Half-lives for α and cluster radioactivity within a Gamow-like model

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A simple phenomenological model based on the Gamow theory for the evaluation of half-lives for α and cluster radioactivity is proposed. The model contains only one adjustable parameter: the nuclear radius constant, common for both kind of decays and an additional hindrance factor to the lifetimes which gives the effect of an odd particle. A good agreement with the experimental data for nuclei with $Z \geqslant 84$ and $N \geqslant 104$ is achieved.

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I. INTRODUCTION

One of the most important decay modes of heavy nuclei, the α radioactivity, was discovered in 1899 by Rutherford [1] as one of three components of the radiation emitted by uranium nuclei. The first law describing the systematics of the α-decay half-life times $T_{1/2}^{\alpha}$ was proposed in 1911 by Geiger and Nuttall [2]. This phenomenological decay law was explained theoretically in 1928 by Gamow [3], who assumed that the α decay was due to the quantum mechanical tunneling of a charged α particle through the nuclear Coulomb barrier. The up-to-date experimental systematics of $T_{1/2}^{\alpha}$ can be found in Ref. [4]. The α-decay half-lives $T_{1/2}^{\alpha}$ of nuclei vary from 10^{-7} to 10^{18} s and kinetic energies of emitted α particles between 4 and 11 MeV. In 1966 Viola and Seaborg proposed a simple formula [5] based on the Gamow model which well approximates these half-lives. Its extended versions with parameters adjusted to the experimental data of heavy nuclei can be found in Refs. [6–8].

Another decay mode, cluster radioactivity, was theoretically predicted in 1980 by Sandulescu, Poenaru, and Greiner [9] and discovered four years later by Rose and Jones [10]. This decay is very rare and corresponds to the emission of a nucleus heavier than ^{4}He, but lighter than a typical mass of a light fragment in binary fission ($A \gtrsim 60$). The half-lives of cluster radioactivity vary from 10^{11} to 10^{29} s. Typical atomic masses of clusters are in the range $14 \leqslant A \leqslant 34$, while the mass of a daughter nucleus is close (± 4 nucleons) to the doubly magic ^{208}Pb isotope. Because of the large mass difference of cluster and daughter nucleus, the cluster radioactivity phenomenon may be described in a similar way as the α decay using Gamow-like theories [11] or as a very mass asymmetric fission process [9,12]. Both approaches, although from the first sight very different, lead nevertheless to similar estimates of the half-lives for the cluster radioactivity. Simple, Viola-Seaborg–like, phenomenological formulas for the cluster decay half-lives were proposed in Refs. [7,13].

A nice regularity in the systematics for the α and cluster decays, was recently discovered by Poenaru *et al.* in Ref. [14], where a single universal curve for both radioactivities was found. Following this idea, we have made an attempt to reproduce the half-lives for both these decays in a model based on the Gamow theory [15]. A simple formula for $T_{1/2}$ is derived using the WKB theory for the penetration of the Coulomb barrier with a square well for the nuclear part. We show in the following that using only one adjustable parameter, the radius constant, it is possible to reproduce with a good accuracy all existing data for decays of even-even nuclei. Similarly as in the other papers (see, e.g., [6,7,13]) an additional parameter, here a hindrance factor h, is introduced to describe the decay of odd systems. The decay half-lives are calculated and compared to the experimental data for the 298 α particle and 26 cluster decay processes of nuclei heavier than ^{208}Pb. A good agreement with the data has been achieved.

II. MODEL

The half-life of a decaying nucleus is then given by

$$T_{1/2} = \frac{\ln 2}{\lambda} 10^h \ , \tag{1}$$

where h is the so-called decay hindrance factor due to the effect of an odd-proton or/and an odd-neutron. Of course for the decays of the even-even nuclei one takes $h = 0$. λ is the decay constant for the α or cluster emission and it is equal to the following product (see, e.g., [14]):

$$\lambda = \nu S P \ , \tag{2}$$

where ν is the number of the collective assaults per time unit of the emitted object on the potential barrier and S is the preformation probability of the α particle or the cluster at the nuclear surface.

The probability of tunneling of an α particle or a cluster through the potential barrier presented in Fig. 1 is given within the WKB theory by the following integral (see, e.g., [16]):

$$P = \exp\left[-\frac{2}{\hbar}\int_R^b \sqrt{2\mu(V(r) - E_{\rm k})}dr\right] . \tag{3}$$

Here $\mu = M_{\rm nuc} A_1 A_2/(A_1 + A_2)$ is the reduced mass, whereas A_1 and A_2 are atomic mass numbers of the emitted cluster and the daughter nucleus, respectively, $M_{\rm nuc} = 931$ MeV$/c^2$ is the nuclear mass unit, and $E_{\rm k}$ the kinetic energy of the emitted particle. The spherical square well radius R is equal to the sum of the radii of both decay fragments

$$R = r_0\big(A_1^{1/3} + A_2^{1/3}\big) \ . \tag{4}$$

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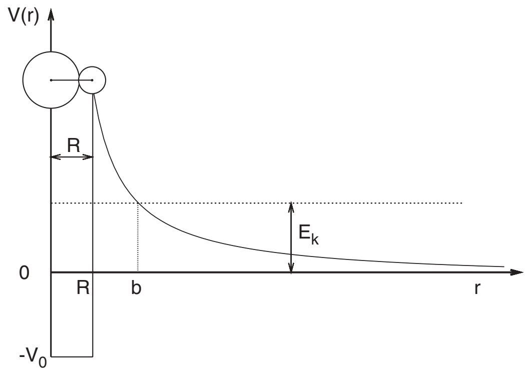

FIG. 1. Schematic plot of the potential energy as a function of the distance between the centers of the decaying nuclei.

One should interpret R as an effective radius for which the action integral (3) is equal to the 'experimental' ones. The parameter r_0 has then to be fitted to the data. The exit point from the barrier b corresponds to point, where the Coulomb potential is equal to the kinetic energy (E_k):

$$b = \frac{Z_1 Z_2 e^2}{E_\mathrm{k}} \,, \tag{5}$$

where Z_1 and Z_2 are the charge numbers of the emitted cluster and the residual nucleus, respectively. The kinetic energies of the emitted particles are taken in the present calculation from Ref. [4].

The potential energy $V(r)$ shown in Fig. 1 is given by

$$V(r) = \begin{cases} -V_0 & 0 \leqslant r \leqslant R\,, \\ \frac{Z_1 Z_2 e^2}{r} & r > R\,. \end{cases} \tag{6}$$

After taking into account Eqs. (4), (5), and (6) one can easily evaluate the integral in the following analytical form (see, e.g., [16]):

$$P = \exp\left\{ -\frac{2}{\hbar}\sqrt{2\mu Z_1 Z_2 e^2 b}\left[\arccos\sqrt{\frac{R}{b}} - \sqrt{\frac{R}{b} - \left(\frac{R}{b}\right)^2}\,\right]\right\}. \tag{7}$$

Usually one evaluates the frequency of assaults on the barrier ν from the following classical expression:

$$\nu = \frac{\sqrt{2(E_\mathrm{k} + V_0)/\mu}}{2R}, \tag{8}$$

where the depth of the potential well V_0 is one of the model parameters. In different approaches it varies from 100 MeV [17] to 1100 MeV [22]. The last value was obtained from the α-scattering data. Blendowske and Walliser have assumed the potential depth for the cluster with the mass number A_1 as $V_0 = 25A_1$ MeV [17].

The preformation probability of the α-particle S_α can be estimated within microscopic theories [18,19] or treated as the penetrability of the internal part of the barrier in a fission theory [20,21], but in the majority of applications it is treated as a free adjustable parameter. The preformation factor of a cluster with mass A_1 is often taken in a form proposed in [17]:

$$S_\mathrm{c} = S_\alpha^{(A_1-1)/3} \,, \tag{9}$$

where S_α is a formation factor for the α particle.

Looking at Eqs. (2) and (8) it is easy to guess that the depth of the nuclear potential well V_0 and the preformation factor S_α be correlated when one is going to reproduce the experimental decay constant λ. The deeper the potential well, the higher the frequency of assaults and, consequently, the lower the value of S_α is needed to be to reproduce the data when the radius R of the square well is fixed. On the other hand, the change of V_0 or S_α can be accommodated by an appropriate shift of the radius constant as one can see in Figs. 2 and 3.

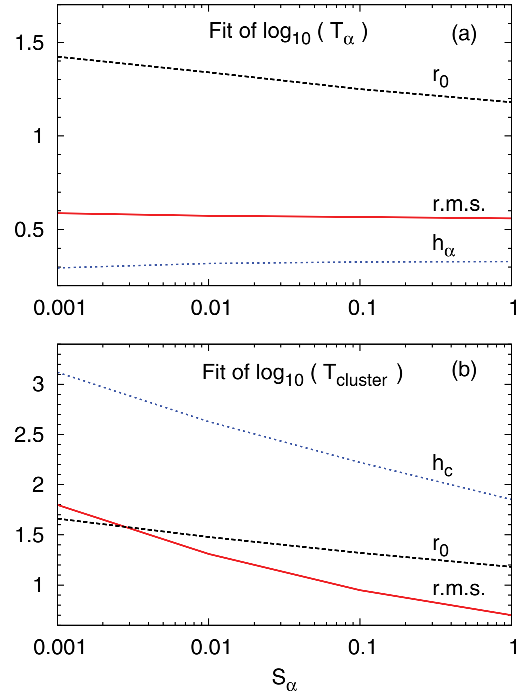

FIG. 2. (Color online) Influence of the formation factor S_α on the rms deviation of the WKB estimates from the data for the α decay (a) and cluster emission (b). The change of the radius constant obtained by the least square fit as well as the hindrance factor for the odd systems are plotted as a function of S_α.

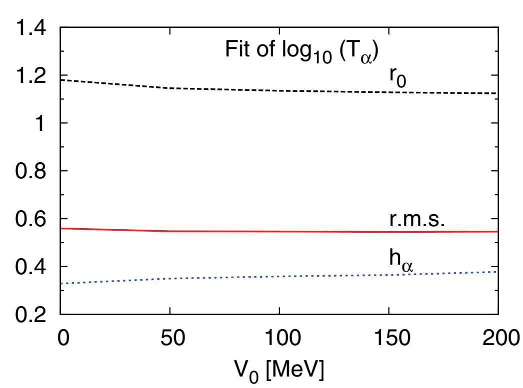

FIG. 3. (Color online) Similar data as in Fig. 2 but as a function of the nuclear potential depth constant V_0 in Eq. (6).

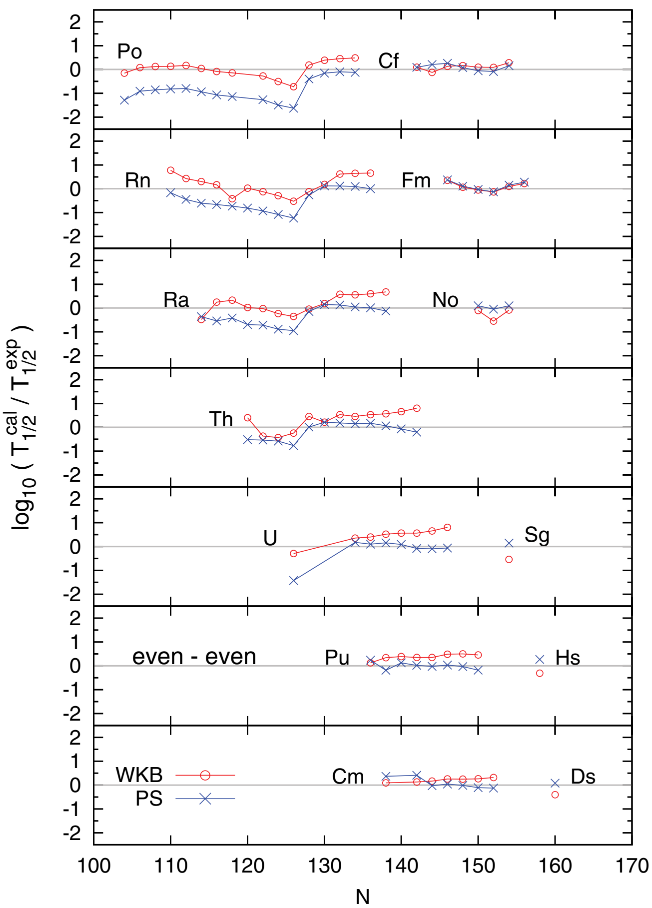

FIG. 4. (Color online) Differences of the theoretical and experimental α decay half-lives on logarithmic scale calculated for even-even nuclei according to the presented model (WKB, circles) and the Parkhomenko and Sobiczewski formula [6] (PS, crosses).

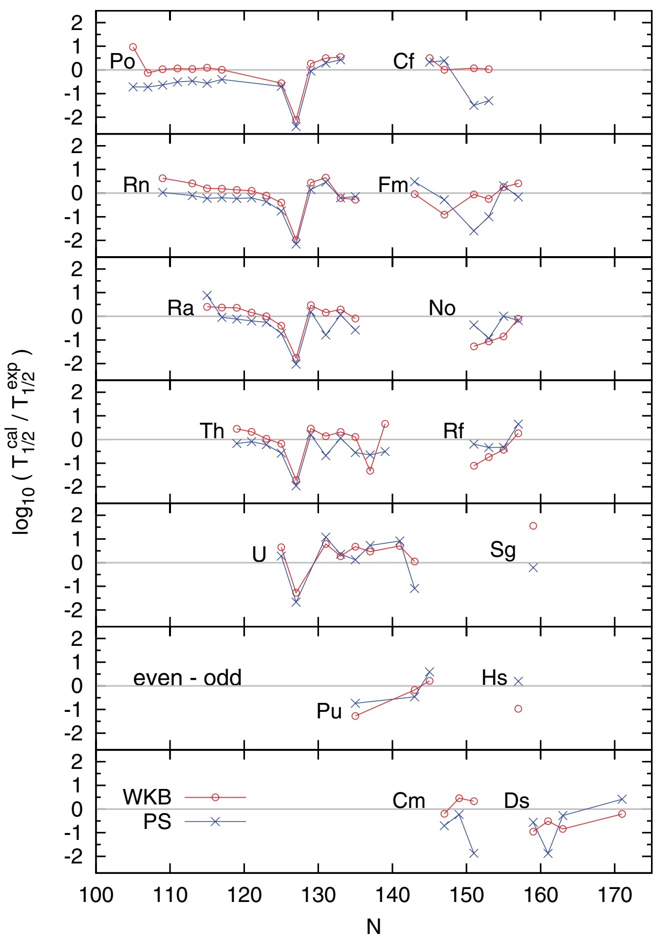

FIG. 5. (Color online) The same as in Fig. 4 but for even-odd nuclei.

Performing a least square fit of the radius constant r_0 (4) (and the hindrance factor h for the odd systems; see the next section) to known experimental half-lives for the α decays (298 cases) and the cluster emission (26 cases) from nuclei heavier than ^{208}Pb as function of S_α and keeping $V_0 = 0$ constant, we get the root-mean-square (rms) deviation of $\log_{10}(T_{1/2}^{\rm cal}/T_{1/2}^{\rm exp})$ as presented in Fig. 2. The cluster preformation probability is related here to S_α according to Eq. (9). Both for the α decay [Fig. 2(a)] and the cluster emission [Fig. 2(b)] the best fit is obtained when $S_\alpha = 1$. The increase of the preformation factor can be compensated by an appropriate reduction of the radius constant. This result confirms the conclusions of Refs. [20,21] that the preformation probability of an α particle or a cluster is related to the penetrability of the internal part of the barrier: a smaller effective radius of the square well (6) gives a smaller tunneling probability (3). It is also worthwhile to stress that the best fit for both the α and the cluster decay is obtained at almost the same value of $r_0 \approx 1.2$ fm which is in line with the results of Ref. [14], where a common universal curve for the α and cluster radioactivities was found.

Similarly, the increase of the nuclear potential depth, and in consequence, the frequency of assaults [via Eq. (8)], can be compensated by a reduction of the radius constant as one can see in Fig. 3 for the α decay half-lives. It is interesting to notice that the quality of the fit (rms) is almost independent of the potential well depth. That figure is made for a constant preformation probability $S_\alpha = 1$.

As the results of the above investigation we have decided to simplify the formula (2) for the decay constant by neglecting the preformation probability as it can be included in the tunneling probability P given by the integral (3):

$$\lambda = \nu P \ , \tag{10}$$

and to evaluate in the following the number of assaults per time unit on the barrier using an approximate equation for the ground state of the spherical square-well potential with the radius given Eq. (4):

$$\nu = \frac{\pi \hbar}{2\mu R^2} \ . \tag{11}$$

It is seen that in this approach the frequency of assaults depends only on the size of the well and the reduced mass μ of the emitted fragment.

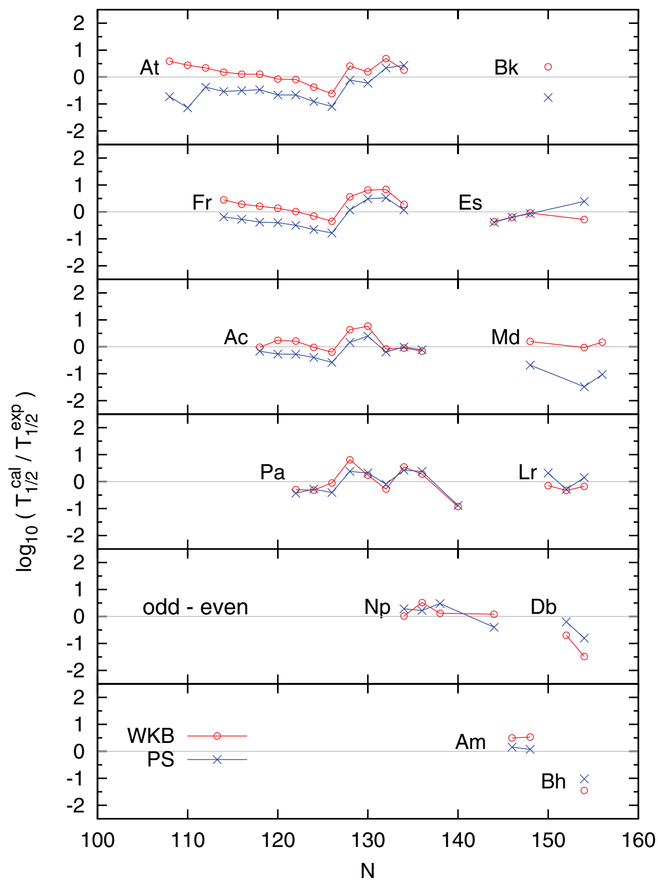

FIG. 6. (Color online) The same as in Fig. 4 but for odd-even nuclei.

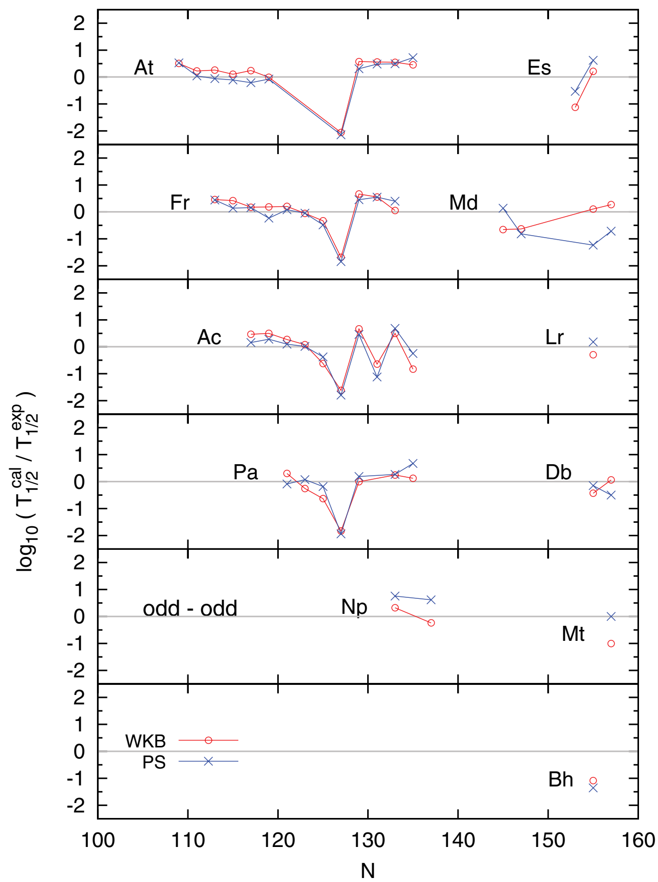

FIG. 7. (Color online) The same as in Fig. 4 but for odd-odd nuclei.

III. α DECAYS

The least square fit of the radius constant r_0 is performed by an adjustment to the logarithms of the measured half-lives for the 298 α decays listed in Ref. [4] for nuclei heavier than ^{208}Pb. The model estimates are obtained using Eqs. (1), (3), and (10), (11). For nuclei with an odd number of nucleons the additional constant *hindrance factor* (h_p, h_n or h_{np}) is fitted. Following the discussion in the previous section we assume here the formation probability $S_\alpha = 1$ when preforming the fitting procedure. As the quality of the fit is almost the same when one assumes only one hindrance constant h instead of three independent hindrance factors for each kind of odd nucleon: h_p, h_n, or h_{np}, we have decided to stay with only two adjustable parameters r_0 and h which we have obtained through the least square fit procedure:

$$r_0 = 1.21 \text{ fm}, \quad h = h_p = h_n = 0.216, \quad h_{np} = 2h\,. \tag{12}$$

The radius constant was simultaneously fitted to the data for the α and the cluster radioactivities.

The least square fit value of the radius constant is $r_0 = 1.21$ fm and its distribution width is only $\sigma_{r_0} = 0.02$ fm. The data obtained from $\log_{10}(T_{1/2})$ for the α decays are presented by the open circles while those obtained from the half-lives for the cluster emission by the filled symbols. The average value of the hindrance factor for the α decays is $h = 0.216$ and its widths is $\sigma_h = 0.546$.

The α-decay half-lives obtained within the present model are compared below with the phenomenological formula developed by Parkhomenko and Sobiczewski [6]:

$$\log_{10} T_{1/2}^{\alpha}(Z,N) = aZ(Q_\alpha(Z,N) - E_i)^{-1/2} + bZ + c\ , \tag{13}$$

where the constants $a = 1.5372$, $b = -0.1607$, and $c = -36.573$ are common for all decaying nuclei. The effect of an odd proton or neutron on $T_{1/2}^{\alpha}$ is obtained in this approach by the energy shifts in the Q_α-value $E_p = 0.113$ MeV, $E_n = 0.171$ MeV, and $E_{pn} = E_p + E_n$. The estimates made using the formula (13) which contains five adjustable parameters, belong to the most precise ones.

The logarithm of the ratio of the calculated half-life ($T_{1/2}^{cal}$) to the measured one ($T_{1/2}^{exp}$) for all examined nuclei is presented in Figs. 4–7 as a function of the neutron number. The estimates obtained with the present WKB model (circles) are compared with the results obtained using the Parkhomenko-Sobiczewski (PS) formula (13) (crosses) [6]. It is seen in Fig. 4 that for even-even nuclei both models give comparable estimates. The WKB results are closer to the data for lighter nuclei than the PS estimates while in the superheavy region the PS results are slightly better. Large deviations of both estimates from the data are observed for even-odd nuclei (Fig. 5) around the magic numbers $N = 126$ and $N = 152$ or 154, where the considered models underestimate the $T_{1/2}^{exp}$ by two orders of magnitude. A slightly better agreement can be observed in Fig. 6 for odd-even

TABLE I. Root-mean-square deviations of $\log_{10}(T_{1/2})$ calculated for the Viola-Seaborg–like formula given in Eq. (13) for the α decay [6] and in Eq. (14) the cluster radioactivity [13] is compared with the results of the present WKB model with the radius constant $r_0 = 1.21$ fm.

$\pi_Z - \pi_N$	n	h	V-S rms	WKB rms
	α decay		[6]	
e-e	96		0.54	0.39
e-o	85	0.216	0.78	0.68
o-e	65	0.216	0.53	0.47
o-o	52	0.432	0.72	0.68
Cluster radioactivity			[13]	
e-e	16		0.80	0.85
odd A	10	1.973	0.64	0.45

isotopes, where typically the estimates deviate from the data by less than one order of magnitude. The effect of the magic number $N = 126$ is also visible in Fig. 7 in which the results for odd-odd nuclei are presented. Here the quality of both fits is comparable. The large deviations of the estimated and the measured half-lives observed in the vicinity of magic numbers demand the inclusion of the shell effects in the model. It is worth to mention here that in Ref. [8] the new Geiger-Nuttal law for the calculations of α-decay half-lives was proposed where the effects of the quantum numbers, in particular the magic number $N = 126$, were taken into account.

The rms deviations of the WKB estimates made for the α and cluster decays using Eqs. (1), (3), and (10), (11) with the same radius constant $r_0 = 1.21$ fm for all α and clusters radioactivities and only one hindrance factor for odd systems are on the average slightly smaller than the rms obtained when using the phenomenological five parameters formula (13) proposed in [6]. The details are summarized in Table I.

IV. CLUSTER RADIOACTIVITY

The same formalism as for the α decay was used to describe the cluster-emission half-lives with the same radius constant $r_0 = 1.21$ fm.

The estimates of the half-lives for the cluster decay are compared in Fig. 8 with the experimental data taken from [4,11,12,23–25]. A good agreement with the data is achieved without any readjustment of the radius constant. For the odd-A systems (ten cases) an additional hindrance factor $h = 1.973$ is introduced. For the majority of cases (22 of 26) the deviation does not exceed one order of magnitude. The rms deviation of $\log_{10}(T_{1/2})$ is equal to 0.85 in the case of the decay of even-even nuclei, while for odd-nuclei it is only 0.45.

Our results are compared in Table I with the estimates obtained using the Viola-Seaborg type formula developed in Ref. [13]:

$$\log_{10}(T_{1/2}) = aZ_1Z_2Q^{-1/2} + cZ_1Z_2 + d + h\ , \quad (14)$$

which contains four adjusted parameters:

$$a = 1.51799,\ c = -0.053387,\ d = -92.91142,\ h = 1.402.$$

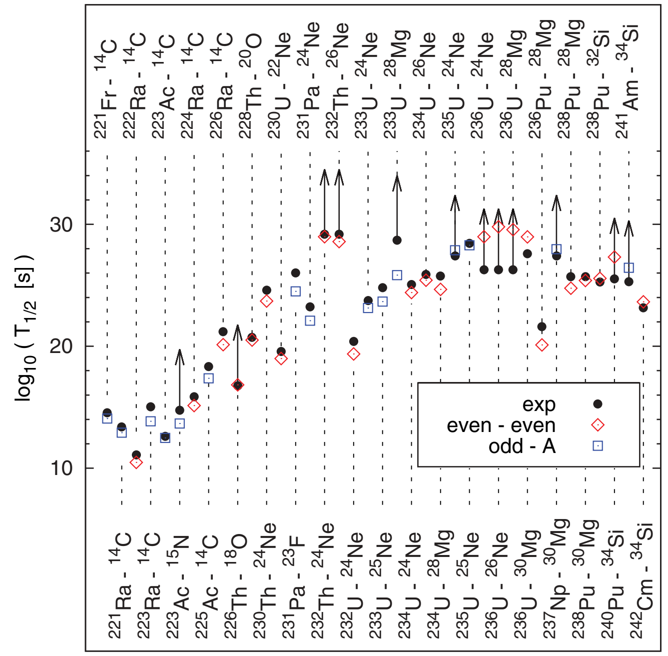

FIG. 8. (Color online) Cluster radioactivity half-lives. The experimental data (dots) are compared to calculated values (open symbols). Arrows are set in cases where observed half-lives have determined only a lower limit.

The last parameter is the hindrance factor for emission of clusters from odd-A nuclei. It is seen from Table I that our estimates obtained with only one fitted parameter to the cluster data reproduce the experimental data with a similar quality as the formula (14).

V. SUMMARY

The following conclusions can be drawn from our investigation:

(i) The effect of the preformation factor can be emulated by an appropriate change of the radius constant which respectively decreases the tunneling probability.

(ii) The depth of the collective nuclear potential well felt by an effective α particle or cluster formed in the decaying nucleus is probably much smaller that one commonly assumes.

(iii) The frequency of assaults of the emitted light fragment can be estimated from the quantum-mechanical ground-state frequency in the spherical square well.

(iv) For all nuclei with $Z \geqslant 84$ and $N \geqslant 104$, i.e., for 298 α and 26 cluster experimentally known decay events, our model describes with rather good accuracy the half-lives of both types of decays.

(v) The model reproduces α and cluster decay half-lives using the same radius constant $r_0 = 1.21$ fm, common for even and odd decaying nuclei.

(vi) The effect of an odd particle on the α-decay lifetimes was introduced in the form of one additional parameter: a hindrance factor h, the same for nuclei with odd proton or neutron number, while for the odd-odd system the hindrance factor is doubled. Also only one hindrance factor was used to reproduce the half-lives of odd-A nuclei with respect to the cluster emission.

We believe the results of our investigations will also be important for other branches of physics and chemistry which are dealing with the potential barrier tunneling.

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