Investigation of 0⁺ states in 198Hg after two-neutron pickup

C. Bernards,^{1,*} R. F. Casten,¹ V. Werner,¹ P. von Brentano,² D. Bucurescu,³ G. Graw,⁴ S. Heinze,² R. Hertenberger,⁴ J. Jolie,²

S. Lalkovski,⁵ D. A. Meyer,¹ D. Mücher,^{2,†} P. Pejovic,² C. Scholl,² and H.-F. Wirth⁶

¹*Wright Nuclear Structure Laboratory, Yale University, New Haven, Connecticut 06520, USA*

³*National Institute for Physics and Nuclear Engineering, Bucharest R-76900, Romania*

⁴*Fakultat f ¨ ur Physik, Ludwig-Maximilians-Universit ¨ at M¨ unchen, D-85748 Garching, Germany ¨*

⁵*Faculty of Physics, University of Sophia, 1164 Sofia, Bulgaria*

⁶*Physik Department, Technische Universitat M¨ unchen, D-85748 Garching, Germany ¨*

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We studied 0^+ states in ¹⁹⁸Hg after the ²⁰⁰Hg(p, t)¹⁹⁸Hg transfer reaction up to 3 MeV excitation energy. The experiment was performed using the high-resolution Q3D magnetic spectrograph at the Maier-Leibnitz Laboratory Tandem accelerator in Munich. In total, only four 0^+ states were observed in ¹⁹⁸Hg, significantly fewer than in other experiments of the (p, t) transfer campaign. We discuss the low-energy 0^+ state density as a function of the valence nucleon number N_{val} and test if the 0^+ density can be used as a signature for the prolate-oblate shape-phase transition in the Hf-Hg region.

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I. INTRODUCTION

In recent years, much effort was invested in systematic studies of low-lying 0^+ excitations in medium- to heavy-mass nuclei throughout the nuclear landscape, ranging from ¹⁵²Gd to 194 Pt $[1-5]$. This region is particularly interesting, as the structure of these nuclei changes from transitional nuclei in the Gd region, over well-deformed nuclei in the Yb region, to γ -soft nuclei in the Pt region [\[6\]](#page-4-0). Experiments at the high-resolution Q3D magnetic spectrograph [\[7,8\]](#page-4-0) in Munich allowed the study of 0^+ states in unprecedented detail using (*p,t*) transfer reactions and started with the discovery of an unforeseen high number of low-lying 0^+ excitations in 158 Gd [\[1\]](#page-4-0). Extending these studies to other nuclei, the enhanced density of low-lying 0^+ states in the Gd region was interpreted as a new signature for the shape-phase transitions from spherical to deformed nuclei [\[9\]](#page-4-0). These observations were followed by various calculations reproducing the density and distribution of 0^+ states $[10-13]$.

These studies of 0^+ states were extended into the *γ*-soft region by investigating Pt isotopes [\[5\]](#page-4-0). The present work probes further towards the end of the proton and neutron shell. In the Hf-Hg region, a prolate-oblate phase transition has been observed by investigating several observables from 180Hf to ²⁰⁰Hg: energy ratios $[R_{4/2}]$, quadrupole moments $[Q(2^+_1)]$, and the reduced transition probabilities $[B(E2; 2^+_2 \rightarrow 2^+_1)]$ [\[14\]](#page-4-0). Extending the 0^+ studies to the Hg isotopes, we can test if the low-lying 0^+ density can be applied as a signature of this shape-phase transition as well.

II. EXPERIMENT

We performed a (p, t) transfer experiment at the Munich MLL (Maier-Leibnitz Laboratory of LMU Munich and TU Munich) Tandem accelerator facility using an unpolarized 25-MeV proton beam to impinge on a $48 \mu g/cm^2$ ²⁰⁰HgS target on a 7 μ g/cm² carbon backing. The outgoing tritons were detected with the Q3D magnetic spectrograph [\[7\]](#page-4-0) and the 1-m-long focal-plane detector [\[8\]](#page-4-0) at three different laboratory angles with respect to the beam axis: 5◦, 17*.*5◦, and 30◦, respectively. For every angle, two measurements with different magnetic settings were necessary to cover the excitation energy range of 0–3 MeV. The HgS target was highly enriched in 200 Hg (96.41%) and contained impurities of 201 Hg (1.46%), 199Hg (0*.*99%), 202Hg (0*.*91%), 198Hg (0*.*13%), 204Hg (0*.*10%), and 196 Hg ($<0.02\%$), respectively.

III. ANALYSIS AND RESULTS

The experimental setup allows straightforward 0^+ state assignments using the data taken at 5◦ and 17*.*5◦, because the (*p,t*) angular distribution strongly peaks in the forward direction only for the $L = 0$ transfer from the 0^+ ground state in ²⁰⁰Hg to 0^+ states in ¹⁹⁸Hg [\[2\]](#page-4-0). Using this characteristic feature of the (p, t) angular distribution, it is sufficient to compare the ratio $R(5/17.5) \equiv \sigma(5^\circ)/\sigma(17.5^\circ)$ of the observed cross sections σ at these angles. The 30 \degree data is not as significant for the determination of $L = 0$ transfers into ¹⁹⁸Hg, because it does not allow a clean separation of 0^+ states from 2^+ and 4^+ states [\[2\]](#page-4-0). However, the 30 \degree data is very helpful as an additional data set for the identification of peaks and the spectra calibration. A safe lower limit for the $R(5/17.5)$ ratio, in order prevent incorrect assignments of 0^+ states, was previously chosen to be $R(5/17.5) > 3$ [\[4\]](#page-4-0).

Figure [1](#page-1-0) shows the complete triton spectrum for the 5° setting from 0 to 3 MeV. The data were calibrated using wellknown excitation energies of ¹⁹⁸Hg states. We normalized all

²Institut für Kernphysik, Universität zu Köln, D-50937 Köln, Germany

^{*}christian.bernards@yale.edu

[†]Present address: Physik Department, Technische Universität München, D-85748 Garching, Germany.

FIG. 1. (Color online) Plot of the complete triton spectrum from 0 to ∼3 MeV, measured at 5◦ laboratory angle. The intensity of the high-energy (HE) part of the spectrum is normalized to the low-energy (LE) part. The arrows mark the assigned 0^+ states in ¹⁹⁸Hg. The 1646 -keV $0⁺$ state is not visible in this plot owing to its small cross section.

experimental runs (different angles and magnetic settings) to the integrated beam current measured behind the target. In Fig. 1, the intensity of the high-energy (HE) part, colored in green, is normalized to the low-energy (LE) part. Owing to the high resolving power (4–6 keV full width at half maximum for 15- to 20-MeV tritons) of the Q3D setup and focal plane detector, a total of ∼70 excited states were analyzed in this work. Figure 2 illustrates the differences between spectra taken at 5° and 17.5° , the relevant angles for the assignment of 0^+ states. Levels corresponding to ¹⁹⁸Hg and peaks arising from target impurities are labeled in the plot. Contaminants from target impurities were identified by comparison with the *Q* values of the corresponding (p, t) transfer reaction.

The *R*(5*/*17*.*5) ratios—for all observed states that were not identified as contaminants—are plotted in Fig. 3. The threshold $R(5/17.5) = 3$ is marked by a green horizontal line. In total,

FIG. 2. (Color online) Logarithmic plot comparing the 5◦ and the normalized 17.5[◦] spectra for the LE setting. States identified as ¹⁹⁸Hg states are labeled with the corresponding transfer $L = 0, 2,$ or 4 from the 200Hg ground state. Identified contaminants are labeled with **c**. They correspond to excited or ground states in 199 Hg, 200 Hg, 197 Hg, and 196 Hg.

FIG. 3. (Color online) $R(5/17.5)$ ratio used to identify 0^+ states in ¹⁹⁸Hg. The green line marks the $R(5/17.5) = 3$ threshold that is used as a lower limit for 0^+ assignments.

we observed only four states that exceed this limit and we discuss these states below.

Table I lists the observed states that exceed the $R(5/17.5) =$ 3 limit seen in Fig. 3. Besides the ground state, we observe three more states showing this typical $L = 0$ transfer characteristic. Two of these states, the 0^+ states at 1401 and 1780 keV, are already known and assigned as 0^+ states [\[15,16\]](#page-4-0). In addition, we found a new, weakly populated state at 1646 keV. We only observe a peak of that energy in the 5◦ data. By carefully testing our detection sensitivity in the 17*.*5◦ spectrum, we derive an upper limit for the 17*.*5◦ cross section and consequentially give a lower limit for the *R*(5*/*17*.*5) ratio. With 3*.*35, this lower limit is clearly distinguishable from all other observed states that do not correspond to a $L = 0$ transfer. Because we have the isotopic analysis on the target purity and were able to identify peaks originating from the most significant impurities in the spectra, we believe that this state does not stem from a target impurity and corresponds to a 0^+ excitation in ¹⁹⁸Hg. In Ref. $[16]$, a further 0^+ state was assigned at 1550 keV, as well, based on a (*p,t*) transfer experiment. We observe a state at 1549 keV, but our much higher resolution data do not support this 0^+ assignment because $R(5/17.5) \approx 0.4$. Our result is consistent with the *γ γ* angular correlation data of Ref. [\[17\]](#page-4-0).

A summary of all observed states and their cross sections for all measured angles is given in Table [II.](#page-2-0) The listed absolute cross sections include a 10% error from the beam-current normalization on the target. In case of very weakly excited states, such as the 0^+ state at 1646 keV, we were not always able to measure the cross sections for all three angles if it was below our detection sensitivity. Nevertheless, none of the other weakly excited states seems to correspond to a $L = 0$

TABLE I. $R(5/17.5)$ ratios and cross sections of assigned 0^+ states, discussed in detail in the text.

| Energy (keV) | R(5/17.5) | $\sigma(5^\circ)$ (mb/sr) | $\sigma(17.5^{\circ})$ (mb/sr) | | |
|----------------|------------|---------------------------|--------------------------------|--|--|
| 0.0(0) | 13.32 (38) | 0.7956(798) | 0.0597(62) | | |
| 1401.0(3) | 9.42(147) | 0.0225(25) | 0.0024(4) | | |
| 1646.4(8) | >3.35 | 0.0009(3) | < 0.0002 | | |
| 1779.6(2) | 4.93(55) | 0.0145(17) | 0.0029(4) | | |

TABLE II. Summary of observed peaks and their cross sections. The cross sections have a 10% systematic uncertainty owing to the beam-current normalization. The uncertainties on the relative *R*(5*/*17*.*5) ratios are often smaller.

| | Cross section (mb/sr) | | | | | | | |
|-------------------------|-----------------------|----------------------|--------------------|--|--|--|--|--|
| Energy (keV) | $\sigma(5^\circ)$ | $\sigma(17.5^\circ)$ | $\sigma(30^\circ)$ | | | | | |
| $-28.4(3)$ ^b | 0.0024(4) | 0.0007(3) | 0.0018(5) | | | | | |
| 0.0(0) ^a | 0.7956(798) | 0.0597(62) | 0.4052(409) | | | | | |
| $322.3(7)^b$ | 0.0007(3) | 0 | 0.0010(5) | | | | | |
| 411.7(2) ^a | 0.0286(31) | 0.0548(57) | 0.0364(40) | | | | | |
| 451.6(3) ^b | 0.0038(6) | 0.0007(3) | 0.0022(6) | | | | | |
| 575.6 $(4)^{b}$ | 0.0018(4) | 0 | 0.0007(4) | | | | | |
| 795.9 $(8)^c$ | 0 | 0.0007(3) | 0 | | | | | |
| $867.4(6)$ ^c | 0.0005(2) | 0.0009(3) | $\boldsymbol{0}$ | | | | | |
| 1048.7(2) ^a | 0.0056(8) | 0.0032(5) | 0.0029(6) | | | | | |
| $1087.8(1)^{a}$ | 0.0368(39) | 0.0692(71) | 0.0546(59) | | | | | |
| 1292.4 (11) | 0 | 0.0004(2) | $\boldsymbol{0}$ | | | | | |
| 1401.0(3) ^a | 0.0225(25) | 0.0024(4) | 0.0054(9) | | | | | |
| 1419.9 (9) | 0 | 0 | 0.0006(2) | | | | | |
| 1548.6(2) | 0.0019(4) | 0.0051(6) | 0.0062(7) | | | | | |
| 1612.3 (2) ^a | 0.0021(4) | 0.0085(10) | 0.0041(5) | | | | | |
| 1635.7(1) ^a | 0.0057(8) | 0.0122(13) | 0.0183(19) | | | | | |
| 1646.4(8) | 0.0009(3) | < 0.0002 | $\boldsymbol{0}$ | | | | | |
| 1683.3(1) ^a | 0.0029(5) | 0.0058(7) | 0.0110(12) | | | | | |
| 1752.8(3) | 0.0008(3) | 0.0020(3) | 0.0026(4) | | | | | |
| 1779.6(2) | 0.0145(17) | 0.0029(4) | 0.0092(10) | | | | | |
| 1816.2(4) | 0 | 0.0006(2) | 0.0011(2) | | | | | |
| 1834.4 (2) | 0.0474(50) | 0.0726(74) | 0.0835(85) | | | | | |
| 1901.3 (4) | 0.0014(3) | 0.0012(2) | 0.0017(3) | | | | | |
| 1911.0(3) | 0.0009(3) | 0.0017(3) | 0.0033(4) | | | | | |
| 1929.1 $(2)^a$ | 0.0078(10) | 0.0088(10) | 0.0077(9) | | | | | |
| 1960.2 (4) | 0.0009(3) | 0.0015(3) | 0.0025(4) | | | | | |
| 1971.9(7) | 0 | 0.0001(1) | 0.0008(2) | | | | | |
| 2005.8(3) | 0.0007(3) | 0.0018(3) | 0.0014(3) | | | | | |
| 2038.9 (8) | 0 | 0.0006(2) | $\overline{0}$ | | | | | |
| 2049.8 (7) | 0.0007(3) | 0.0015(3) | 0.0012(2) | | | | | |
| 2056.2 (9) | 0.0008(3) | 0.0009(5) | 0.0004(2) | | | | | |
| 2071.3(2) ^a | 0.0047(7) | 0.0069(8) | 0.0032(4) | | | | | |
| 2125.7(2) | 0.0031(5) | 0.0057(7) | 0.0076(9) | | | | | |
| 2134.2 (2) | 0.0055(8) | 0.0120(13) | 0.0104(12) | | | | | |
| 2176.6 (22) | 0 | 0.0007(2) | 0.0005(2) | | | | | |
| 2203.4 (4) | 0.0008(3) | 0.0012(2) | 0.0015(3) | | | | | |
| 2254.6(6) | 0 | 0 | 0.0011(2) | | | | | |
| 2287.9 (2) | 0.0050(7) | 0.0099(11) | 0.0061(7) | | | | | |
| 2296.3(2) | 0.0040(6) | 0.0042(6) | 0.0044(6) | | | | | |
| 2318.6 (10) | 0.0024(4) | 0.0030(4) | 0.0051(6) | | | | | |
| 2331.3(1) ^a | 0.0114(13) | 0.0195(21) | 0.0346(36) | | | | | |
| 2354.5 (7) | 0.0035(5) | 0.0027(4) | 0.0056(7) | | | | | |
| 2401.1(2) | 0.0046(7) | 0.0071(8) | 0.0076(9) | | | | | |
| 2431.5 (1) | 0.0084(10) | 0.0138(15) | 0.0207(22) | | | | | |
| 2450.4 (8) | 0 | 0.0006(2) | $\boldsymbol{0}$ | | | | | |
| 2465.5(2) | 0.0044(6) | 0.0171(19) | 0.0117(13) | | | | | |
| 2488.6 (2) | 0.0149(17) | 0.0186(20) | 0.0173(19) | | | | | |
| 2516.2(2) | 0.0010(3) | 0.0020(3) | 0.0033(5) | | | | | |
| 2535.8(3) | 0.0018(4) | 0.0021(4) | 0.0026(4) | | | | | |
| 2557.6(5) | $\boldsymbol{0}$ | 0.0021(3) | 0.0032(4) | | | | | |
| 2564.9 (7) | 0 | 0.0008(2) | 0 | | | | | |
| 2582.1(7) | 0 | 0.0007(2) | 0.0006(2) | | | | | |
| 2608.5 (15) | 0.0022(4) | 0.0029(4) | 0.0033(5) | | | | | |
| 2620.3(3) | 0.0027(5) | 0.0031(4) | 0.0041(5) | | | | | |

TABLE II. (*Continued.*)

| | Cross section (mb/sr) | | | | | | | |
|-----------------|-------------------------|----------------------|--------------------|--|--|--|--|--|
| Energy (keV) | $\sigma(5^\circ)$ | $\sigma(17.5^\circ)$ | $\sigma(30^\circ)$ | | | | | |
| 2640.1(6) | 0.0026(4) | 0.0051(7) | 0.0047(6) | | | | | |
| 2651.7(3) | 0.0054(7) | 0.0059(7) | 0.0076(9) | | | | | |
| 2678.5(3) | 0.0039(6) | 0.0077(9) | 0.0063(8) | | | | | |
| 2691.3(3) | 0.0044(6) | 0.0070(8) | 0.0147(16) | | | | | |
| 2711.3(3) | 0.0021(4) | 0.0025(4) | 0.0033(5) | | | | | |
| $2731.5(4)^{a}$ | 0.0008(2) | 0.0008(2) | 0.0012(2) | | | | | |
| 2741.1(7) | 0 | 0.0006(2) | 0 | | | | | |
| 2755.2(4) | 0.0037(6) | 0.0078(9) | 0.0106(12) | | | | | |
| 2772.4(3) | 0.0054(7) | 0.0099(11) | 0.0111(13) | | | | | |
| 2782.5 (7) | 0.0018(9) | 0.0035(4) | 0.0031(5) | | | | | |
| 2800.5(5) | 0.0009(2) | 0.0011(2) | 0.0020(3) | | | | | |
| 2826.3(5) | 0.0015(3) | 0.0036(5) | 0.0041(5) | | | | | |
| 2844.7 (4) | 0.0016(3) | 0.0016(3) | 0.0018(3) | | | | | |
| 2877.5 (13) | 0.0011(3) | 0.0012(2) | 0.0007(2) | | | | | |

^aThis state was used for energy calibration using the published ¹⁹⁸Hg excitation energy from the Nuclear Data Sheets [\[15\]](#page-4-0).

bIdentified contaminant from target impurity.

c Very likely an unidentified contaminant.

transfer in 198Hg. The origin of the low-intensity peaks at 796 and 867 keV is not clear. However, considering the fact that they would correspond to the second and third excited states in 198 Hg, we assume that they most likely stem from unidentified contaminants.

To rule out the possibility that the low number of observed $L = 0$ transfers in this experiment is simply limited by statistics, we compare our measured cross sections with cross sections from other already analyzed experiments of this Q3D (p, t) campaign. The 0^+ state at 1646 keV is the weakest excited 0^+ state we observe in ¹⁹⁸Hg. The cross section $\sigma(5^\circ) = 0.0009$ mb/sr is very similar to the absolute cross sections $\sigma(5^\circ)$ for the weakest populated 0^+ states observed in Refs. [\[2,5\]](#page-4-0). With $\sigma(5^\circ) = 0.0009$ mb/sr, the cross section of the 0^+ state at 1646 keV is about 0.1% of the ground-state cross section at 5[°]. This variation in excitation cross sections is in good agreement with the variation of cross sections reported in Ref. [\[5\]](#page-4-0), where the least populated assigned 0^+ states in 192Pt and 194Pt have about 0*.*1%–0*.*2% of the ground-state cross section measured at 5◦ laboratory angle. Therefore, we conclude that the sensitivity to identify 0^+ states in this data set is similar to the sensitivity of other experiments performed within this (p, t) campaign $[2,4,5]$.

IV. DISCUSSION

With the investigation of ¹⁹⁸Hg, we extend our search for 0^+ states from the rare-earth region towards the ²⁰⁸Pb shell closure. The density of low-energy 0^+ states differs for the different nuclei investigated in this (*p,t*) campaign and is shown in histogram form in Fig. [4.](#page-3-0) With only four assigned 0^+ states up to 3 MeV excitation, ¹⁹⁸Hg clearly has the lowest number of low-energy 0^+ states among the investigated nuclei. In Fig. [4,](#page-3-0) the nuclei 154 Gd, 194 Pt, and 198 Hg are highlighted.

FIG. 4. (Color online) Histogram of the cumulative number of definite 0^+ states up to 3 MeV excitation energy using the results of 13 different Q3D (p, t) experiments. Plotted are the definite 0^+ assignments for 152 Gd, 154 Gd, 158 Gd, 162 Dy, 168 Er, 170 Yb, 176 Hf, 180 W, $184W$, $190Os$, $192Pt$, $194Pt$, and $198Hg$. The data for the different nuclei are taken from Refs. [\[1–5\]](#page-4-0) and the present work. The enveloping lines for the 0^+ states in ¹⁵⁴Gd and ¹⁹⁸Hg—and the nucleus ¹⁹⁴Pt—are highlighted. Nuclei in the $A \approx 190-200$ mass region investigated in Ref. [\[14\]](#page-4-0) are labeled on the right-hand side of the graph.

Their position in the graph is particularly interesting when interpreting the 0^+ density as a signature for shape-phase transitions. In Ref. [\[9\]](#page-4-0), the large number of low-lying 0^+ states in 154Gd was interpreted as a signature for the shape-phase transition from spherical to deformed nuclei, with 154Gd being closest to the critical point.

Now, having data available on ¹⁹⁸Hg from a comparably sensitive experiment, we can test how the 0^+ density evolves at the prolate-oblate phase-shape transition observed in the Hf-Hg region [\[14\]](#page-4-0). For this shape-phase transition, it was concluded that the nucleus 194Pt is closest to the critical point.

The corresponding mass $A \approx 190-200$ nuclei and their 0^+ state densities are marked by arrows in Fig. 4. Their evolution does not show an enhancement in 194 Pt. The 0⁺ density seems to depend rather on the proximity of the ²⁰⁸Pb shell closure. Indeed, 198 Hg has the fewest number of 0^+ states. This behavior seems to be a simple consequence of the number of configurations available to create 0^+ states. This number decreases rapidly as the closed shell at 208Pb is approached.

The interacting boson model (IBM) [\[18\]](#page-4-0) has turned out to be very useful for the description of collective properties of nuclei by using a simple Hamiltonian, only considering valence nucleons (holes), and treating them pairwise as bosons. Using the *sd* IBM-1 version of the model, only spin $l = 0$ or $l = 2$ bosons are considered and no distinction between protons or neutrons is made. Thus, the *N*_{val} valence nucleons are approximated by *N*val*/*2 *sd* bosons. The decrease in the number of 0^+ states in Fig. 4 is in qualitative agreement with IBM calculations performed for these nuclei [\[10\]](#page-4-0). We investigate this further in Fig. 5.

In Fig. 5 we plot the number of 0^+ state assignments up to 3 MeV as a function of the number of valence nucleons (holes) *N*val. Tentative assignments are accounted for in the upper error bars. In addition, we show the total number of 0^+ states that can be formed within the *sd* IBM-1. By comparing this maximum number of IBM 0^+ states with the number of observed 0^+ states, one notes an upward trend, similar in the data and

FIG. 5. (Color online) Number of 0^+ states up to 3 MeV for the nuclei investigated in this Q3D (p, t) campaign as a function of the number of valence nucleons N_{val} . The error bars include tentative assignments. In addition, the maximum number of *sd* IBM 0⁺ states and the calculated number of 0^+ states below 3 MeV in this model using realistic parameters fitted according to Refs. [\[20,21\]](#page-4-0) are plotted.

calculations, up to $N_{val} \sim 22$ valence nucleons, corresponding to ¹⁵⁴Gd. The total number of low-lying 0^+ states for the individual nuclei, as well as the increasing number of 0^+ states with N_{val} are reasonably reproduced. Of course, the IBM is a highly truncated model that includes only a small part of the full shell-model space. Moreover, some of the IBM 0^+ states may well lie above 3 MeV so one would expect to observe only a subset of the possible configurations and states in the calculation. Thus, one should view the concordance in the number of 0^+ states in the IBM and the experimentally observed ones up to $N_{val} \sim 22$ only in a qualitative way as indicating the general trends and the rapid increase in the number of allowed 0^+ configurations.

Above $N_{\text{val}} \sim 22$ the number of experimentally observed low-lying 0^+ states tends to saturate and is relatively constant, taking uncertainties owing to tentative assignments into account. For $N_{val} > 22$, even the truncated number of IBM 0^+ states far exceeds the number seen experimentally. In strong contrast to the maximum number of 0^+ states in the *sd* IBM space, we plot the number actually calculated to lie below 3 MeV in the IBM using realistic parameters and the extended consistent *Q* formalism [\[19\]](#page-4-0) as in Refs. [\[20,21\]](#page-4-0). The parameters were fitted to a number of key observables in the ground, 0^+_2 , and quasi- 2^+_γ band by considering contour plots [\[20–22\]](#page-4-0). For the energies, emphasis was placed on the ratios $R_{4/2} = E(4^+_1)/E(2^+_1), E(0^+_2)/E(2^+_1)$, and $E(2^+_\gamma)/E(2^+_\gamma)$. For electromagnetic transition probabilities, the *B*(*E*2) ratios $B_{2\gamma} = B(E2; 2^+_{\gamma} \rightarrow 0^+_{1})/B(E2; 2^+_{1} \rightarrow 0^+_{1})$ and $R_{2\gamma} = B(E2; 2^{+}_{\gamma} \rightarrow 0^{+}_{1})/B(E2; 2^{+}_{\gamma} \rightarrow 2^{+}_{1})$ were considered. The resulting parameters were found to allow a realistic description of the 0^+_2 level energies [\[20\]](#page-4-0) and are now used to calculate the low-energy 0^+ states up to 3 MeV excitation energy. The calculated 0^+ densities below 3 MeV do not show an increase in the number of 0^+ states with N_{val} and result in a rather constant number $(4–6)$ of $0⁺$ states in the energy range investigated, similar to the experimentally observed saturation, but naturally with fewer states than observed. This hints at the fact that the number of experimentally observed 0^+ states

TABLE III. Investigated nuclei plotted in Fig. [5](#page-3-0) sorted by their number of valence nucleons N_{val} . The numbers of 0^+ states correspond to states up to 3 MeV excitation energy.

| Nucleus: | 198 Hg | 194 Pt | 192Pt | ^{190}Os | 152 Gd | ${}^{154}\mathrm{Gd}$ | 184 W | ${}^{158}\mathrm{Gd}$ | $^{180}\mathrm{W}$ | 162 Dy | 170Yb | 168 Er | 176 Hf |
|--------------------|----------|-------------|-------|------------|---------------|-----------------------|---------|-----------------------|--------------------|-------------|-------|----------|----------|
| N_{val} | 10 | 14 | 16 | 18 | 20 | 22 | 24 | 26 | 28 | 30 | 30 | 32 | 32 |
| Exp. 0^+ states | | | | | | 13 | | | | | 14 | | 8 |
| Calc. 0^+ states | | | | | | | | | | 4 | | | 5 |

includes many noncollective 0^+ states that are not described in the IBM. The data shown in Fig. [5](#page-3-0) are listed in Table III.

The overall conclusion from Fig. [5](#page-3-0) is the strong increase in the number of observed low-lying 0^+ states as the valence space expands, followed by a saturation in this number near midshell. These features should be characteristic of any realistic theoretical interpretation of these results.

V. CONCLUSION

To summarize, in a high-resolution experiment at the Tandem accelerator at LMU and TU Munich, we have identified the 0^+ states in ¹⁹⁸Hg up to about 3 MeV, finding far fewer than in other investigated nuclei. We note that recent experiments [5] on ¹⁹⁴Pt and ¹⁹²Pt show more 0^+ states than ¹⁹⁸Hg but fewer than in the nuclei near midshell. Thus, there is an approximately linear increase in the number of 0^+ states below 3 MeV as a function of valence-nucleon

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number up to about 22 valence nucleons. This is reasonable owing to the increasing number of possible 0^+ shell-model configurations. However, a second, contrasting, feature of the data is a saturation in the number of low-lying 0^+ states as one goes further into the shell (larger numbers of valence nucleons). This runs counter to the obvious increase in the total number of available 0^+ configurations with increasing valence-space size and is most likely attributable to the fact that many of the allowed 0^+ configurations lie well above 3 MeV in excitation energy.

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