Anomalous gluon production and condensation in glasma

Aiichi Iwazaki
Faculty of International Politics and Economics, Nishogakusha University, 6-16 3-bantyo Tiyoda, Tokyo 102-8336, Japan
(Received 27 August 2012; revised manuscript received 26 November 2012; published 6 February 2013)

Collinear color electric and magnetic fields are thought to be produced immediately after high-energy heavy-ion collisions. We discuss anomalous gluon production under the background gauge fields. The gluons are Nielsen-Olesen unstable modes. The production rate of the modes by the Schwinger mechanism has recently been found to be anomalously larger than the rate of quarks or other stable gluons. Analyzing classical evolutions of the modes with initial conditions given by vacuum fluctuations, we find that their production makes the color electric field decay very rapidly. The lifetime of the field is approximately given by the inverse of saturation momentum in the collisions. We also show that the mode with zero momentum forms a Bose condensate, and its gluon number density grows to be of the order of $1/\alpha_g$. After the saturation of the gluon number density, the condensate melts into quark-gluon plasma owing to nonlinear interactions in QCD.

DOI: 10.1103/PhysRevC.87.024903 PACS number(s): 24.85.+p, 12.38.Mh, 25.75.−q, 03.75.Nt

I. INTRODUCTION

Recently attention has been paid to how color electric (E) and magnetic (B) fields produced in high-energy heavy-ion collisions [1] decay by producing quarks and gluons. They form quark-gluon plasma (QGP). One of the most efficient driving forces for the decay is the Schwinger mechanism [2–4]. The massless quark production rate is similar to the gluon production rate in the mechanism. The difference simply comes from the difference in the color charges, spins, and statistics. However, when the color magnetic field (B) is present, the gluon production rate becomes anomalously larger than the quark production rate. This important fact was recently shown by Itakura and Tanji [5]. They found that the gluon production rate is proportional to $\exp(\pi B/E)$, while the quark production rate is proportional to $\exp(-\pi B/E)$ or unity.

As is well known, the gluons can have imaginary energies under the color magnetic field. Namely, the states with magnetic moments parallel to $\vec{B}$ in the lowest Landau level are unstable; their energies ($\propto\sqrt{p_3^2 - gB}$) are imaginary when $p_3^2 < gB$, where p_3 is a momentum component in the direction parallel to $\vec{B} = (0, 0, B)$ (g is a gauge coupling constant). Thus, their amplitudes exponentially grow with time. The amplitudes of the other states of the gluons with real energies simply oscillate with time. The presence of such unstable modes (Nielsen-Olesen unstable modes [6]) has prevented us from properly quantizing the gluons. Hence, the production of the Nielsen-Olesen unstable gluons by the Schwinger mechanism could not be discussed. But the above authors pointed out that the production rate can be obtained. This is because the unstable modes become stable owing to the acceleration by the electric field. The momentum $p_3(t) = \int^t dt'\, gE$ becomes sufficiently large such that the imaginary energy becomes real for $p_3^2(t) > (gB)^2$. Consequently they have derived the gluon production rate.

The importance of their result is that the production rate, in other words, the occupation number [$\propto \exp(\pi B/E)$] of the gluon state is much larger than unity. This should be compared with the typical occupation number [$\propto \exp(-\pi B/E)$] associated with other stable gluons or massless quarks [3]. It is much less than unity. The large gluon production rate leads to the rapid decay of the electric field. In particular, the decay process accelerates because the production rate $\propto \exp(\pi B/E)$ increases more as the electric field E becomes weaker. Hence, when we discuss the decay, it is important to include the back reaction of the electric field to the gluon production. Furthermore, we may classically analyze the production of the unstable gluons because of the large occupation number of the gluons. In the classical treatment, we can easily take into account the back reaction of the electric field to the gluon production. The back reaction has not yet been discussed by the above authors [5].

In this paper, by assuming that the unstable modes are initially produced by vacuum fluctuations, we analyze the classical evolution of the modes and the color electric field. We find that the electric field rapidly decays owing to the acceleration of the large amount of the unstable gluons. The lifetime of the electric field is approximately given by Q_s^{-1} where Q_s denotes saturation momentum in high-energy heavy-ion collisions. (We assume in this paper that gB and gE are of the order of Q_s^2.) We show that the color electric current carried by the gluons is much larger than that of the quarks when they are produced by the Schwinger mechanism. Thus, the effect of the quarks on the decay of the electric field is negligible compared with that of the gluons. We also show that the unstable gluons form a Bose-Einstein condensate because of the large occupation number of the gluons. We find that the number density of the gluons in the condensate grows to be of the order of $1/g^2$. After the formation of the condensate with such a large occupation number of gluons, it would melt into QGP with the equipartition of the momentum by the nonlinear interactions of QCD.

In the next section we briefly review the Nielsen-Olesen unstable modes and discuss anomalous production of the modes. In Sec. III, we discuss field configurations of the Nielsen-Olesen unstable modes and find basic equations governing the temporal behaviors of the modes as well as the color electric field. In Sec. IV, we find that the gluons of the modes are dominantly produced and their production leads to the rapid decays of the electric field. In tSec. V we show that the Bose-Einstein condensation of the gluons arises in which the gluon number density grows to be of the order of $1/g^2$. We discuss that the condensation with such large occupation number would melt, owing to nonlinear interactions of the gauge fields. In the final section we summarize our results.

II. NIELSEN-OLESEN UNSTABLE MODES

We first explain our formalism and briefly review the Nielsen-Olesen unstable modes. We also explain the anomalous production of the unstable modes under the electric field. We consider SU(2) gauge theory with the background color electric and magnetic fields given by $\vec{E}_a = \delta_{a,3}(0, 0, E)$ and $\vec{B}_a = \delta_{a,3}(0, 0, B)$. They are supposed to be spatially homogeneous and collinear both in the real and color spaces. The gauge fields are represented by the gauge potential $A_\mu \equiv A_\mu^{a=3}$. Under the background fields, the gauge potentials $\Phi_\mu \equiv (A_\mu^1 + iA_\mu^2)/\sqrt{2}$ perpendicular to A_μ^3 behave as charged vector fields. When we represent SU(2) gauge potentials A_μ^a using the variables A_μ and Φ_μ, the Lagrangian of SU(2) gauge potentials is writtenas

$$L = -\frac{1}{4}F_{\mu,\nu}^2 - \frac{1}{2}|D_\mu\Phi_\nu - D_\nu\Phi_\mu|^2 - ig(\partial_\mu A_\nu - \partial_\nu A_\mu) \times \Phi^{\dagger\mu}\Phi^\nu + \frac{g^2}{4}(\Phi_\mu^\dagger\Phi_\nu - \Phi_\nu^\dagger\Phi_\mu)^2, \tag{1}$$

with $F_{\mu,\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $D_\mu = \partial_\mu - igA_\mu$, where we used a gauge $D_\mu\Phi^\mu = 0$. The gauge field A_μ represents both the background gauge field $A_{\mu,b}$ and fluctuations δA_μ. We find that the fields Φ_μ represent charged vector fields with the anomalous magnetic moment described by the term $-ig(\partial_\mu A_\nu - \partial_\nu A_\mu)\Phi^{\dagger\mu}\Phi^\nu$. Therefore, it is easy to see that when the background magnetic field $B = \partial_1 A_{2,b} - \partial_2 A_{1,b}$ is present, but $E = 0$, the particles represented by the fields Φ occupy the Landau levels and interact with each other through the term $\frac{g^2}{4}(\Phi_\mu^\dagger\Phi_\nu - \Phi_\nu^\dagger\Phi_\mu)^2$. The energies of the states in the Landau levels denoted by integer $N \geqslant 0$ are given by $E_N = \sqrt{2gB(N+1/2) \pm 2gB + p_3^2}$, where $\pm$ denotes magnetic moment parallel $(-)$ or antiparallel $(+)$ to $\vec{B}$.

Among them we notice the states in the lowest Landau level ($N = 0$) with the magnetic moment parallel to $\vec{B}$. Their energies can be imaginary: $E_{N=0} = \sqrt{p_3^2 - gB}$. Thus, the modes with the imaginary energies exponentially increase or decrease with time. That is, the field Φ representing the modes evolves with time such that $\Phi \propto \exp(-iE_{N=0}t) = \exp(\pm|E_{N=0}|t) = \exp(\pm|\sqrt{gB - p_3^2}|\,t)$. The states are called as Nielsen-Olesen unstable modes. In particular, the mode with $p_3 = 0$ increases or decreases most rapidly. The presence of such unstable modes implies the instability of the vacuum state, i.e., $\langle\Phi\rangle = 0$, when the background color magnetic field B is present. This is similar to the case that the state $\psi = 0$ is unstable in a model of a complex scalar field with the double-well potential $-m^2|\psi|^2 + \frac{\lambda}{2}|\psi|^4$ $(m^2 > 0)$. In this model unstable modes exist around the state $\psi = 0$ and exponentially grow such as $\psi \propto \exp(t\sqrt{m^2 - \vec{p}^{\,2}})$ where $\vec{p}$ denotes a momentum. Similarly the background gauge fields involving color magnetic fields are unstable. Indeed, the classical simulations [7] have been performed to show the instability of the states with the background gauge fields. The instability in the simulation is considered [8,9] to be caused by the Nielsen-Olesen unstable modes: Their amplitudes exponentially grow and then saturate when nonlinear interactions are effective, owing to the growth of the amplitudes.

The spontaneous production [10] of the unstable modes (or gluons) is caused by the color magnetic field. Thus, it is not the Schwinger mechanism. When we analyze the Schwinger mechanism of the unstable modes, the presence of the imaginary energy was an obstacle because we cannot properly quantize the modes. But we should note that, in order to obtain the gluon production rate by the Schwinger mechanism [3], we only need the in-state (in the infinite past) and the out-state (in the infinite future) under the electric field. Although the Nielsen-Olesen modes are unstable, they are stable in the infinite past and future in the presence of the electric field. This is because the square of the momentum $p_3(t) = \int^t dt'\, gE$ is sufficiently large in the past and future such that $p_3^2(t) - gB > 0$. The stability of the unstable modes in the infinite past and future allows us to estimate the production rate [5] of the modes.

In general, charged fields oscillate with the frequency $\propto gEt$ in the past ($t \to -\infty$) and future ($t \to +\infty$) under the electric field E. The frequency is real and depends on time. They oscillate even in the period between the past and the future. In this case the production rate of the fields is less than unity; it is proportional to, for example, $\exp(-\pi B/E)$ when the magnetic field is present. But, in the case of the unstable modes they pass a period in which their frequency becomes imaginary so that their amplitudes exponentially grow. Before and after the period they simply oscillate with the real frequency. In other words, the modes oscillate and their amplitudes smoothly change in the far past, but once they enter the period, their amplitudes exponentially grow with time. After passing the period, they oscillate again but with much larger amplitudes than those before passing the period. These behaviors are peculiar to the unstable modes. In particular, the exponential growth [5] of the amplitudes leads to the anomalous production rate $\propto \exp(\pi B/E)$.

We may understand a naive physical reason why the rate increases more as the electric field becomes weaker. The large production rate comes from the fact that the modes pass the period in which they exponentially grow. They stay in the period approximately for $\Delta t = \sqrt{gB}/gE$ because the momentum increases in the period such that $\Delta p_3 = \Delta t\, gE = \sqrt{gB}$. Hence, the amplitude grows by $\exp(\sqrt{gB}\Delta t) = \exp(B/E)$. The weaker electric field causes the longer stay in the period and hence the larger growth of the amplitude. Although it is a rough estimation, it explains why the gluon production rate $\propto \exp(\pi B/E)$ becomes large as the electric field becomes weak.

Hereafter we take only the unstable modes and analyze their production under the electric field. The unstable modes are described by the field $\Phi \equiv (\Phi_1 + i\Phi_2)/\sqrt{2}$ and are governed by the following Hamiltonian:

$$H = \int d^3x \big(\tfrac{1}{2}(\partial_0 A_{3,b})^2 + |\partial_0 \Phi|^2 + |(\vec{\partial} - ig\vec{A}_b)\Phi|^2 - 2gB|\Phi|^2\big) = \int d^3x \big(\tfrac{1}{2}(\partial_0 A_{3,b})^2 + |\partial_0 \Phi|^2 + |(\partial_3 - igA_{3,b})\Phi|^2 - gB|\Phi|^2\big), \quad (2)$$

where we neglected the nonlinear interactions $\frac{g^2}{4}(\Phi_\mu^\dagger \Phi_\nu - \Phi_\nu^\dagger \Phi_\mu)^2$ in Eq. (1). The color electric and magnetic fields are given such that $\vec{E} = \partial_0 \vec{A}_b = (0, 0, \partial_0 A_{3,b})$ and $\vec{B} = \vec{\partial} \times \vec{A}_b = (0, 0, B)$. The nonlinear interactions are not effective as long as the amplitude Φ is small. When the field Φ grows large such that the nonlinear interactions are effective, the unstable modes couple with the other modes in higher Landau levels as well as themselves.

We note that the last term in the Hamiltonian represents a negative potential. When the magnetic field forms a flux tube, the term represents a negative potential with finite width given by the width of the flux tube. Whether or not the field Φ possesses unstable modes depends on the existence of the states trapped in the negative potential.

It is interesting to see the analogy between our model and the model of the complex scalar field with the double-well potential mentioned above. The model describes Cooper-pair condensates. Thus, the decay of the color electric field corresponds to the decay of an external electric field imposed on superconductors. The question is how fast the external electric field decays, just after a metal in a normal state is supercooled below the critical temperature at which the normal state ($\langle\psi\rangle = 0$) and superconducting state ($\langle\psi\rangle = \sqrt{m^2/\lambda}$) are separated. The normal state $\psi = 0$ decays by producing the Cooper pairs ψ, which condense to form the state $\langle\psi\rangle = \sqrt{m^2/\lambda}$. Since they are accelerated by the electric field, the electric field loses its energy and vanishes.

III. PRODUCTION OF NIELSEN-OLESEN UNSTABLE MODES

Because the production rate is much larger than unity (this implies that the occupation number in a state is much larger than unity), the production of the unstable gluons may be classically analyzed. Then, we can easily take into account the back reaction of the electric field to the gluon production. In this section we will formulate basic equations governing the back reaction.

First, we discuss the assumption that the background gauge fields are homogeneous in the transverse plane perpendicular to the collinear fields $\vec{B}$ and $\vec{E}$. When the unstable modes are excited, they destroy the homogeneity because of the localization of the wave functions of the modes:

$$\phi \equiv (x_1 - ix_2)^n \exp\left(-\frac{gB|z|^2}{4} + ip_3x_3\right), \quad (3)$$

with $z \equiv x_1 + ix_2$ and integer $n \geqslant 0$ where we used a gauge potential $\vec{A}_b = (-Bx_2/2, Bx_1/2, 0)$. The effect of the back reaction induces the inhomogeneity in the background gauge fields; the currents carried by the modes are not homogeneous so that the background gauge fields affected by the currents are also not homogeneous.

But, by taking the appropriate linear combination of the unstable modes we can form almost homogeneous field configurations in the transverse plane. Then, their currents are also almost homogeneous. Such field configurations are given by

$$\Phi = \sum_{l=1\sim N} \phi_l(\vec{x}), \quad \phi_l(\vec{x}) = \int dp_3\, c(p_3) \exp\left(-\frac{gB|z - z_l|^2}{4} + ip_3x_3\right), \quad (4)$$

with $c(p_3)$ being a dimensionless function of the longitudinal momentum p_3, and $z_l = x_{1,l} + ix_{2,l}$, where each component ϕ_l satisfies the condition $\phi_l\phi_{l'} \simeq \delta_{l,l'}\phi_l^2$ because we impose that $|z_l - z_{l'}| \geqslant l_B \equiv \frac{1}{\sqrt{gB}}$. Namely, a configuration ϕ_l is separated from the nearest neighbors approximately by the distance l_B. Furthermore, we assume that the area L^2 of the transverse plane is given by $L^2 = Nl_B^2$. Thus, we find that the field configuration Φ is approximately uniform in the transverse space. This kind of configuration of the unstable modes was analyzed [11] to discuss so-called "spaghetti vacuum."

Using the field configuration, we rewrite the Hamiltonian of the unstable modes,

$$H = \int d^3x \big(\tfrac{1}{2}(\partial_0 A_b)^2 + |\partial_0 \Phi|^2 + |(i\partial_3 - gA_b)\Phi|^2 - gB|\Phi|^2\big) \simeq N \int d^3x \big(\tfrac{1}{2}(\partial_0 A_b)^2 + |\partial_0 \phi|^2 + |(i\partial_3 - gA_b)\phi|^2 - gB|\phi|^2\big), \quad (5)$$

with $\phi = \int dp_3\, c(p_3) \exp(ip_3x_3 - gB|z|^2/4)$, where the color electric field is given by $E = \partial_0 A_b$ with the homogeneous gauge potential $A_b \equiv A_{3,b}$. The Hamiltonian describes the unstable modes under the homogeneous background electric and magnetic fields. The first term represents the energy of the electric field and the other terms represent the energy of the unstable modes. We can see that the last term with the magnetic field gB represents a negative potential for the unstable modes ϕ. Thus, it gives rise to the imaginary energy of the field $\phi \propto \exp(-i\epsilon t)$ with $\epsilon^2 = -gB < 0$.

If the magnetic field forms a flux tube with a finite width, it gives a negative potential with the finite width. Thus, if the field is trapped by the potential, the energy ϵ can be imaginary, but its absolute value is smaller than $\sqrt{gB}$; it depends on the width of the tube. When the width is infinite, the energy ϵ is given by $\sqrt{-gB}$. On the other hand, when the width is finite, the absolute value of energy ϵ becomes smaller as the width becomes smaller. The flux tubes of the background gauge fields are more realistic field configurations produced in high-energy heavy-ion collisions than the homogeneous ones under consideration. Because the gauge fields are homogeneous in the longitudinal direction, they can be viewed as an ensemble of electric and magnetic flux tubes with various widths. Based on this view, we have discussed [8,9] the results of the numerical simulations [7]. Although the flux tubes are realistic ones, it is meaningful to analyze the anomalous gluon production in the homogeneous gauge fields, in order to see physical essences of the production.

We proceed to analyze the back reaction. For this purpose, we decompose the field $\phi(\vec{x})$ into the components of the momentum eigenstate,

$$\phi = \frac{1}{\sqrt{L^3}} \sum_{\vec{p}} \phi_p \exp(i\vec{p}\cdot\vec{x}) \tag{6}$$

with

$$\phi_p = \frac{8\pi^2 c(p_3)}{gBL^{3/2}} \exp\left(-\frac{p_T^2}{gB}\right), \tag{7}$$

where the transverse momentum p_T is given such that $p_T^2 \equiv p_1^2 + p_2^2$. Then, it follows that

$$H = L^2\left(\frac{L}{2}(\partial_0 A_b)^2 + \frac{1}{l_B^2}\sum_{\vec{p}}\big(|\partial_0\phi_p|^2 + |(p_3+gA_b)\phi_p|^2 - gB|\phi_p|^2\big)\right), \tag{8}$$

where we have used the following formula:

$$\int d^3x \exp(i\vec{p}\vec{x}) = (2\pi)^3\delta^3(p) = L^3\delta^3_{p,0}. \tag{9}$$

The equations of motions of the fields ϕ_p and the gauge field A_b are respectively given by

$$\partial_0^2\phi_p = gB\phi_p - (p_3+gA_b)^2\phi_p \quad \text{and} \quad L\partial_0^2 A_b = -\frac{2g}{l_B^2}\sum_{\vec{p}}(p_3+gA_b)|\phi_p|^2 \tag{10}$$

where the second equation represents a Maxwell equation $\partial_0 E = -J$ with the current $J = \frac{2g}{l_B^2}\sum_{\vec{p}}(p_3+gA_b)|\phi_p|^2$. It describes how the electric field changes by the effect of the current J.

We should point out that the largest amplitude of the unstable modes is given by the mode with the vanishing transverse and longitudinal momentum; that is, the mode with $p_T = 0$ in Eq. (7) and $p_3 + gA_b = 0$ in Eq. (10). The mode grows most rapidly compared with the other modes with $p_3 + gA_b \neq 0$. Here, the momentum $p_3(t) \equiv p_3 + gA_b(t) = p_3 + g\int_0^t dt' E(t')$ denotes that of the mode with the initial momentum $p_3(t=0) = p_3$. We naively expect from the similarity to the model of the complex scalar field that the mode with zero momentum forms a stable Bose condensate. But as we discuss later, although the mode forms a Bose condensate, the condensate becomes unstable when its amplitude grows to the order of $1/g$.

In order to solve the equations we need to impose initial conditions. The initial condition of the electric field is given such that $E(t=0) = \partial_0 A_b(t=0) = E_0$ and $A_b(t=0) = 0$. This corresponds to the fact that we switch on the electric field $E = E_0$ at $t = 0$. In other words, we consider the situation that high-energy heavy-ion collisions occur at $t = 0$ and the color background gauge fields are produced at that instant.

How should we choose initial conditions of the field ϕ? Before the collisions, there are no color electric and magnetic fields. The gluons with small x form color glass condensates in nuclei. Just after the collisions the gluons form the coherent gauge fields E and B, but there is no classical field ϕ. Thus, we may naively choose the initial conditions such that $\partial_0\phi(t=0) = 0$ and $\phi(t=0) = 0$. But these initial conditions lead to the trivial result $\phi(t) = 0$ for any time $t > 0$. Therefore, we need to find other appropriate initial conditions. As we explained in the previous section, the unstable modes are spontaneously generated owing to the instability of the state with the homogeneous magnetic field. Thus, it is reasonable to take the initial conditions given by the vacuum fluctuations,

$$\phi_p(t=0) \equiv \sqrt{\langle\hat{\phi}_p^2\rangle} \quad \text{and} \quad \partial_t\phi_p(t=0) \equiv \sqrt{\langle(\partial_t\hat{\phi}_p)^2\rangle}, \tag{11}$$

where $\hat{\phi}_p$ denotes the momentum component of the free massless scalar field $\hat{\phi}$ with no background fields. The state $|\ \rangle$ represents the vacuum state $\langle\hat{\phi}\rangle = 0$. The vacuum can be represented by the following wave functionals:

$$W(\phi_p) \propto \exp\left(-\sum_{\vec{p}}|p||\phi_p|^2\right) \quad \text{and} \quad W(\partial_0\phi_p) \propto \exp\left(-\sum_{\vec{p}}\frac{|\partial_0\phi_p|^2}{|p|}\right), \tag{12}$$

with $|p| \equiv \sqrt{p_T^2 + p_3^2}$. Namely, we assume that the initial conditions are given by the vacuum fluctuations in the vacuum without E and B.

It apparently seems that we should use vacuum fluctuations when the color magnetic field is present. But, the vacuum fluctuations cannot be defined when B is present, because there is no stable vacuum owing to the Nielsen-Olesen instability. This is the reason why we use the vacuum fluctuations in the vacuum without E and B. Furthermore, what we need to obtain as an initial condition is the longitudinal momentum distribution at $t = 0$. The transverse momentum distribution of the unstable modes has been found since we are only concerned with the states in the lowest Landau level. In this way we take into account the initial conditions given by vacuum fluctuations in the vacuum without E and B. With regard to the initial conditions for the unstable modes, intriguing research [12] has been performed in expanding glasma. In the final section we make a comment on the relation between the initial condition in our paper and the one dictated by the reference.

If we consider the case that the double-well potential, $-m^2|\psi|^2 + \frac{\lambda}{2}|\psi|^4 (m^2 > 0)$, is added to massless scalar field at $t = 0$, it is reasonable to think that the vacuum fluctuations in the vacuum of the massless scalar field evolve to make a stable state $\langle\psi\rangle \neq 0$ just after the addition of the potential. The situation is similar to the case of the glasma.

Using the wave functionals, we shall find the distribution of the Nielsen-Olesen unstable modes in the vacuum. In order to do so, we rewrite the field ϕ_p such that

$$\phi_p = \frac{[\tilde{\phi}_1(p_3) + i\tilde{\phi}_2(p_3)]\sqrt{\pi}}{(gB)^{3/4}L\sqrt{T(p_3 l_B)}} \exp\left(-\frac{p_T^2}{gB}\right) \tag{13}$$

with dimensionless real functions $\tilde{\phi}_i(p_3)$. Then, the wave functionals are given by

$$W(\tilde{\phi}) \propto \exp\left(-\sum_{p_3} \frac{[\tilde{\phi}_1(p_3)]^2 + [\tilde{\phi}_2(p_3)]^2}{4}\right) \quad \text{and}$$
$$W(\partial_0\tilde{\phi}) \propto \exp\left(-\sum_{p_3} \frac{[(\partial_0\tilde{\phi}_1)^2 + (\partial_0\tilde{\phi}_2)^2]U(p_3 l_B)}{4gBT(p_3 l_B)}\right), \tag{14}$$

where

$$T(x) \equiv \int_0^\infty dy\sqrt{y+x^2}\,\exp(-2y) \quad \text{and}$$
$$U(x) \equiv \int_0^\infty dy \frac{\exp(-2y)}{\sqrt{y+x^2}}. \tag{15}$$

This is the distribution of the Nielsen-Olesen unstable modes in the vacuum of the massless scalar field; that is, the distribution of $\tilde{\phi}$. We note that the distribution of the real part $\tilde{\phi}_1$ is identical to that of the imaginary part $\tilde{\phi}_2(p_3)$. Using the distribution, we obtain the expectation values of the unstable modes,

$$\langle \tilde{\phi}_i(p_3)^2\rangle = \frac{\int d\tilde{\phi}_i |W(\tilde{\phi})|^2 \tilde{\phi}_i^2}{\int d\tilde{\phi}_i |W(\tilde{\phi})|^2} = 1 \quad \text{and}$$
$$\langle (\partial_0\tilde{\phi}_i(p_3))^2\rangle = \frac{\int d(\partial_0\tilde{\phi}_i)|W(\partial_0\tilde{\phi})|^2(\partial_0\tilde{\phi}_i)^2}{\int d(\partial_0\tilde{\phi}_i)|W(\partial_0\tilde{\phi}_i)|^2} = \frac{gBT(p_3 l_B)}{U(p_3 l_B)}, \tag{16}$$

with $i = 1, 2$.

Therefore, we find that the equations of motion are given by

$$\partial_0^2\tilde{\phi}(p_3) = gB\,\tilde{\phi}(p_3) - (p_3 + gA_b)^2\tilde{\phi}(p_3) \quad \text{and}$$
$$\partial_0^2 gA_b = -\frac{g^2\sqrt{gB}}{4\pi}\int_{-\infty}^{+\infty} dp_3 \frac{(p_3 + gA_b)\tilde{\phi}(p_3)^2}{T(p_3 l_B)} \tag{17}$$

with $\tilde{\phi} \equiv \tilde{\phi}_i$ and the initial conditions

$$A_b(t=0) = 0, \quad \partial_0 A_b(t=0) = E_0 \quad \text{and}$$
$$\tilde{\phi}(p_3, t=0) = 1, \quad \partial_0\tilde{\phi}(p_3, t=0) = \sqrt{\frac{gBT(p_3 l_B)}{U(p_3 l_B)}}, \tag{18}$$

where we used the relation $\sum_{p_3} = \frac{L}{2\pi}\int_{-\infty}^{+\infty} dp_3$.

We should stress that the initial conditions of $\tilde{\phi}$ were determined with the distribution of the unstable modes in the vacuum without E and B. The initial condition gives the longitudinal momentun distribution $\tilde{\phi}$ at $t = 0$. Namely, we have simply determined the dependence on p_3, i.e., $\tilde{\phi}(p_3)$ of the Nielsen-Olesen unstable modes ϕ_p in Eq. (13) by taking the vacuum functional in Eq. (14) and using it as the initial condition.

Obviously both the equations of motion and initial conditions are independent on the system size L. Furthermore, the color electric current J is given by

$$J = \frac{g\sqrt{gB}}{4\pi}\int_{-\infty}^{+\infty} dp_3 \frac{(p_3 + gA_b)\tilde{\phi}(p_3)^2}{T(p_3 l_B)}, \tag{19}$$

which is also independent on L. The current vanishes at $t = 0$ because $T(x) = T(-x)$ and $A_b(t=0) = 0$. Using these equations of motion, we can discuss the temporal behaviors of the electric field $E(t) = \partial_0 A_b$ and the unstable modes $\tilde{\phi}(p_3, t)$ by taking into account the back reaction of the electric field to the production of the modes.

For convenience, we write down the field ϕ of the unstable modes in terms of the variable $\tilde{\phi}$,

$$\phi = \frac{1}{\sqrt{L^3}}\sum_{\vec{p}} \phi_p \exp(i\vec{p}\cdot\vec{x})$$
$$= \frac{1}{\sqrt{L^3}}\sum_{\vec{p}} \frac{[\tilde{\phi}(p_3) + i\tilde{\phi}(p_3)]\sqrt{\pi}}{(gB)^{3/4}L\sqrt{T(p_3 l_B)}}\exp\left(-\frac{p_T^2}{gB}\right)\exp(i\vec{p}\cdot\vec{x}), \tag{20}$$

where $\tilde{\phi}(p_3)$ is a dimensionless real function. We note that the vacuum fluctuation $\phi(t=0)$ of the unstable modes is of the order unity, while the background gauge fields B and $E(t=0) = E_0$ are of the order of $\sim O(1/g)$.

IV. NUMERICAL RESULTS

Now we wish to discuss the production of the Nielsen-Olesen gluons and quarks. Especially, we would like to discuss the ratio between the amount of the gluons and that of the quarks produced by the electric field. In order to discuss the amounts of the particles we compare the color electric current of the quarks with that of the gluons. We show that the amount of the gluons is about a hundred times larger than that of the quarks. As a result the color electric field rapidly decays owing to this anomalous gluon production.

First we derive a relevant equation describing the quark production by the Schwinger mechanism. The equation has been previously derived [4]. We would like to explain it briefly. We assume that the quarks are massless and that they form a SU(2) doublet. Then, the color charges of the quarks coupled with A_μ are given by $g/2$ and $-g/2$. Both of them possess their antiparticles with their charges given by $-g/2$ and $g/2$, respectively. Therefore, we have four massless fermions: a pair of quarks (q_+ and q_-) in a SU(2) doublet and their antiquarks ($\bar{q}_+$ and $\bar{q}_-$). The quarks (q_+ and $\bar{q}_-$) have the positive charge $g/2$ and the quarks (q_- and $\bar{q}_+$) have the negative charge $-g/2$. Their number densities are identical to each other because a pair of positive and negative charged quarks is created at the same moment under the electric field.

It was recently shown [4,13] that the evolution of the number density n_q of the massless fermions is governed by the chiral anomaly when collinear strong electric and magnetic fields are present. In particular, the anomaly equation becomes very simple when the magnetic field is sufficiently strong such that the particles produced occupy only the states with the lowest energy. Namely, the equation of the chiral anomaly is given by

$$\partial_0 J_0^5 = 4\partial_0 n_q = 2\frac{(g/2)^2 E(t) B}{2\pi^2}, \tag{21}$$

where we assumed homogeneity of the chiral current in the transverse and longitudinal directions $\vec{\partial} \cdot \vec{J}^5 = 0$. The equality $\partial_0 J_0^5 = 4\partial_0 n_q$ in Eq. (21) comes from the fact that all of the four fermions have the positive chirality when $\vec{E}$ is parallel to $\vec{B}$. This is because the positive (negative) charged fermions are accelerated in the direction parallel (antiparallel) to $\vec{E}$ and their spins are pointed in the direction (antiparallel) parallel to $\vec{B}$ when they occupy the states in the lowest Landau level.

Obviously, the chiral anomaly in Eq. (21) describes how the number density n_q evolves with time under the effect of the electric and magnetic fields. The electric field loses its energy owing to the acceleration of the quarks as well as the gluons. Hence, we add the contribution of the quarks to the Maxwell equation. Consequently, the equations describing the evolution of the numbers of the quarks and the gluons as well as the evolution of the electric field are given by

$$\begin{aligned} \partial_0 n_q &= \frac{g^2 E(t) B}{16\pi^2}, \\ \partial_0^2 \tilde{\phi}(p_3) &= gB\,\tilde{\phi}(p_3) - (p_3 + gA_b)^2 \tilde{\phi}(p_3), \\ \partial_0^2 gA_b &= -4g^2 n_q - \frac{g^2\sqrt{gB}}{4\pi}\int_{-\infty}^{+\infty} dp_3 \frac{(p_3 + gA_b)\tilde{\phi}(p_3)^2}{T(p_3 l_B)}, \end{aligned} \tag{22}$$

where $4gn_q$ denotes the current of the four kinds of massless quarks. The initial conditions are given by

$$\begin{aligned} n_q(t=0) &= 0, \quad \tilde{\phi}(p_3, t=0) = 1, \\ \partial_0 \tilde{\phi}(p_3, t=0) &= \sqrt{\frac{gB\,T(p_3 l_B)}{U(p_3 l_B)}}, \\ A_b(t=0) &= 0, \quad \text{and} \quad \partial_0 A_b(t=0) = E_0. \end{aligned} \tag{23}$$

By solving these equations we can see how the electric field vanishes owing to the production of the unstable modes and the quarks. Furthermore, we can obtain the temporal behaviors of the electric current densities of the quarks $J_q = 4gn_q$ and the gluons $J_g = \frac{g\sqrt{gB}}{4\pi}\int_{-\infty}^{+\infty} dp_3 \frac{(p_3+gA_b)\tilde{\phi}(p_3)^2}{T(p_3 l_B)}$. Both of them vanish at $t = 0$. After the electric field is switched on at $t = 0$, the pair production of the quarks arises and their electric current flows. Similarly, the gluons, as the unstable modes, are produced as the vacuum fluctuations and their electric current flows along the electric field. Owing to the production of the quarks and the gluons, the electric field decreases and vanishes at $t = t_c > 0$. Hence we compare the electric current of the quarks with that of the gluons at $t = t_c$ when the electric field vanishes,

$$R(t = t_c) = \frac{J_g}{J_q} = \frac{\frac{g\sqrt{gB}}{4\pi}\int_{-\infty}^{+\infty} dp_3 \frac{(p_3 + gA_b(t=t_c))\tilde{\phi}(p_3, t=t_c)^2}{T(p_3 l_B)}}{4gn_q(t = t_c)}, \tag{24}$$

where all of the integration range of the momentum p_3 has been taken. But the relevant modes we should take into account are the unstable modes. The integration range should be limited to the range in which each mode $\tilde{\phi}(p_3, t)$ can exponentially increase. For example, the modes $\tilde{\phi}(p_3, t)$ with $p_3 > \sqrt{gB}$ never exponentially increases, so we should not include those modes. On the other hand, the mode $\tilde{\phi}(p_3, t)$ with $|p_3 + gE_0 t_c| > \sqrt{gB}$ and $p_3 < 0$ can pass the period in which

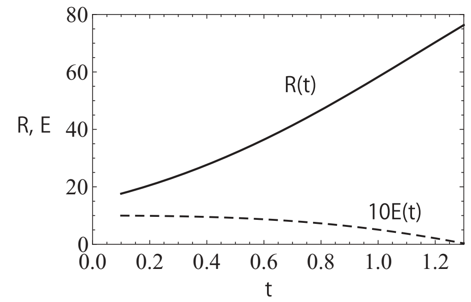

FIG. 1. Temporal behaviors of the ratio $R(t)$ (solid line) and ten times the electric field, $10 \times E(t)$ (dashed line).

it exponentially increases until the electric field vanishes. Hence, the relevant integration range is approximately given such that $|p_3 + gE_0 t_c| > \sqrt{gB}$ for $p_3 < 0$ and $p_3 < \sqrt{gB}$ for $p_3 > 0$. As our analysis of the gluons is classical, our estimation of the ratio is not rigorous. Thus, rigorously speaking, we do not know the appropriate integration range: If we take all of the integration range, modes irrelevant to Nielsen-Olesen instability are taken into account. Therefore, we may limit the integration range such as $|p_3| < \sqrt{gB}$, in order to see roughly how large is the amount of the gluons produced compared with that of the quarks.

In Fig. 1 we show the temporal behaviors of the electric field and the ratio R with the parameters $g = B = E_0 = 1$. We have taken the integration range $|p_3| < \sqrt{gB}$ and checked that the result does not change even if the integration range $|p_3| < 1.2\sqrt{gB}$ is taken. We find that, when the electric field vanishes, the electric current of the gluons is approximately 80 times larger than that of the quarks. Owing to this fact, the lifetime of the electric field is much shorter than the life-time only when the quark production is taken into account. Actually, the above equations can be explicitly solved if the contribution of the gluons is neglected, i.e., $\tilde{\phi} = 0$. The solution of the electric field is given by

$$E = E_0 \cos\left(\frac{\sqrt{g^3 B}\,t}{2\pi}\right). \tag{25}$$

The solution represents a plasma oscillation [3,4]. Hence, the lifetime t_c at which E vanishes is given by $\pi^2/\sqrt{g^3 B}$, which is equal to $\pi^2 \sim 10$ with $g = B = 1$. It is roughly 8 times longer than the lifetime shown in the figure. The lifetimes are given by $1.3Q_s^{-1}$ and $10Q_s^{-1}$ respectively in the physical unit Q_s^{-1}; Q_s (=1–2 GeV) denotes the saturation momentum of high-energy heavy_ ion collisions in the Relativistic Heavy Ion Collider (RHIC) or Large Hadron Collider (LHC). In this way the decay of the electric field is mainly caused by the anomalous gluon production; that is, the production of the unstable modes. The contribution of the quarks is negligible. As we show in next section, the lifetime t_c is of the order of Q_s^{-1}, while it is of the order of $Q_s^{-1}g^{-1}$ in the case of no gluon production. Thus, R becomes larger as g becomes smaller, because the quark production is suppressed as g becomes small.

The unstable modes are generated at $t = 0$ by the vacuum fluctuations and are amplified by the magnetic field. At the same time, they are accelerated by the electric field. In Fig. 2 we show the temporal behavior of the momentum distribution

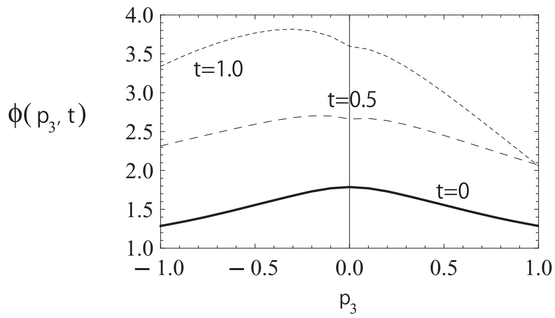

FIG. 2. $\phi(p_3,t) \equiv \tilde{\phi}(p_3,t)/\sqrt{T(p_3 l_B)}$ at $t=0$ (solid line), $t=0.5$ (dashed line), and $t=1.0$ (short-dashed line).

ϕ_p or $\tilde{\phi}(p_3,t)/\sqrt{T(p_3 l_B)}$, that is, the growth of the amplitude with time. We can see that the vacuum fluctuations (shown by the curve at $t=0$ in Fig. 2) give the momentum distribution symmetric in p_3 with the peak at $p_3=0$. The peak moves to points with negative momentum with time. This is because the gluons are accelerated by the electric field so that the momentum with which the amplitude $\tilde{\phi}(p_3,t)/\sqrt{T(p_3 l_B)}$ has the largest growth rate is given by $p_3 = -\int_0^t dt' gE(t') = -gA_b(t) < 0$.

We can see, however, from the figure that the peak is not located at $p_3 = -\int_0^t dt' gE(t')$, but at momentum near $p_3=0$ as long as t is small. This is because the initial condition $\frac{\partial_0\tilde{\phi}(p_3,t=0)}{\sqrt{T(p_3 l_B)}} = \sqrt{\frac{gB}{U(p_3 l_B)}}$ becomes larger as $|p_3|$ becomes larger. Thus, as long as t is small, the peak stays at $p_3=0$ by the effect of the initial condition. The effect diminishes with time. Indeed, the maximum of the amplitude approaches the value $\phi_p(p_3 = -\int_0^t dt' gE(t'))$ as t goes beyond 0.5, while it is given by $\phi_p(p_3 \simeq 0)$ at $t < 0.5$.

V. GLUON CONDENSATION

We have mentioned the similarity between the Nielsen-Olesen unstable modes and unstable modes in the model of the complex scalar field. The model describes Cooper pairs in superconductors. Obviously, the unstable modes with zero momentum in the model form a Bose condensate $\langle\psi\rangle = \sqrt{m^2/\lambda}$ of the Cooper pairs. That is, once the modes are produced, the amplitudes of the modes exponentially grow and take the value of the stable state. They are stabilized by the nonlinear interaction $\lambda|\psi|^4$. Hence, it is natural to expect from the similarity that the unstable modes of the gluons may form a Bose condensate [14].

The unstable modes typically carry the momentum $\vec{p} = (p_T \simeq \sqrt{gB},\, p_3 \simeq \sqrt{gB})$. This is because the modes have the spatially transverse extension given by $1/\sqrt{gB}$ and their longitudinal momentum p_3 given by $\Delta p_3 = -gE\Delta t$, owing to the acceleration by the electric field with the interval $\Delta t = \sqrt{gB}/gE$. Among the unstable modes, the mode with the vanishing transverse and longitudinal momenta ($p_T=0$ and $p_3 + gA_b = 0$), has the largest amplitude and growth rate. Furthermore, as we will show below, the amplitude of the mode can grow to the order of $1/g$ until the nonlinear interactions among the unstable modes are effective. Hence, we may think that the mode forms a Bose condensate of gluons with zero momentum.

We have classically discussed the gluon production in the previous sections; the formulas may be regarded as the ones concerning the gluon condensate. We now show using the formulas that the number density n_g of the gluons produced is of the order of Q_s^3/g^2, i.e., $n_g \sim Q_s^3/g^2$ for small $g \ll 1$. The most relevant mode for the production is the one with zero momentum, $p_3=0$, as shown in Fig, 2. As long as $gA_b \ll \sqrt{gB} = Q_s$, we can approximately obtain a solution $\tilde{\phi}(0) = \exp(Q_s t)$ from the second equation with $p_3=0$ in Eq. (22). The solution holds near $t=0$ because of the initial condition $gA_b(t=0)=0$. The solution approximately holds until gA_b is comparable with Q_s. In order to see how gA_b increases with time, we rewrite the Maxwell equation in Eq. (22) such that

$$\begin{aligned}\partial_0^2 gA_b &= -\frac{g^2\sqrt{gB}}{4\pi}\int_{-\sqrt{gB}}^{+\sqrt{gB}} dp_3 \frac{(p_3+gA_b)\tilde{\phi}(p_3)^2}{T(p_3 l_B)}\\ &\sim -\frac{g^2\sqrt{gB}}{4\pi}\frac{2\sqrt{gB}\,gA_b\,\tilde{\phi}(0)^2}{T(0)}\\ &= -g^2\frac{Q_s^2\,\tilde{\phi}(0)^2 gA_b}{2\pi T(0)},\end{aligned} \tag{26}$$

with $T(0) = (\sqrt{\pi/2})/4$, where we neglected the contribution of the quarks. As long as $g^2\tilde{\phi}(0)^2$ is very small, the electric field $E = g\partial_0 A$ slowly decreases. But, once $g^2\tilde{\phi}(0)^2$ reaches the order of 1, the electric field $E = g\partial_0 A_b$ rapidly decreases and vanishes. Thus, we may approximately estimate the lifetime t_c of the electric field such that $g^2\tilde{\phi}(0)^2 = g^2\exp(Q_s t_c) = 1$, i.e., $t_c = -Q_s^{-1}\ln(g^2)$. In this way we find that $\tilde{\phi}(0, t=t_c)^2 \sim g^{-2}$ and $gA_b \sim gE_0 t_c \sim Q_s$ when the electric field vanishes. The unstable mode $\phi \propto \tilde{\phi}(0)$ grows to the order of g^{-1}. Using the result we can show that the number density of the condensed gluons is of the order of g^{-2}. Here we remember that the current carried by the four kinds of massless quarks is given by $4gn_q$, where n_q denotes the number density of a kind of the quarks. Thus, we may define the number density n_g of gluons such as $n_g = J_g/g$ in the classical approximation. Then, it follows that

$$n_g = \frac{gB\tilde{\phi}(0, t=t_c)^2 gA_b(t)}{2\pi T(0)} \sim \frac{Q_s^3}{g^2}. \tag{27}$$

Therefore, we find that the gluon condensate arises owing to the anomalous gluon production and that the number density of the gluons increases up to the order of $Q_s^3 g^{-2}$. The result is expected [14] when the number of gluons is conserved; in other words, nonlinear interactions violating the gluon number conservation are not effective. The expectation comes from the fact that the number density of the gluons forming the background gauge fields A_b is of the order of $Q_s^3 g^{-2}$, while the number density of thermalized gluons produced by the decay of A_b is of the order of Q_s^3. (The energy density ϵ_g of the thermalized gluons is given such that $\epsilon_g \propto n_g^{4/3}$ in terms of n_g or $\epsilon_g \propto Q_s n_g$ since the typical energy of the gluon is Q_s. Thus, we find that $n_g \propto Q_s^3$.) In order to derive our result, we simply use the fact that the initial magnitude of the unstable modes is of the order unity. Thus, our result does not depend on the details of the initial conditions for the unstable mode $\tilde{\phi}(0, t = 0)$ used in our paper.

In our calculations we have not included interactions within the condensate, namely, we have neglected the four-point interactions of Φ. (The field Φ involves modes in higher Landau levels as well as the unstable modes in the lowest Landau level.) Once the interactions are effective, the condensate would melt. Contrary to the expectation from the similarity to the model of the complex scalar field, the nonlinear interactions in the gauge theory do not stabilize the condensate. The interactions cause the momentum transfer from the mode ϕ to the other stable modes in higher Landau levels or they produce new type of unstable modes. Indeed, as we have shown in the previous paper [9], seconday Nielsen-Olesen unstable modes are induced after the primary unstable modes ϕ grow sufficiently large for the nonlinear interactions to be effective. That is, the localized electric currents of each ϕ_l in Eq. (4) become large and induce a magnetic field surrounding the current of ϕ_l. We called it the azimuthal magnetic field in the paper. Under the azimuthal magnetic field, the secondary Nielsen-Olesen unstable modes are induced. The modes carry [7] larger momentum p_3 than Q_s, while the unstable modes primarily induced carry smaller momentum than Q_s. The primary unstable modes form the Bose condensate with zero momentum, which induces the secondary unstable modes with large momentum. In this way, the momentum transfer occurs from the condensate to the secondary unstable modes. As a result the condensate would melt. The excitations of the secondary unstable modes are caused by the nonlinear interactions. The result has also been expected in Ref. [14]. We would like to point out that the cascade from small momentum to large momentum shown above can be seen in a model of scalar fields [15].

We can estimate when the nonlinear interactions are effective. We note that the nonlinear interactions are given schematically by $g^2\Phi^4$ or $g(A_b + \delta A)\Phi^2$ in Eq. (1); the background gauge fields A_b are assumed to be of the order of $1/g$. The interactions $g^2\Phi^4$ or $g\delta A\Phi^2$ are smaller than the kinetic terms of Φ and A_b if the amplitude of the unstable mode ϕ or Φ is much less than the order of $1/g$. But when Φ reaches the order of $1/g$, all of the interaction terms become of the same order of magnitude as the kinetic terms. (Note that the fluctuation δA is proportional to the term $g\Phi^2$ in the equation of motion of δA and becomes the same order of magnitude as Φ when $\Phi \sim g^{-1}$.) Hence, the nonlinear interactions become effective when $\Phi \sim g^{-1}$ or the number density of the condensate reaches the order of g^{-2}.

Consequently, owing to the anomalous gluon production, the Bose condensate appears in the process of the decay of the background gauge fields E and B. However, the condensate melts after the number density of the gluons in the condensate grows to the order of g^{-2}. Eventually, thermalized QGP would be realized.

VI. SUMMARY AND DISCUSSION

Motivated by the recent study of the anomalous production of the Nielsen-Olesen unstable modes by the Schwinger mechanism, we have discussed the decay of the color electric field in the classical approximation by taking into account the back reaction of the electric field to the gluon and quark production. We have found that the electric field rapidly decays owing to the anomalous production of the gluons. It has turned out that the contribution of the quarks to the decay is negligible. We have also found that the amount of the produced gluons is about a hundred times larger than that of the quarks.

A model of color glass condensate predicts that the color electric and magnetic fields are produced immediately after high-energy heavy-ion collisions. Fluid dynamical simulations of thermalized QGP suggest that the fields should decay into the plasma within the time 1 fm$/c$. Our analysis indicates that such a very fast decay is caused by the anomalous gluon production. Actually, our analysis shows that the decay is completed within a time of the order of Q_s^{-1}.

We have also shown that the gluons of the Nielsen-Olesen unstable mode form a Bose condensate with zero momentum. The number of the gluons in the condensate rapidly increases and is saturated when it becomes of the order of $1/g^2$. After the saturation, the nonlinear interactions are effective so that the rapid momentum transfer from the condensate to modes with large momentum ($>\sqrt{gB}$) arises. Hence the equipartition of the momentum and the thermalization of QGP would be achieved.

We have discussed the decay of the color electric field in the glasma. The color magnetic field in the glasma also decays as follows. In general the longitudinal color magnetic fields form flux tubes, which expands with time. Owing to the expansion, electric field δE_T perpendicular to the magnetic field is induced according to Faraday's law of induction. On the other hand, owing to the expansion of the longitudinal electric flux tubes, magnetic field δB_T is induced, which is parallel or antiparallel to the electric field δE_T. Thus, under the field δB_T, Nielsen-Olesen unstable modes $\delta\phi_T$ are excited and make the electric field δE_T decay rapidly. Eventually, the expansion of the magnetic flux tube induces the electric field δE_T, which decays owing to the acceleration of the unstable modes $\delta\phi_T$. This implies the decay of the magnetic flux tube. This is the decay mechanism of the magnetic field.

We have discussed the gluon production in the nonexpanding glasma. When we treat it in the expanding glasma, a similar analysis is possible. But, we should use initial conditions for the unstable modes shown in the recent paper, Ref. [12]. We can show that the "free fluctuations" in the reference are identical to the vacuum fluctuations in our discussion when the free fluctuations are formulated in the Cartesian coordinates. On the other hand, relevant fluctuations are "fluctuations in the Glasma." Thus, the initial condition in our paper apparently seems not to be appropriate. But, the fluctuations in Glasma are typically represented by field configurations in the lowest Landau level. We have used such a typical field configuration as the initial condition. In that sense, our choice of the initial condition is not necessarily inappropriate, although the longitudinal momentum distribution in the initial condition is different from the ones of the fluctuations in the Glasma. In both cases, the magnitudes of the initial unstable modes are of order unity. Our results do not depend on the details of the initial conditions. Thus, our results might hold in general. We wish to discuss the decay of glasma by using initial conditions dictated in Ref. [12] in the near future.

ACKNOWLEDGMENTS

We would like to express thanks to Dr. K. Fukushima for calling attention to the recent paper by Berges and Sexty.

[1] E. Iancu, A. Leonidov, and L. McLerran, arXiv:hep-ph/0202270; E. Iancu and R. Venugopalan, in *Quark Gluon Plasma 3*, edited by R. C. Hwa and X.-N. Wang (World Scientific, Singapore, 2004), p. 249.

[2] J. Schwinger, Phys. Rev. **82**, 664 (1951); A. Casher, H. Neuberger, and S. Nussinov, Phys. Rev. D **20**, 179 (1979); K. Kajantie and T. Matsui, Phys. Lett. B **164**, 373 (1985); M. Gyulassy and A. Iwazaki, *ibid*. **165**, 157 (1985).

[3] N. Tanji, Ann. Phys. (NY) **324**, 1691 (2009), and references therein.

[4] A. Iwazaki, Phys. Rev. C **80**, 052202 (2009); **84**, 065203 (2011); **85**, 034909 (2012).

[5] N. Tanji and K. Itakura, Phys. Lett. B **713**, 117 (2012).

[6] N. K. Nielsen and P. Olesen, Nucl. Phys. B **144**, 376 (1978).

[7] P. Romatschke and R. Venugopalan, Phys. Rev. Lett. **96**, 062302 (2006); Phys. Rev. D **74**, 045011 (2006); J. Berges, S. Scheffler, and D. Sexty, *ibid.* **77**, 034504 (2008); K. Fukushima and F. Gelis, Nucl. Phys. A **874**, 108 (2012).

[8] A. Iwazaki, Phys. Rev. C **77**, 034907 (2008); Prog. Theor. Phys. **121**, 809 (2009); H. Fujii and K. Itakura, Nucl. Phys. A **809**, 88 (2008).

[9] H. Fujii, K. Itakura, and A. Iwazaki, Nucl. Phys. A **828**, 178 (2009).

[10] A. Iwazaki, Phys. Rev. C **77**, 034907 (2008).

[11] H. B. Nielsen and M. Ninomiya, Nucl. Phys. B **156**, 1 (1979).

[12] K. Dusling, F. Gelis, and R. Venugopalan, Nucl. Phys. A **872**, 161 (2011).

[13] Y. Hidaka, T. Intani, and H. Suganuma, AIP Conf. Proc. **1388**, 516 (2011).

[14] J. Blaizot, F. Gelis, J. Liao, L. McLerran, and R. Venugopalan, arXiv:1107.5296.

[15] J. Berges and D. Sexty, Phys. Rev. Lett. **180**, 161601 (2012).