Empirical evidence for magic numbers of superdeformed shapes

Neha Sharma,[1,3] H. M. Mittal,[1] Suresh Kumar,[2] and A. K. Jain[3]
[1]*Dr. B. R. Ambedkar National Institute of Technology, Jalandhar 144011, India*
[2]*Department of Physics and Astrophysics, University of Delhi, Delhi 110007, India*
[3]*Department of Physics, Indian Institute of Technology, Roorkee 247667, India*
(Received 17 July 2012; revised manuscript received 31 December 2012; published 27 February 2013)

We present for the first time many types of empirical evidence that point to the existence of preferred neutron/proton numbers for superdeformed (SD) shapes. We use a simple premise based on the pairing correlations to obtain the proposed empirical evidence. In particular, plots of γ-ray energy ratio such as $R(I) = E_\gamma(I \rightarrow I\text{-}2)/E_\gamma(I\text{-}2 \rightarrow I\text{-}4)$ vs N and Z, $R(I)$ vs I plots, nuclear softness parameter values, and the number of SD bands for a given N and Z are used to pinpoint the N, Z numbers that are most favored as the deformed magic numbers. The proton and neutron magic numbers so obtained not only confirm the earlier theoretical predictions made for the chain of particle numbers corresponding to the SD shapes but also verify the increase in deformation with the particle number within each chain. The analysis also leads to several new predictions for the occurrence of the SD bands.

DOI: 10.1103/PhysRevC.87.024322 PACS number(s): 21.60.−n, 21.10.Re, 21.10.Pc, 27.70.+q

I. INTRODUCTION

Historically, the spherical magic numbers have played a key role in the development of nuclear physics and nuclear models [1]. Many pieces of empirical evidence support the existence of spherical magic numbers (N or Z = 2, 8, 20, 28, etc.). Prominent among these are the behavior of the neutron separation energies and binding energies, the number of isotones and isotopes for a given N and Z, respectively, the excitation energies of the first 2^+ states, and the ratio of the first 4^+ to 2^+ level energies in the even-even nuclei. The relationship of shapes and shell structure has played a vital role in the development of nuclear structure physics [2–5], because the idea of magic numbers has been generalized to nonspherical shapes. It is now well known that the shell closures at the 2:1 axes ratio in the deformed oscillator potential lead to a good understanding of the phenomenon of high-spin superdeformation and the fission isomers. Hundreds of superdeformed (SD) bands in different mass regions [6] and the fission isomers in the heavy-mass nuclei have been observed. Their understanding in terms of the shell-correction-based models testifies to the success of the shell-correction-based approaches. For detailed discussion on SD bands we refer the reader to Refs. [7,8]. In the past, much attention has been paid to the SD bands in the $A = 150$ mass region, where ^{152}Dy ($Z = 66$, $N = 86$) has been used as a doubly magic core to obtain some of these unique features such as the incremental alignment and the identical band nature of the SD bands [9–12]. An empirical analysis of the SD bands [13], based on the dynamical features of a semiclassical analysis of the cranking and the particle rotor models [14,15], also pointed out many general features such as the rigid-body nature of the moment of inertia and a negative alignment. It was suggested that the SD bands are unique structures located around twin stable fixed points in a sea of chaos. A detailed numerical analysis of the semiclassical model in the nonlinear regime and a comparison with the experimental data were then carried out to obtain the features such as the $\Delta I = 2$ staggering [16].

In searching for the SD configurations and their theoretical ramifications, Ragnarsson *et al.* [4], in 1978, pointed out the existence of chains of particle numbers for SD configurations by using the harmonic oscillator potential and also the modified oscillator potential. Later, Bengtsson *et al.* [17] and Dudek *et al.* [18] pointed out the same with the Woods-Saxon potential. It was also pointed out [4] that the increasing mass number A leads to increasing deformation of the SD bands, which was later established by several calculations in the mass region 110 to 150 [17–22]. However, no direct empirical evidence has so far been presented to confirm these predictions by using the simple premise as mentioned above for the spherical magic numbers. Sufficient data on SD bands now allows us to carry out such an analysis in many ways. In this paper, we present many types of empirical evidence and reconfirm many of the previous theoretical predictions [4,17,18]. We show that the SD configurations occur for chains of the particle numbers, and that the deformation increases with the particle number. In doing so, we are able to identify the most probable SD magic numbers.

This paper is organized in the following way. A description of the various possible signatures/criteria for the preferred SD particle numbers is presented in Sec. II. We apply these criteria to the SD bands throughout the nuclear chart in Sec. III; the results are presented for the SD bands, broadly classified into three groups according to the increasing magnitude of deformation. In doing so, we have excluded the few cases of the highly deformed bands observed in the $A = 40$ mass region [23,24]. These bands have softness parameters almost two orders of magnitude larger than those for the SD bands and are well connected to the normal deformed bands. Conclusions are presented in Sec. IV.

II. SIGNATURES OF THE SUPERDEFORMED MAGIC NUMBERS

Existence of a rotational motion is intimately linked to the presence of deformation. The magic numbers for the SD shapes

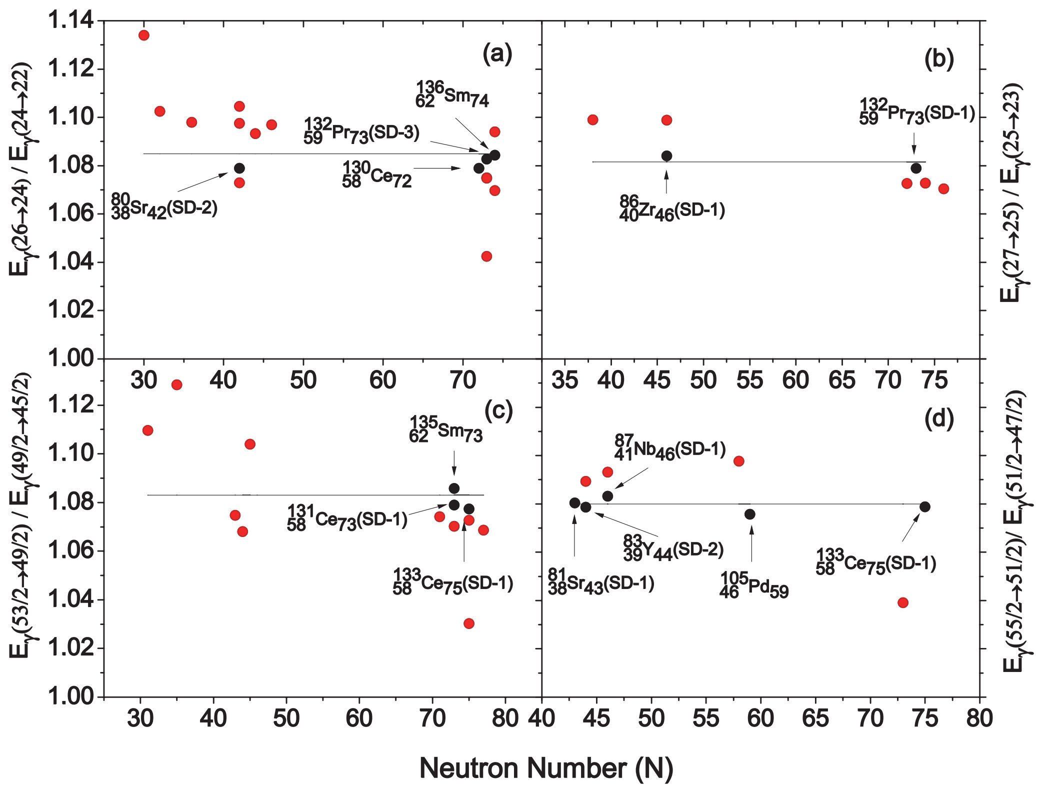

FIG. 1. (Color online) (a)–(d) γ-ray energy ratio $R(I)$ vs neutron number N for deformation 1.5:1 group of SD bands for the $A = 57$–137 mass region. The line in the graph shows the rigid rotor value for the ratio. The nuclides approaching the rigid rotor line have been identified and are pointed by dark circles.

thus support rotation in contrast to the magic numbers for the spherical shape. Hence, the signatures of magic numbers for the spherical and the SD shapes cannot be the same. The observed high-spin SD bands are known to display a near-rigid rotor behavior. Because the "magic" numbers correspond to the positions of low-level density or gaps in the single-particle level scheme, we surmise that the SD bands corresponding to the SD magic numbers will be more exact rigid rotors than their neighbors. This is because of the reduced pairing, which is weaker because of low-level density near the Fermi energy. We then use three different criteria to pinpoint those SD bands that are most rigid; this in turn leads to a set of particle numbers for neutrons and protons where the SD bands are expected to be clustered. This is further supported by a fourth signature: The N and Z values where maximum numbers of SD bands have been observed just like the observation of a large number of isotopes/isotones for the spherical magic numbers. A chain of particle numbers emerges consisting of these deformed magic numbers.

A. The energy ratio $R(I)$

One of the most common signatures of rigidity can be a quantity like the $(E4/E2)$ energy ratio in the deformed even-even nuclei. Since the SD bands are high-spin bands and their band-head energies are generally not known, we cannot use such an energy ratio. Instead, we use the ratios of the γ-ray transition energies. We calculate the ratios like $R(I) = E_\gamma(I \rightarrow I\text{-}2)/\ E_\gamma(I\text{-}2 \rightarrow I\text{-}4)$ and plot these values vs N and Z for all the SD bands where the same ratio can be calculated (this is only possible in those SD bands where the same set of spins are known). The SD bands, therefore, are divided into several groups so that the same ratio can be calculated for all the members of a group. A typical ratio, for example, may be $E_\gamma(53/2 \rightarrow 49/2)/E_\gamma(49/2 \rightarrow 45/2)$. This ratio, when plotted as a function of N or Z, must be closest to the rigid rotor value for those neutron or proton numbers that are like magic numbers for the SD shapes. We find that this approach to identifying the SD magic numbers is simple yet robust.

B. Plots of the energy ratio $R(I)$ vs I

The idea of the energy ratio may be extended further to look at the behavior of the ratio $R(I)$ for the whole range of spins of a SD band. These $R(I)$ vs I plots are then compared with similar plots for an ideal rigid rotor over the same spin range. The SD band closest to the rigid rotor behavior can be easily identified and helps us in confirming the SD magic numbers.

C. Softness parameter for the SD bands

The softness parameter, which is a small parameter in the expansion of energy in terms of $I(I+1)$, can be a very good

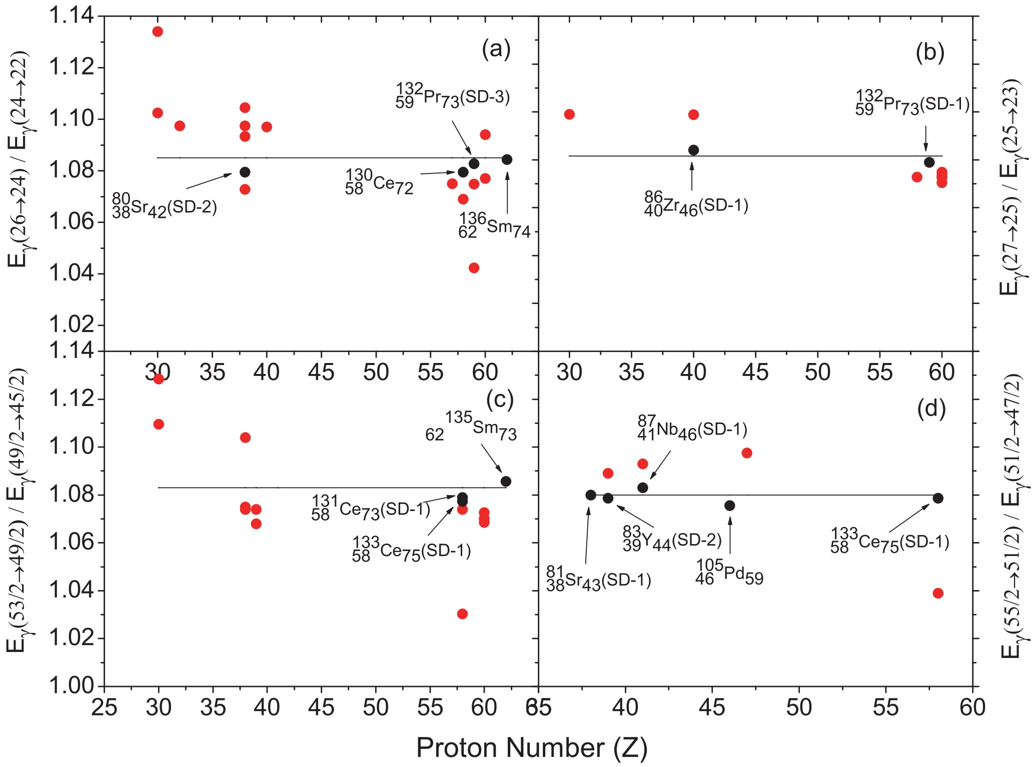

FIG. 2. (Color online) (a)–(d) γ-ray energy ratio $R(I)$ vs proton number (Z) for deformation (1.5:1) group of SD bands for the $A = 57$–137 mass region. The line in the graph shows the rigid rotor value for the ratio. The nuclides approaching or lying on the rigid rotor line have been identified and pointed by dark circles.

measure of the rigidity of bands [2]. The softness parameter values can be obtained, for example, by fitting a four-parameter formula to the SD bands. The softness parameter is expected to be the least for those SD bands that correspond to the neutron and proton magic numbers for the SD shapes. Plots of the softness parameter vs N and Z can, therefore, reveal the favored SD magic numbers.

D. Number of SD bands for a given N or Z

It is well known that the nuclei having spherical magic numbers of neutrons or protons exhibit maximum number of isotones/isotopes. We use a similar approach by plotting the number of SD bands vs the neutron or proton number. The number of SD bands for those neutron or proton numbers is expected to be the largest that are more favored or magic. It may be argued that all the SD bands may not be known in a given nuclide; but it is expected that more SD bands would be discovered in those nuclides which are magic in nature.

It may be pointed out that the first two criteria, i.e., Secs. II A and II B, cannot be applied to all the SD bands because the range of the known spins varies from band to band.

III. RESULTS

A. Classification of the SD bands over the whole chart of nuclides

In the present paper, we have considered only those SD bands for which spin assignments (although tentative) have been listed in the table of SD bands [6] and the ENSDF and the XUNDL databases [25]. All the data for the γ-ray transition energies have been taken from these two sources [6,25]. It may be emphasized that these data bases are reasonably up to date and a search of additional papers on SD bands has also been made so that no significant data are left out. Further, a broad classification of the bands into the three categories of deformation from small elongation to large elongation (axes ratios 1.5:1, 1.7:1, and 2:1) has been used for discussion although this classification is approximate in nature.

B. The γ-ray energy ratios $R(I)$ vs N and Z for the deformation 1.5:1

We first discuss the γ-ray energy ratios $R(I)$ vs N and Z. We apply this criterion to nearly 59 SD bands belonging to 30 nuclei in the mass region $A = 57$–137 having a deformation corresponding to the axis ratio 1.5:1. Because different spins are known in different SD bands, we have grouped the bands according to spins which are common to this group. A ratio $R(I) = E_\gamma(I \rightarrow I\text{-}2)/E_\gamma(I\text{-}2 \rightarrow I\text{-}4)$ is then plotted as a function of N and Z along with the rigid rotor values for the group of nuclei [Figs. 1(a)–1(d) and 2(a)–2(d)]. The ratio is chosen in such a way that we can get maximum number of SD bands for that particular ratio. On the basis of the plots of Figs. 1(a)–1(d) and 2(a)–2(d), we find the SD bands having $N = 43$, 46, 59, and 72–75 and $Z = 38$, 40, 41, 46, 58, 59, and 62 are closest to the rigid rotor line.

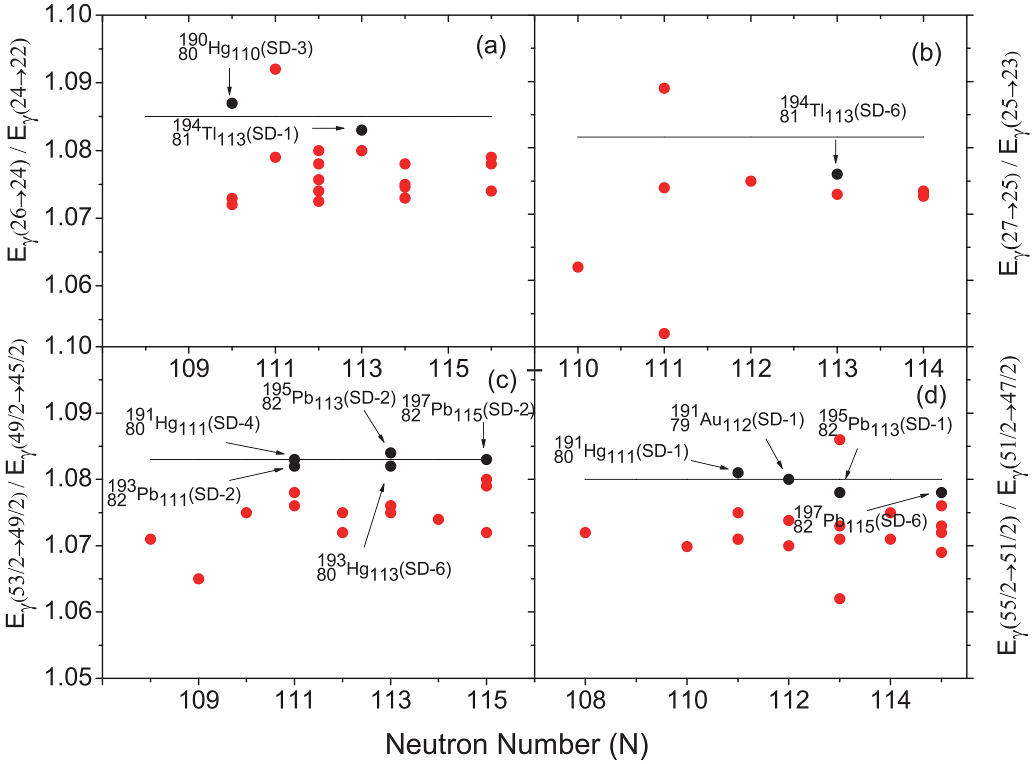

FIG. 3. (Color online) (a)–(d) Same as in Fig. 1, but for the deformation 1.7:1 group of SD bands for the $A = 189$–198 mass region.

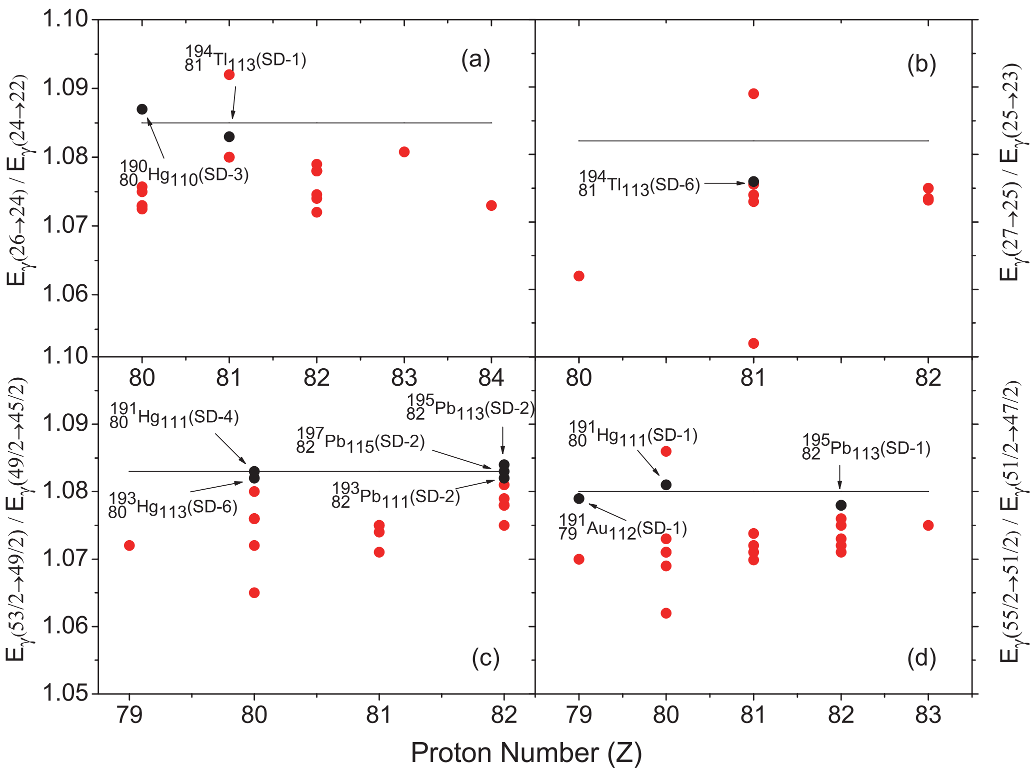

FIG. 4. (Color online) (a)–(d) Same as in Fig. 2, but for the deformation 1.7:1 group of SD bands for the $A = 189$–198 mass region.

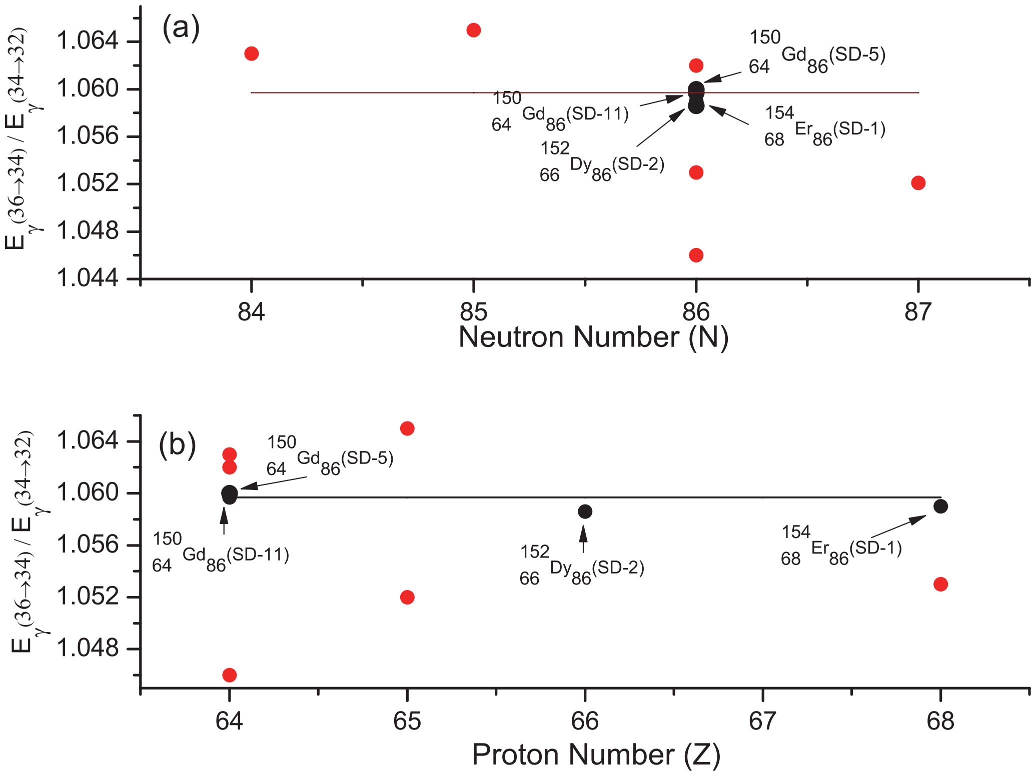

FIG. 5. (Color online) (a),(b) Same as in Fig. 1, but for the deformation 2:1 group of SD bands for the $A = 148$–154 mass region. The top panel shows the graph for the neutron number (N), while the bottom panel is for the proton number (Z).

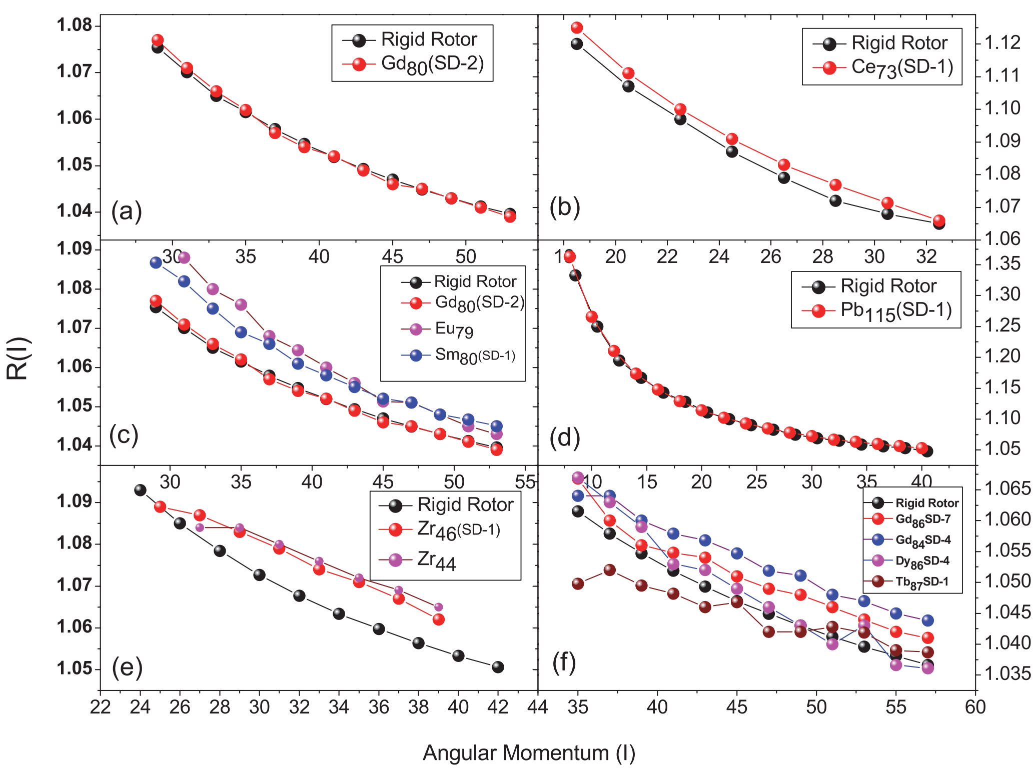

FIG. 6. (Color online) (a)–(f) Plot of $R(I)$ vs the angular momentum I. The solid black line is the rigid rotor behavior for the particular shape of the SD band.

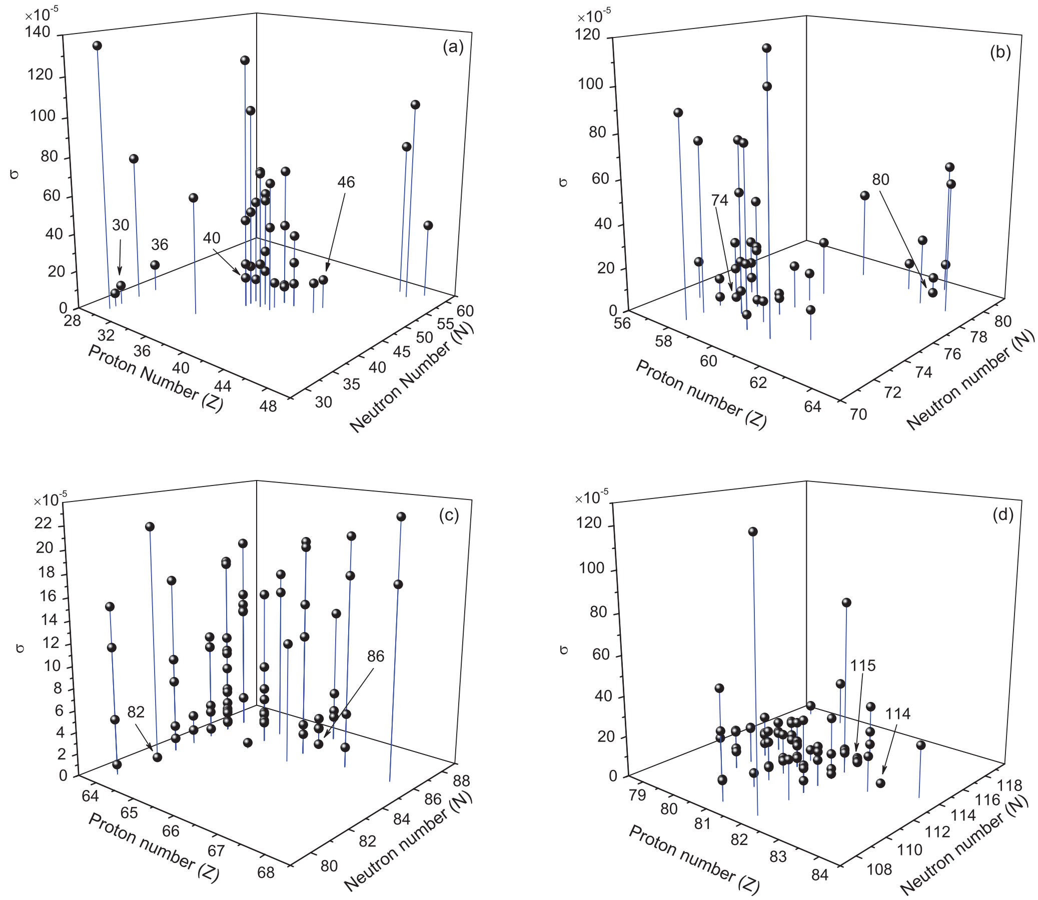

FIG. 7. (Color online) (a)–(d) Softness parameter vs the proton number (Z) and the neutron number (N) for the SD bands (a) in the $A = 80$ mass region, (b) in the $A = 130$ mass region, (c) in the $A = 150$ mass region, (d) in the $A = 190$ mass region. The arrows point to the neutron numbers which have the smallest value for the softness parameter.

C. The γ-ray energy ratios $R(I)$ vs N and Z for the deformation 1.7:1

We also apply this criterion to nearly 97 SD bands belonging to 25 nuclei in the mass region $A = 189$–190 with the deformation 1.7:1 and plot the ratio $R(I) = E_\gamma(I \to I\text{-}2)/E_\gamma(I\text{-}2 \to I\text{-}4)$ as a function of N and Z along with the rigid rotor values [Figs. 3(a)–3(d) and 4(a)–4(d)]. From these plots, we find that $N = 110$, 111, 112, 113, and 115 and $Z = 79$, 80, 81, 82, and 90 are closest to the rigid rotor line.

D. The γ-ray energy ratios $R(I)$ vs N and Z for the deformation 2:1

We have grouped the 63 bands from 13 nuclei in the mass region $A = 148$–154 having a deformation 2:1 according to the spins that are common to this group. A ratio $R(I) = E\gamma(I \to I\text{-}2)/E\gamma(I\text{-}2 \to I\text{-}4)$ is then plotted as a function of N and Z along with the rigid rotor values [Figs. 5(a) and 5(b)]. From these plots, we find that $N = 86$ and $Z = 64$ lies on the rigid rotor line.

E. The γ-ray energy ratios $R(I)$ vs the angular momentum I

We further extend this criterion to compare the behavior of the SD bands over the whole spin range with that expected for the corresponding rigid rotor band. The γ-ray energy ratios $R(I)$ as a function of spin I are plotted for the SD bands along with the rigid rotor values. Some typical plots are displayed in Figs. 6(a)–6(f). We have chosen to show in Figs. 6(a)–6(d) the SD bands of Ce, Gd, and Pb isotopes which nearly overlap with the respective rigid rotor plots. In Fig. 6(c) we also compare the SD band of ^{144}Gd with the other SD bands in neighboring nuclei ($Z = 62, 63$), the SD band of $Z = 64$, $N = 80$; i.e., ^{144}Gd stands out as the best rigid rotor. Therefore, the proton number 64 and the neutron number 80 emerge as the best possible SD magic numbers. We can see that the smoothness of the bands as well as the overlap with the rigid rotor behavior decreases as we move to the lighter mass regions [Fig. 6(e)]; it is not as good for ^{84}Zr, which is otherwise a deformed magic nucleus from other criteria. Figure 6(f) shows the variation of γ-ray energy ratios $R(I)$ vs the angular momentum I for the mass region $A = 148$–155 having deformation 2:1. This criterion leads to the following particle numbers: 40–41, 58 (1.5:1), 64,

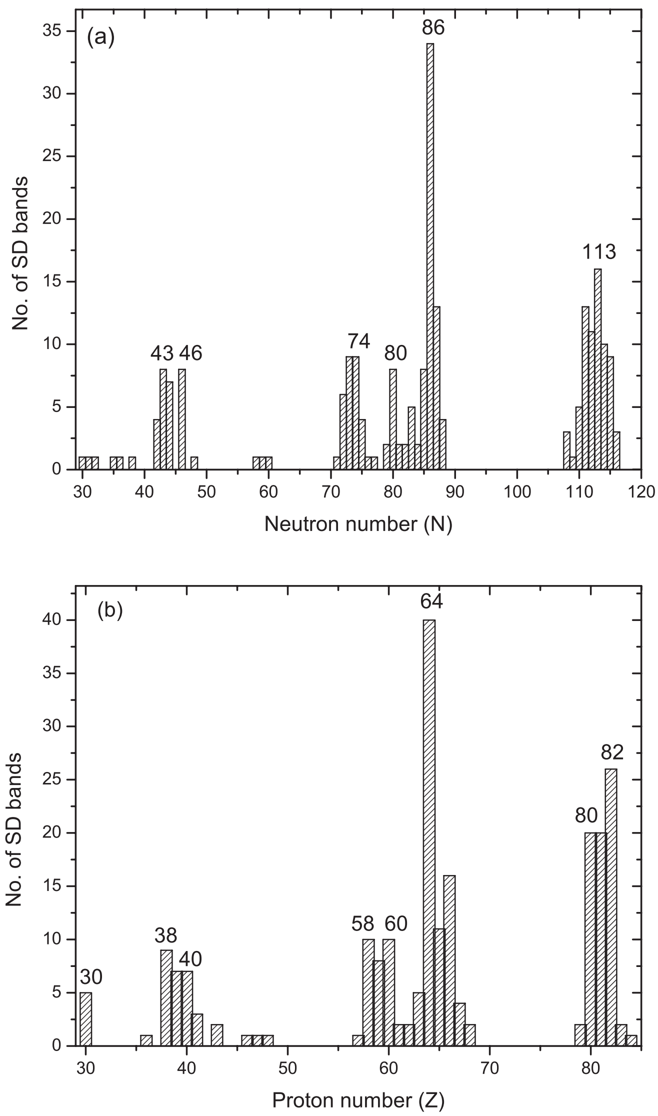

FIG. 8. (a),(b) Number of SD bands vs (a) the neutron number N and (b) the proton number Z.

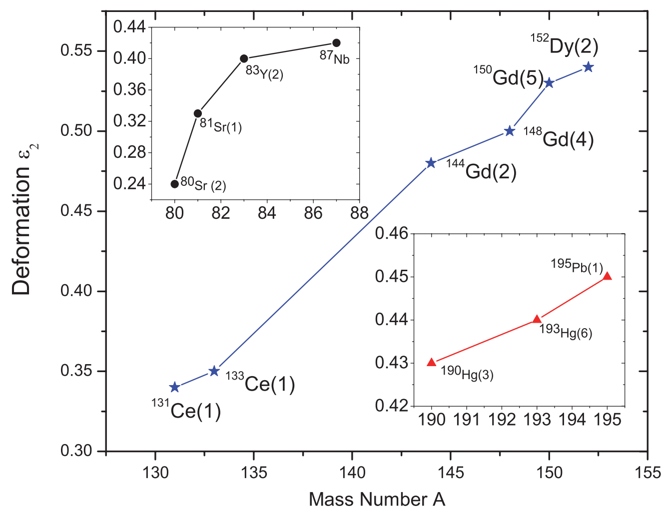

FIG. 9. (Color online) Plot of the measured deformation vs mass number for the SD bands corresponding to the magic numbers of protons/neutrons from $A = 80$ to $A = 190$.

82 (1.7:1), and 66 (2:1) for the protons and 46, 73 (1.5:1), 80, 115 (1.7:1), and 86 (2:1) for the neutrons.

F. Softness parameter vs N and Z

We next consider the values of the softness parameter for the SD bands as obtained by a fitting of the γ-ray energies to a simple four-parameter formula [26]. Plots of the softness parameter vs N and Z are shown in Figs. 7(a)–7(d). A minimum value of the softness parameter is a reasonably good way to identify the most rigid behavior. We note that the softness parameter takes minimum value at precisely the same N and Z value that is found to be magic from the previous criteria. In particular, the softness parameter is observed to have a minimum value at the neutron numbers $N = 30$, 36, 46, 73–75, 80, 82, 86, and 114–115. Similarly, the proton numbers $Z = 30$, 40, 58, 62, 64, 66, 80, and 82 emerge as the possible SD magic numbers.

G. Number of the SD bands vs N and Z

We plot the total number of SD bands known for various Z and N values in the Figs. 8(a) and 8(b). This approach reveals that the neutron numbers $N = 31$, 35, 43–46, 60, 72–74, 80, 86, and 114–115 and the proton numbers $Z = 30$, 38, 40, 58, 60, 64, and 80–82 possess relatively large number of SD bands. In fact, $Z = 58$, 64, and 80–82 and $N = 46$, 84–86, and 111–115 stand out as the most prominent magic numbers defining a meta stable valley in the SD region.

We can now look at the various possible (N, Z) combinations which lead to the presently observed SD bands. We find that the N/Z ratio of the presently observed SD bands ranges from 1.21 (in $A = 80$–90 region) to 1.40 (in $A = 187$–200 region). This may be compared with the N/Z ratio of stable isotopes in the same mass regions which varies from 1.28 to 1.53. All the SD bands observed so far, therefore, tend to lie on the neutron deficient side. Predictions can be made about the observation of more SD bands based on these particle numbers, for both the neutron deficient and the neutron rich side. On the neutron deficient side, very small number of SD bands (only 5) can be seen in the mass region $A = 102$–112 corresponding to $Z = 44$–48 and $N = 58$–64; we therefore, expect many more SD bands in this region. It should also be possible to observe SD bands towards the neutron rich side for the combination of (N, Z) values lying around $Z = 56$–60 and $N = 78$–84. Similarly, one can look for SD bands in the nuclei lying around $Z = 64$, $N = 96$, and $Z = 71$, $N = 112$.

Finally, we plot in Fig. 9 the measured deformation of only those SD bands which have a magic number of protons or neutrons as a function of the mass number A. The plot confirms the theoretical finding that increasing particle number leads to increasing deformation [4,19]. We are able to confirm this feature in all the mass regions, right from $A = 80$ to $A = 190$.

TABLE I. Comparison of the empirical SD particle numbers with the magic numbers for the spherical (1:1) shape and the SD (2:1) shape. The spherical and SD magic numbers correspond to the isotropic harmonic oscillator without and with spin-orbit term (in parentheses).

Deformation ($\omega_\perp : \omega_z$)							
Theory		Empirical					
1:1	2:1	1.5:1		1.7:1		2:1	
		Z	N	Z	N	Z	N
2	4						
8	10						
	16						
20(28)	28	30	30–31				
			35–36				
40(50)	40(40–46)	38–41,46	42–46				
	60(60–68)	58,59,62	58–60	64		64–68	
70(82)	80		72–75	80–82	80		86
112(126)	110(110–118)				111–115		

IV. CONCLUSIONS

On the basis of the various kinds of evidence, we identify $Z = 30$, 38–41, 46, 58, 59, and 62 and $N = 30$, 36, 42–46, 59, 60, and 72–75 as the most favored magic numbers for the 1.5:1 shape. Similarly, we identify $Z = 64$ and 80–82 and $N = 80$ and 111–115 as the most favored magic numbers for the 1.7:1 shape. The most favored magic numbers for the 2:1 shape are obtained as $Z = 64$–66 and $N = 86$. We summarize these results in Table I, where a comparison is made with the magic numbers for the spherical shape (with modification due to the spin-orbit term) and for the 2:1 shape (and their modification due to the spin-orbit term) from the oscillator model. It is interesting to note that the chains of the particle numbers as obtained from the empirical analysis are nearly identical to those predicted by the theoretical works cited previously. We also observe that the deformation increases with the particle number which confirms the earlier theoretical predictions. These conclusions appear to justify the initial premise about the correlation between the level density and the pairing.

It may be noted that the range of nuclei where the SD bands are observed has been defined indirectly by many null results. This is because in many cases, SD bands have been searched but not observed and these null results have not been highlighted in the literature. We predict that many more SD bands may be observed in the mass region $A = 102$–112 corresponding to $Z = 44$–48 and $N = 58$–64. We also predict that many SD bands could exist for the combination of (N, Z) values lying around $Z = 58$ and $N = 80$; these could be the nuclides having $Z = 56$–60 and $N = 78$–84 which are mostly naturally abundant. Similarly, one could look for SD bands in the nuclides lying around $Z = 64$, $N = 96$ and $Z = 71$, $N = 112$ (towards the neutron rich side).

ACKNOWLEDGMENTS

N.S. would like to thank the Ministry of Human Resource and Development (Government of India) for the assistantship at NITJ. Financial support from the Department of Science and Technology and Department of Atomic Energy (Government of India) at IITR is gratefully acknowledged.

[1] A. De Shalit and H. Feshbach, *Theoretical Nuclear Physics, Volume 1: Nuclear Structure* (Wiley & Sons, New York, 1974).
[2] A. Bohr and B. R. Mottelson, *Nuclear Structure, Volume I: Single Particle Motion* (W. A. Benjamin, New York, 1969); *Nuclear Structure Volume, II: Nuclear Deformations* (W. A. Benjamin, New York, 1975).
[3] M. Brack, J. Damgaard, A. S. Jensen, H. C. Pauli, V. M. Strutinsky, and C. Y. Wong, Rev. Mod. Phys. **44**, 320 (1972).
[4] I. Ragnarsson, S. G. Nilsson, and R. K. Sheline, Phys. Rep. **45**, 1 (1978).
[5] S. G. Nilsson and I. Ragnarsson, *Shapes and Shells in Nuclear Structure* (Cambridge University Press, Cambridge, 1995).
[6] B. Singh, R. Zywina, and R. B. Firestone, Nucl. Data Sheets **97**, 241 (2002), and references therein.
[7] P. J. Nolan and P. J. Twin, Annu. Rev. Nucl. Part. Sci. **38**, 533 (1988).
[8] R. V. F. Janssens and T. L. Khoo, Annu. Rev. Nucl. Part. Sci. **41**, 321 (1991).
[9] W. Satula, J. Dobaczewski, J. Dudek, and W. Nazarewicz, Phys. Rev. Lett. **77**, 5182 (1996).
[10] B. Kharraja and U. Garg, Phys. Rev. Lett. **80**, 1845 (1998).
[11] I. Ragnarsson, Nucl. Phys. A **557**, 167c (1993).
[12] L. Pauling, Proc. Natl. Acad. Sci. U.S.A. **89**, 7277 (1992).
[13] M. Dudeja, S. S. Malik, and A. K. Jain, Phys. Lett. B **412**, 14 (1997).
[14] S. R. Jain, A. K. Jain, and Z. Ahmed, Phys. Lett. B **370**, 1 (1996).
[15] A. K. Jain, M. Dudeja, S. S. Malik, and Z. Ahmed, Phys. Lett. B **392**, 243 (1997).
[16] S. S. Malik, M. Dudeja, Amita, and A. K. Jain, Int. J. Mod. Phys. E **9**, 487 (2000).
[17] T. Bengtsson *et al.*, Phys. Scr. **24**, 200 (1981).
[18] J. Dudek, W. Nazarewicz, Z. Szymanski, and G. A. Leander, Phys. Rev. Lett. **59**, 1405 (1987).

[19] E. S. Paul *et al*., Phys. Rev. Lett. **98**, 012501 (2007).
[20] A. Neusser *et al*., Phys. Rev. C **70**, 064315 (2004).
[21] A. V. Afanasjev and I. Ragnarsson, Nucl. Phys. A **608**, 176 (1996).
[22] I. Ragnarsson, Acta Phys. Pol. B **27**, 33 (1996).
[23] E. Ideguchi *et al*., Phys. Rev. Lett. **87**, 222501 (2001).
[24] E. Ideguchi *et al*., Phys. Lett. B **686**, 18 (2010).
[25] The ENSDF and XUNDL databases available at http://www.nndc.bnl.gov
[26] N. Sharma, H. M. Mittal, S. Kumar, and A. K. Jain (unpublished).