PHYSICAL REVIEW C **87**, 021302(R) (2013)

Partial dynamical symmetry as a selection criterion for many-body interactions

A. Leviatan,[1] J. E. García-Ramos,[2] and P. Van Isacker[3]
[1]*Racah Institute of Physics, The Hebrew University, Jerusalem 91904, Israel*
[2]*Department of Applied Physics, University of Huelva, 21071 Huelva, Spain*
[3]*Grand Accélérateur National d'Ions Lourds, CEA/DSM–CNRS/IN2P3, B.P. 55027, F-14076 Caen Cedex 5, France*
(Received 11 January 2013; revised manuscript received 4 February 2013; published 20 February 2013)

We propose the use of partial dynamical symmetry (PDS) as a selection criterion for higher-order terms in situations when a prescribed symmetry is obeyed by some states and is strongly broken in others. The procedure is demonstrated in a first systematic classification of many-body interactions with SU(3) PDS that can improve the description of deformed nuclei. As an example, the triaxial features of the nucleus ^{156}Gd are analyzed.

DOI: 10.1103/PhysRevC.87.021302 PACS number(s): 21.60.Fw, 21.10.Re, 21.60.Ev, 27.70.+q

Many-body forces play an important role in quantum many-body systems [1]. They appear either at a fundamental level or as effective interactions which arise due to restriction of degrees of freedom and truncation of model spaces. A known example is the structure of light nuclei, where two-nucleon interactions are insufficient to achieve an accurate description and higher-order interactions between the nucleons must be included [2]. Given the difficulty in constraining the nature of such higher-order terms from experiments, one is faced with the problem of their determination. One way, currently the subject of active research [3], is to determine them from chiral effective field theory applied to quantum chromodynamics. This establishes a hierarchy of inter-nucleon interactions according to their order. In light-medium nuclei, these interactions serve as input for *ab initio* methods [e.g., the no-core shell model (NCSM) [4]] to generate, by means of similarity transformations, A-body effective Hamiltonians in computational tractable model spaces.

The situation is more complex in heavy nuclei, where *ab initio* methods are limited by the enormous increase in size of the model spaces required to accommodate correlated collective motion of many nucleons. One possible approach to circumventing this problem is to augment the NCSM method through a symplectic symmetry-adapted choice of basis [5]. A second approach is to employ energy density functionals and incorporate beyond mean-field effects by mapping to collective Hamiltonians [6], e.g., the interacting boson model (IBM) [7]. In both approaches the Hilbert spaces are based on particular dynamical algebras which lead to a dramatic reduction of the basis dimension. Nevertheless, even with such simplification, the number of possible interactions in the effective Hamiltonians grows rapidly with their order, and a selection criterion is called for. In this Rapid Communication, we suggest a method to select possible higher-order terms which is based on the idea of partial dynamical symmetry (PDS).

The concept of PDS [8] is a generalization of that of a dynamical symmetry (DS) [9] where the conditions of the latter (solvability of the complete spectrum, existence of exact quantum numbers for all eigenstates, and pre-determined structure of the eigenfunctions) are relaxed and apply to only *part* of the eigenstates and/or of the quantum numbers. PDSs have been identified in various dynamical systems involving bosons and fermions (for a review, see Ref. [8]). They play a role in diverse phenomena including nuclear and molecular spectroscopy [10–12], quantum phase transitions [13] and mixed regular and chaotic dynamics [14]. Here we consider the SU(3) symmetry in view of its significance for deformed nuclei, as recognized in the Elliott and symplectic shell models [15,16] and the IBM. We use the mathematical algorithm to construct, order by order, all possible interactions with a given PDS [17,18], apply it to the SU(3) limit of the IBM, and illustrate with a concrete example how the PDS and data constrain the form and strength of higher-order interactions.

The IBM describes low-energy collective states of the nucleus in terms of N monopole (s) and quadrupole (d) bosons representing pairs of nucleons. The dynamical algebra is U(6) with generators in terms of which operators of all physical observables can be written. The classification of states in the SU(3) limit is [19]

$$\begin{array}{ccccccc} \mathrm{U(6)} & \supset & \mathrm{SU(3)} & \supset & \mathrm{SO(3)} & \supset & \mathrm{SO(2)} \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ [N] & & (\lambda,\mu)\ K & & L & & M \end{array}, \tag{1}$$

where underneath each algebra the associated labels are given [K is a multiplicity label needed in the SU(3) $\supset$ SO(3) reduction]. These define the Elliott basis [15], $|[N](\lambda,\mu)KLM\rangle$, from which the Vergados basis [20], $|[N](\lambda,\mu)\tilde{\chi}LM\rangle$, is obtained by a standard orthogonalization procedure. The classification (1) assumes a symmetric U(6) irreducible representation (irrep) $[N]$ which is appropriate for the IBM. Apart from terms involving the conserved total boson number operator $\hat{N}$, a rotational-invariant Hamiltonian with SU(3) DS has the form

$$\hat{H}_{\mathrm{DS}} = \alpha_1\hat{C}_2[\mathrm{SU(3)}] + \alpha_2\hat{C}_2[\mathrm{SO(3)}] + \alpha_3\hat{C}_3[\mathrm{SU(3)}], \tag{2}$$

where $\hat{C}_n[G]$ is the nth-order Casimir operator of the Lie algebra G and α_i are coefficients. This form exhausts all independent Casimir operators of SU(3) and SO(3), that is, any other commuting operator can be written as a function of those appearing in Eq. (2). $\hat{H}_{\mathrm{DS}}$ is completely solvable with eigenenergies

$$E_{\mathrm{DS}} = \alpha_1 f_2(\lambda,\mu) + \alpha_2 L(L+1) + \alpha_3 f_3(\lambda,\mu), \tag{3}$$

where $f_2(\lambda,\mu) = \lambda^2 + (\lambda+\mu)(\mu+3)$ and $f_3(\lambda,\mu) = (\lambda-\mu)(2\lambda+\mu+3)(\lambda+2\mu+3)$. The spectrum resembles that of

TABLE I. Normalized two- and three-boson SU(3) tensors.

n	(λ,μ)	$\tilde{\chi}$	ℓ	$\hat{B}^\dagger_{[n](\lambda,\mu)\tilde{\chi}\ell m}$
2	(4,0)	0	0	$\sqrt{\frac{5}{18}}(s^\dagger)^2+\sqrt{\frac{2}{9}}(d^\dagger d^\dagger)^{(0)}_0$
2	(4,0)	0	2	$\sqrt{\frac{7}{9}}s^\dagger d^\dagger_m-\sqrt{\frac{1}{9}}(d^\dagger d^\dagger)^{(2)}_m$
2	(4,0)	0	4	$\sqrt{\frac{1}{2}}(d^\dagger d^\dagger)^{(4)}_m$
2	(0,2)	0	0	$P^\dagger_0\equiv-\sqrt{\frac{2}{9}}(s^\dagger)^2+\sqrt{\frac{5}{18}}(d^\dagger d^\dagger)^{(0)}_0$
2	(0,2)	0	2	$P^\dagger_{2m}\equiv\sqrt{\frac{2}{9}}s^\dagger d^\dagger_m+\sqrt{\frac{7}{18}}(d^\dagger d^\dagger)^{(2)}_m$
3	(6,0)	0	0	$\sqrt{\frac{7}{162}}(s^\dagger)^3+\sqrt{\frac{14}{45}}s^\dagger(d^\dagger d^\dagger)^{(0)}_0-\sqrt{\frac{8}{405}}[(d^\dagger d^\dagger)^{(2)}d^\dagger]^{(0)}_0$
3	(6,0)	0	2	$\sqrt{\frac{7}{30}}(s^\dagger)^2d^\dagger_m-\sqrt{\frac{2}{15}}s^\dagger(d^\dagger d^\dagger)^{(2)}_m+\sqrt{\frac{2}{21}}[(d^\dagger d^\dagger)^{(0)}d^\dagger]^{(2)}_m$
3	(6,0)	0	4	$\sqrt{\frac{11}{30}}s^\dagger(d^\dagger d^\dagger)^{(4)}_m-\sqrt{\frac{14}{165}}[(d^\dagger d^\dagger)^{(2)}d^\dagger]^{(4)}_m$
3	(6,0)	0	6	$\sqrt{\frac{1}{6}}[(d^\dagger d^\dagger)^{(4)}d^\dagger]^{(6)}_m$
3	(2,2)	0	0	$W^\dagger_0\equiv-\sqrt{\frac{1}{9}}(s^\dagger)^3+\sqrt{\frac{1}{20}}s^\dagger(d^\dagger d^\dagger)^{(0)}_0-\sqrt{\frac{7}{180}}[(d^\dagger d^\dagger)^{(2)}d^\dagger]^{(0)}_0$
3	(2,2)	0	2	$V^\dagger_{2m}\equiv-\sqrt{\frac{14}{65}}(s^\dagger)^2d^\dagger_m-\sqrt{\frac{1}{130}}s^\dagger(d^\dagger d^\dagger)^{(2)}_m+\sqrt{\frac{18}{91}}[(d^\dagger d^\dagger)^{(0)}d^\dagger]^{(2)}_m$
3	(2,2)	2	2	$W^\dagger_{2m}\equiv\sqrt{\frac{2}{39}}(s^\dagger)^2d^\dagger_m+\sqrt{\frac{14}{39}}s^\dagger(d^\dagger d^\dagger)^{(2)}_m+\sqrt{\frac{5}{78}}[(d^\dagger d^\dagger)^{(0)}d^\dagger]^{(2)}_m$
3	(2,2)	2	3	$W^\dagger_{3m}\equiv\sqrt{\frac{7}{30}}[(d^\dagger d^\dagger)^{(2)}d^\dagger]^{(3)}_m$
3	(2,2)	2	4	$W^\dagger_{4m}\equiv\sqrt{\frac{2}{15}}s^\dagger(d^\dagger d^\dagger)^{(4)}_m+\sqrt{\frac{7}{30}}[(d^\dagger d^\dagger)^{(2)}d^\dagger]^{(4)}_m$
3	(0,0)	0	0	$\Lambda^\dagger\equiv-\sqrt{\frac{1}{81}}(s^\dagger)^3+\sqrt{\frac{5}{36}}s^\dagger(d^\dagger d^\dagger)^{(0)}_0+\sqrt{\frac{35}{324}}[(d^\dagger d^\dagger)^{(2)}d^\dagger]^{(0)}_0$

a quadrupole axially deformed rotor with eigenstates arranged in SU(3) multiplets, and K corresponds geometrically to the projection of the angular momentum on the symmetry axis. The Hamiltonian $\hat{H}_{\rm DS}$ is genuinely many-body (with interactions that are up to third order in the bosons). Its applicability is limited, however, since only three independent operators exist, and states in different K bands with the same $(\lambda,\mu)L$ are degenerate. Flexibility can be considerably increased by introducing interactions with PDS. The method to construct such interactions is based on an expansion of the Hamiltonian in terms of tensors which annihilate prescribed set of states [17,18]. In the present study, the tensors involve n-boson creation and annihilation operators with definite character under the SU(3) chain (1),

$$\hat{B}^\dagger_{[n](\lambda,\mu)\tilde{\chi}\ell m},\quad \tilde{B}_{[n^5](\mu,\lambda)\tilde{\chi}\ell m}\equiv(-)^m(\hat{B}^\dagger_{[n](\lambda,\mu)\tilde{\chi}\ell,-m})^\dagger. \qquad (4)$$

The SU(3) tensor operators for $n=2$ and 3 are given in Table I. Of particular interest are the operators with $(\lambda,\mu)\neq(2n,0)$ because the corresponding annihilation operators yield zero when acting on the ground-band members $|[N](2N,0)K=0,LM\rangle$ (and possibly other states). Interactions involving these operators can be added to the Hamiltonian (2) without destroying solvability of part of its spectrum. Two such operators, $P^\dagger_0$ and $P^\dagger_{2m}$, exist for $n=2$, and allow the construction of an IBM Hamiltonian with up to two-boson interactions that have a solvable ground band $|[N](2N,0)K=0,LM\rangle$ and a solvable γ band $|[N](2N-4,2)K=2,LM\rangle$. A two-body Hamiltonian with SU(3) PDS can be applied to ^{168}Er, and the excellent SU(3) description of the energies and $E2$ properties of these bands can be retained while lifting the degeneracy of the β and γ bands [10].

The most general $(2+3)$-body Hamiltonian with SU(3) PDS can be written in terms of the operators given in Table I,

$$\begin{aligned}\hat{H}_{\rm PDS}=&\,h_2P^\dagger_2\cdot\tilde{P}_2+h_0P^\dagger_0P_0+g_4W^\dagger_4\cdot\tilde{W}_4+g_3W^\dagger_3\cdot\tilde{W}_3\\&+g^a_2V^\dagger_2\cdot\tilde{V}_2+g^b_2W^\dagger_2\cdot\tilde{W}_2+g^c_2(V^\dagger_2\cdot\tilde{W}_2+W^\dagger_2\cdot\tilde{V}_2)\\&+g^a_0\Lambda^\dagger\Lambda+g^b_0W^\dagger_0W_0+g^c_0(\Lambda^\dagger W_0+W^\dagger_0\Lambda),\end{aligned} \qquad (5)$$

with $2+8$ interaction strengths h_i and g^x_i. Terms involving the operator $\hat{C}_2[{\rm SO}(3)]\equiv\hat{L}^2$ can be added to this Hamiltonian, as is done in Eq. (2). To illustrate the increase in flexibility of a Hamiltonian with SU(3) PDS, we list in Table II the number of interactions under the different scenarios. Up to third order, a general rotationally invariant Hamiltonian has 26 independent interactions, decreasing to 16 if one is only interested in

TABLE II. Number of interactions in the IBM.

Order	Number of interactions[a]		
	General	SU(3) DS	SU(3) PDS
1	$2\mapsto1$	$1\mapsto0$	$1\mapsto0$
2	$7\mapsto5$	$3\mapsto2$	$4\mapsto3$
3	$17\mapsto10$	$4\mapsto1$	$10\mapsto6$
$1+2+3$	$26\mapsto16$	$8\mapsto3$	$15\mapsto9$

[a]On the left of $\mapsto$ is the number of interactions of a given order; this reduces to the number on the right of $\mapsto$ if one is only interested in excitation energies in a single nucleus.

excitation energies in a single nucleus. (This excludes terms involving $\hat{N}$). A Hamiltonian with SU(3) DS has, up to third order, eight independent terms, but five of them [$\hat{N}$, $\hat{N}^2$, $\hat{N}^3$, $\hat{N}\hat{L}^2$, and $\hat{N}\hat{C}_2$[SU(3)]] are constant in a single nucleus or can be absorbed in an interaction of lower order, leaving only the three genuinely independent terms shown in Eq. (2). The corresponding numbers for a Hamiltonian with SU(3) PDS are 15 and 9. The latter number agrees with the 10 terms in the Hamiltonian (5) which lacks $\hat{L}^2$ but includes the combinations $\hat{N}P_2^\dagger \cdot \tilde{P}_2$ and $\hat{N}P_0^\dagger P_0$. We conclude from Table II that more than half of all possible interactions in the IBM have in fact an SU(3) PDS.

Several SU(3)-preserving interactions are contained in expression (5). Specifically, $\hat{\theta}_2 \equiv 2\hat{N}(2\hat{N}+3) - \hat{C}_2$ corresponds to $h_0 = h_2 = 18$; $(\hat{N}-2)\hat{\theta}_2$: $g_0^a = 54$, $g_0^b = g_2^b = g_2^a = g_3 = g_4 = 30$; $\hat{C}_3 + (2\hat{N}+3)[3\hat{\theta}_2 - 2\hat{N}(4\hat{N}+3)]$: $g_0^a = 648$; and $\hat{\Omega} - (4\hat{N}+3)\hat{L}^2$: $h_2 = -108$, $g_0^a = 9g_0^b = -3g_0^c = 270$, $g_2^a/5 = g_2^b/21 = g_2^c/\sqrt{105} = 24/13$, $g_4 = -120$. The three terms involving $\hat{C}_n$[SU(3)] are included in $\hat{H}_{\rm DS}$ [Eq. (2)]. The (integrity basis) term $\hat{\Omega} = -4\sqrt{3}\hat{Q}\cdot(\hat{L}\times\hat{L})^{(2)}$ is composed of SU(3) generators, hence is diagonal in (λ,μ), but breaks the K degeneracy of the exact DS. Its impact on nuclear spectroscopy has been well studied in the symplectic shell model and the IBM [21–23]. The PDS notion goes a step further by allowing SU(3) mixing in most (but not all) of the eigenstates of the Hamiltonian.

As noted, in general, $\hat{H}_{\rm PDS}$ [Eq. (5)] does not preserve SU(3); yet, by construction, for *any* choice of parameters the ground-band members $|[N](2N,0)K=0,LM\rangle$ are solvable. For specific choices, additional solvable states are obtained. In particular, by choosing only the h_0, g_0^a, g_0^b, g_0^c terms and $\hat{H}_{\rm DS}$ [Eq. (2)], the states $|[N](2N-4,2k)K=2k,LM\rangle$, $k=1,2,\ldots$ (among which are the γ-band members with $k=1$) remain solvable with energies $E_{\rm DS}$ [Eq. (3)]. This case therefore has the same solvable states as the two-body Hamiltonian with SU(3) PDS considered in Ref. [10]; the additional three-body terms lead to a different mixing of the non-solvable states.

Another class of Hamiltonians with SU(3) PDS exists which has solvable β-band members $|[N](2N-4,2)K=0,LM\rangle$ with energies $E_{\rm DS}$ [Eq. (3)]. This follows from the structure of the relevant Hamiltonian,

$$\hat{H}'_{\rm PDS} = \hat{H}_{\rm DS} + \eta_2 W_2^\dagger \cdot \tilde{W}_2 + \eta_3 W_3^\dagger \cdot \tilde{W}_3, \qquad (6)$$

and the fact that W_{2m} and W_{3m} annihilate the intrinsic state of the β band, $|\beta\rangle \propto (\sqrt{2}P_0^\dagger - P_{20}^\dagger)(s^\dagger + \sqrt{2}d_0^\dagger)^{N-2}|0\rangle$. The property of solvability of the ground and β bands can be exploited in the following way. The Hamiltonian $\hat{H}_{\rm DS}$ [Eq. (2)] has a rotor spectrum with characteristic $L(L+1)$ splitting for all bands. Deviations from this pattern are often observed for the γ band of deformed nuclei and are indicative of γ-soft or triaxial behavior [24]. We illustrate the procedure with an application to ^{156}Gd. The parameters $\alpha_1 = -7.6$ keV, $\alpha_2 = 12.0$ keV, and $\alpha_3 = 0$ in $\hat{H}_{\rm DS}$ are fixed from the excitation energy of the β-band head and the moments of inertia of the ground and β bands. This completely determines the SU(3) DS spectrum, shown on the left of Fig. 1, which is characterized by degenerate β and γ bands. In the observed spectrum these

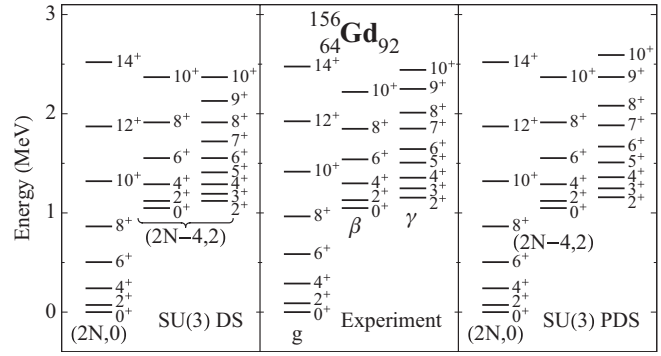

FIG. 1. Observed spectrum of ^{156}Gd [25] compared with the calculated spectra of $\hat{H}_{\rm DS}$ (2) with SU(3) DS and of $\hat{H}'_{\rm PDS}$ (6) with SU(3) PDS for $N=12$ and parameters given in the text. Bands are labeled with the SU(3) quantum numbers (λ,μ).

bands are not degenerate and, more importantly, the γ-band energies display an odd-even staggering. This effect can be visualized by plotting the quantity [26]

$$Y(L) = \frac{2L-1}{L} \times \frac{E(L)-E(L-1)}{E(L)-E(L-2)} - 1, \qquad (7)$$

where $E(L)$ is the excitation energy of a γ-band level with angular momentum L. For a rotor this quantity is flat, $Y(L)=0$, as illustrated in Fig. 2 with the SU(3) DS calculation. The data, however, show considerable odd-even staggering which can be well described by a combination of three-body interactions with $\eta_2 = -18.1$ keV and $\eta_3 = 46.2$ keV. The calculated staggering increases with L which agrees with the experiment up to $L=10$. For $L>10$ the observed staggering changes character, a phenomena requiring higher angular momentum pairs, which are beyond the scope of the standard (s,d) IBM description. The two interactions $W_2^\dagger \cdot \tilde{W}_2$ and $W_3^\dagger \cdot \tilde{W}_3$ induce a mixing of the γ band with higher-lying excited bands. Other approaches advocating the coupling of the γ band to the β band [23] or to the ground band [27] fail to describe the odd-even staggering in ^{156}Gd. For the PDS calculation, the wave functions of the states in the γ band involve 15% SU(3) admixtures into the dominant $(2N-4,2)$ component. Higher bands exhibit larger SU(3) mixing and their wave functions are spread over many SU(3) irreps, as shown for the $K=0_3$ band in Fig. 3. This complex SU(3) decomposition is in marked contrast to the SU(3) purity of

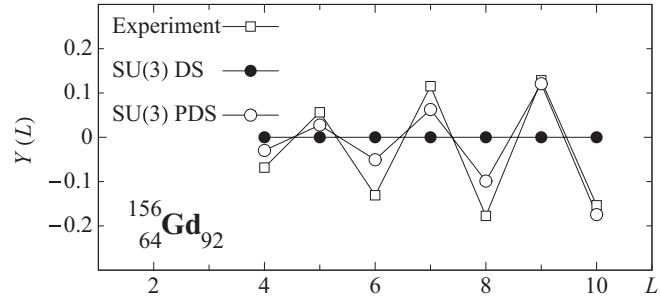

FIG. 2. Observed and calculated [SU(3) DS and PDS] odd-even staggering of the γ band in ^{156}Gd.

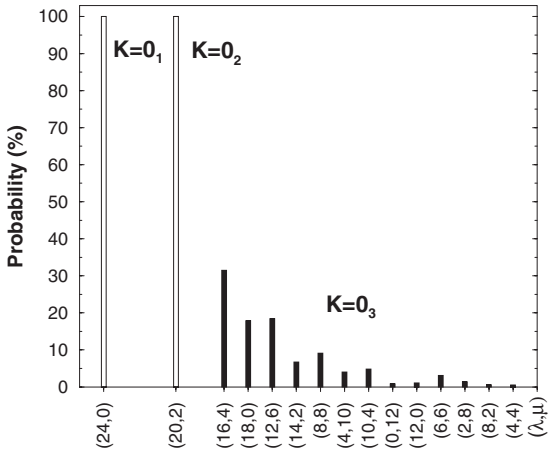

FIG. 3. SU(3) decomposition of wave functions of $L = 0$ states in the $K = 0_1,\ 0_2,\ 0_3$ bands for the PDS calculation.

the ground ($K = 0_1$) and β ($K = 0_2$) bands. Such strong symmetry-breaking cannot be treated in perturbation theory.

Previous studies of triaxiality in the IBM framework have employed only the cubic η_3 term of Eq. (6) [28,29]. The current work hints that both η_2 and η_3 terms are necessary for an accurate description of odd-even staggering in deformed nuclei. This highlights the capacity of the PDS approach to identify novel relevant terms of a given order. We emphasize that the PDS results for the γ band are obtained without altering the good agreement for the ground and β bands, already achieved with the SU(3) DS calculation. This is further illustrated with the $E2$ transitions in ^{156}Gd. The observed $B(E2)$ values between ground, β, and γ bands are shown in Table III and compared to the results of the SU(3) DS and PDS calculations. The effective boson charge $e_b = 0.166\ e$b in the electric quadrupole operator $e_b[s^\dagger\tilde{d} + d^\dagger s + \chi(d^\dagger\tilde{d})^{(2)}]$ and the value $\chi = -0.168$ are fitted to the $B(\mathrm{E2}; 2_1^+ \to 0_1^+)$ and $B(\mathrm{E2}; 2_\beta^+ \to 4_1^+)$ values. The $E2$ transitions between ground and β bands can be calculated analytically [30] and remain valid in SU(3) PDS. Transitions involving γ-band members are different in SU(3) DS and PDS and are computed numerically for the latter. It is seen from Table III that the mixing of the γ band with higher-lying excited bands improves the agreement with the data in most cases.

In summary, we have identified several classes of $(2 + 3)$-body IBM Hamiltonians with SU(3) PDS and obtained an improved description of signature splitting in the γ band of ^{156}Gd. The analysis serves to highlight the merits gained by using the notion of PDS as a tool for selecting higher-order terms in systems where a prescribed symmetry is not obeyed uniformly. On one hand, the PDS approach allows more flexibility by relaxing the constraints of an exact DS. On the other hand, the PDS picks particular symmetry-breaking terms which do not destroy results previously obtained with a DS for a segment of the spectrum. The PDS construction is implemented order by order, yet the scheme is non-perturbative in the sense that the non-solvable states experience strong symmetry-breaking. These virtues can be exploited in attempts to extend the ab-initio and beyond-mean-field methods to heavy nuclei. The present work motivates and sets the stage for further exploring the impact of PDS with higher-order terms on the dynamics in quantum many-body systems.

TABLE III. Observed and calculated $B(E2)$ values in ^{156}Gd. The parameters of the $E2$ operator are given in the text.

Transition	$B(\mathrm{E2}; L_i^\pi \to L_f^\pi)$ $(e^2\,\mathrm{b}^2)$		
	Expt. [25]	SU(3) DS	SU(3) PDS
$2_1^+ \to 0_1^+$	0.933 *25*	0.933	0.933
$4_1^+ \to 2_1^+$	1.312 *25*	1.313	1.313
$6_1^+ \to 4_1^+$	1.472 *40*	1.405	1.405
$8_1^+ \to 6_1^+$	1.596 *85*	1.409	1.409
$10_1^+ \to 8_1^+$	1.566 *70*	1.364	1.364
$2_\beta^+ \to 0_\beta^+$	0.26 *11*	0.679	0.679
$4_\beta^+ \to 2_\beta^+$	1.40 *75*	0.951	0.951
$0_\beta^+ \to 2_1^+$	0.04 *2*	0.034	0.034
$2_\beta^+ \to 0_1^+$	0.0031 *3*	0.0055	0.0055
$2_\beta^+ \to 2_1^+$	0.0165 *15*	0.0084	0.0084
$2_\beta^+ \to 4_1^+$	0.0204 *20*	0.020	0.020
$4_\beta^+ \to 2_1^+$	0.0065 *35*	0.0067	0.0067
$4_\beta^+ \to 4_1^+$		0.0067	0.0067
$4_\beta^+ \to 6_1^+$	0.0105 *55*	0.021	0.021
$2_\gamma^+ \to 0_1^+$	0.0233 *8*	0.035	0.030
$2_\gamma^+ \to 2_1^+$	0.0361 *12*	0.056	0.048
$2_\gamma^+ \to 4_1^+$	0.0038 *2*	0.0037	0.0031
$3_\gamma^+ \to 2_1^+$	0.0364 *70*	0.062	0.053
$3_\gamma^+ \to 4_1^+$	0.0254 *50*	0.032	0.028
$4_\gamma^+ \to 2_1^+$	0.0090 *25*	0.017	0.015
$4_\gamma^+ \to 4_1^+$	0.050 *15*	0.067	0.057
$4_\gamma^+ \to 6_1^+$		0.0089	0.0076
$4_\gamma^+ \to 2_\beta^+$	0.0214 *80*	0.0033	0.0096

This work was supported in part (A.L.) by the Israel Science Foundation and in part (J.E.G.R.) by the Spanish Ministerio de Economía y Competitividad and the European regional development fund (FEDER) under Project No. FIS2011-28738-C02-02, by Junta de Andalucía under Project No. P07-FQM-02962, and by Spanish Consolider-Ingenio 2010 (CPANCSD2007-00042). J.E.G.R. and P.V.I. acknowledge support from the Spanish-French collaboration project AIC-D-2011-0676.

[1] H.-W. Hammer, A. Nogga, and A. Schwenk, Rev. Mod. Phys. **85**, 197 (2013).

[2] S. C. Pieper and R. B. Wiringa, Annu. Rev. Nucl. Part. Sci. **51**, 53 (2001); S. C. Pieper, Nucl. Phys. A **751**, 516 (2005).

[3] R. Machleidt and D. R. Entem, Phys. Rep. **503**, 1 (2011).

[4] B. R. Barrett, P. Navrátil, and J. P. Vary, Prog. Part. Nucl. Phys. **69**, 131 (2013).

[5] T. Dytrych, K. D. Sviratcheva, C. Bahri, J. P. Draayer, and J. P. Vary, Phys. Rev. Lett. **98**, 162503 (2007); Phys. Rev. C **76**, 014315 (2007).
[6] K. Nomura, T. Nikšić, T. Otsuka, N. Shimizu, and D. Vretenar, Phys. Rev. C **84**, 014302 (2011); K. Nomura, N. Shimizu, D. Vretenar, T. Nikšić, and T. Otsuka, Phys. Rev. Lett. **108**, 132501 (2012).
[7] F. Iachello and A. Arima, *The Interacting Boson Model* (Cambridge University, Cambridge, England, 1987).
[8] A. Leviatan, Prog. Part. Nucl. Phys. **66**, 93 (2011).
[9] F. Iachello, *Lie Algebras and Applications* (Springer, Berlin, 2006).
[10] A. Leviatan, Phys. Rev. Lett. **77**, 818 (1996).
[11] J. Escher and A. Leviatan, Phys. Rev. Lett. **84**, 1866 (2000); D. J. Rowe and G. Rosensteel, *ibid.* **87**, 172501 (2001); P. Van Isacker and S. Heinze, *ibid.* **100**, 052501 (2008).
[12] J. L. Ping and J. Q. Chen, Ann. Phys. **255**, 75 (1997).
[13] A. Leviatan, Phys. Rev. Lett. **98**, 242502 (2007).
[14] N. Whelan, Y. Alhassid, and A. Leviatan, Phys. Rev. Lett. **71**, 2208 (1993); A. Leviatan and N. D. Whelan, *ibid.* **77**, 5202 (1996).
[15] J. P. Elliott, Proc. R. Soc. London A **245**, 128 (1958); **245**, 562 (1958).
[16] G. Rosensteel and D. J. Rowe, Phys. Rev. Lett. **38**, 10 (1977).
[17] Y. Alhassid and A. Leviatan, J. Phys. A **25**, L1265 (1992).
[18] J. E. García-Ramos, A. Leviatan and P. Van Isacker, Phys. Rev. Lett. **102**, 112502 (2009).
[19] A. Arima and F. Iachello, Ann. Phys. (NY) **111**, 201 (1978).
[20] J. D. Vergados, Nucl. Phys. A **111**, 681 (1968).
[21] G. Rosensteel, J. P. Draayer, and K. J. Weeks, Nucl. Phys. A **419**, 1 (1984); J. P. Draayer and G. Rosensteel, *ibid.* **439**, 61 (1985).
[22] G. Vanden Berghe, H. E. De Meyer and P. Van Isacker, Phys. Rev. C **32**, 1049 (1985); J. Vanthournout, *ibid.* **41**, 2380 (1990).
[23] D. Bonatsos, Phys. Lett. B **200**, 1 (1988).
[24] N. V. Zamfir and R. F. Casten, Phys. Lett. B **260**, 265 (1991).
[25] C. W. Reich, Nucl. Data Sheets **99**, 753 (2003).
[26] R. F. Casten, N. V. Zamfir, P. von Brentano, F. Seiffert, and W. Lieberz, Phys. Lett. B **265**, 9 (1991).
[27] N. Minkov, S. B. Drenska, P. P. Raychev, R. P. Roussev, and D. Bonatsos, Phys. Rev. C **61**, 064301 (2000).
[28] K. Heyde, P. Van Isacker, M. Waroquier, and J. Moreau, Phys. Rev. C **29**, 1420 (1984).
[29] R. F. Casten, P. Von Brentano, K. Heyde, P. Van Isacker, and J. Jolie, Nucl. Phys. A **439**, 289 (1985).
[30] P. Van Isacker, Phys. Rev. C **27**, 2447 (1983).