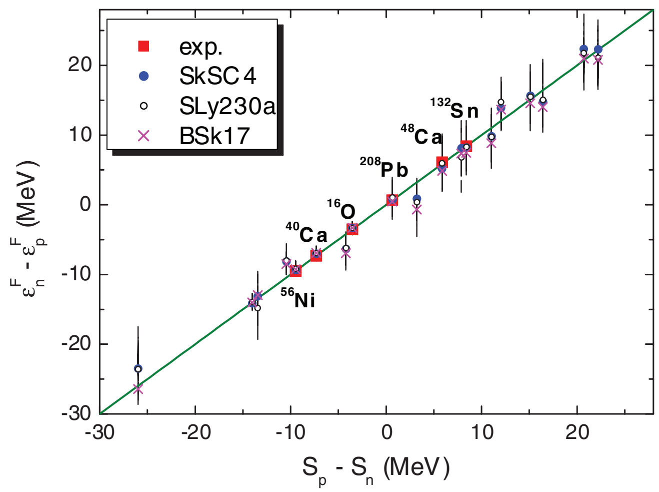

FIG. 2. (Color online) Fermi-energy difference as a function of separation-energy difference. The red squares denote the experimental data for six doubly magic nuclei [34,35]. Others are the calculated results with difference Skyrme forces. The solid circles, open circles, and crosses denote the results of three Skyrme forces SkSC4 [40], SLy230a [41], and BSk17 [30], respectively. The error bars denote the uncertainty of the model calculations from 54 different Skyrme forces in which the corresponding incompressibility coefficient for symmetry nuclear matter is $K_\infty = 230 \pm 30$ MeV and the saturation density $\rho_0 = 0.16 \pm 0.005$ fm^{-3}. One sees that the calculated results from the traditional 10-parameter Skyrme forces support the relation $\Delta S \simeq \Delta\varepsilon$, even for the extremely neutron-rich and proton-rich nuclei. The correlation between the Fermi-energy difference and the separation energy difference for neutrons and protons could be helpful for constraining the equation of state for asymmetry nuclear matter. Here, we would like to emphasize that the Fermi-energy difference can effectively remove the influence of isospin-independent terms in the nuclear forces.

III. NUCLEAR SYMMETRY ENERGY AT SUBSATURATION AND SATURATION DENSITIES

Based on the calculated Fermi-energy difference $\Delta\varepsilon$ with the 54 different Skyrme forces, the average deviation

$$\langle\sigma\rangle = \frac{1}{m}\sum_{i=1}^{m}|\Delta\varepsilon^{(i)} - \Delta S^{(i)}| \qquad (7)$$

from the $m = 19$ nuclei mentioned previously is calculated. Figures 3(a) and 3(b) show the average deviation as a function of symmetry energy $E_{\rm sym}$ at the density of $\rho = 0.09$ and 0.11 fm^{-3}, respectively. The nuclear symmetry energy in the Skyrme energy density functional is expressed as

$$\begin{aligned} E_{\rm sym}(\rho) &= \frac{1}{2}\left[\frac{\partial^2(E/A)}{\partial I^2}\right]_{I=0} \\ &= \frac{1}{3}\frac{\hbar^2}{2m}\left(\frac{3\pi^2}{2}\right)^{2/3}\rho^{2/3} - \frac{1}{8}t_0(2x_0+1)\rho \\ &\quad -\frac{1}{24}\left(\frac{3\pi^2}{2}\right)^{2/3}\Theta_{\rm sym}\rho^{5/3} - \frac{1}{48}t_3(2x_3+1)\rho^{\sigma+1}, \end{aligned} \qquad (8)$$

with $\Theta_{\rm sym} = 3t_1x_1 - t_2(4+5x_2)$. The quantities t_0, t_1, t_2, t_3, x_0, x_1, x_2, x_3, and σ are the Skyrme parameters. The squares and solid curves denote the results of 54 Skyrme forces and the parabolic fit, respectively. One sees that the minimal deviations are located around $E_{\rm sym}(0.09) = 23.6$ MeV and $E_{\rm sym}(0.11) = 26.2$ MeV, respectively. We also note that the obtained symmetry energies do not change appreciably if only the six doubly magic nuclei are involved in the calculation of the average deviation $\langle\sigma\rangle$. It indicates that the Fermi-energy difference is a useful observation for studying the symmetry energy at subsaturation densities. The dashed line shows the minimal value of the parabolic curve, which is 1.19 MeV. The minimal average deviation from the 54 Skyrme forces is about 1.0 MeV. Considering the systematic error of the WS3$^{\rm RBF}$ mass model, which is about 0.19 MeV [37], the Skyrme forces with

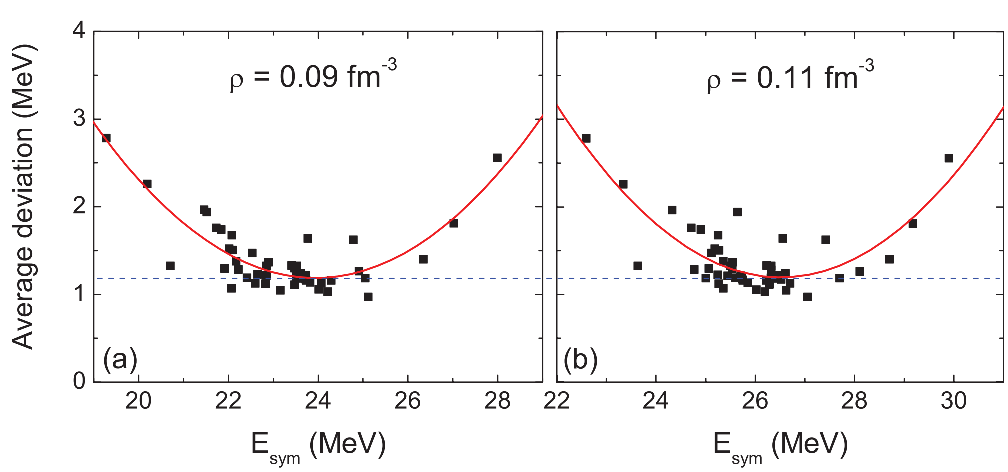

FIG. 3. (Color online) Average deviation as a function of nuclear symmetry energy $E_{\rm sym}$ at the density of (a) $\rho = 0.09$ fm^{-3} and (b) 0.11 fm^{-3}, with 54 different Skyrme forces. The solid curves denote the parabolic fit to the squares. The dashed line show the position of 1.19 MeV.

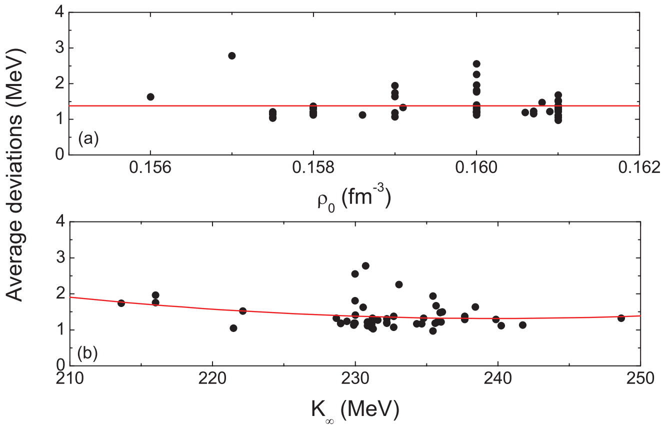

FIG. 4. (Color online) Average deviation as a function of (a) saturation density ρ_0 and (b) incompressibility coefficient K_∞ of symmetric nuclear matter, based on the calculations of the 54 Skyrme forces.

$\langle\sigma\rangle \leqslant 1.19$ MeV reasonably well describe the Fermi-energy difference for the 19 nuclei mentioned previously.

As one of the key properties of nuclear matter, the symmetry energy is particularly important in modeling nuclear matter and finite nuclei because it probes the isospin part of the Skyrme interaction. For a sensitive observation to investigate the nuclear symmetry energy, the influence of isospin-independent terms in the nuclear forces should be removed as clean as possible. In Fig. 4, we show the average deviation as a function of nuclear saturation density ρ_0 and the incompressibility coefficient K_∞ for symmetry nuclear matter. One cannot evidently obtain the optimal values of ρ_0 and K_∞ according to the average deviations from the 54 Skyrme forces, since these two quantities are determined by the isospin-independent parts of the Skyrme interactions. We also note that the strength of the spin-orbit interaction W_0 in the Skyrme forces does not affect the value of $\Delta\varepsilon$, generally, due to the cancellation between protons and neutrons, which is helpful to remove the influence of the shell effect on the symmetry energy. For example, the value of $\Delta\varepsilon$ for ^{132}Sn only changes by 0.7% with a variation of the strength of the spin-orbit interaction by 32% according to the SLy7 calculations.

From the 54 Skyrme forces, 17 parameter sets with $\langle\sigma\rangle \leqslant$ 1.19 MeV are selected for the description of $E_{\rm sym}(\rho)$. These selected Skyrme forces can well reproduce the experimental data for the Fermi-energy difference of the six doubly magic nuclei. We note that the symmetry energies obtained from these 17 Skyrme interactions are close to each other at densities around $\rho_c = 0.11$ fm^{-3}, and the value of $E_{\rm sym}(\rho_c) = 26.2 \pm 1.0$ MeV with 1.5σ uncertainty. It could be much more useful if the slope parameter of the symmetry energy at the saturation density ρ_0,

$$L = 3\rho_0 \left(\frac{\partial E_{\rm sym}}{\partial \rho}\right)_{\rho=\rho_0}, \tag{9}$$

can be also well constrained with this approach. Unfortunately, we find that the uncertainties of the symmetry energy obtained at densities lower and higher than ρ_c gradually increase. Figure 5 shows the average deviation as a function of nuclear symmetry energy $E_{\rm sym}$ at the density of $\rho = 0.13$ and 0.16 fm^{-3}. The parabolic behavior of the average deviation becomes unclear with increasing of density and even disappears at the saturation density. The corresponding uncertainties of the symmetry energy at $\rho = 0.13$ and 0.14 fm^{-3} increase to 1.5 and 2.0 MeV, respectively. It indicates that the slope parameter L cannot be well constrained by using the neutron-proton Fermi-energy difference uniquely, although the nuclear symmetry energy at subsaturation density can be well described.

To further analyze the density dependence of the symmetry energy at the saturation density, the values of the symmetry energy at ρ_c and ρ_0, the slope parameters L, and the neutron-skin thickness $\Delta R_{\rm np}$ of ^{208}Pb are listed in Table I. One sees that for the seven Skyrme forces, i.e., SkSC4, SkSC1, v075, v080, v090, v105, MSk3, the corresponding slope parameters L are very small and even negative. In Ref. [31], it is thought that the BSk, SkSC, and MSk families under-predict both the symmetry energy and its derivative at the saturation density. To further check these selected Skyrme forces for the description of other physical quantities, we study the neutron-skin thickness of ^{208}Pb. The linear relationship between the

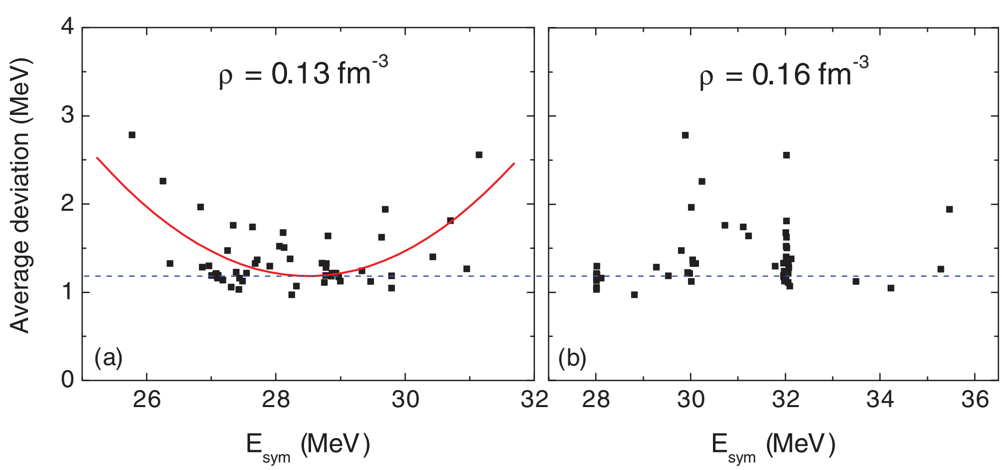

FIG. 5. (Color online) The same as Fig. 3, but at the density of (a) $\rho = 0.13$ and (b) 0.16 fm^{-3}.

TABLE I. Nuclear symmetry energy and neutron-skin thickness ΔR_{np} of ^{208}Pb with the selected 17 Skyrme forces. $\langle\sigma\rangle$ denote the calculated average deviation according to Eq. (7) for the 19 nuclei. The units of ΔR_{np} are fm, and those of the other quantities are MeV. The bold-face entries denote the four forces with the smallest average deviation and reasonable neutron-skin thickness of ^{208}Pb.

Label	$\langle\sigma\rangle$	$E_{sym}(\rho_c)$	$E_{sym}(\rho_0)$	L	ΔR_{np} (^{208}Pb)	Ref.
Ska25s20	**1.05**	**26.6**	**34.2**	**65.1**	**0.20**	[31]
SV-sym32	**1.07**	**25.4**	**32.1**	**57.4**	**0.19**	[42]
SLy2	**1.11**	**26.3**	**32.1**	**47.5**	**0.16**	[43]
Ska35s20	**1.12**	**26.3**	**33.5**	**64.4**	**0.20**	[31]
BSk17	1.13	25.3	30.0	36.3	0.15	[30]
SLy230a	1.13	26.7	32.0	44.3	0.15	[41]
SLy7	1.17	26.5	32.0	46.9	0.16	[36]
SLy6	1.18	26.4	32.0	47.5	0.16	[36]
Skz1	1.19	27.7	32.0	27.7	0.15	[44]
SkT7	1.19	25.0	29.5	31.1	0.15	[45]
SkSC4	0.97	27.1	28.8	-2.4	0.11	[40]
v075	1.03	26.2	28.0	-0.3	0.11	[46]
v080	1.06	26.0	28.0	2.2	0.11	[46]
v090	1.14	25.8	28.0	5.1	0.11	[46]
MSk3	1.16	25.7	28.0	6.8	0.11	[47]
SkSC1	1.16	26.2	28.1	0.1	0.11	[48]
v105	1.18	25.7	28.0	7.1	0.11	[46]

slope parameter L and the ΔR_{np} of ^{208}Pb was observed in Refs. [8,22]. Figure 6 shows the slope parameter L of these Skyrme forces as a function of the corresponding neutron-skin thickness of ^{208}Pb. The corresponding values of $\Delta R_{np}(^{208}\text{Pb})$ from the seven Skyrme forces with $L < 10$ MeV are about 0.11 fm. The recent experimental measurements [23–26] for the ΔR_{np} of ^{208}Pb show $0.131 \leqslant \Delta R_{np}(^{208}\text{Pb}) \leqslant 0.218$ fm. It implies that the seven Skyrme forces with small L are not suitable for the description of the symmetry energy at around saturation densities. The other ten Skyrme forces with $28 \leqslant L \leqslant 65$ MeV, i.e., Ska25s20, Ska35s20, SV-sym32, SLy2, SLy6, SLy7, SLy230a, BSk17, Skz1, and SkT7, describe reasonably well both the Fermi-energy difference and the neutron-skin thickness of ^{208}Pb. The obtained central value of L with this approach is generally consistent with the result from neutron star mass and radius measurements [9]. Out of the ten forces, four with the smallest average deviation ($\langle\sigma\rangle \leqslant 1.12$ MeV, see Table I) have values of $L = 56 \pm 9$ MeV for the slope parameter, and the Skyrme forces Ska25s20, Ska35s20, and SV-sym32 were also recommended in a recent study [31].

Figure 7 shows the calculated nuclear symmetry energy as a function of density. The squares (with 1.5σ uncertainty

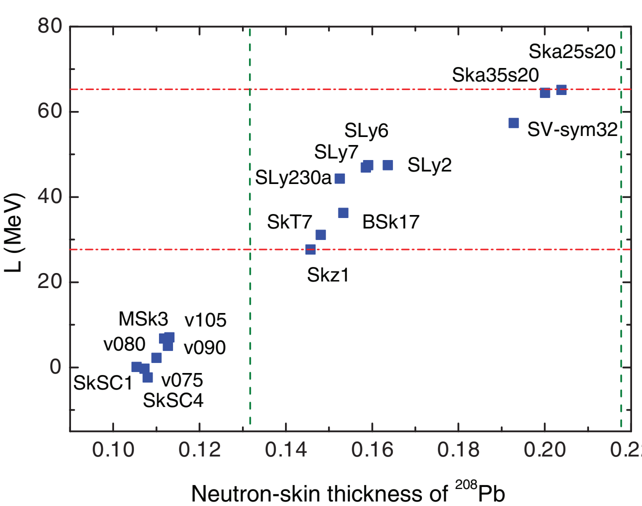

FIG. 6. (Color online) Slope parameter of nuclear symmetry energy as a function of neutron-skin thickness of ^{208}Pb. The region between the two dashed vertical lines denotes the measured neutron-skin thickness of ^{208}Pb, $0.131 \leqslant \Delta R_{np}(^{208}\text{Pb}) \leqslant 0.218$ fm [23–26].

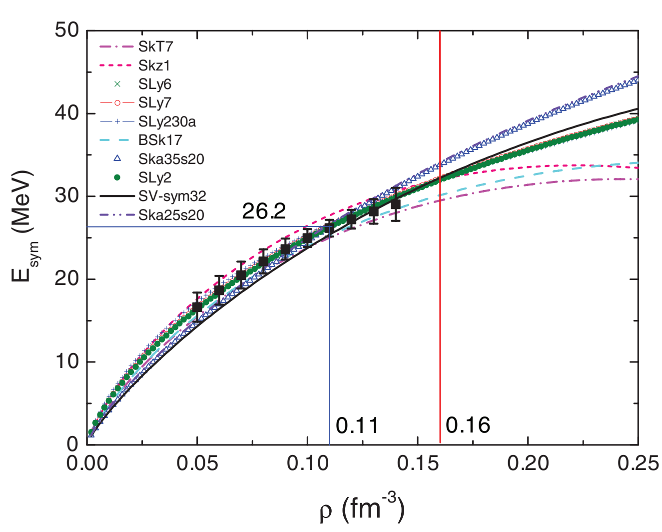

FIG. 7. (Color online) Nuclear symmetry energy as a function of density. The squares denotes the results from the 17 selected Skyrme forces based on the Fermi-energy difference. Other symbols and curves denote the results of ten selected Skyrme forces with which both the Fermi-energy difference and the neutron-skin thickness of ^{208}Pb can be reasonably well described.

as the error bar) denote the results from the 17 selected Skyrme forces listed in Table I. Other symbols and curvatures denote the results from the ten selected forces which can simultaneously well describe the neutron-skin thickness of ^{208}Pb. The symmetry energy at the saturation density from the ten selected Skyrme forces is $E_{\rm sym}(\rho_0) = 31.9 \pm 2.1$ MeV with 1.5σ uncertainty.

IV. SUMMARY

The correlation between the neutron-proton Fermi-energy difference $\Delta\varepsilon$ and the separation energy difference ΔS for some doubly magic and semimagic nuclei is analyzed with the Skyrme energy density functionals and nuclear masses, with which nuclear symmetry energy at subsaturation densities is constrained from 54 different Skyrme forces. The experimental data and the Skyrme Hartree-Fock calculations show $\Delta S \simeq \Delta\varepsilon$ for closed-shell nuclei, even for the extremely neutron-rich and proton-rich cases. The correlation between $\Delta\varepsilon$ and ΔS is a good observation for studying the nuclear symmetry energy at subsaturation densities, which probes the isospin-dependent part of the Skyrme interaction, since the cancelation between protons and neutrons directly removes the influence of isospin-independent terms. The extracted symmetry energy from 17 selected Skyrme forces at the density of 0.11 fm^{-3} is about 26.2 ± 1.0 MeV. The slope parameter of symmetry energy is also studied by further combining the neutron-skin thickness of ^{208}Pb. Out of 54 Skyrme forces, ten with $28 \leqslant L \leqslant 65$ MeV can reasonably well describe both the Fermi-energy difference and the neutron-skin thickness of ^{208}Pb. Within the ten forces, four Skyrme forces with the smallest deviation (i.e., Ska25s20, Ska35s20, SV-sym32, and SLy2) have values of $L = 56 \pm 9$ MeV for the slope parameter. The structures and masses of finite nuclei at the ground states is helpful to obtain information on symmetry energy at the subsaturation densities. One should also note that the uncertainty of the symmetry energy extracted from nuclear structures significantly increases with density in the region $\rho > 0.16$ fm^{-3}. It implies that more-isospin-sensitive observations from heavy-ion collisions at intermediate and high energies and neutron stars are still required for further constraining the symmetry energy at the saturation and supersaturation densities.

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